High precision QCD for LHC physics

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Physics Challenges in the face of LHC-14
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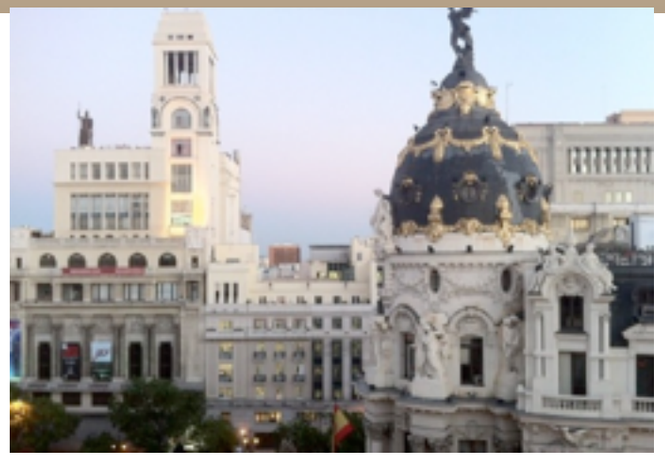
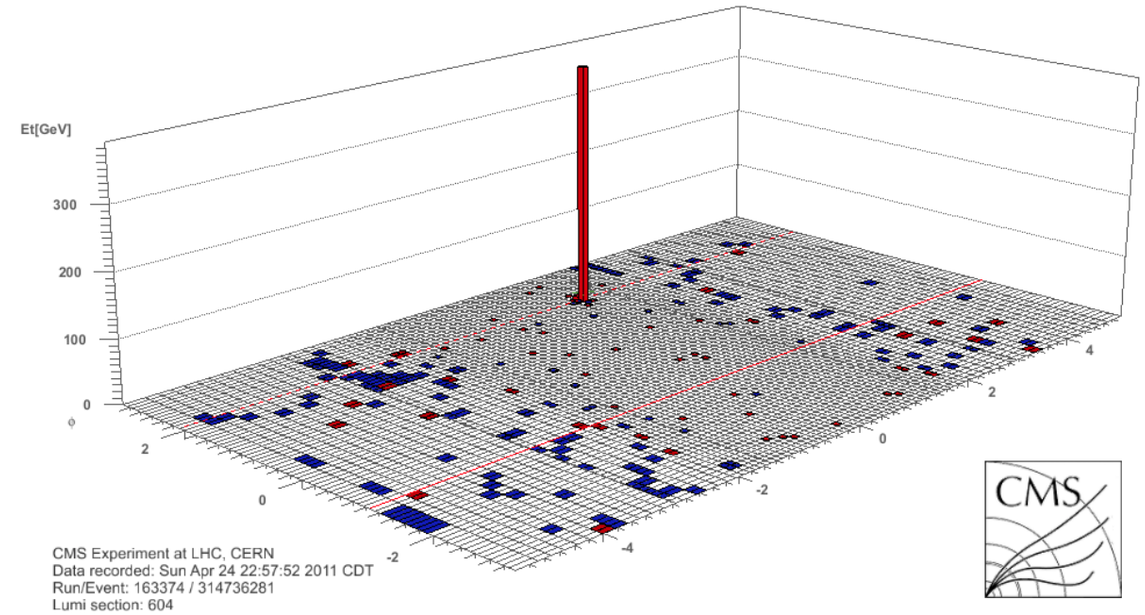
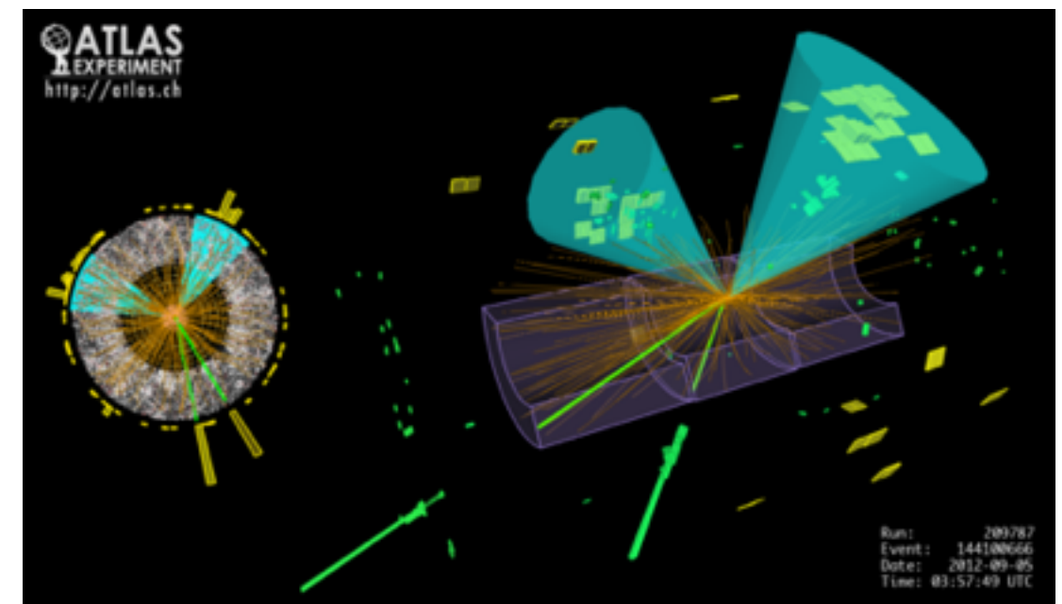


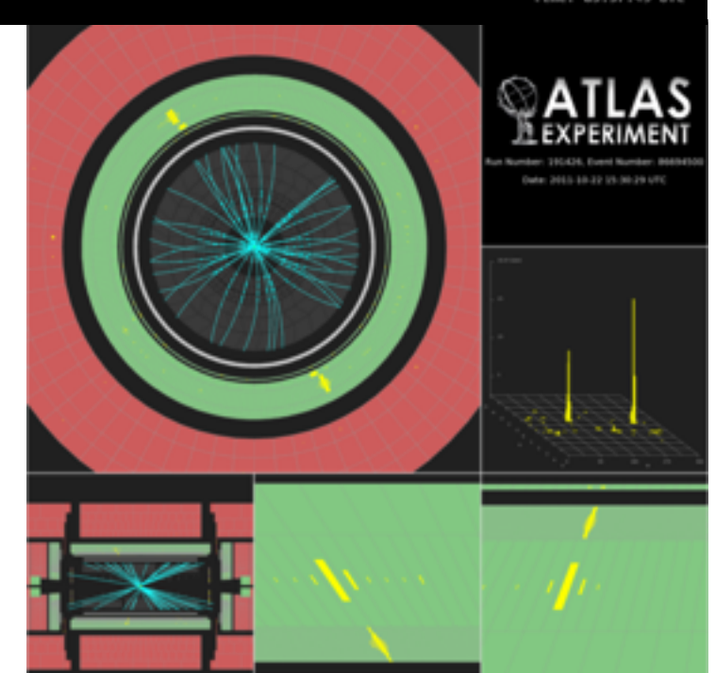
Main objectives of LHC experiments

- A major objective achieved: the discovery of the Higgs boson
- LHC-13..14 has a lot more to do
- Precision Higgs physics
- Discover or exclude particle candidates of dark matter.
- Precision studies of particles making up dark matter, or other particles if we are lucky.
- LHC is a precision physics machine: experimental and theoretical uncertainties are comparable.

$$(\sim 1\%)\mathrm{stat} + (\sim 3\%)\mathrm{syst} + (\sim 4\%)\mathrm{lumi} \approx \frac{\alpha_s(M)}{\pi} \log\left(\frac{\mu}{M}\right)$$

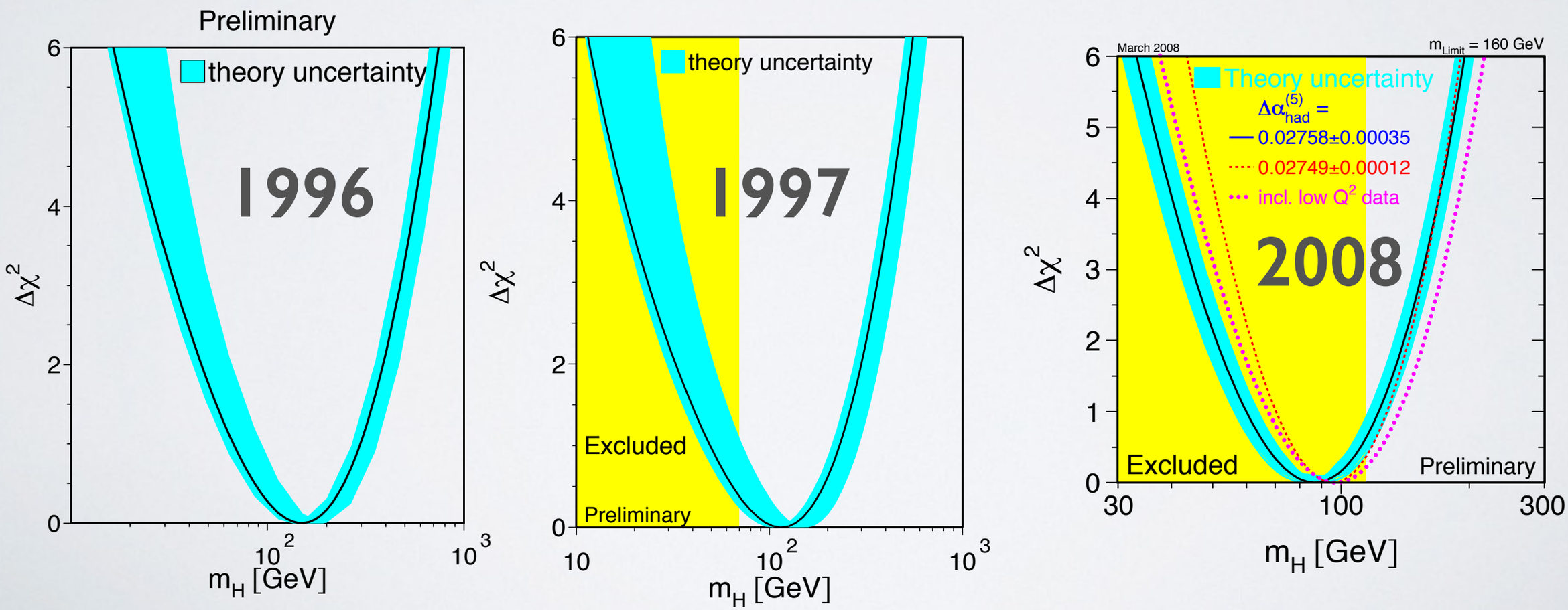






PRECISION BEFORE DISCOVERY

Precision measurements, precision theory predictions and a Standard Model assumption gave before the discovery estimates of the Higgs mass.



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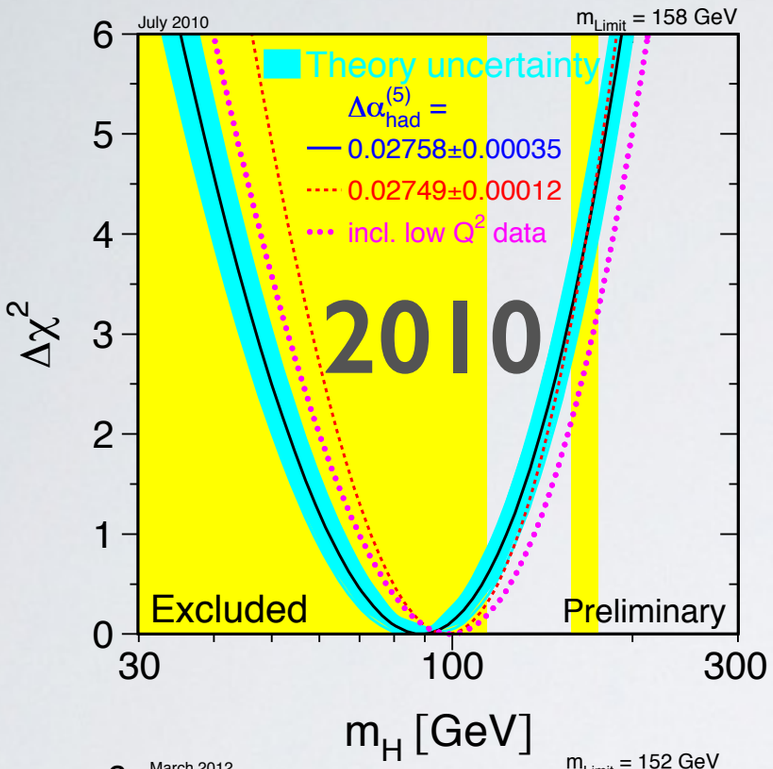
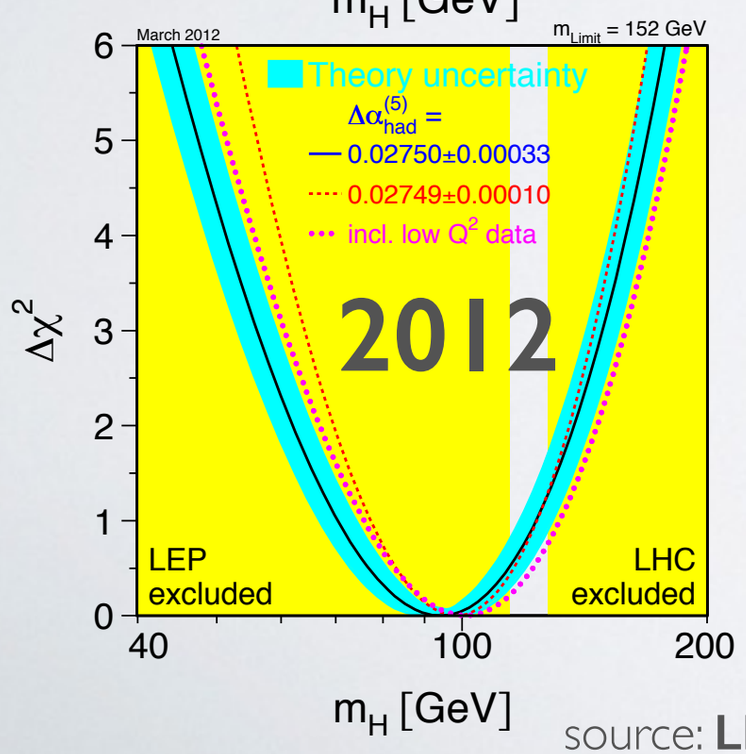
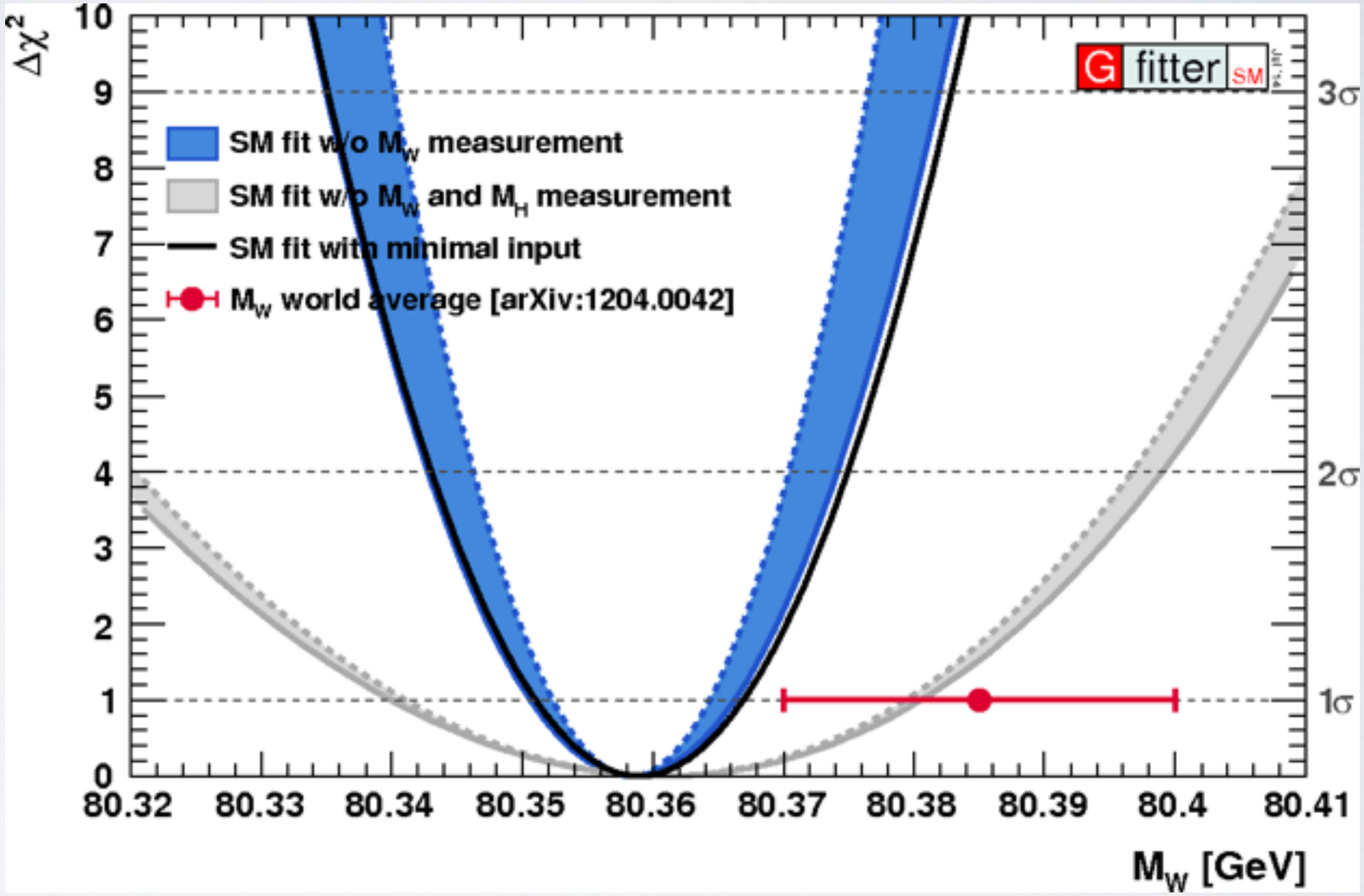
BEFORE/AFTER DISCOVERY







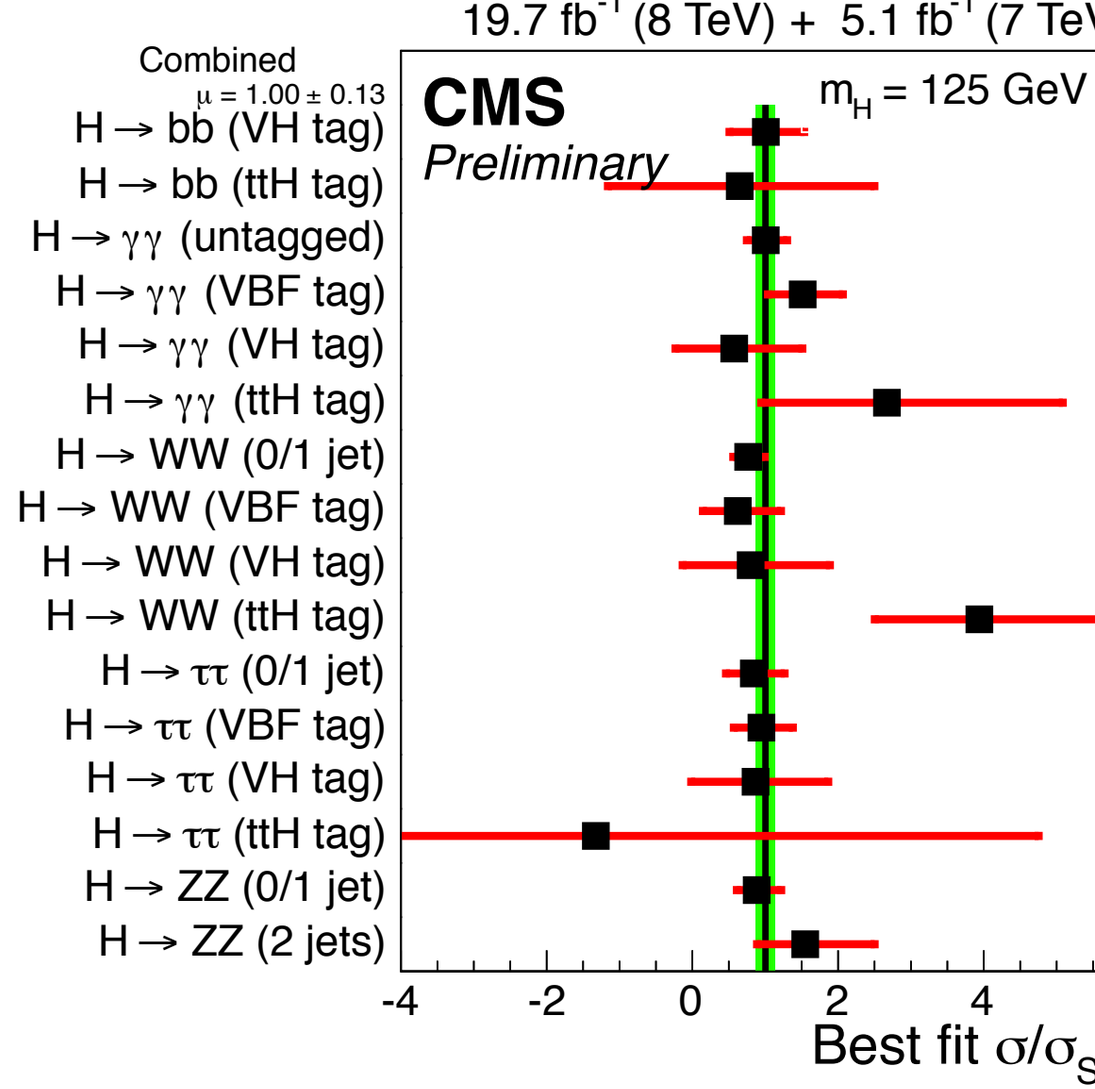
checking self-consistency of the Standard Model

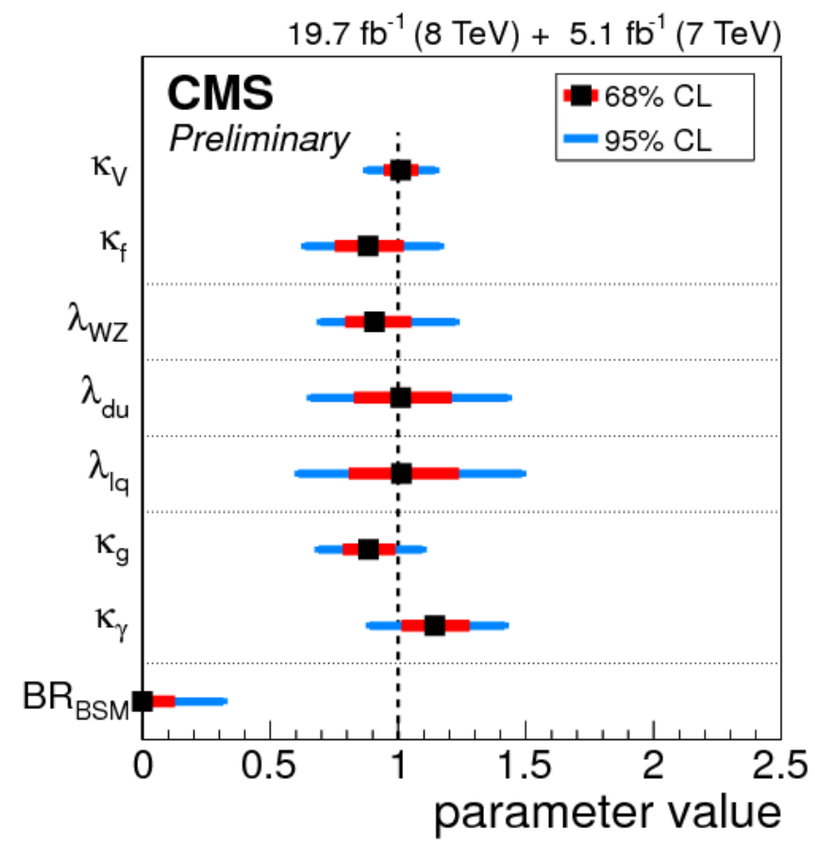
source: **LEPEWWG**

Precision after the discovery

- From the measurement in many channels, the experiments can infer a “total” Higgs cross-section.
- Similarly, they can infer the couplings of the Higgs boson and SM fermions and bosons.
- The theoretical uncertainty is already at the same level of accuracy as statistical and systematic uncertainties.
- Agreement with the Standard Model would have been very difficult to establish had we not have precise QCD predictions for the signal.

$$\sigma/\sigma_{\rm SM} = 1.00 \pm 0.13 \left[\pm 0.09(\text{stat.})^{+0.08}_{-0.07}(\text{theo.}) \pm 0.07(\text{syst.})\right]$$





- There is no dark matter in the Standard Model.
  But it is easy or necessary to have in extensions of it.

D1	$\bar{\chi}\chi\bar{q}q$	m_q/M_*^3
D2	$\bar{\chi}\gamma^5\chi\bar{q}q$	im_q/M_*^3
D3	$\bar{\chi}\chi\bar{q}\gamma^5 q$	im_q/M_*^3
D4	$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	m_q/M_*^3
D5	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$1/M_*^2$
D6	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$1/M_*^2$
D7	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/M_*^2$
D8	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/M_*^2$
D9	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$	$1/M_*^2$
D10	$\bar{\chi}\sigma_{\mu\nu}\gamma^5\chi\bar{q}\sigma_{\mu\nu}q$	i/M_*^2
D11	$\bar{\chi}\chi G_{\mu\nu}G^{\mu\nu}$	$\alpha_s/4M_*^3$
D12	$\bar{\chi}\gamma^5\chi G_{\mu\nu}G^{\mu\nu}$	$i\alpha_s/4M_*^3$
D13	$\bar{\chi}\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$	$i\alpha_s/4M_*^3$
D14	$\bar{\chi}\gamma^5\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$	$\alpha_s/4M_*^3$
D15	$\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$	M
D16	$\bar{\chi}\sigma_{\mu\nu}\gamma^5\chi F_{\mu\nu}$	D
M1	$\bar{\chi}\chi\bar{q}q$	$m_q/2M_*^3$
M2	$\bar{\chi}\gamma^5\chi\bar{q}q$	$im_q/2M_*^3$

M3	$\bar{\chi}\chi\bar{q}\gamma^5 q$	$im_q/2M_*^3$
M4	$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	$m_q/2M_*^3$
M5	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$1/2M_*^2$
M6	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/2M_*^2$
M7	$\bar{\chi}\chi G_{\mu\nu}G^{\mu\nu}$	$\alpha_s/8M_*^3$
M8	$\bar{\chi}\gamma^5\chi G_{\mu\nu}G^{\mu\nu}$	$i\alpha_s/8M_*^3$
M9	$\bar{\chi}\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$	$i\alpha_s/8M_*^3$
M10	$\bar{\chi}\gamma^5\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$	$\alpha_s/8M_*^3$
C1	$\chi^\dagger\chi\bar{q}q$	m_q/M_*^2
C2	$\chi^\dagger\chi\bar{q}\gamma^5 q$	im_q/M_*^2
C3	$\chi^\dagger\partial_\mu\chi\bar{q}\gamma^\mu q$	$1/M_*^2$
C4	$\chi^\dagger\partial_\mu\chi\bar{q}\gamma^\mu\gamma^5 q$	$1/M_*^2$
C5	$\chi^\dagger\chi G_{\mu\nu}G^{\mu\nu}$	$\alpha_s/4M_*^2$
C6	$\chi^\dagger\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$	$i\alpha_s/4M_*^2$
R1	$\chi^2\bar{q}q$	$m_q/2M_*^2$
R2	$\chi^2\bar{q}\gamma^5 q$	$im_q/2M_*^2$
R3	$\chi^2 G_{\mu\nu}G^{\mu\nu}$	$\alpha_s/8M_*^2$
R4	$\chi^2 G_{\mu\nu}\tilde{G}^{\mu\nu}$	$i\alpha_s/8M_*^2$

$$\{Q_\alpha, Q_\beta\} = 0$$

$$\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \qquad [P^\mu, Q_\alpha] = 0$$

$$[M^{\mu\nu}, Q_\alpha] = -i\,(\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta$$

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\,(\sigma^\mu)_{\alpha\dot\beta}\, P_\mu$$

$$\sigma^\mu \equiv (1, \sigma^i) \quad ; \quad \overline{\sigma}^\mu \equiv (1 - \sigma^i)$$

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 = \left(\frac{z_{\rm UV}}{z}\right)^2 (\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

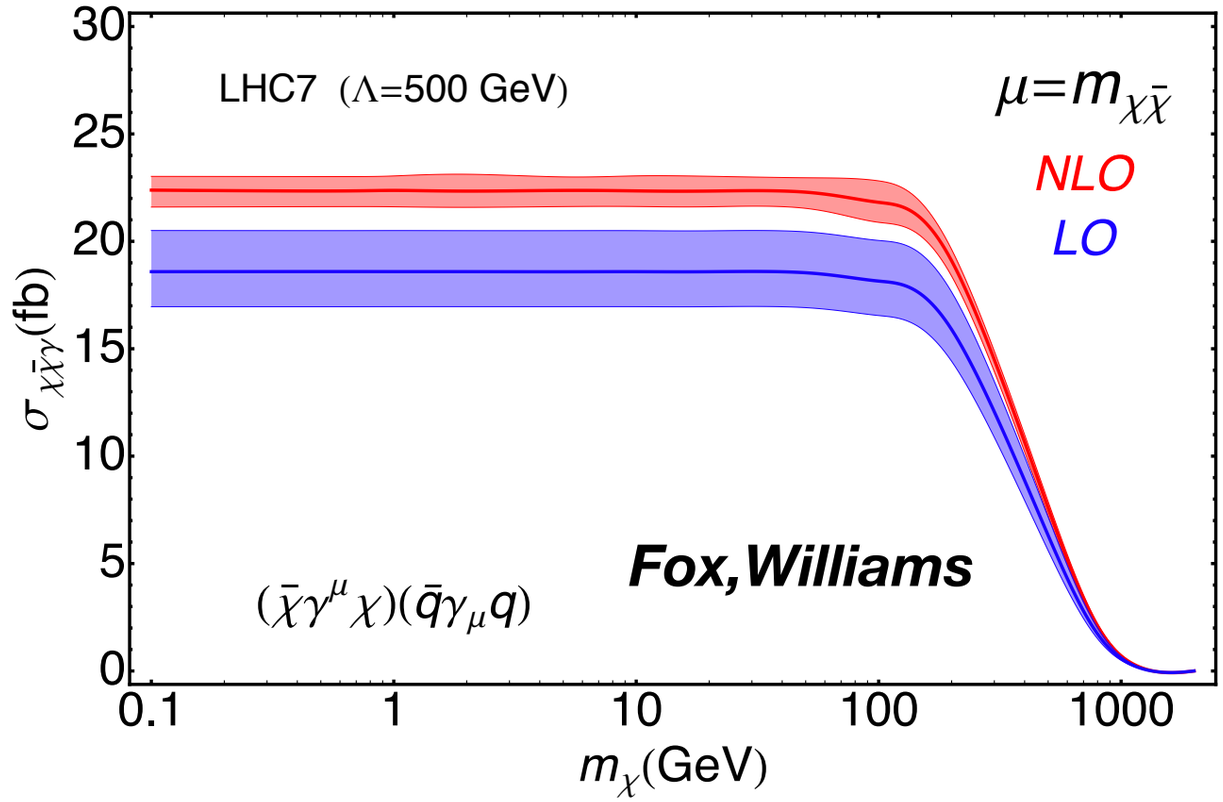
$$e^{-1}\mathcal{L} = -\frac{1}{4}(F_+^2 + F_-^2) + \bar\psi_+(\mathrm{i}\gamma^M\mathcal{D}_M - m)\psi_+ + \bar\psi_-(\mathrm{i}\gamma^M\mathcal{D}_M - m)\psi_-$$
$$- g_X Q_X \gamma^M(\bar\psi_+ X_{+\,M}\psi_+ + \bar\psi_- X_{+\,M}\psi_- + \bar\psi_+ X_{-\,M}\psi_- + \bar\psi_- X_{-\,M}\psi_+)$$

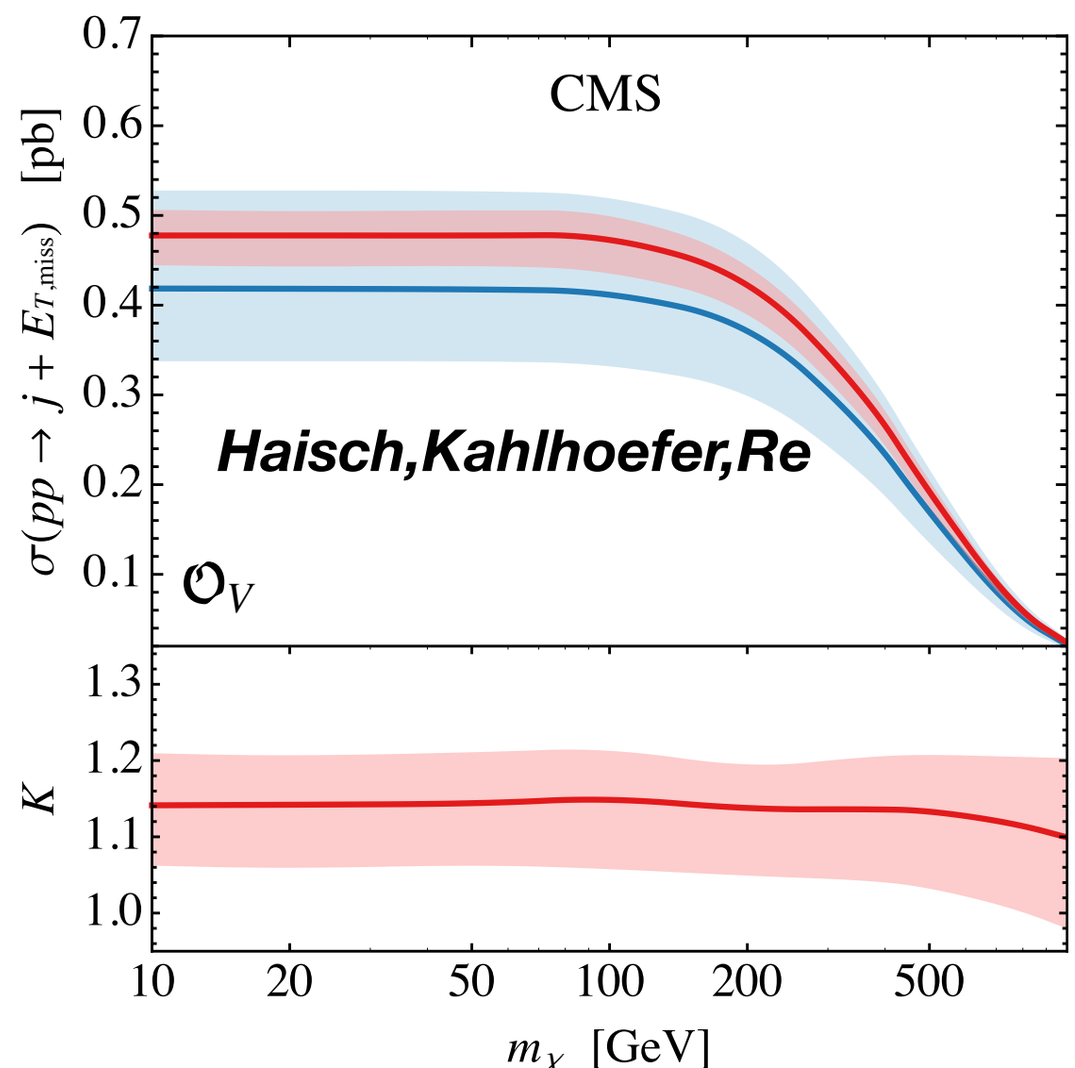
What is the model? Can we be agnostic of it and still discover dark matter?

- How strong are the interactions of DM particles and Standard Model particles?

Precision before dark matter particle discovery

- Precise theory predictions and precise data are our main tools to constrain the mass and couplings of particle candidates of dark matter.
- As for Higgs, QCD corrections can be large and tricky.
- Lead to smaller uncertainties and sharper limits on new physics scales.





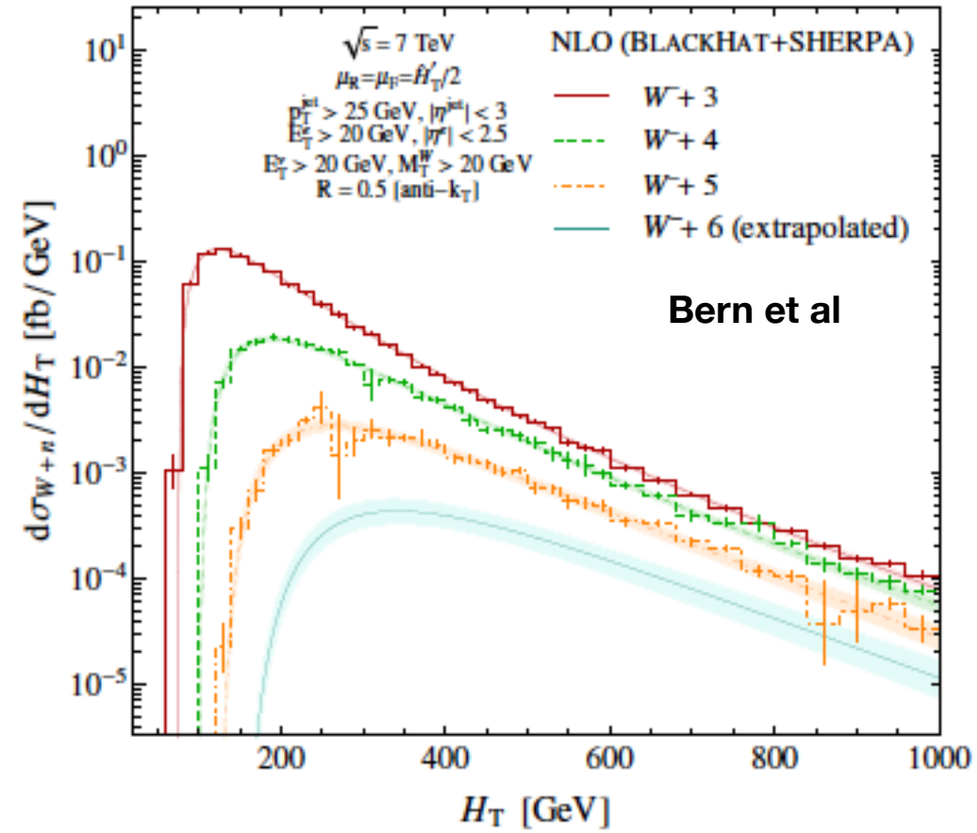
In this talk

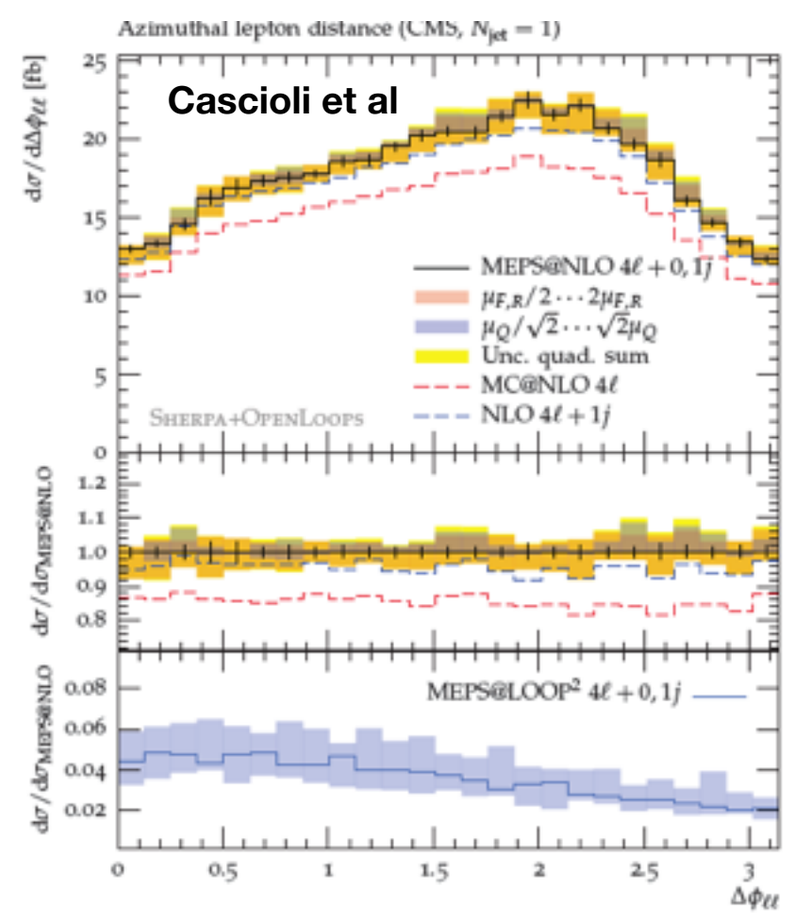
- Fixed order perturbative QCD
- A few highlights in phenomenology from recent breakthroughs from NLO through NNNLO in Higgs physics.
- Mathematical puzzles/methods: strategy of regions.
- The temptation and dangers of **N..NLO*approx***.

Fixed order perturbation theory

The NLO revolution

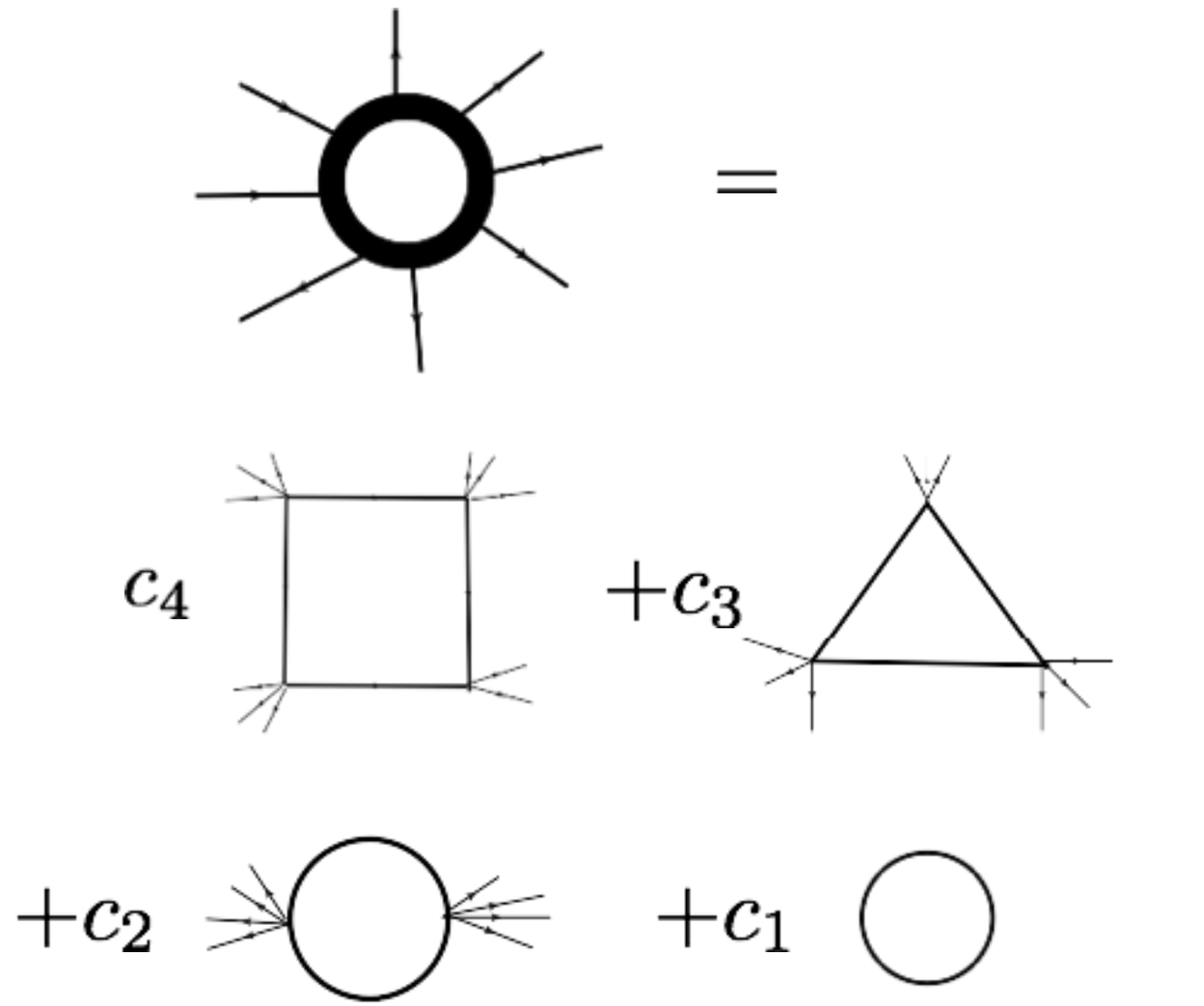
- The field of perturbative QCD is in one of its finest moments.
- NLO is reaching maturity with very well automated tools *(aMC@NLO, Blackhat, Rocket, GoSam, OpenLoops,…)*
- Helps the experiments to investigate confidently potential signals over backgrounds of multi-particle final states.

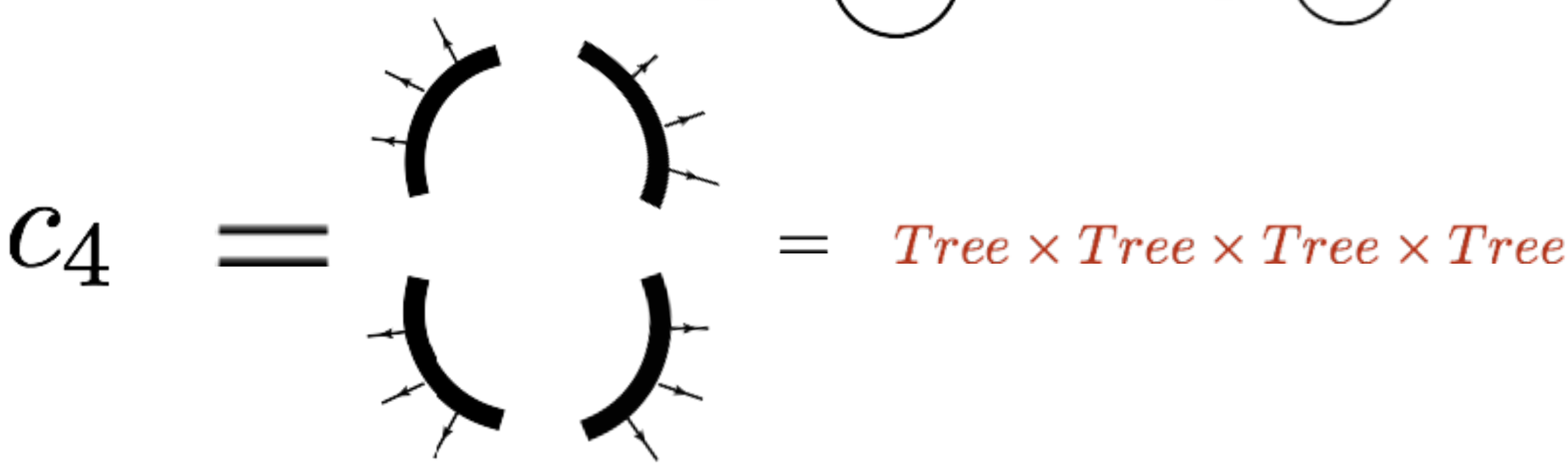




Why is NLO automated?

- Great understanding of QCD radiation and one-loop amplitudes.
- One-loop amplitudes in gauge theories = (Tree-amplitudes in gauge-theories) and (Integrals in scalar field theories)
- Singularities of tree-amplitudes due to radiation of a single parton understood for arbitrary processes.





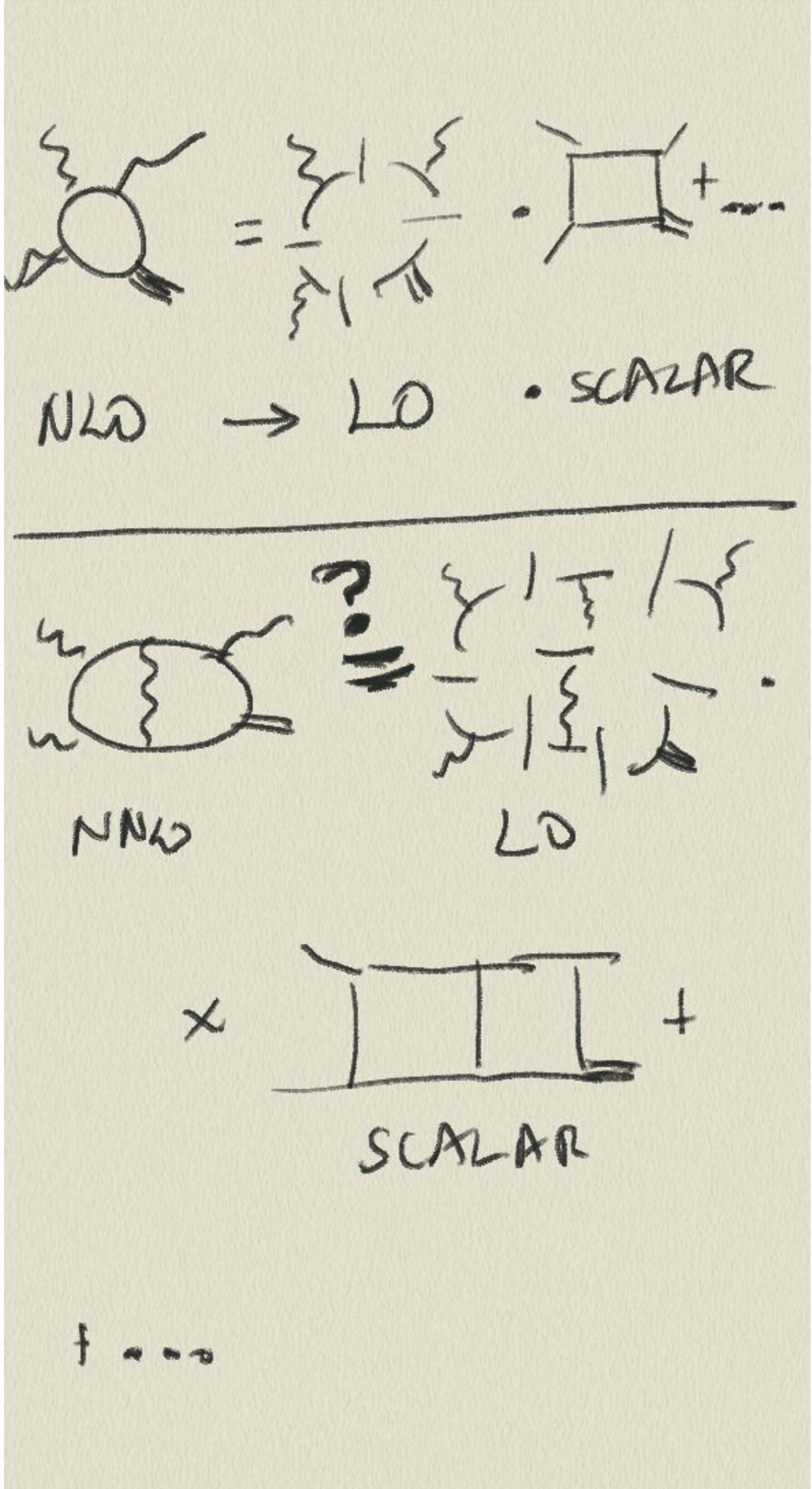
From NLO to NNLO

- A very beautiful structure of perturbation theory at NLO, where we can reduce the cross-section calculations to a few scalar integrals and LO calculations (infrared limits, master integral coefficients)

- It makes one dream that also higher orders NNLO, NNNLO, etc may be reduced to a few scalar integrals and LO calculations.

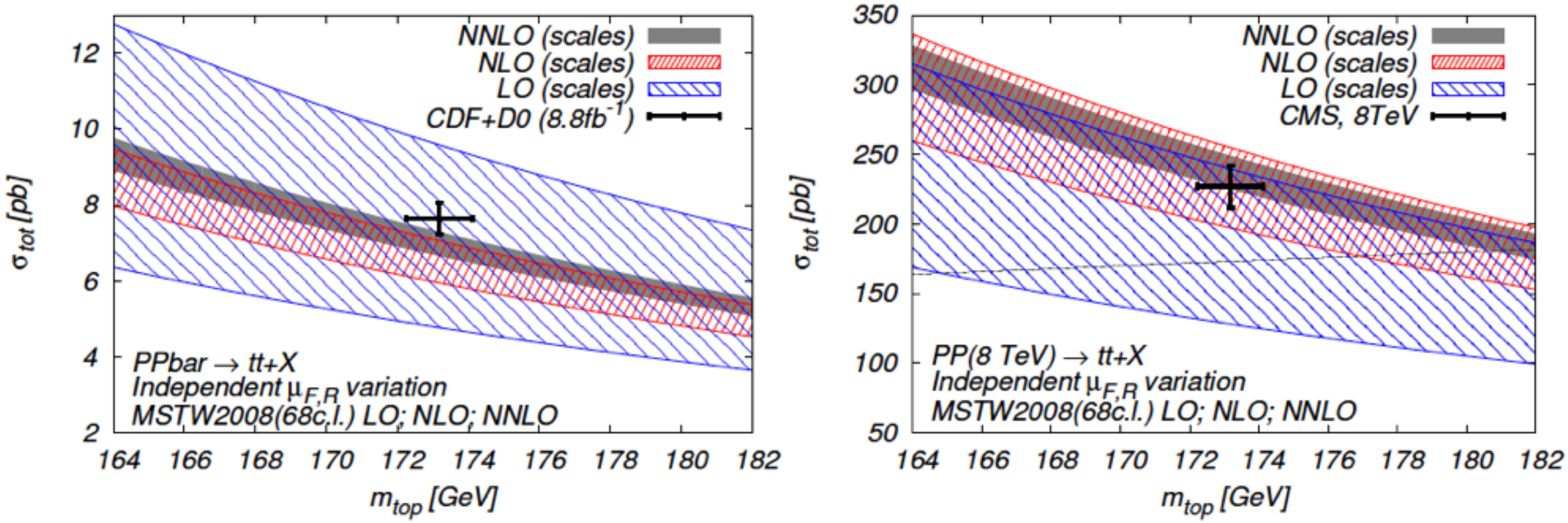
- Such a structure has not arisen yet, but it is promising.

- Progress is fast over the last decade with increasingly sophisticated methods.

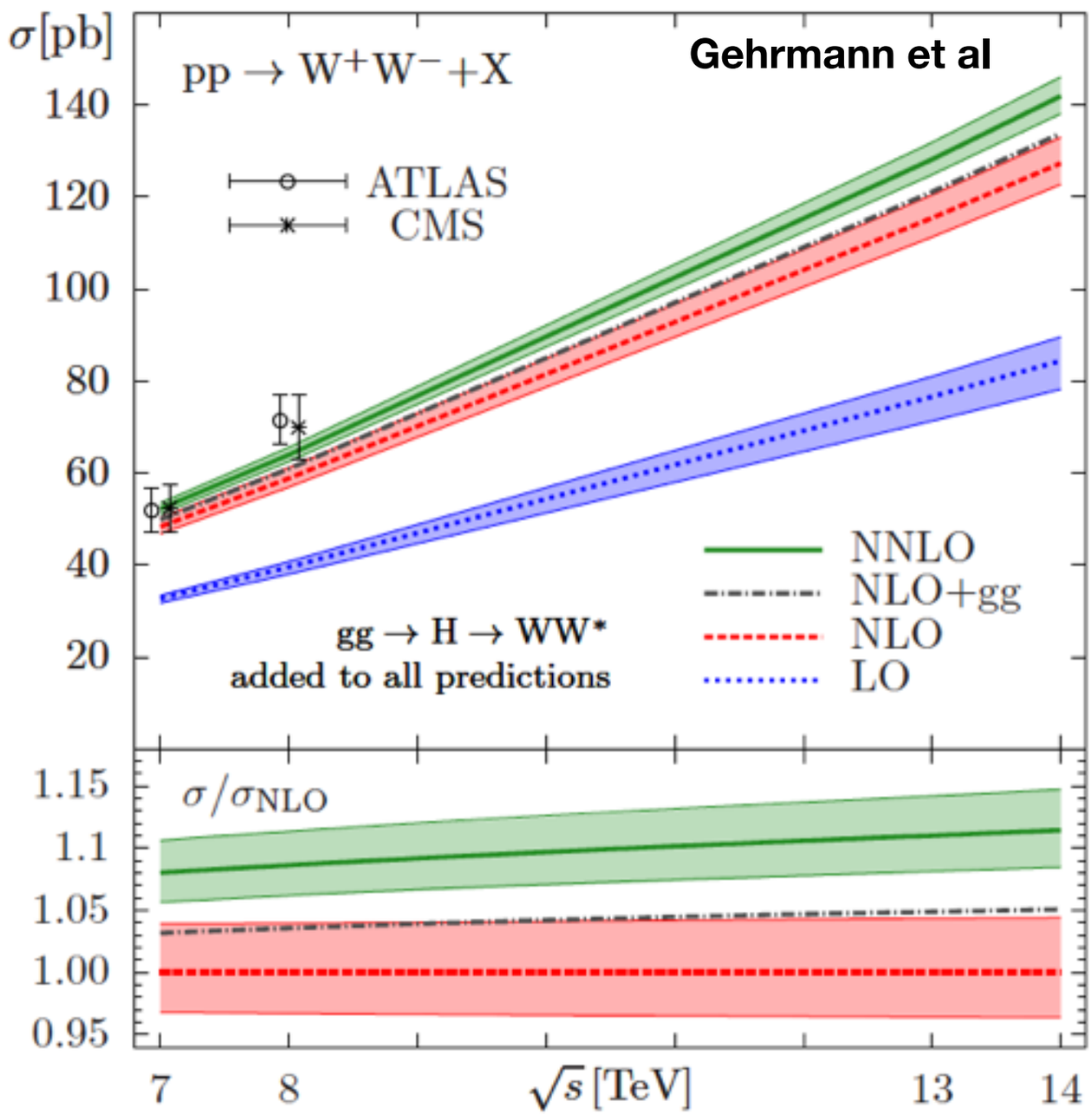


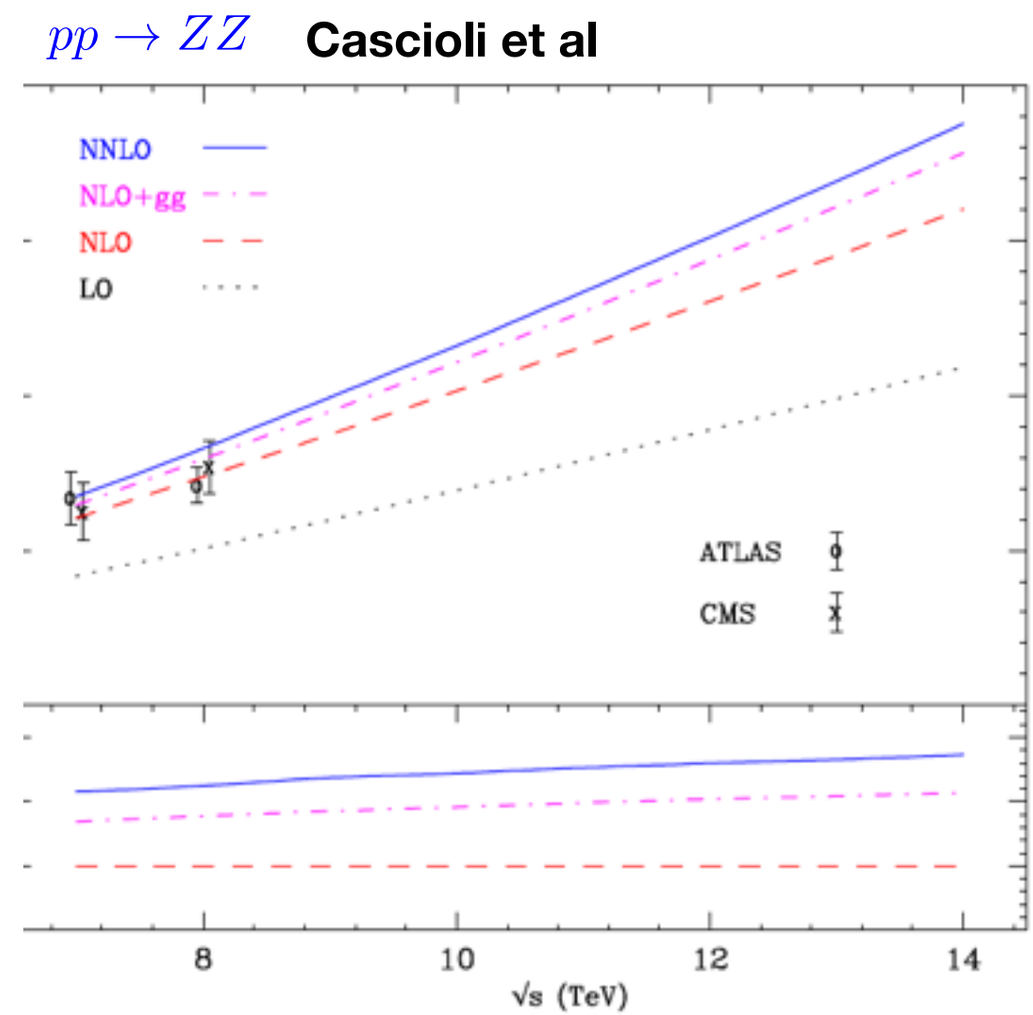
Recent NNLO computations

Czakon et al



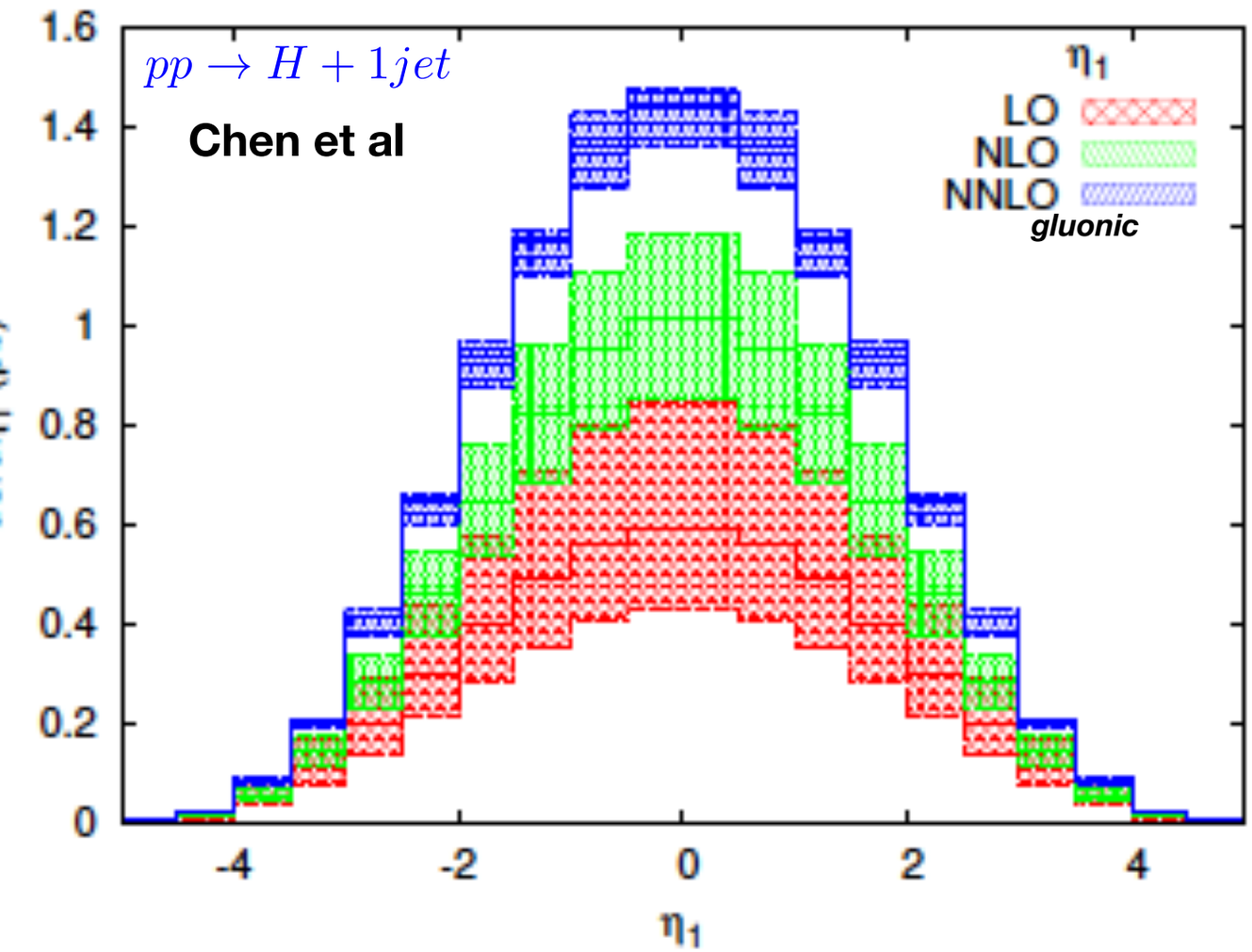
Recent NNLO computations

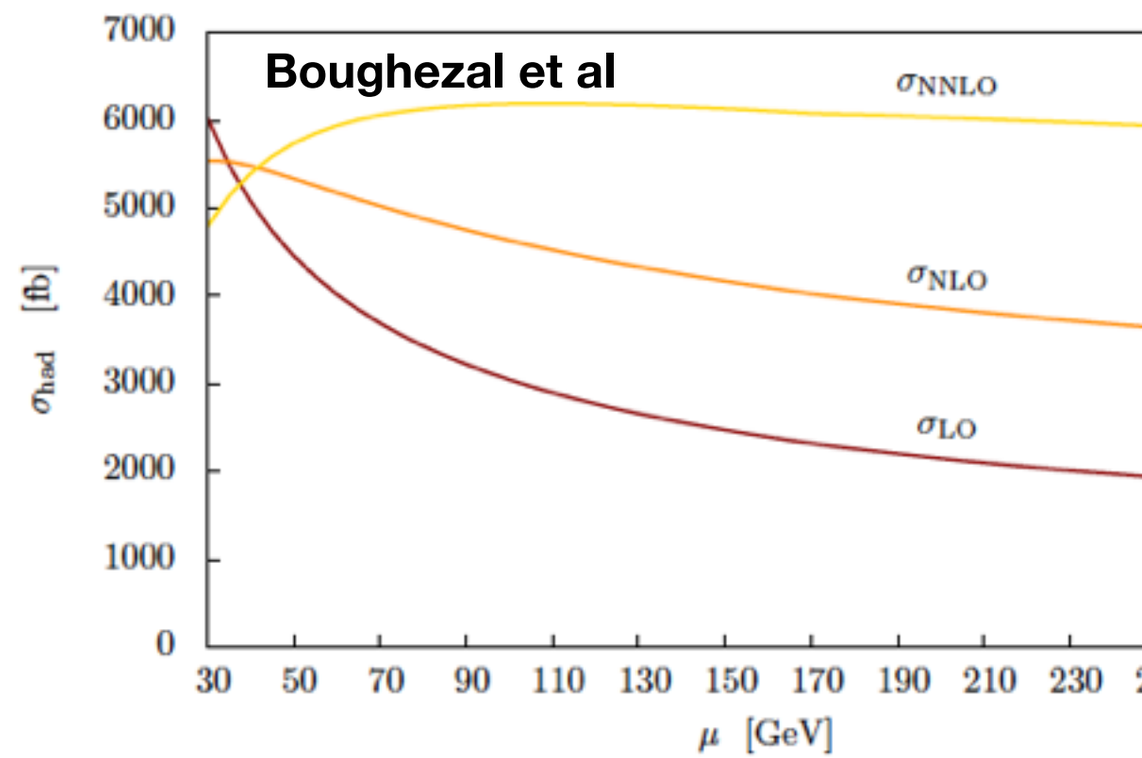


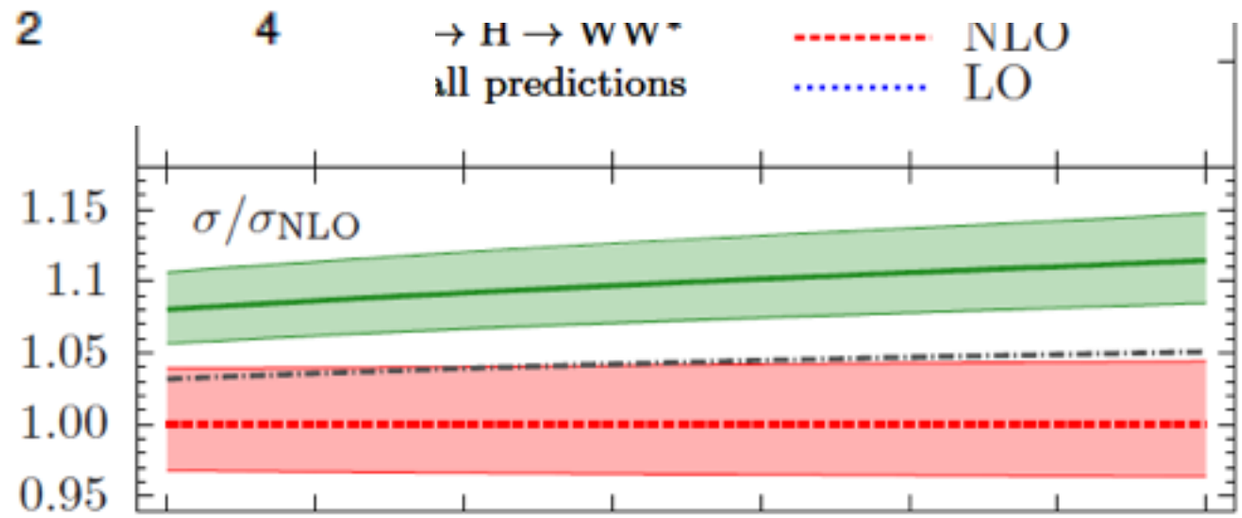


Recent NNLO computations

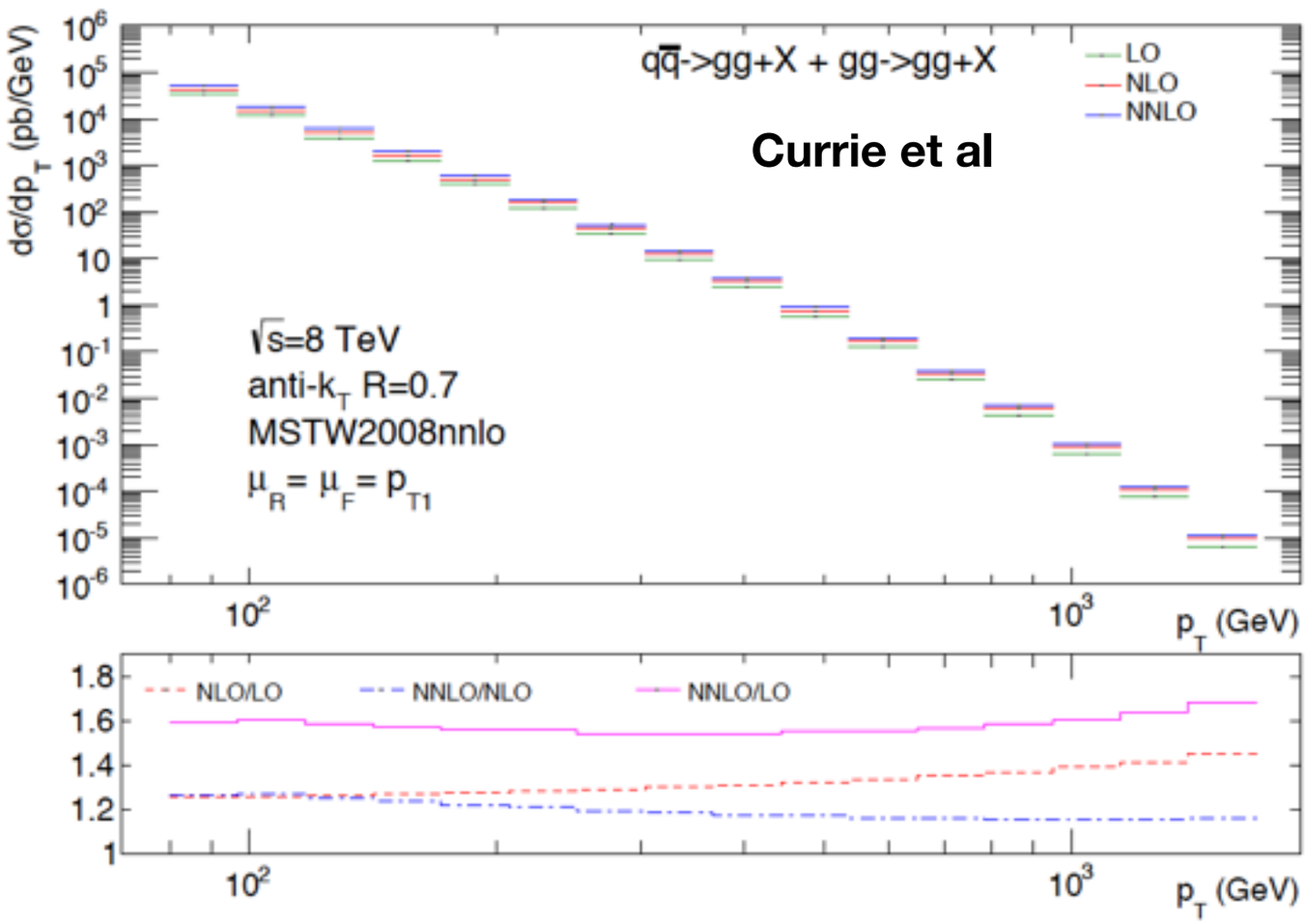
Gehrmann et al

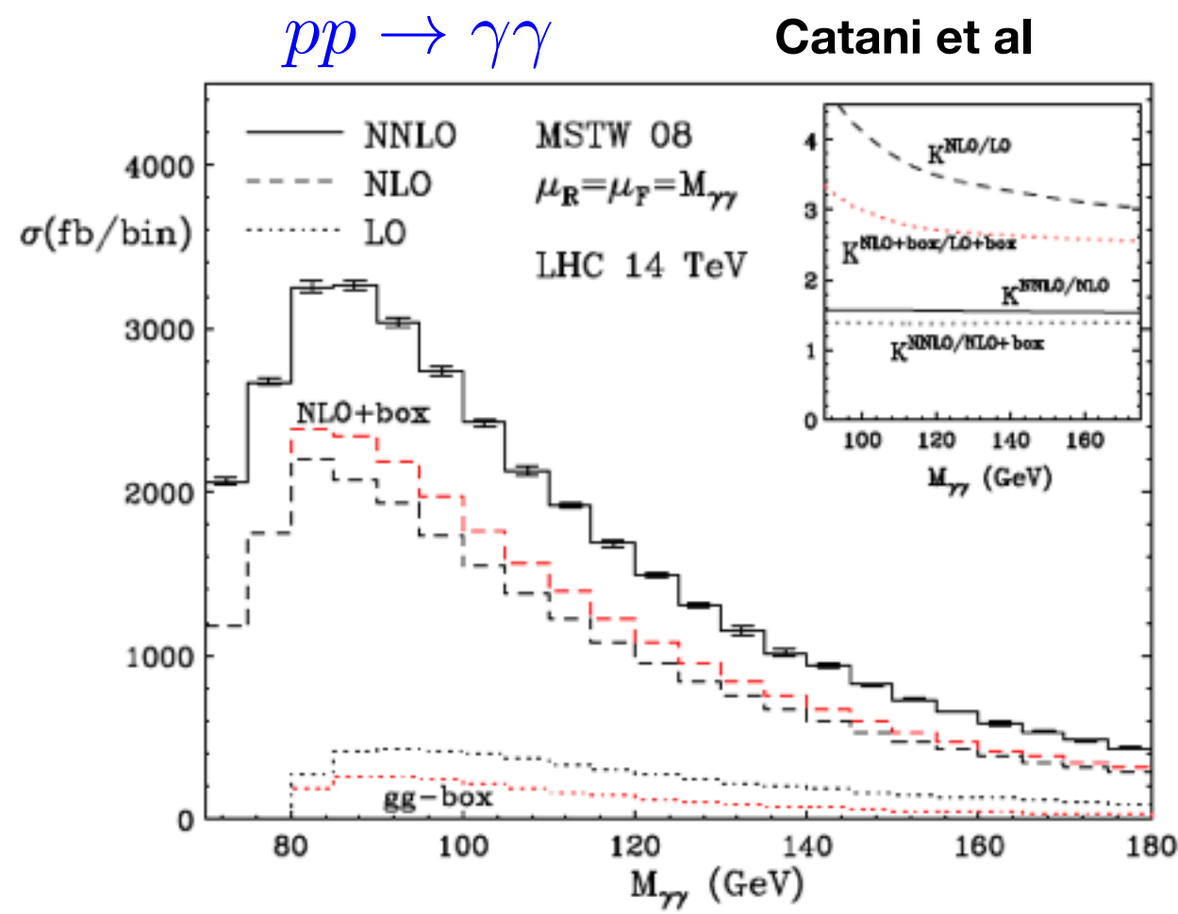






Recent NNLO computations





The start of the NNLO revolution

- Subtraction methods at NNLO are reaching maturity. It is now clearer how to combine real corrections and virtual corrections, cancelling infrared singularities.

- Result of persistence to solve a difficult problem and very good ideas. Solutions are numerical and applicable to general processes.

- Now focus turns back to developing even better methods for the computation of virtual amplitudes.

- This is a problem which is customarily attacked on a process by process basis. But with powerful new mathematics! (Talk by Claude Duhr)

- Bright future! This is just the beginning….

Strategy of regions

A small puzzle

An integral depending on a small parameter ($\delta \to 0^+$):

$$I = \int_{-\infty}^{\infty} dx \frac{1}{(x^2+\delta^2)\left[(x-1)^2+\delta^2\right]} = \frac{2\pi}{\delta(1+4\delta^2)}$$

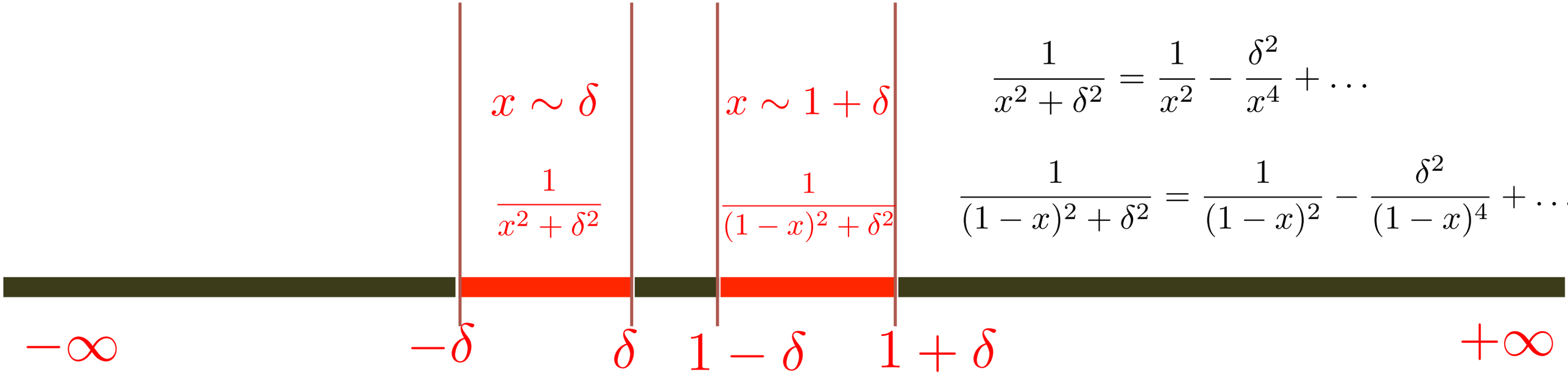
Knowing the analytic result it is easy to expand:

$$I = \frac{2\pi}{\delta}\left[1-(4\delta^2)+(4\delta^2)^2-(4\delta^2)^3+\ldots\right]$$

If we did not know the answer, could we at least get a few terms of the expansion in the small parameter?

NOT EASY: As $\delta \to 0^+$ the integral diverges!

Need to expand around an infinite value!



$$\frac{1}{x^2+\delta^2} = \frac{1}{x^2} - \frac{\delta^2}{x^4} + \dots$$

$$\frac{1}{(1-x)^2+\delta^2} = \frac{1}{(1-x)^2} - \frac{\delta^2}{(1-x)^4} + \dots$$

$$I = \int_{-\infty}^{-\delta} + \int_{-\delta}^{+\delta} + \int_{+\delta}^{1-\delta} + \int_{1-\delta}^{1+\delta} + \int_{1+\delta}^{\infty}$$

- Slice the integration region: **Hard+SoftA+SoftB**
- **Hard**: no denominator becomes singular. **SoftA,B**: one denominator becomes singular.
- In each region, Taylor expansions are legitimate! **We can expand the integrand and integrate within the boundaries of the region.**

Strategy of regions

Let's introduce some regulators

$$I = \int_{-\infty}^{\infty} dx \frac{1}{\left[(x^2+\delta^2)\right]^{1+a}\left[(x-1)^2+\delta^2\right]^{1+b}} \qquad a, b \text{ small}$$

and expand the ***integrand*** *around the first singular point,*
without restricting the integration *within the SoftA region:*

$$x \sim \delta : \qquad I_{x\sim\delta} = \sum_{n=0}^{\infty} \frac{(1+b,n)}{n!} \int_{-\infty}^{\infty} dx \frac{\left(2x - x^2 - \delta^2\right)^n}{\left[x^2+\delta^2\right]^{1+a}}$$

Exchanging the summation and integration is illegitimate.
But let's go on! The a,b regulators allow to perform all integrations.
After we do so we find that we can set the regulators to zero. We obtain:

$$I_{x\sim\delta} = \frac{\pi}{\delta}\left[1 - (4\delta^2) + (4\delta^2)^2 - (4\delta^2)^3 + \ldots\right] = \frac{I}{2}$$

Now expand around the second singular point: $x \sim 1 + \delta$

$$I_{x\sim 1+\delta} = \sum_{n=0}^{\infty} \frac{(1+a,n)}{n!} \int_{-\infty}^{\infty} dx \frac{\left(2(1-x) - (1-x)^2 - \delta^2\right)^n}{\left[(1-x)^2 + \delta^2\right]^{1+b}}$$

Performing, an unrestricted integration and setting the regulators to zero:

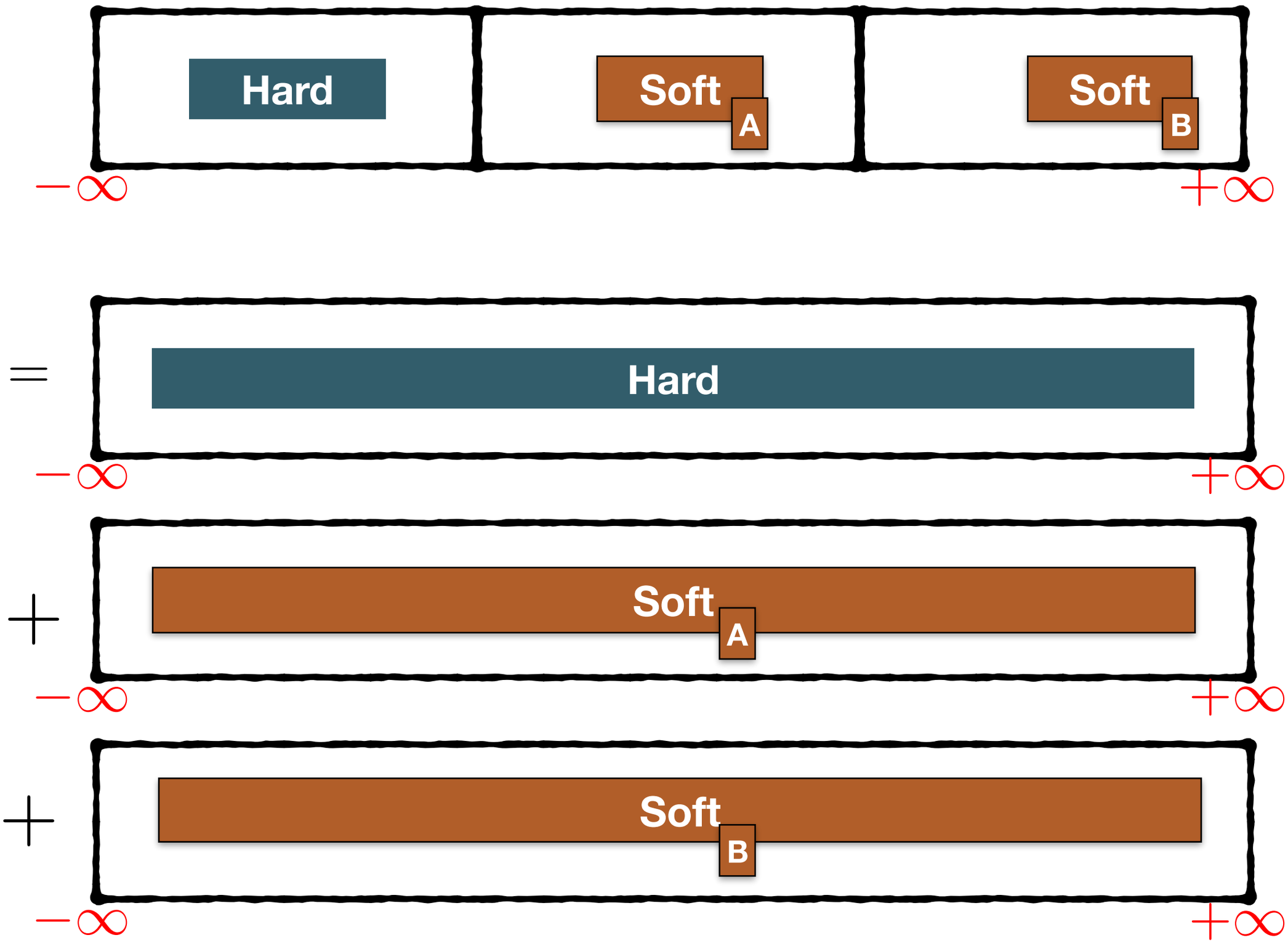
$$I_{x\sim 1+\delta} = \frac{\pi}{\delta}\left[1 - (4\delta^2) + (4\delta^2)^2 - (4\delta^2)^3 + \ldots\right] = \frac{I}{2}$$

Now expand around a point away from the singularities:

$$I_{x\nsim 0,1} = \sum_{n,m=0}^{\infty} \frac{(1+a,n)}{n!} \frac{(1+a,m)}{m!} \int_{-\infty}^{\infty} dx \frac{(\delta^2)^{n+m}}{(x^2)^{1+a+n}\left[(1-x)^2\right]^{1+b+m}} = 0,$$

as $a, b \to 0$.

Strategy of regions



Strategy of regions

- The total integral is the sum of all of its regions, where each one of them is extended to cover the full integration domain.
- Counterintuitive: the integration domains are overlapping! Apparently, the overlaps vanish.
- Not clear why. Regularisation seems to play a role.
- This observation appears to hold in general for all sorts of Feynman integrals.
- Basis for the formulation of effective theories such as SCET and the proof of factorisation theorems (***Beneke,Smirnov,…***)

Application in Higgs physics

- The equality of the full integral with the sum of regions is a statement valid at all orders in the small-parameter expansion, not just the zero limit.

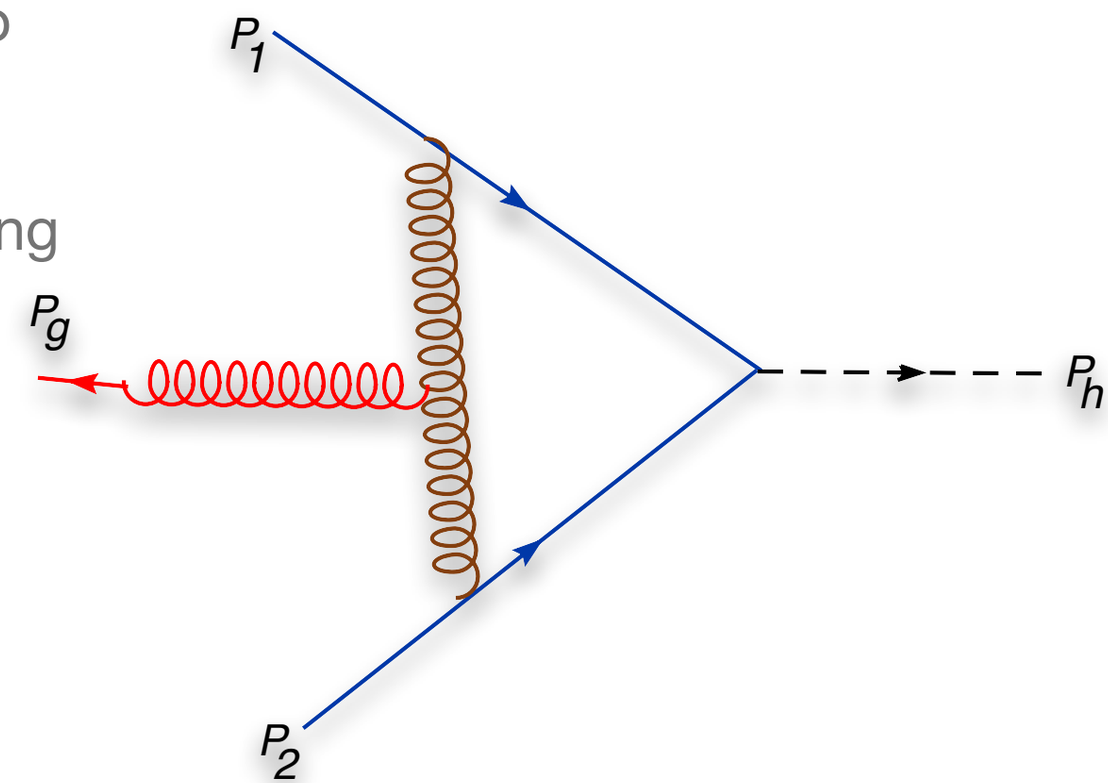
- Can be used to calculate as many sub-leading terms in the small parameter as we can technically calculate.

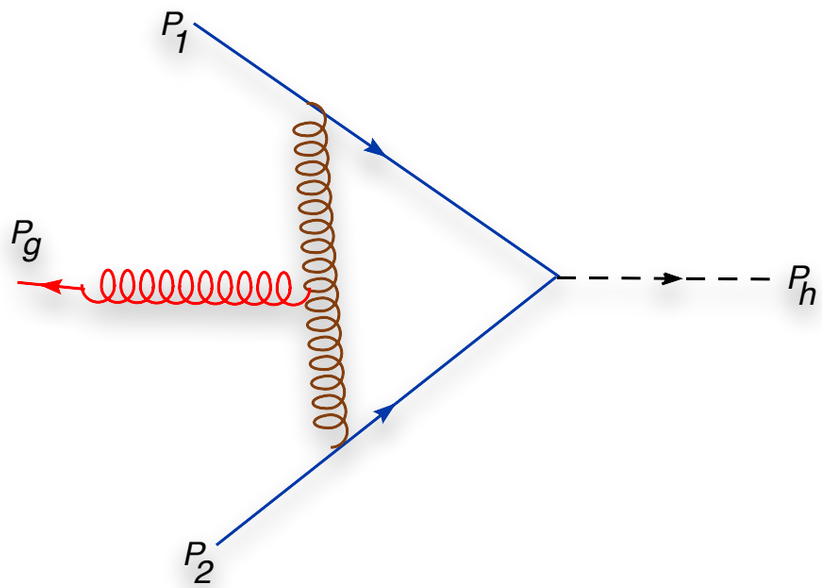
- Small parameter in Higgs boson production: Recoil energy of the Higgs boson:

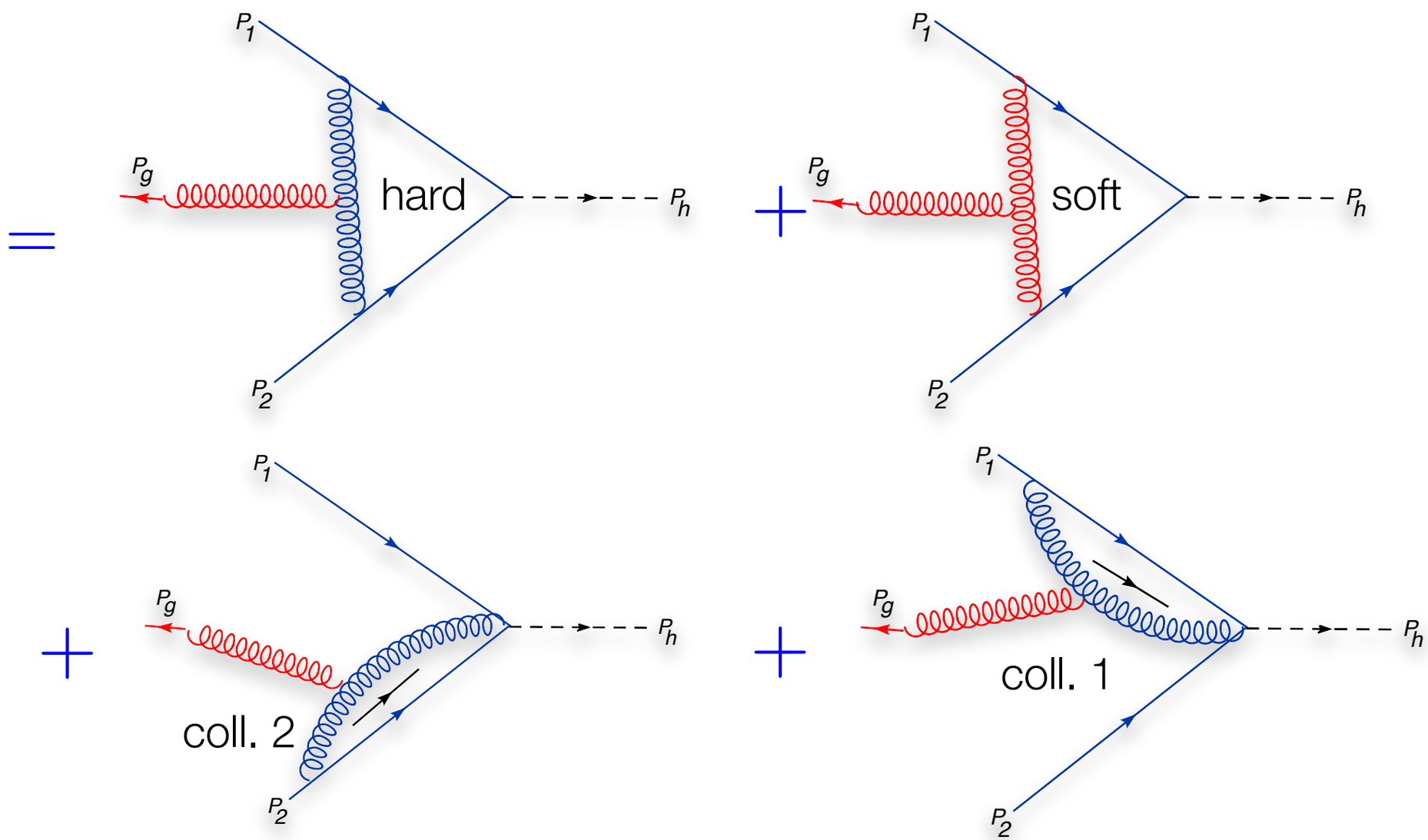
$$\delta = 1 - \frac{(P_1 + P_2)^2}{M_h^2}$$

- Final state gluon radiation is suppressed by this factor:

$$P_g \propto \delta$$



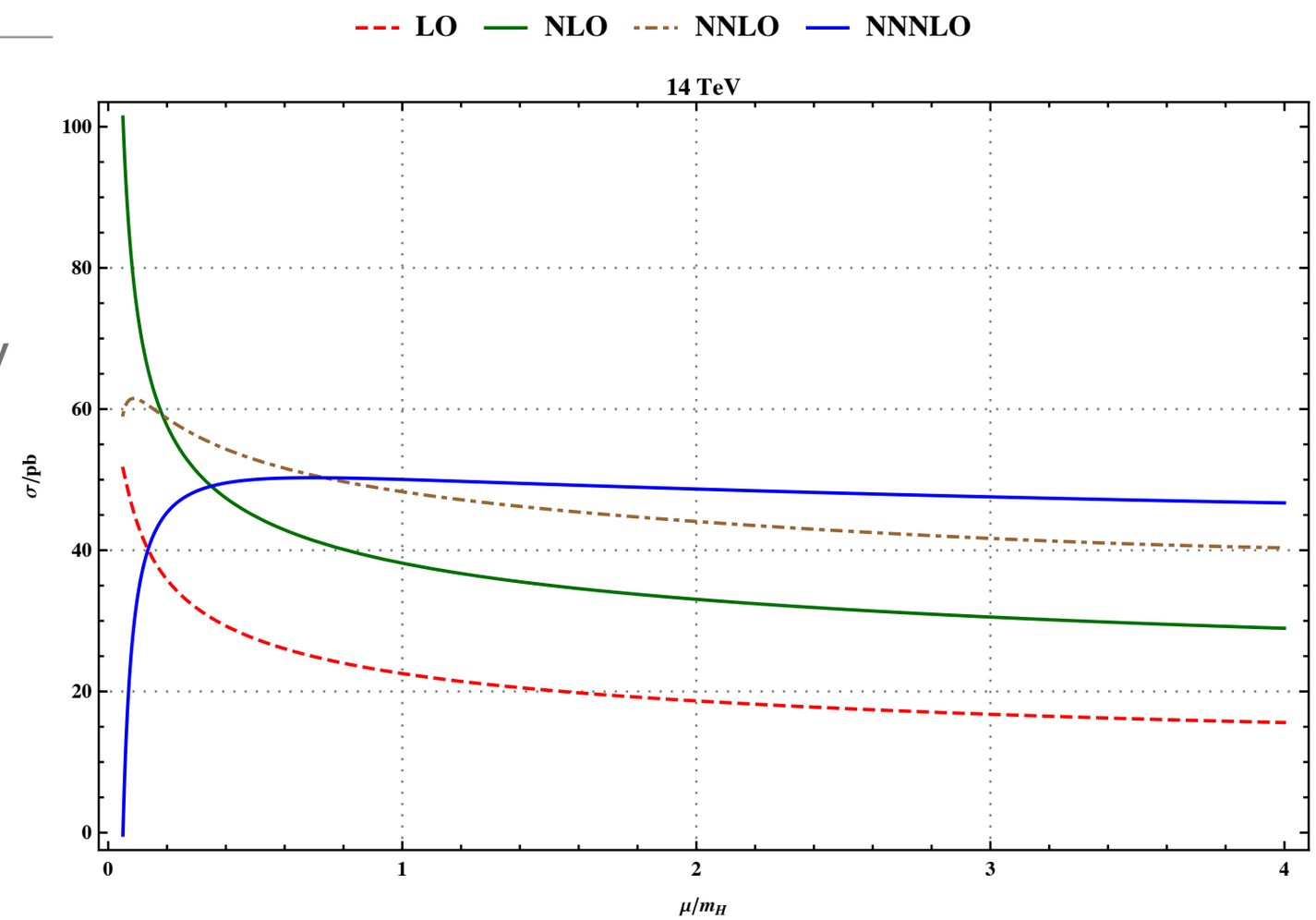

P_1
P_g
P_h
P_2

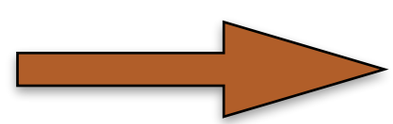

=
P_1
P_g
hard
P_h
P_2
+
P_1
P_g
soft
P_h
P_2
+
P_1
P_g
P_h
coll. 2
P_2
+
P_1
P_g
P_h
coll. 1
P_2

Higgs cross-section at NNNLO

CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

- We have devised a method to compute the Higgs cross-section at NNNLO as an expansion in the Higgs energy recoil.
- Based on the strategy of regions in combination with other techniques for dealing with the algebraic complexity of such a computation.
- Novel techniques for performing multidimensional integrations.





talk by Claude Duhr

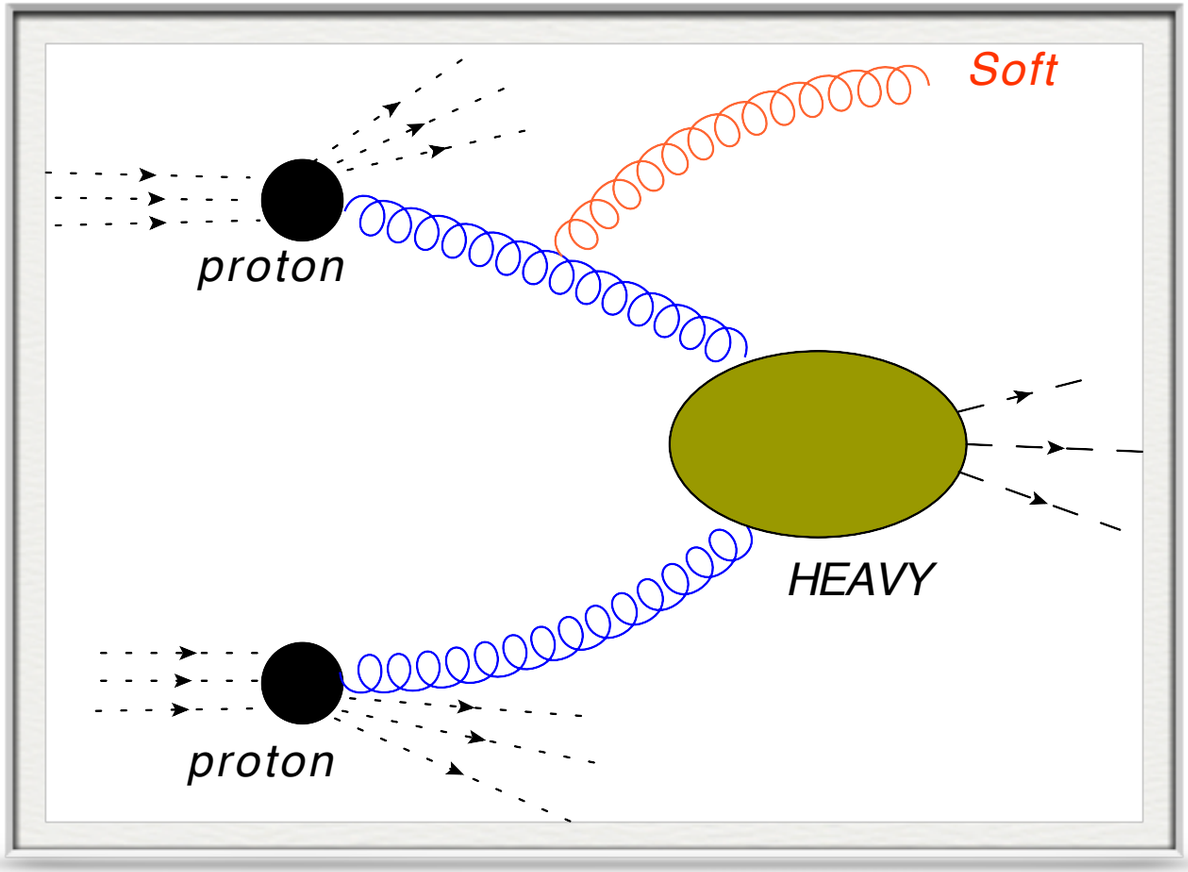
Temptations of the threshold approximation and Higgs production via gluon fusion

Production of a heavy system

- A major purpose of the LHC is to produce particles or systems of particles with a large invariant mass.

- At threshold, the partonic centre of mass energy is converted fully to the invariant mass of the heavy system. Radiation from the initial state is emitted with zero energy (soft radiation).

- It is tempting to compute higher order corrections to the production cross-section in the threshold limit.



Threshold expansion

- Hadronic cross-section:

$$\sigma = \int dx_1 dx_2 \mathcal{L}_{ij}(x_1, x_2)\sigma_{ij}(z) \qquad z \equiv \frac{M^2_{\text{Heavy}}}{x_1 x_2 S}$$

- Threshold expansion:

$$\sigma_{ij}(z) = \sigma_{ij}^{(-1)}(z) + \sigma_{ij}^{(0)}(z) + (1-z)\sigma_{ij}^{(1)}(z) + (1-z)^2\sigma_{ij}^{(2)}(z) + \ldots$$

- with

$$\sigma_{ij}^{(-1)} = c_0(\alpha_s)\delta(1-z) + \sum_{i=0}^{\infty} d_i(\alpha_s)\left[\frac{\log(1-z)^i}{1-z}\right]_+$$

$$\sigma_{ij}^{(k)} = \sum_{i=0}^{\infty} f_i(\alpha_s)\log(1-z)^i$$

Threshold limit

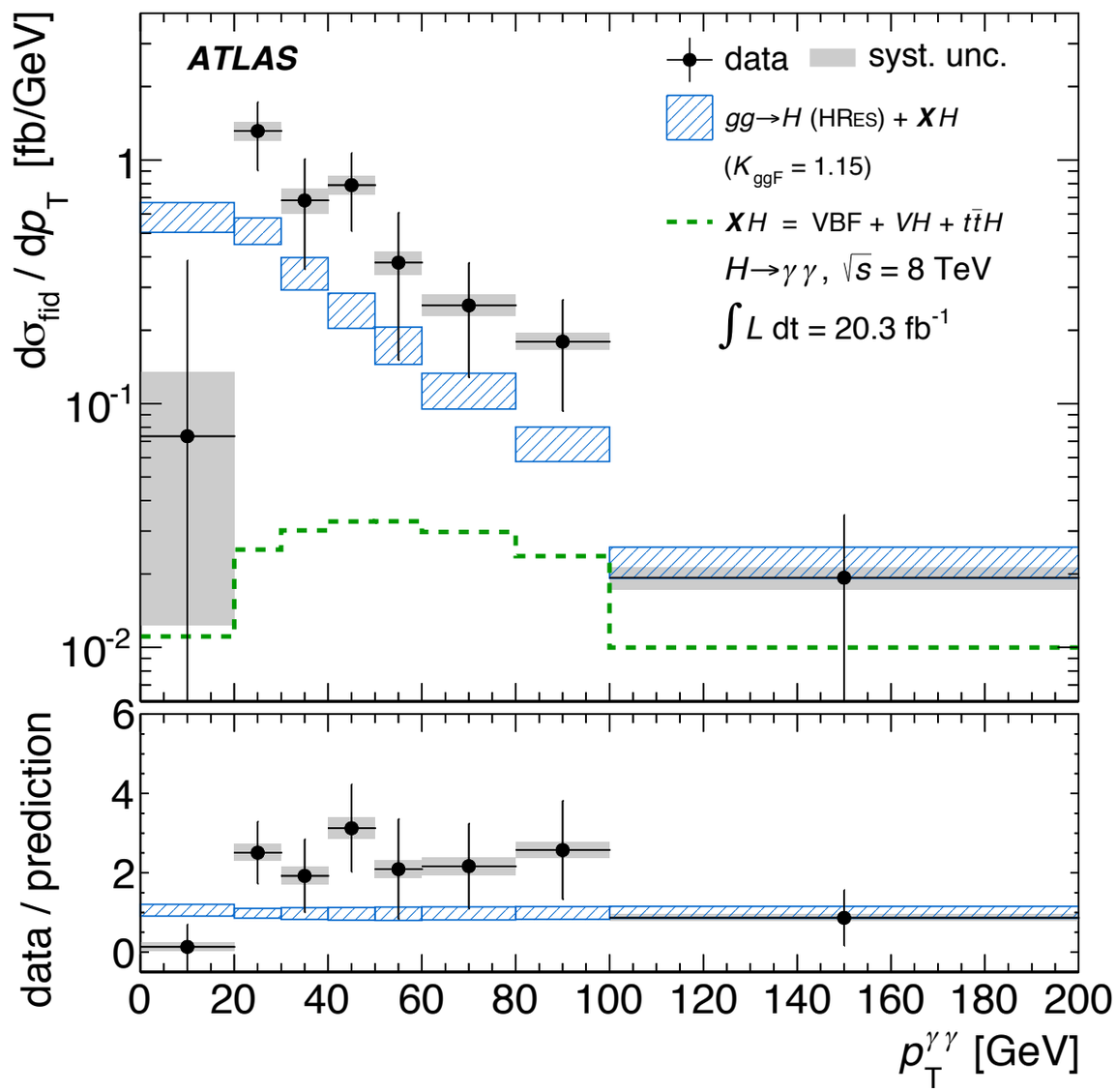
- Threshold logarithms (and plus distributions) are the remnants of cancelations of infrared divergences.
- They follow a universal pattern.
- Easier to calculate and can be predicted at a higher order in the strong-coupling expansion than the last fully-known perturbative order.
- They can be re-summed at all perturbative orders with various methods (SCET,direct QCD,…)
- Universality of threshold logs tempts us to extrapolate phenomenology results from one process to others.

Threshold limit in Higgs production

- Threshold corrections at NNLO in 2001 **(Catani, de Florian, Grazzini; Harlander, Kilgore)**
- Full NNLO computation followed in 2002.
- NNLL threshold resummation matched to full NNLO result in 2003 **(Catani, de Florian, Grazzini, Nason)**
- Threshold plus-distributions were inferred from splitting functions in 2005 **(Moch, Vogt)**
- SCET resummation of threshold logs and Pi^2 in 2008 **(Ahrens, Becher, Neubert, Lin Yang)**)
- "Combination" of threshold and high energy limits 2013 **(Ball, Bonvini, Forte, Marzani, Ridolfi)**
- Threshold corrections at N3LO 2014 **(CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger)**
- Some sub-leading logs beyond threshold 2014 **(De Florian, Mazzitelli, Moch, Vogt)**
- Comparison of methods for threshold resummation 2014 **(Sterman, Zeng; Bonvini, Forte, Ridolfi)**

Usefulness of the threshold limit

- The threshold limit is a non-trivial and important part numerically of the total cross-section
- If an NxLO correction at threshold is large, this is a very important indicator that the full NxLO correction is large.
- Is threshold dominant?
- A light Higgs boson is produced with a significant recoil, but not a very large one.
- Threshold limit is important but sub-leading corrections are sizeable.



N3LO at threshold: z-space

$$
\begin{aligned}
\hat{\eta}^{(3)}(z) \simeq\ & \delta(1-z)\, 1124.308887\ldots && (\to 5.1\%)\\
& + \left[\frac{1}{1-z}\right]_+ 1466.478272\ldots && (\to -5.85\%)\\
& - \left[\frac{\log(1-z)}{1-z}\right]_+ 6062.086738\ldots && (\to -22.88\%)\\
& + \left[\frac{\log^2(1-z)}{1-z}\right]_+ 7116.015302\ldots && (\to -52.45\%)\\
& - \left[\frac{\log^3(1-z)}{1-z}\right]_+ 1824.362531\ldots && (\to -39.90\%)\\
& - \left[\frac{\log^4(1-z)}{1-z}\right]_+ 230 && (\to 20.01\%)\\
& + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 216\,. && (\to 93.72\%)
\end{aligned}
$$

2005 (Moch, Vogt)

2014 (CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger)

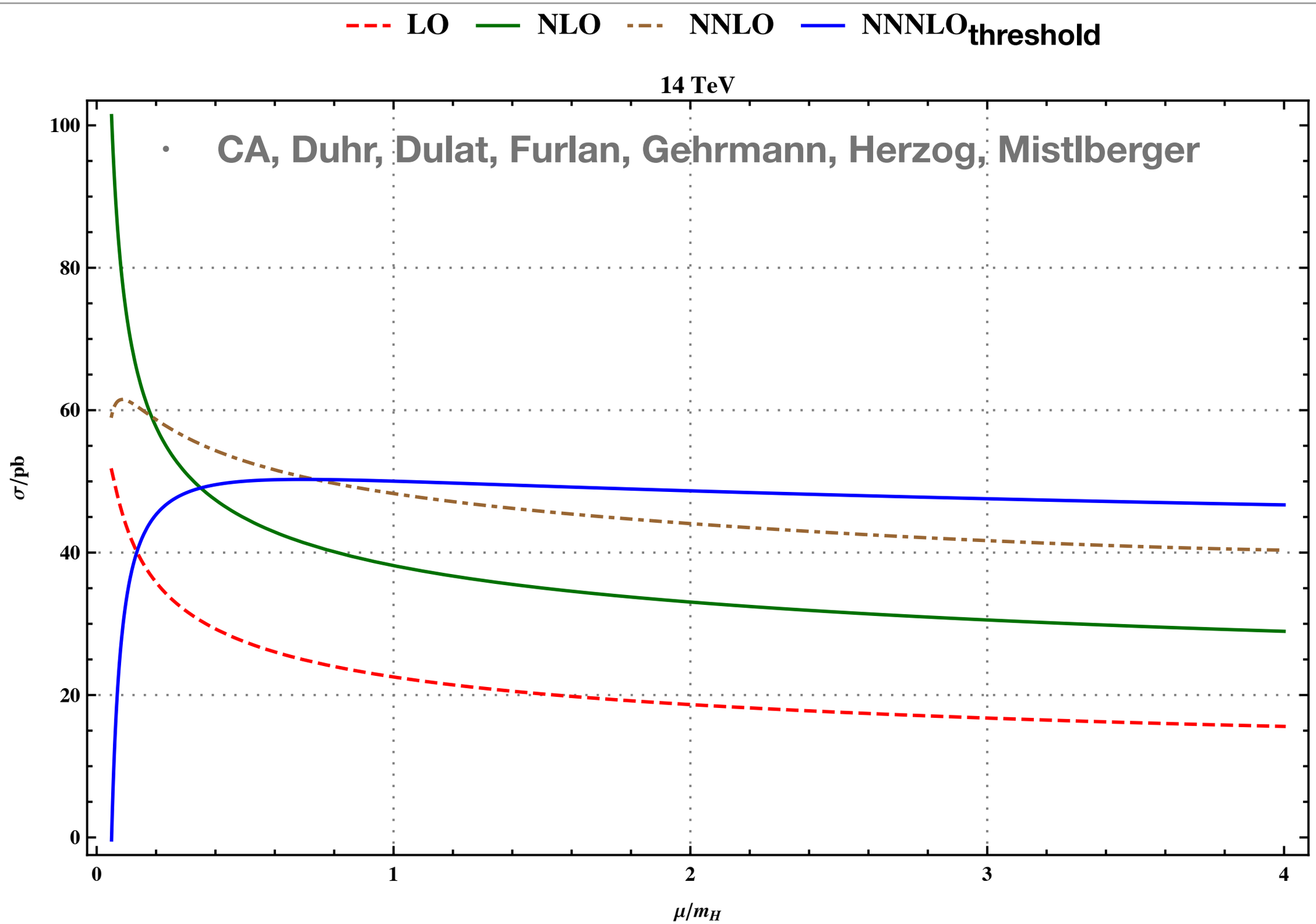
N3LO at threshold: z-space

- Formal Log hierarchy in the threshold limit is not reflected numerically, indicating that kinematically the process is not taking place predominantly at threshold.

- Large cancelations of formally leading against formally sub-leading terms.

- The new delta(1-x) term is as large and of opposite sign (+) as all the previously known plus distributions (-).

Numerical impact



Mellin space

- Hadronic cross-section is a convolution of parton luminosities and the par tonic cross-sections

$$\sigma(\tau) = \tau \int dydz\delta(\tau - zy)\mathcal{L}_{ij}(y)\frac{\sigma_{ij}(z)}{z} \qquad \tau \equiv \frac{Mh^2}{S}$$

- It factorises if we take its Mellin-transform:

$$\sigma(N) = \int_0^1 d\tau\tau^{N-1}\frac{\sigma(\tau)}{\tau} = \mathcal{L}_{ij}(N)\sigma_{ij}(N)$$

$$\mathcal{L}_{ij}(N) \equiv \int_0^1 dyy^{N-1}\mathcal{L}_{ij}(y) \qquad \sigma_{ij}(N) \equiv \int_0^1 dyz^{N-1}\frac{\sigma_{ij}(z)}{z}$$

- **Suited for resummation. Different (equivalent) soft expansion**

$$\frac{\log(1-z)}{1-z} \to \log(N)$$

N3LO at threshold: Mellin-space

	LO	NLO	NNLO	N^3LO
constant	100	77.4	32.2	8.04
(delta)	(100)	(35.1)	(1.72)	(5.07)
$\ln N$		14.8	12.0	5.14
$\ln^2 N$		7.16	7.56	4.04
$\ln^3 N$			1.07	1.09
$\ln^4 N$			0.18	0.27
$\ln^5 N$				0.025
$\ln^6 N$				0.002
SV	100	99.4	53.0	18.6
$C^2(m_H^2)$	100	19.6	2.05	0.12

(De Florian, Mazzitelli, Moch, Vogt)

N3LO at threshold: Mellin-space

- Formal Log hierarchy in the threshold limit is not reflected numerically, indicating that kinematically the process is not taking place predominantly at threshold.
- No cancelations of formally leading against formally sub-leading terms.
- The new delta(1-x) has a larger impact than what had been previously assumed (**Moch, Vogt)**
- Sub-leading terms?

IS THE THRESHOLD LIMIT RELIABLE?

- It is **ambiguous**

$$\sigma = \int dx_1 dx_2 f(x_1) f(x_2) \left[z g(z) \right] \left[\frac{\hat{\sigma}(z)}{z g(z)} \right]_{\text{threshold}}$$

We can choose "freely" $g(z)$ as long as $\lim_{z \to 1} g(z) = 1$

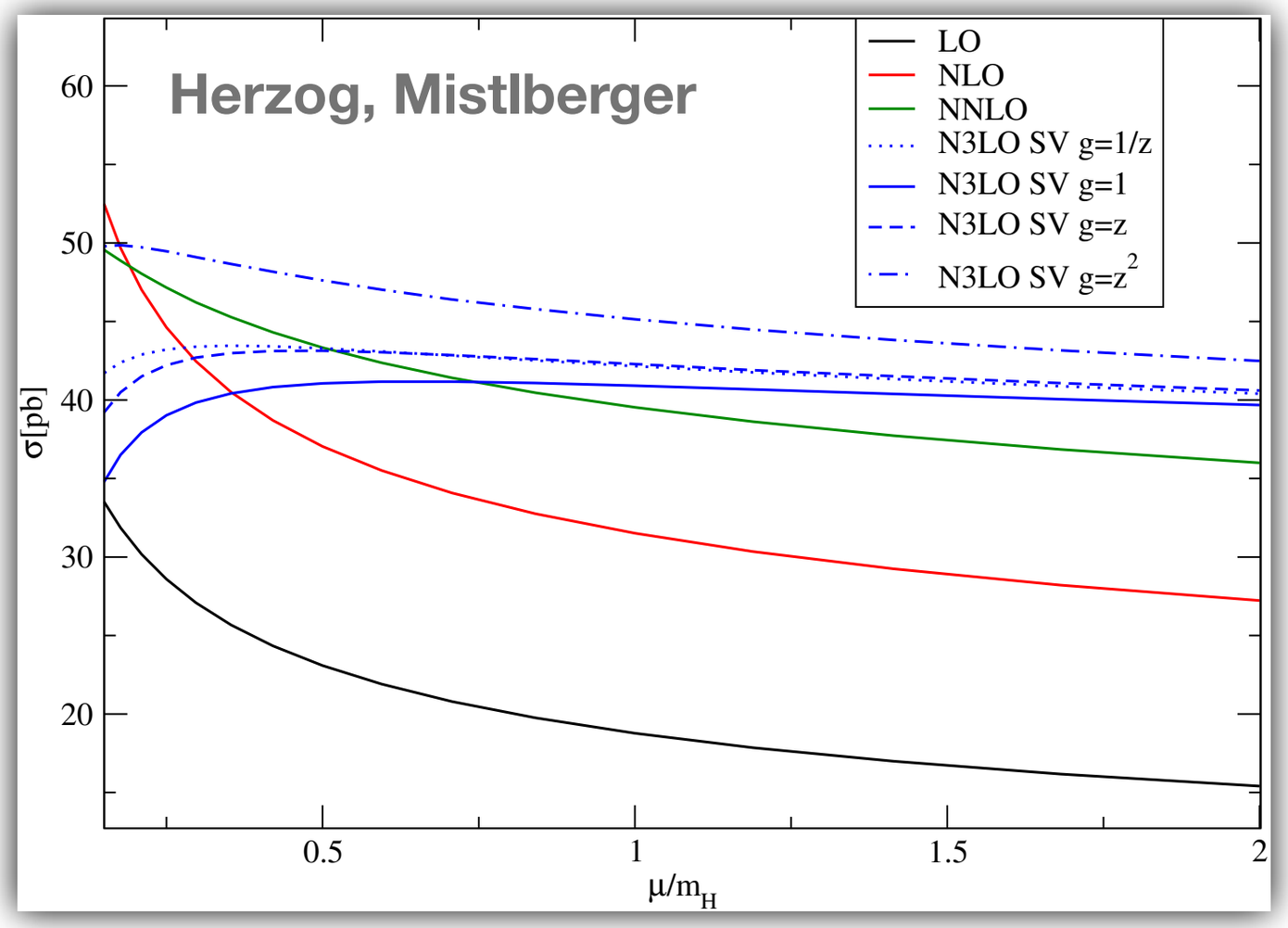
- Favorite choices of the past:

$$g(z) = \frac{1}{z}, 1$$

- "Best" choice at NNLO:

$$g(z) = z$$

- Is there a "correct" **g(z)**?



SCET vs direct QCD resummation

- This type of ambiguity causes the bulk of the difference in the numerical predictions between SCET and direct QCD resummation. **(Sterman, Zeng; Bonvini, Forte, Ridolfi)**

$$g(z) = 1 \text{ vs } g(z) \approx \frac{1}{\sqrt{z}}$$

- SCET calculations showed a negligible threshold resummation correction with respect to NNLO. **(Ahrens, Becher, Neubert, Lin Yang)**

- direct QCD resummations showed a 10% effect from threshold resummation. **(de Florian, Grazzini)**

- Difference originates from the different methods to invert the Mellin transform (before or after folding with parton densities)

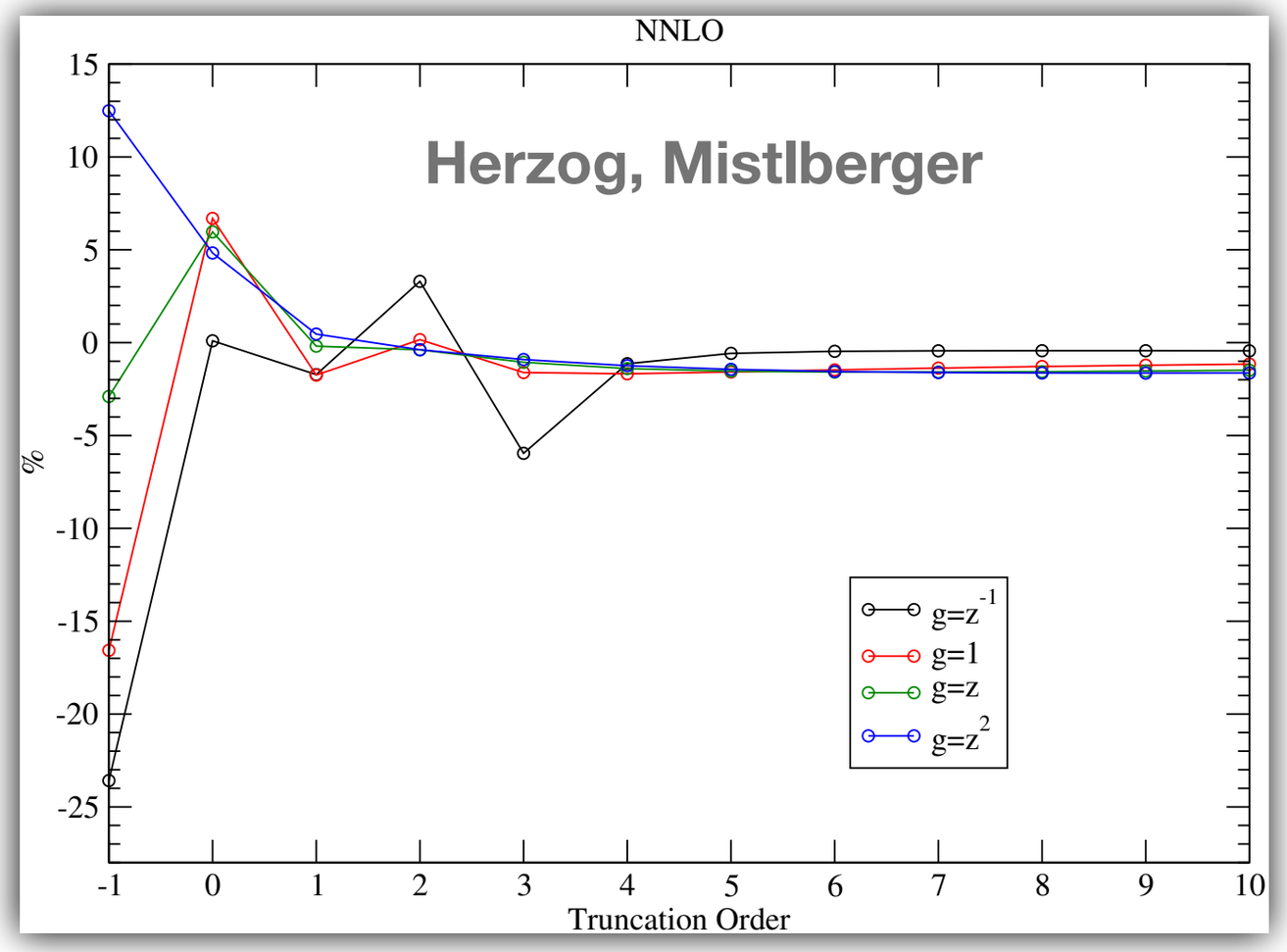
TREATMENT OF THE AMBIGUITY

- Let's try to learn from the NNLO, where we know the full result.

- We truncate the threshold expansion to the order indicated in the x-axis and compare to the full NNLO.

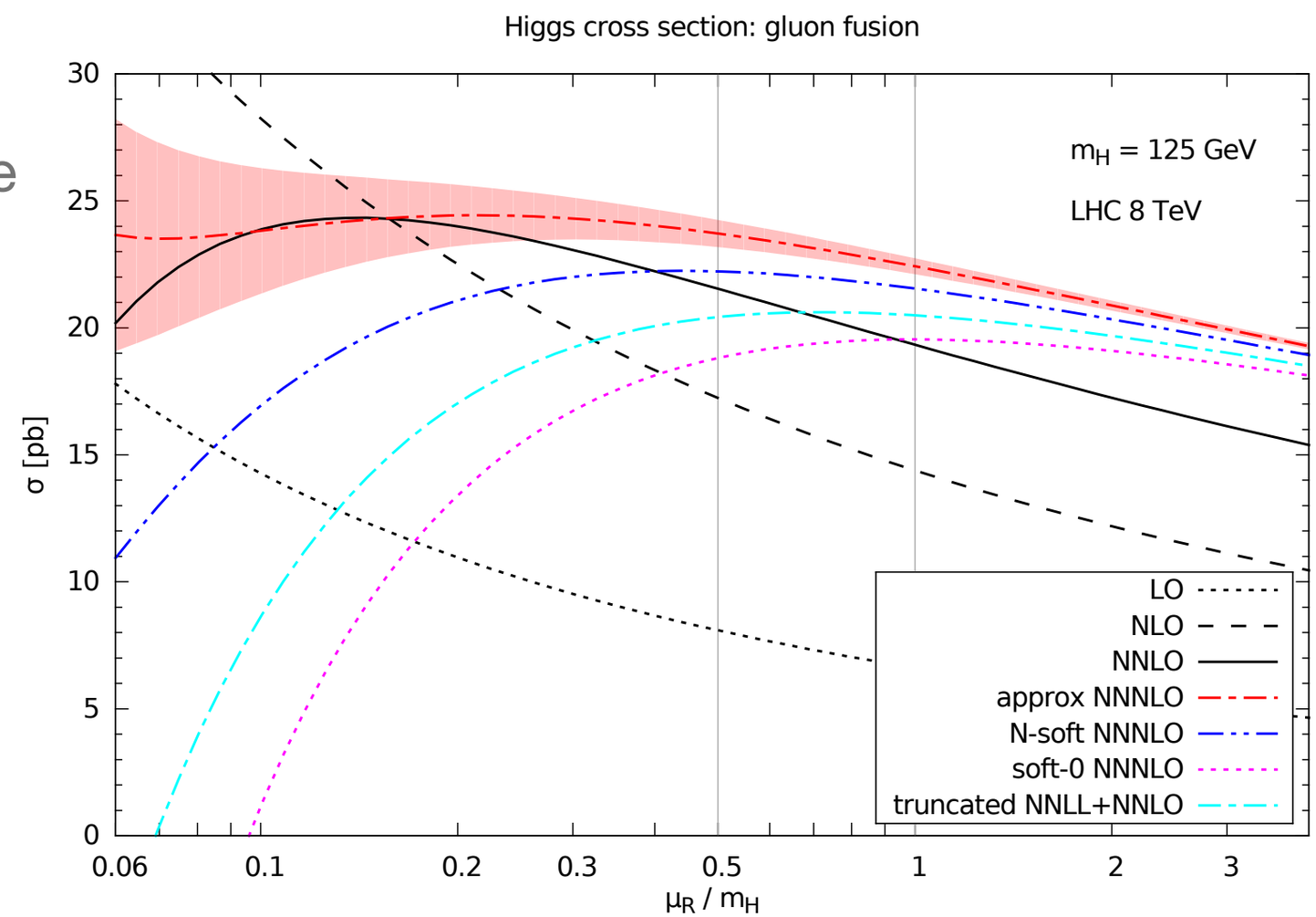
- Sub-leading terms reduce the ambiguity!

- Threshold expansion converges but slowly!



A guess of structure of sub-leading terms

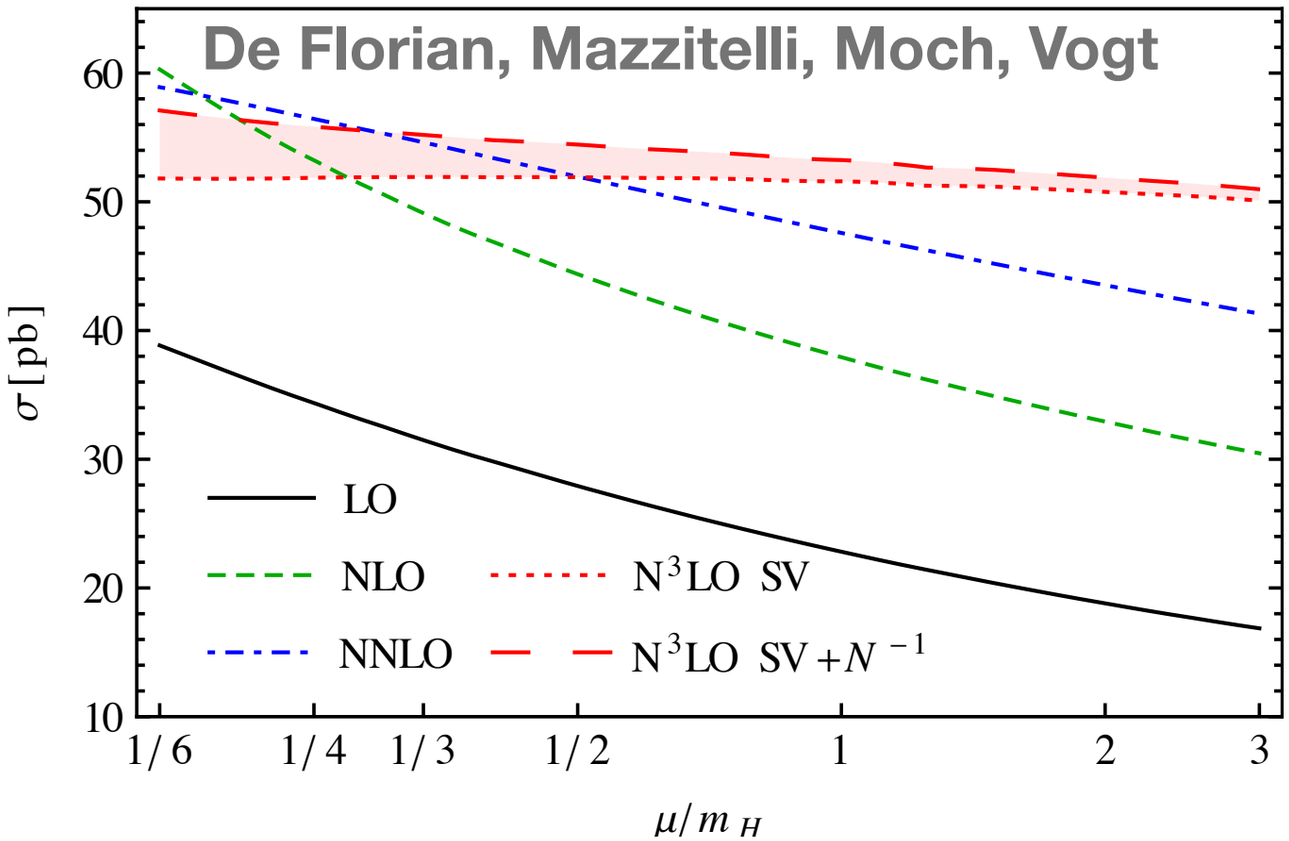
- Resum combinations of logs which do not spoil the analytic properties of the cross-section in Mellin-space in the opposite (high energy) limit.
- This criterion does not fix the sub-leading terms uniquely.
- It is “validated” numerically at NNLO.
- But the same holds for other prescriptions (g(z)=z, SVC-approximation, naive N-space, SCET) which can claim a similar success at NNLO, diverging from each other in their N3LO predictions.



- **Ball, Bonvini, Forte, Marzani, Ridolfi)**

N3LO beyond threshold

- We are performing a calculation of the full N3LO cross-section without using the threshold approximation.
- In the mean time, some sub-leading logs for the next term in the threshold expansion were determined using QCD factorization.
- Other undetermined logs have been guessed, based on other processes and previous orders.



Remarks on the status of the precision of the Higgs cross-section at N3LO.

- Threshold limit is too ambiguous.
- Need sub-leading terms.
- Guessing them is risky, given that we are aiming to a precision of below 5%.
- Guessing cannot substitute genuine calculations.
- We should not fall in love with our formalisms and approximations. Need to be critical of their validity.
- Need **complete calculations** of sub-leading terms in the threshold expansion or even better the full un-approximated N3LO correction.

outlook

The achievements in perturbative QCD of the last few years are amazing.

We have fulfilled some of our wildest dreams (especially at NLO)

This is the dawn of precision QCD at the LHC!