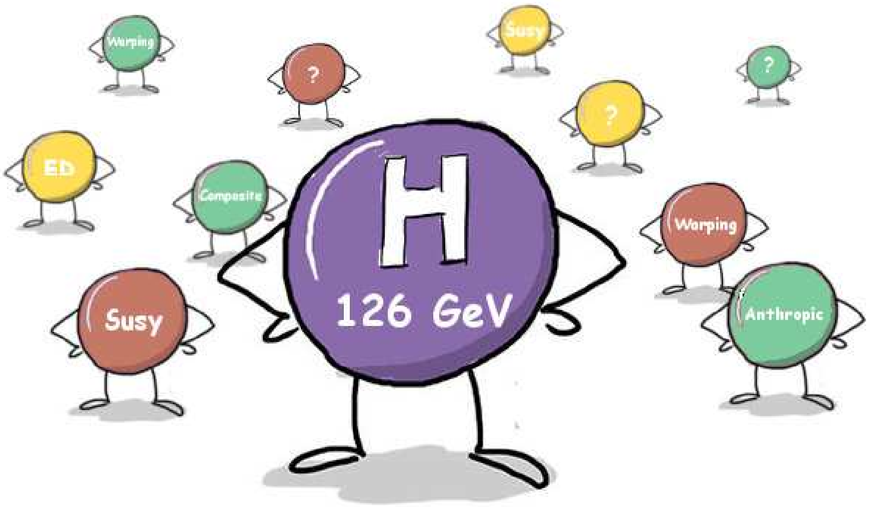
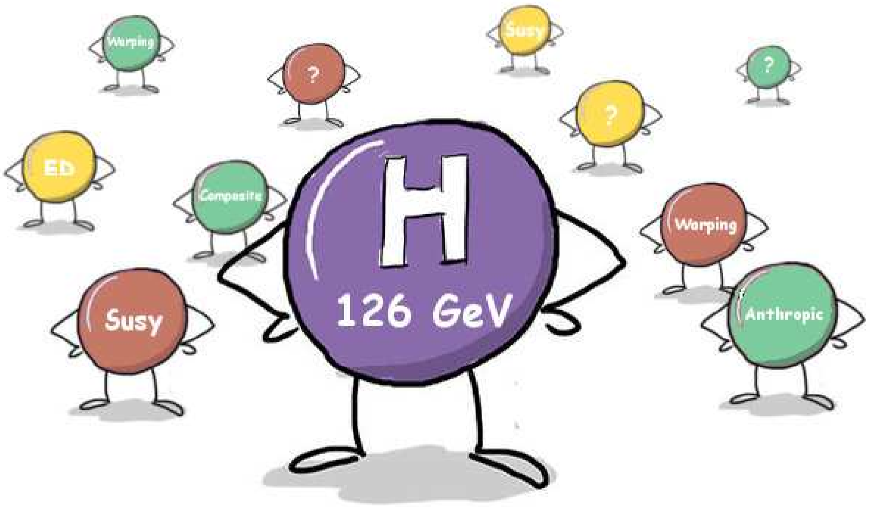


P. Cámara, adapted from phdcomics.com

Mikhail Shaposhnikov

Madrid, 25 September 2013

Why $m_H = 126$ GeV ?



P. Cámara, adapted from phdcomics.com

Mikhail Shaposhnikov

Madrid, 25 September 2013

Why $m_H = 126$ GeV ?

$$\mathbf{125.7 \pm 0.4 = 129.3 \pm 2.0}$$

What does the LHC and $M_H = \mathbf{125.7 \pm 0.4}$ tell us?

What does the LHC and $M_H = 125.7 \pm 0.4$ tell us?

The main LHC result: SM is a consistent effective theory all the way up to the Planck scale

What does the LHC and $M_H = 125.7 \pm 0.4$ tell us?

The main LHC result: SM is a consistent effective theory all the way up to the Planck scale

- No signs of new physics beyond the SM are seen

What does the LHC and $M_H = 125.7 \pm 0.4$ tell us?

The main LHC result: SM is a consistent effective theory all the way up to the Planck scale

- No signs of new physics beyond the SM are seen
- $M_H < 175$ GeV : SM is a weakly coupled theory up to Planck energies

What does the LHC and $M_H = 125.7 \pm 0.4$ tell us?

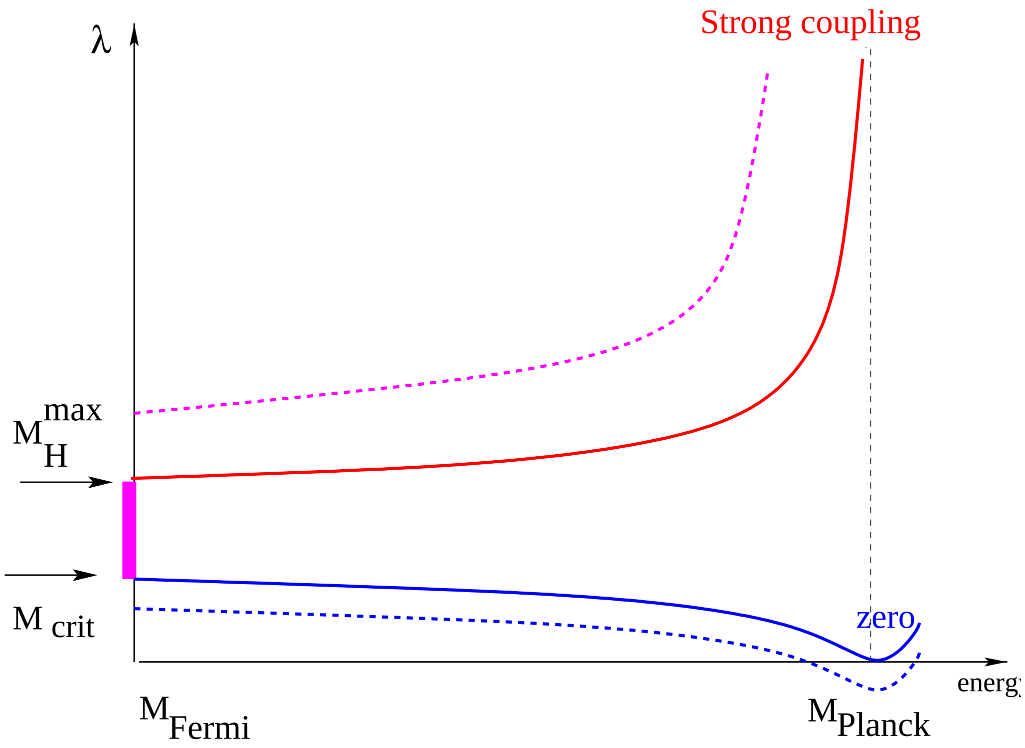
The main LHC result: SM is a consistent effective theory all the way up to the Planck scale

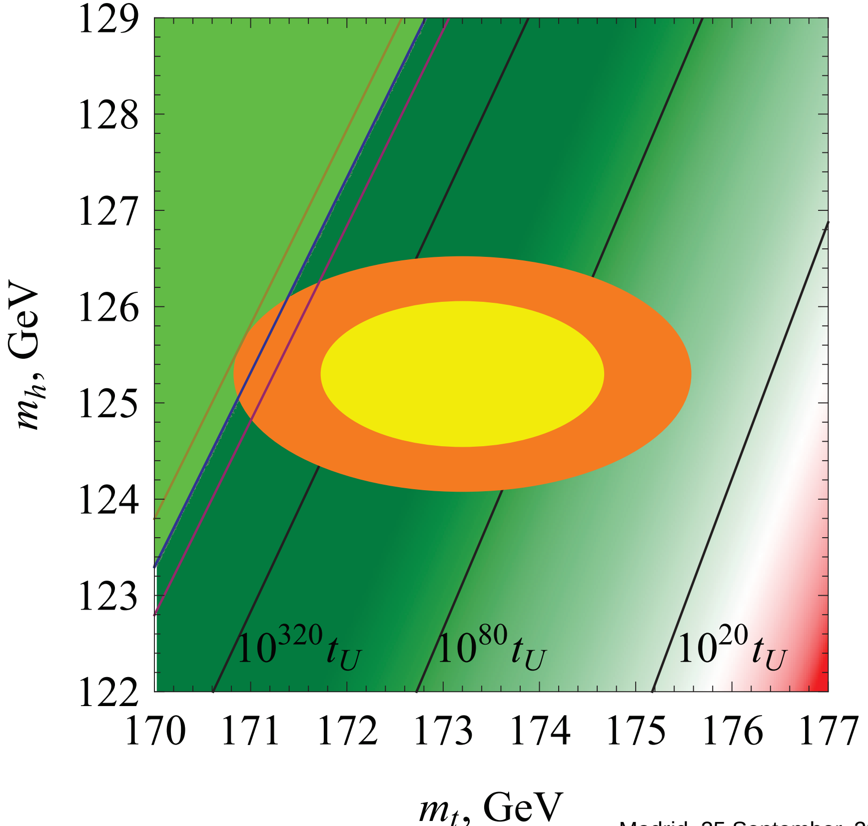
- No signs of new physics beyond the SM are seen
- $M_H < 175$ GeV : SM is a weakly coupled theory up to Planck energies
- $M_H > 111$ GeV: Our EW vacuum is stable or metastable with a lifetime greatly exceeding the Universe age. Espinosa et al

What does the LHC and $M_H = 125.7 \pm 0.4$ tell us?

The main LHC result: SM is a consistent effective theory all the way up to the Planck scale

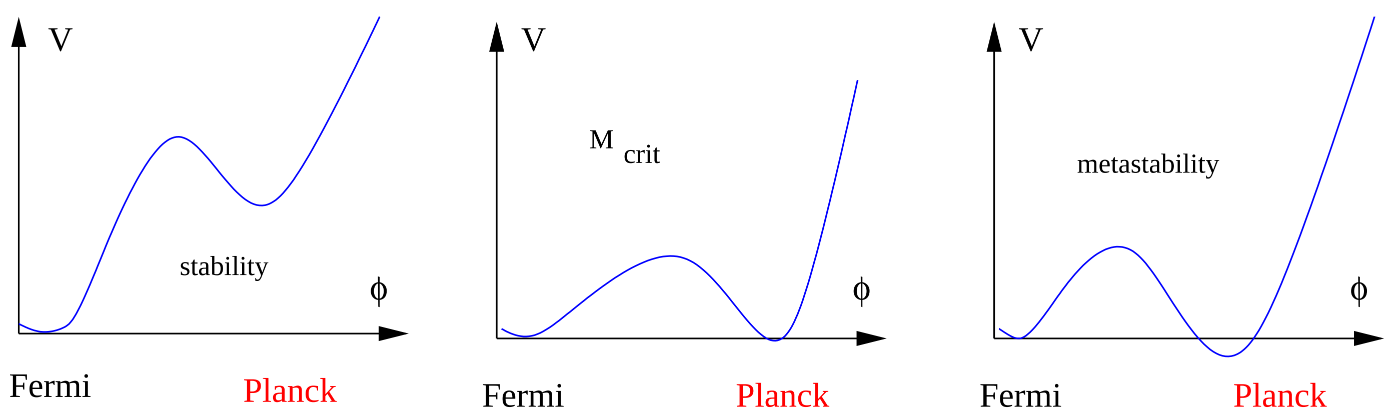
- No signs of new physics beyond the SM are seen
- $M_H < 175$ GeV : SM is a weakly coupled theory up to Planck energies
- $M_H > 111$ GeV: Our EW vacuum is stable or metastable with a lifetime greatly exceeding the Universe age. Espinosa et al





What is $M_H = 129.3 \pm 2.0$?

- Absolute vacuum stability bound on the Higgs mass
- Prediction of the Higgs mass from "multiple point principle"* (95'),
- Higgs inflation bound** (08'),
- asymptotic safety value for M_H *** (09')



* Froggatt, Nielsen

** Bezrukov MS,

De Simone et al

*** Wetterich, MS

Outline

- Absolute stability bound
- Asymptotically safe SM+gravity
- New physics between the Fermi and Planck scales?
- Higgs mass: cosmological inflation
- Conclusions

Computation of absolute stability bound

- Choose the renormalisation scheme for the SM: $\overline{\mathrm{M}S}$
- Compute the effective potential $V(\phi)$ for the Standard Model in tree, one-loop, two-loop,... approximation. It will be a function of the scalar field and $\overline{\mathrm{M}S}$ parameters $\alpha_s(\mu),\ y_t(\mu),\ \lambda(\mu)$ etc.
- Find the relation between $\overline{\mathrm{M}S}$ parameters of the SM at low energy scale (e.g. $\mu = M_Z$ and experimentally measured quantities, such as masses of weak bosons, the Higgs and the pole top masses, etc in tree, one-loop, two-loop,... approximation.
- Make the renormalisation group improvement of the effective potential with the use of RG equations for the SM couplings in one-loop, two-loop, three-loop,... approximation.
- Find the parameters at which the effective potential has two degenerate minima:

$$V(\phi_{SM}) = V(\phi_1), \quad V'(\phi_{SM}) = V'(\phi_1) = 0,$$

Simplified procedure

Instead of computing effective potential, solve "criticality equations":

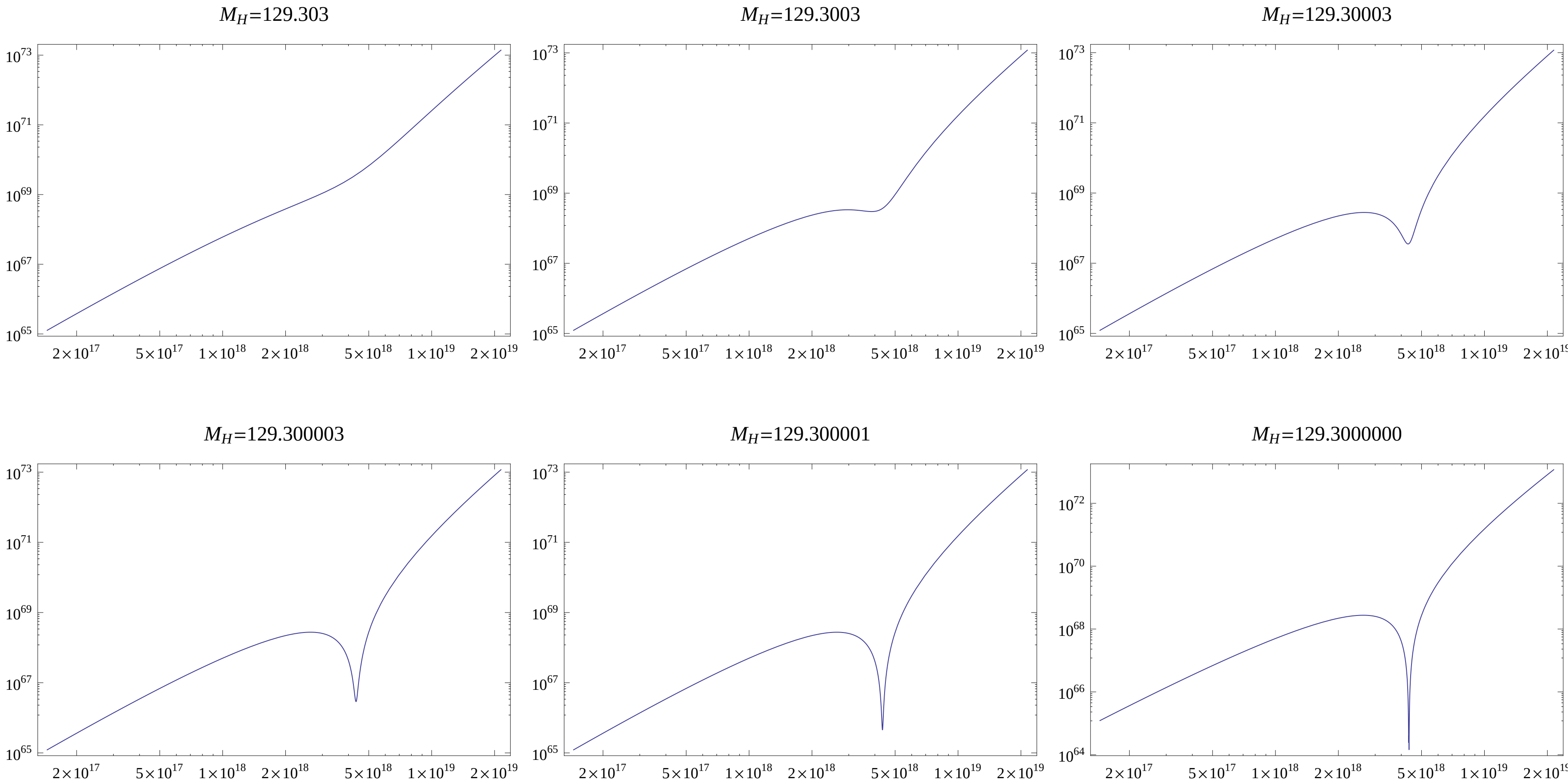
$$\lambda(\mu_0) = 0, \quad \beta_\lambda^{\rm SM}(\mu_0) = 0$$

The reason:

$$V(\phi) \propto \lambda(\phi)\phi^4 \left[1 + O\left(\frac{\alpha}{4\pi}\log(M_i/M_j)\right)\right],$$

where α is here the common name for the SM coupling constants, and M_i are the masses of different particles in the background of the Higgs field. If $O(\alpha)$ corrections are neglected two sets of equations are equivalent. Works with accuracy $\simeq 0.15\,\mathrm{GeV}$ for the masses of the Higgs and of the top.

Effective potential, $M_t = 173.2$ GeV



x axis: ϕ, GeV ; y axis : $V(\phi)$, GeV^4

Critical Higgs mass

Partial or complete two loop matching, three loop running:

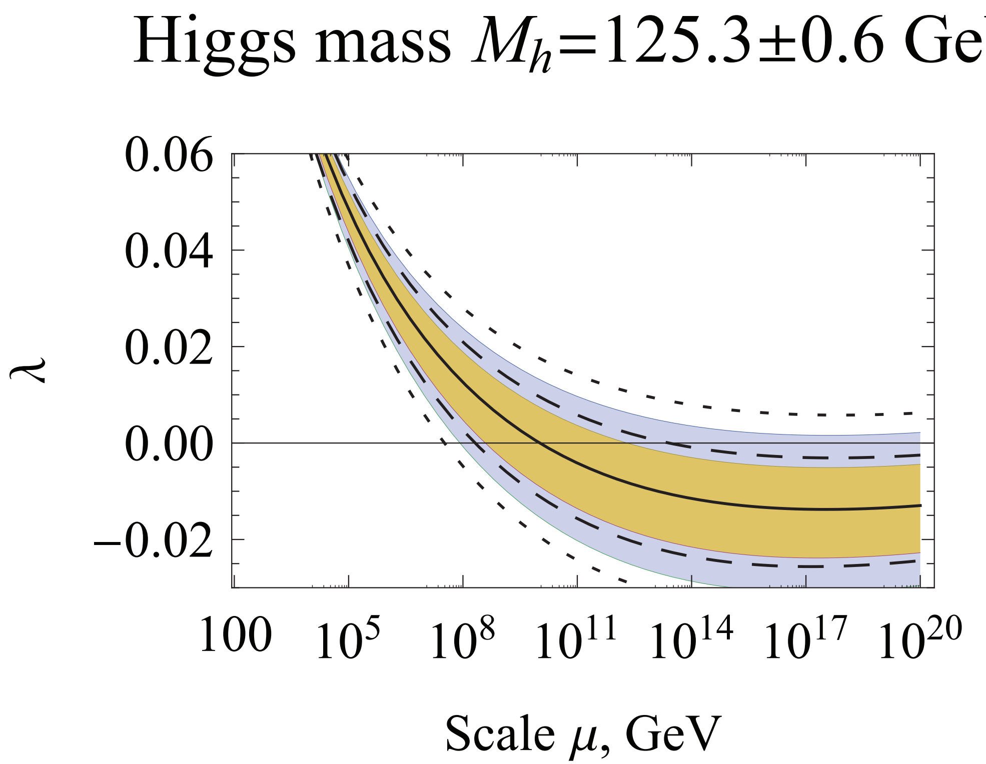
$$M_{crit} = [129.3 + \frac{y_t(M_t) - 0.9361}{0.0058} \times 2.0 - \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.5] \text{ GeV}$$

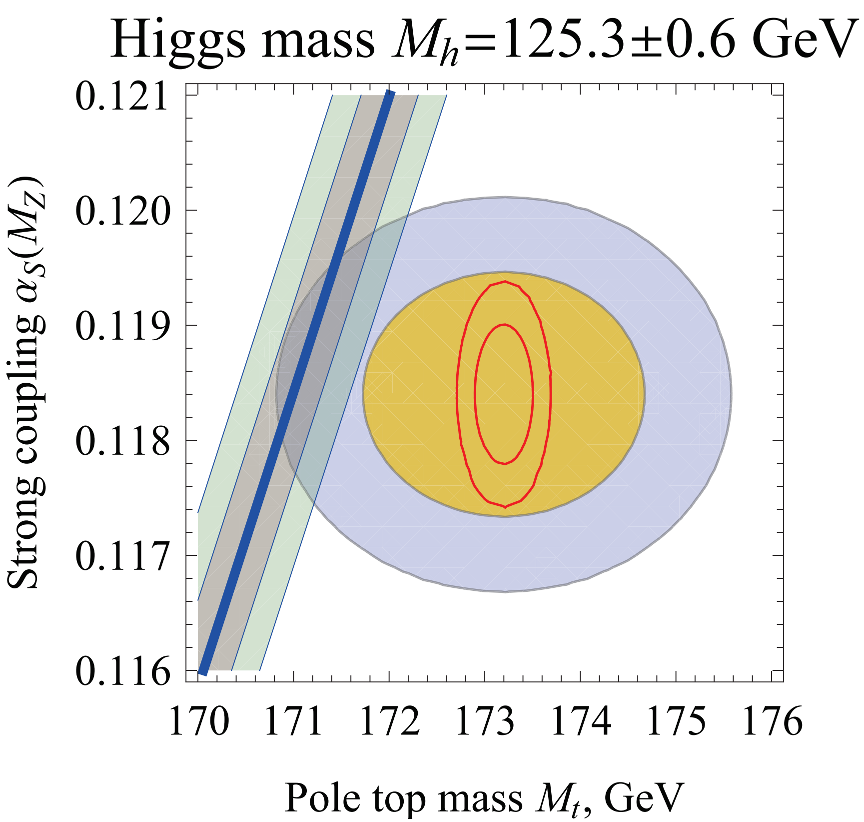
$y_t(M_t)$ - top Yukawa in $\overline{\mathbf{MS}}$ scheme

Matching at EW scale	Central value	theor. error
Bezrukov et al, $\mathcal{O}(\alpha\alpha_s)$	129.4 GeV	1.0 GeV
Degrassi et al, $\mathcal{O}(\alpha\alpha_s, y_t^2\alpha_s, \lambda^2, \lambda\alpha_s)$	129.6 GeV	0.7 GeV
Buttazzo et al, complete 2-loop	129.3 GeV	0.07 GeV

Chetyrkin et al, Mihaila et al, Bednyakov et al, 3 loop running to high energies

Comparison with experiment





errors in y_t: theory + experiment

Tevatron: $M_t = 173.2 \pm 0.51 \pm 0.71$ GeV

ATLAS and CMS: $M_t = 173.4 \pm 0.4 \pm 0.9$ GeV

$\alpha_s = 0.1184 \pm 0.0007$

We do not know whether our vacuum is stable or metastable!

Main uncertainty - top Yukawa coupling.

- Perturbation theory, $\mathcal{O}(\alpha_s^4)$. Estimate of Kataev and Kim: $\delta y_t / y_t \simeq -750(\alpha_s/\pi)^4 \simeq -0.0015$, $\delta M_{crit} \simeq -0.5$ GeV
- Non-perturbative QCD effects, $\delta M_t \simeq \pm\Lambda_{QCD} \simeq \pm 300$ MeV, $\delta M_{crit} \simeq \pm 0.6$ GeV
- 1 GeV experimental error in M_t leads to 2 GeV error in M_{crit}.
- Alekhin et al. Theoretically clean is the extraction of y_t from $t\bar{t}$ cross-section. However, the experimental errors in $p\bar{p} \to t\bar{t} + X$ are quite large, leading to $\delta M_t \simeq \pm 2.8$ GeV, $\delta M_{crit} \simeq \pm 5.6$ GeV.

Precision measurements of m_H, y_t and α_s are needed!

Is it a pure coincidence that the value of M_H is amazingly close to the critical value?

Or, this is a very important message about the structure of high energy theory?

Top and Higgs: asymptotically safe SM+gravity

Asymptotic safety = existence of non-Gaussian UV fixed point for gravity Weinberg '79. Though the theory is non-renormalizable, it is predictive and self-consistent.

To be true: all the couplings of the SM must be asymptotically safe or asymptotically free

Problem for:

- U(1) gauge coupling g_1, $\mu\frac{dg_1}{d\mu} = \beta_1^{\rm SM} = \frac{41}{96\pi^2}g_1^3$
- Scalar self-coupling λ, $\mu\frac{d\lambda}{d\mu} = \beta_\lambda^{\rm SM} =$

$$= \frac{1}{16\pi^2}\left[(24\lambda + 12h^2 - 9(g_2^2 + \frac{1}{3}g_1^2))\lambda - 6h^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2g_1^2\right]$$

- Fermion Yukawa couplings, t-quark in particular h, $\mu\frac{dh}{d\mu} = \beta_h^{\rm SM} =$

$$= \frac{h}{16\pi^2}\left[\frac{9}{2}h^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right]$$

Landau pole behaviour

Gravity contribution to RG running

Let x_j is a SM coupling. Gravity contribution to RG:

$$\mu \frac{dx_j}{d\mu} = \beta_j^{\mathrm{SM}} + \beta_j^{grav} \ .$$

On dimensional grounds

$$\beta_j^{grav} = \frac{a_j}{8\pi} \frac{\mu^2}{M_P^2(\mu)} x_j \ .$$

where

$$M_P^2(\mu) = M_P^2 + 2\xi_0 \mu^2 \ ,$$

with $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV, ξ_0 is some number

Asymptotic safety scenario for the SM model (MS, Wetterich) is realised when:

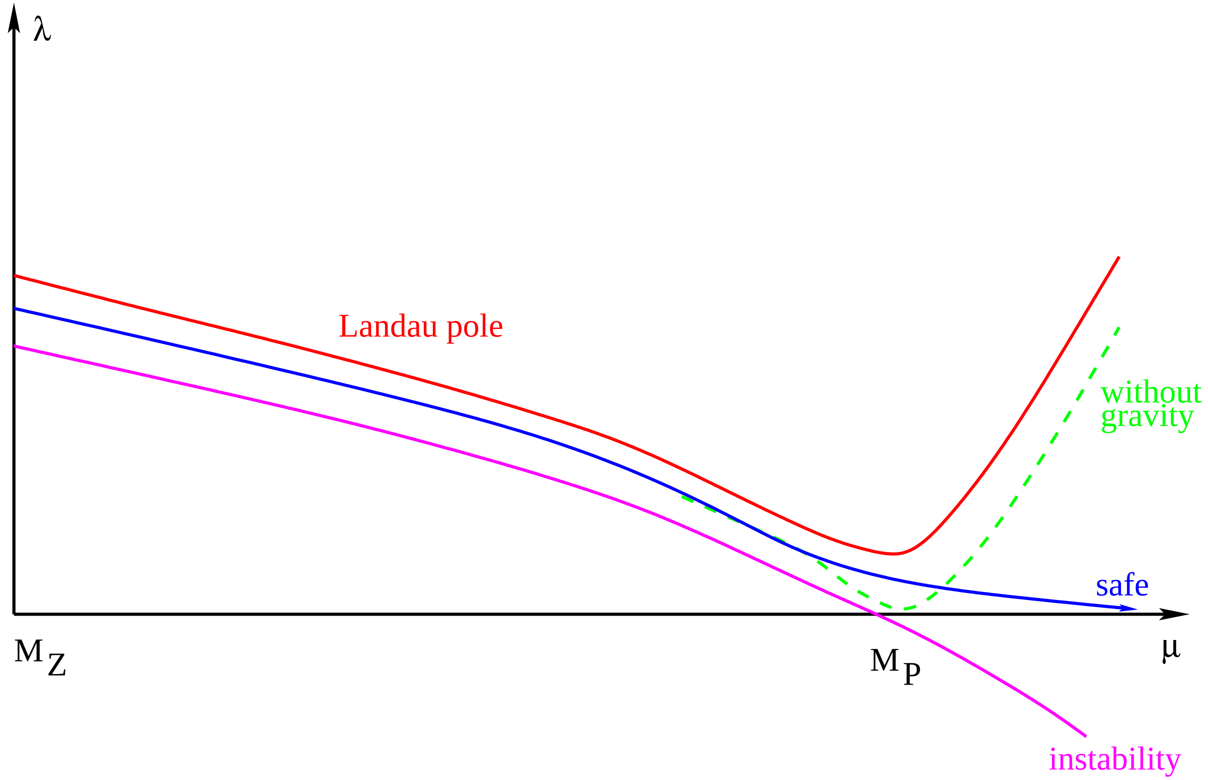
- Gravity contribution to gauge beta functions and that for the top Yukawa are negative, leading to asymptotic safety for U(1) and for y_t.
- If $a_\lambda > 0$ (according to Percacci and Narain '03) $\Longrightarrow$ Higgs mass prediction, $M_H = M_{crit}$
- If $a_\lambda < 0 \Longrightarrow M_{crit} < M_H < 175$ GeV

Computations of a_i: Robinson and Wilczek '05, Pietrykowski '06, Toms '07&'08, Ebert, Plefka and Rodigast '07, Narain and Percacci '09, Daum, Harst and Reuter '09, Zanusso et al '09, Folkerts, Litim and Pawlowski '11, Ellis, Mavromatos '12 ...

Suppose that indeed $a_1 < 0,\ a_h < 0,\ a_\lambda > 0$, what is found in a number of computations. Then the Higgs mass is predicted to be coming from solution of equation

$$\lambda(M_P) = 0$$

with uncertainty of few hundreds of MeV. Simultaneously, it is required that $\beta_\lambda(M_P) \ll 1$.

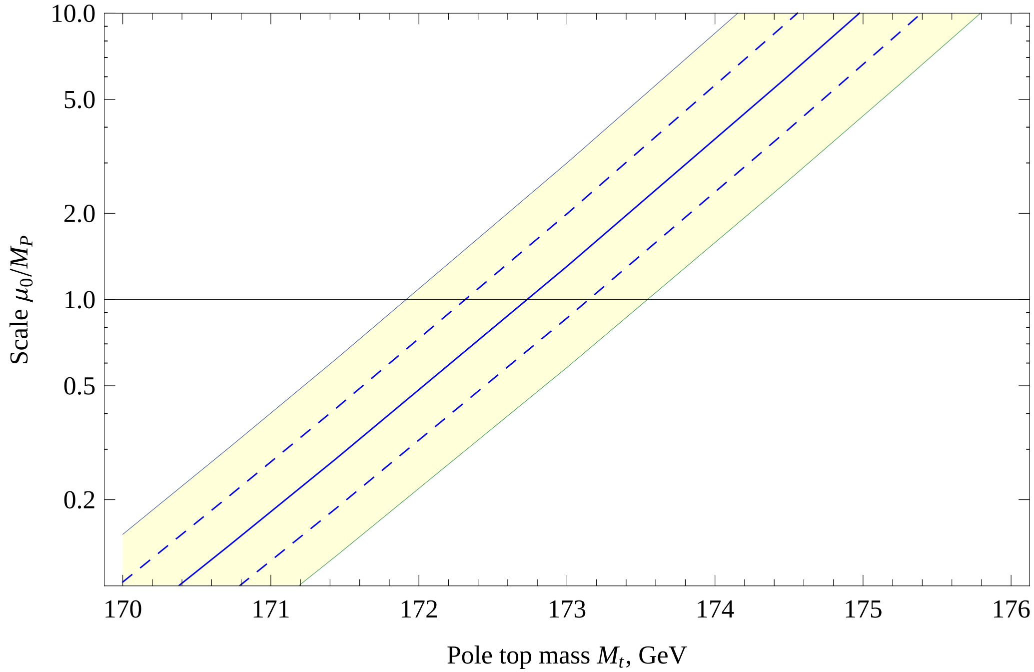


New Physics between the Fermi and Planck scales?

From two equations

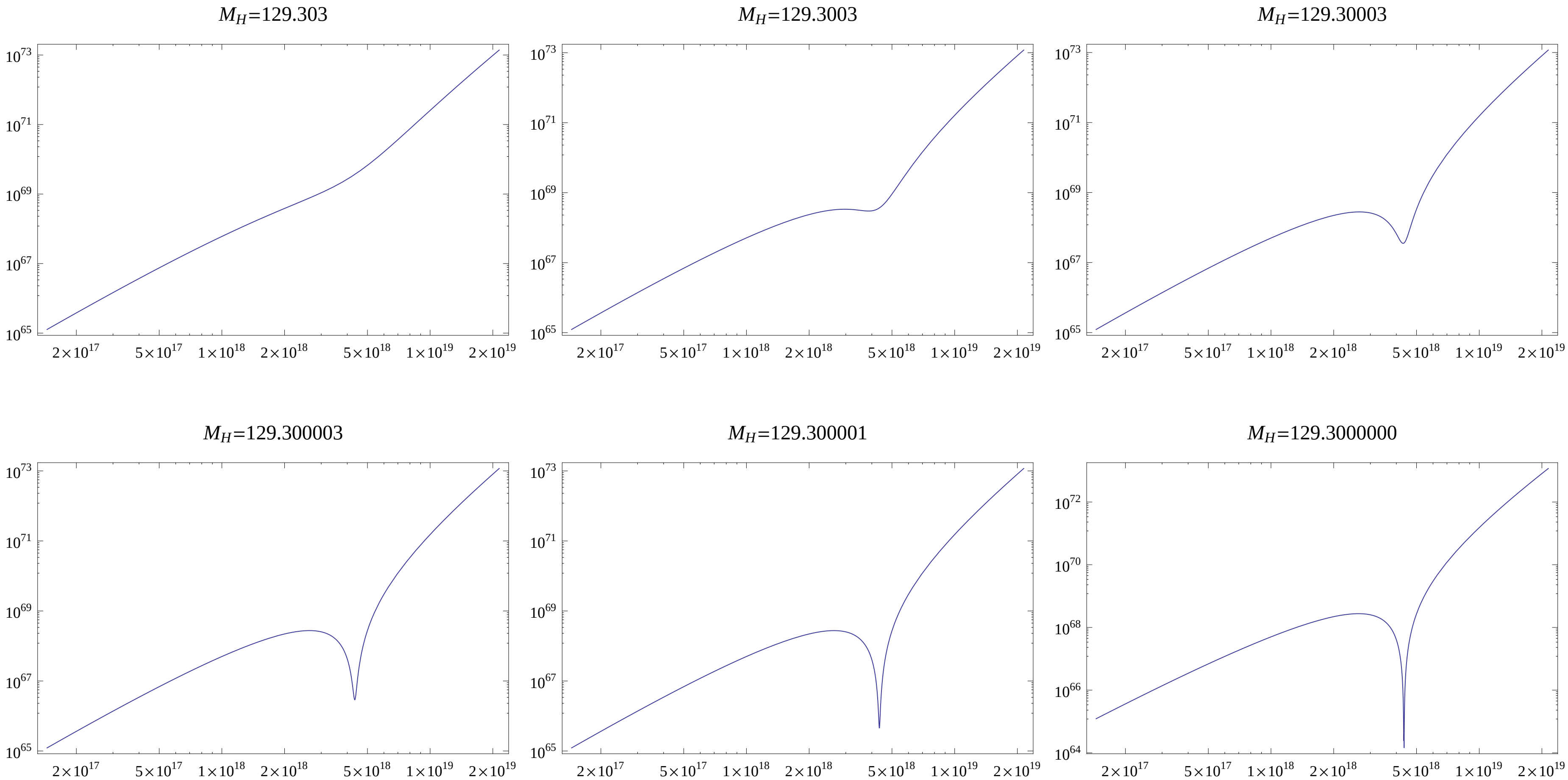
$$\lambda(\mu_0) = 0, \qquad \beta_\lambda^{\text{SM}}(\mu_0) = 0$$

one can determine not only the Higgs mass, but also the scale μ_0.



μ_0 determined by the EW physics gives the Planck scale, $\mu_0 \simeq M_P$!

Effective potential, $M_t = 173.2$ GeV



x axis: ϕ, GeV ; y axis : $V(\phi)$, GeV^4

Fermi scale is determined by the Planck scale (or vice versa)?

This relation is generically spoiled if new physics exists between the Fermi and Planck scales.

$$\Downarrow$$

Argument in favour of absence of new physics scales between Fermi and Planck.

Higgs boson: cosmological inflation

Inflation: solution to a number of problems:

- Horizon: Why the universe is so uniform and isotropic?
- Structure formation : What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant?
- Flatness : Why $\Omega_M + \Omega_\Lambda + \Omega_{\rm rad}$ is so close to 1 now and was immensely close to 1 in the past?

Inflation: solution to a number of problems:

- Horizon: Why the universe is so uniform and isotropic?
- Structure formation : What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant?
- Flatness : Why $\Omega_M + \Omega_\Lambda + \Omega_{\rm rad}$ is so close to 1 now and was immensely close to 1 in the past?

For inflation we better have some bosonic field, which drives it. At last, the Higgs boson has been discovered! Can it make the Universe flat, homogeneous, and isotropic, and produce the necessary spectrum of fluctuations for structure formation?

Inflation: solution to a number of problems:

- Horizon: Why the universe is so uniform and isotropic?
- Structure formation : What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant?
- Flatness : Why $\Omega_M + \Omega_\Lambda + \Omega_{\rm rad}$ is so close to 1 now and was immensely close to 1 in the past?

For inflation we better have some bosonic field, which drives it. At last, the Higgs boson has been discovered! Can it make the Universe flat, homogeneous, and isotropic, and produce the necessary spectrum of fluctuations for structure formation?

Yes: Higgs inflation

Bezrukov, MS

Higgs inflation

non-minimal coupling of Higgs field to gravity

$$S_G = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \frac{\xi h^2}{2} R \right\}$$

Jordan, Feynman, Brans, Dicke,...

Consider large Higgs fields h.

- Gravity strength: $M_P^{\text{eff}} = \sqrt{M_P^2 + \xi h^2} \propto h$
- All particle masses are $\propto h$

For $h > \frac{M_P}{\sqrt{\xi}}$ (classical) physics is the same (M_W / M_P^{eff} does not depend on h)!

Existence of effective flat direction, necessary for successful inflation.

Inflaton potential and observations

If inflaton potential is known one can make predictions and compare them with observations.

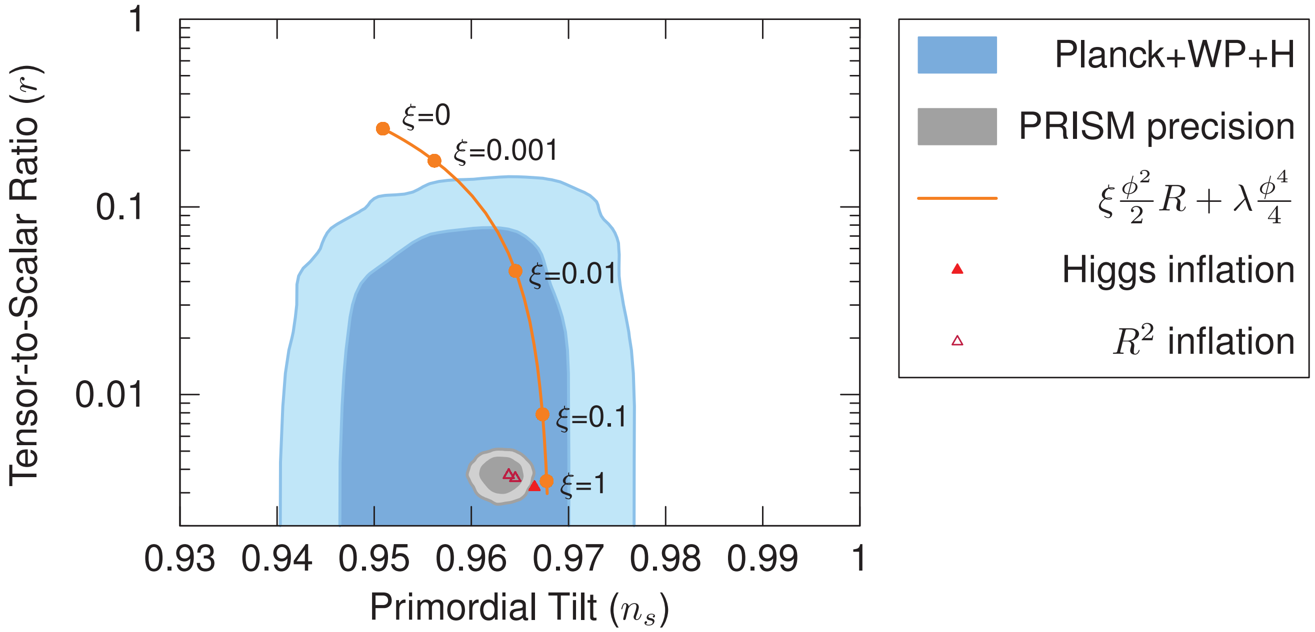
- $\delta T/T$ at the WMAP normalization scale ~ 500 Mpc.
- The value of spectral index n_s of scalar density perturbations

$$\left\langle \frac{\delta T(x)}{T} \frac{\delta T(y)}{T} \right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n_s - 1}$$

- The amplitude of tensor perturbations $r = \frac{\delta \rho_s}{\delta \rho_t}$

These numbers can be extracted from WMAP observations of cosmic microwave background. Higgs inflation: one new parameter, $\xi \Longrightarrow$ two predictions. From WMAP normalization $\xi \sim 700$.

CMB parameters—spectrum and tensor modes

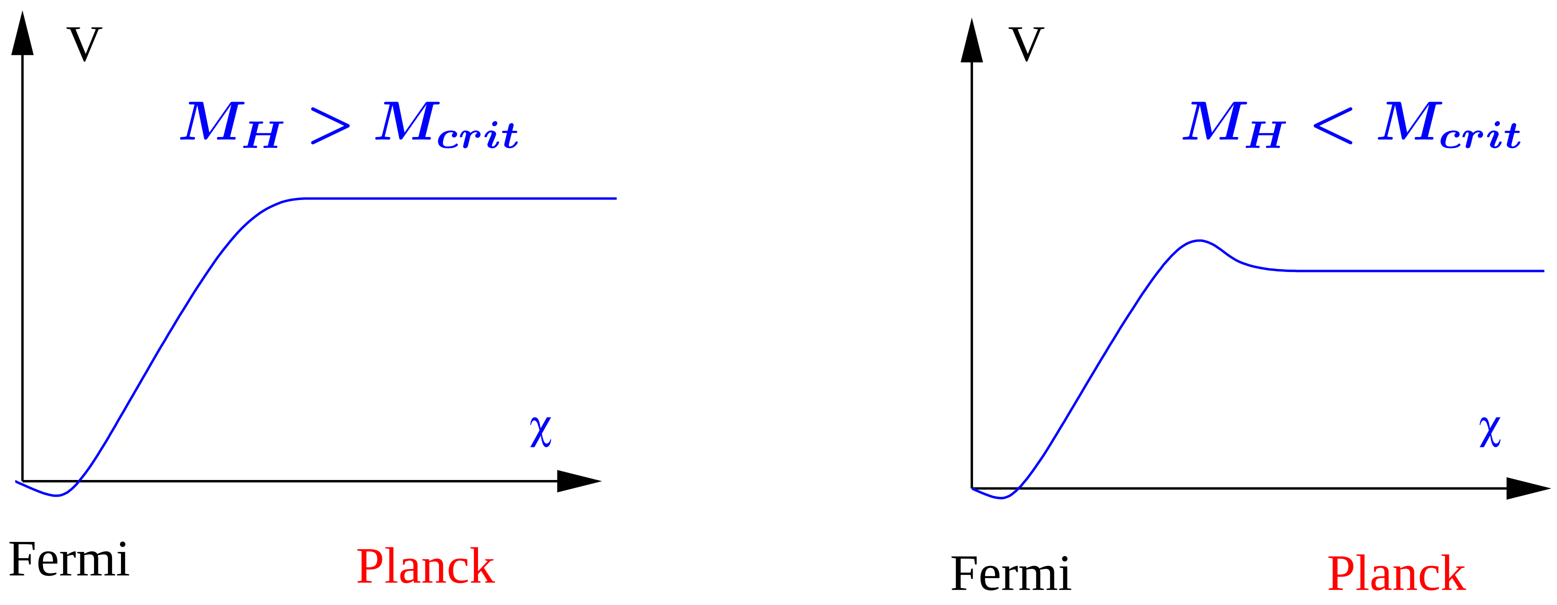


- Higgs inflation: $T_{reh} \sim 10^{13-14}$ GeV, $N \simeq 58$

Perturbations are Gaussian, in accordance with Planck.

Inflation and the Higgs mass

Radiative corrections to inflationary potential: Higgs inflation works only for $\lambda(M_P/\sqrt{\xi}) > 0$. Numerically, $M_H > M_{crit}$ with extra theoretical uncertainty of $\delta M_H \sim 1$ GeV.



χ - canonically normalized Higgs field in Einstein frame.

Peculiarity of $M_H = M_{crit}$

At large ξ the tree amplitudes of scattering of scalars above electroweak vacuum with small ϕ hit the tree unitarity bound at energies

$$E > \Lambda \sim \frac{M_P}{\xi}$$

Burgess, Lee, Trott, '09; Barbon and Espinosa, '09

This does not invalidate the Higgs inflation as all typical energy scales are below the Higgs-dependent cutoff Bezrukov et al, Ferrara et al.

Still, the region of weak coupling above the EW vacuum is the largest at smallest possible ξ, meaning at $M_H = M_{crit}$.

Conclusions

- LHC experiments provide a strong evidence that the SM is a self-consistent effective theory all the way up to the Planck scale.
- The case of $M_H = M_{crit}$ is very peculiar: if this is indeed the case, this is an indication for the absence of new energy scales between the Fermi and Planck scales
- The relation $M_H = M_{crit}$ may come from asymptotic safety scenario for the SM and/or from non-minimal coupling of the Higgs field to gravity
- To have a decisive statement on the relation between M_H and M_{crit}, we should know:
  - Higgs mass with highest possible precision (LHC, $\mathbf{200}$ MeV?)
  - Top Yukawa coupling with accuracy $\mathbf{5 \times 10^{-4}}$ ($\delta M_t \simeq \mathbf{100}$ MeV) (future e^+e^- collider? LHC with new theory input?)
  - α_s with uncertainty $\delta\alpha_s \simeq \mathbf{2 \times 10^{-4}}$