Generalized Focus point in the MSSM

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- Introduction
- Conditions for a focus point
- Conditions for a ‘double' focus point
- Implications on the nature of EWSB
- Conclusions

Work based on:
AD, M. García & M. Quirós arXiv:13123235
AD, M. Quirós & C. Wagner arXiv: 1402.1735-1406.2027

Introduction

- With the discovery of the Higgs with a mass of 125 GeV we are left with the following three possibilities:

1. The Higgs is a fundamental scalar and it is fine-tuned

2. The Higgs is a fundamental scalar and SUSY explains is lightness

3. The Higgs is some type of composite object.

- I will take the avenue of SUSY and devote myself to the MSSM.
- The lack of signals at the LHC is pushing the spectrum of colored sparticles to around 1 TeV.
- On the other hand the soft mass of the Higgs is related to the mass of the Z unless there is a cancellation with the μ-term.

$$m_z^2 \simeq -m_{H_U}^2 - |\mu|^2$$

- Since the soft mass of the Higgs gets corrected through the RGE evolution one has two possibilities:

  1. The value of the soft mass of the Higgs is much smaller than the rest in order not to reintroduce fine-tuning.

  2. There is a ‘little hierarchy’ problem that requires a large value of μ.

- This solution to the RGE evolution where the soft mass of the Higgs vanishes at low energies was called by Feng et. al a focus point.

- It requires that the different contributions coming from squarks, gauginos, A-terms and Higgses to cancel.

- Since the solution of a RGE is homogenous in the different masses, one can rescale the boundary conditions retaining the effect.

- In this talk I will suppose that SUSY is broken at a high scale M.

- I will then analyze for which boundary conditions and value of M I can have a vanishing soft mass for the Higgs at low energy.

- I will also study the possibility of having also a very light stop as a consequence of the RGE.

General Focus Point Solution

- The general solution for the soft mass of the Higgs as a function of a scale Q can be written as:

$$
\begin{aligned}
m_{H_U}^2(\mathcal{Q}) &= m_{H_U}^2 + \eta_Q[\mathcal{Q}, M](m_Q^2 + m_U^2 + m_{H_U}^2) + \sum_a \eta_a[\mathcal{Q}, M]M_a^2 \\
&+ \sum_{a \neq b} \eta_{ab}[\mathcal{Q}, M]M_a M_b + \sum_a \eta_{aA}[\mathcal{Q}, M]M_a A_t + \eta_A[\mathcal{Q}, M]A_t^2 + \Delta_{Y,H_U}
\end{aligned}
$$

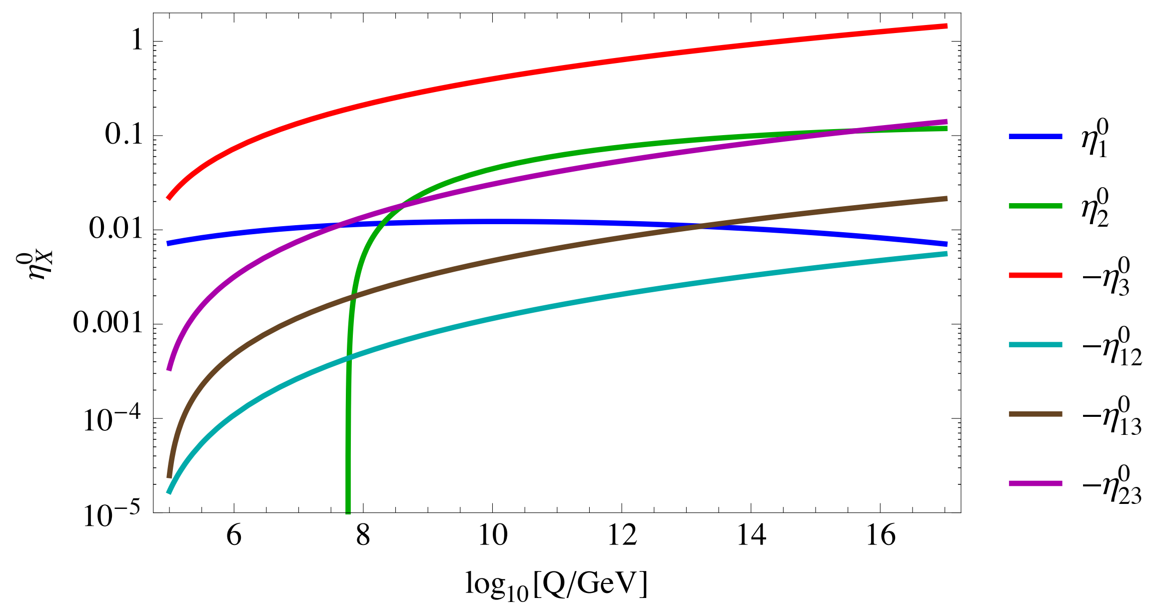
- So the focus point solution is written as:

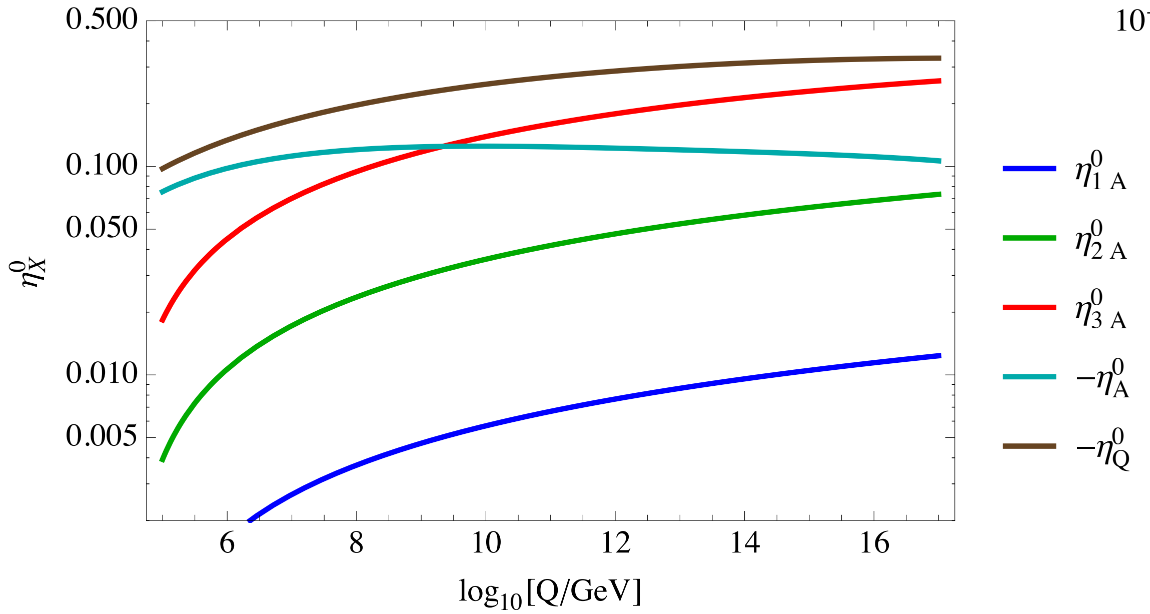
$$
\begin{aligned}
0 &= m_{H_U}^2 + \eta_Q^0(M)(m_Q^2 + m_U^2 + m_{H_U}^2) + \sum_a \eta_a^0(M)M_a^2 \\
&+ \sum_{a \neq b} \eta_{ab}^0(M)M_a M_b + \sum_a \eta_{aA}^0(M)M_a A_t + \eta_A^0(M)A_t^2
\end{aligned}
$$

- The focus point stays the same if one rescale all boundary conditions by the same factor. But the caveat is the value of Q.
- The value of Q is chosen so as one can generate the value of 125 GeV relying on heavy squarks and possibly an A-term Q=2 TeV

- The coefficients η can be calculated numerically and fitted to a polynomial. Here are the plots as a function of M for Q=2 TeV.

The gluino gives the most important contribution

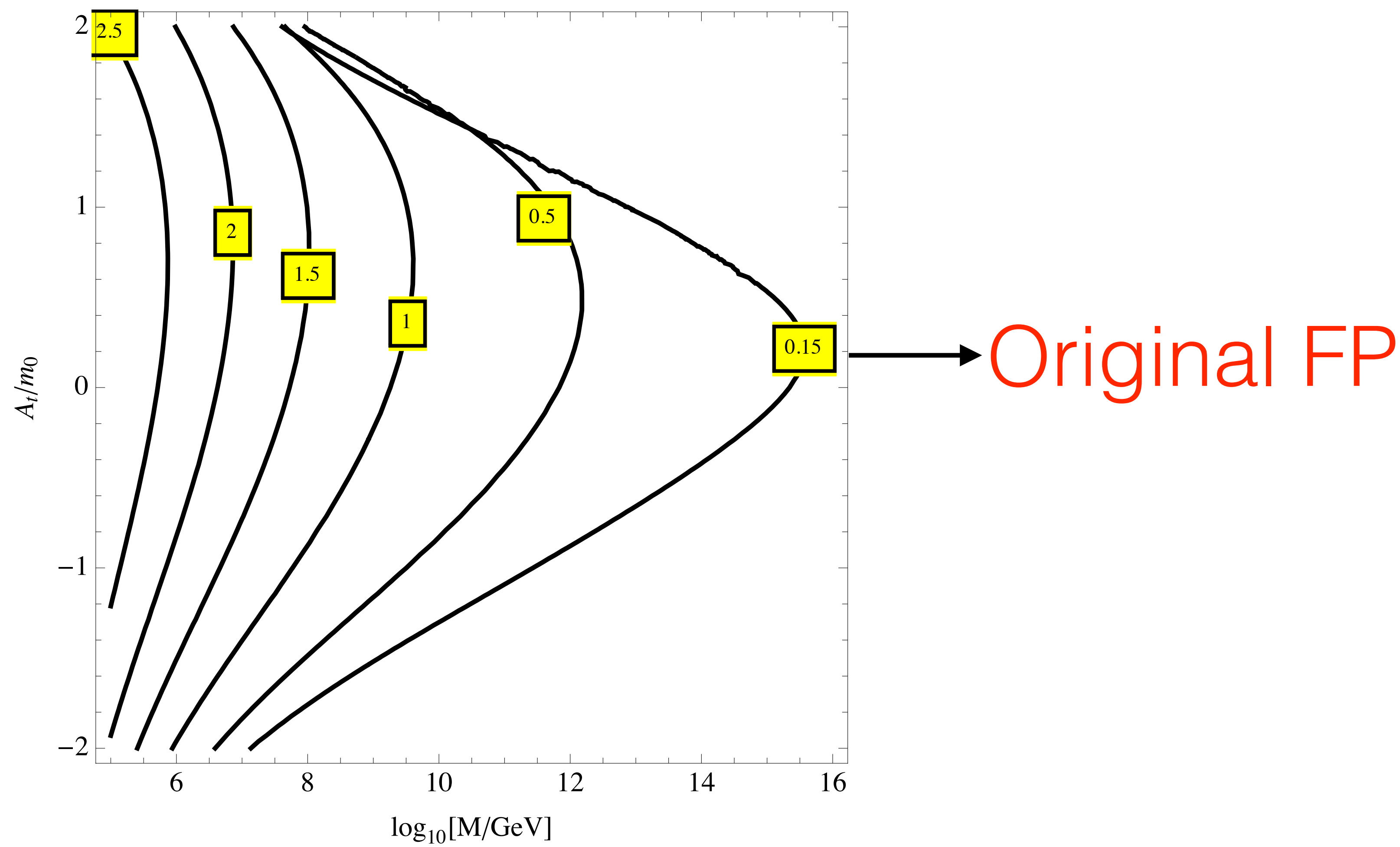




- One can now study the focus point for different boundary conditions:

CMSSM: $M_Q = M_U = M_{H_U} = m_0 \quad M_a = m_{1/2}$

Contour lines of $m_{1/2}/m_0$

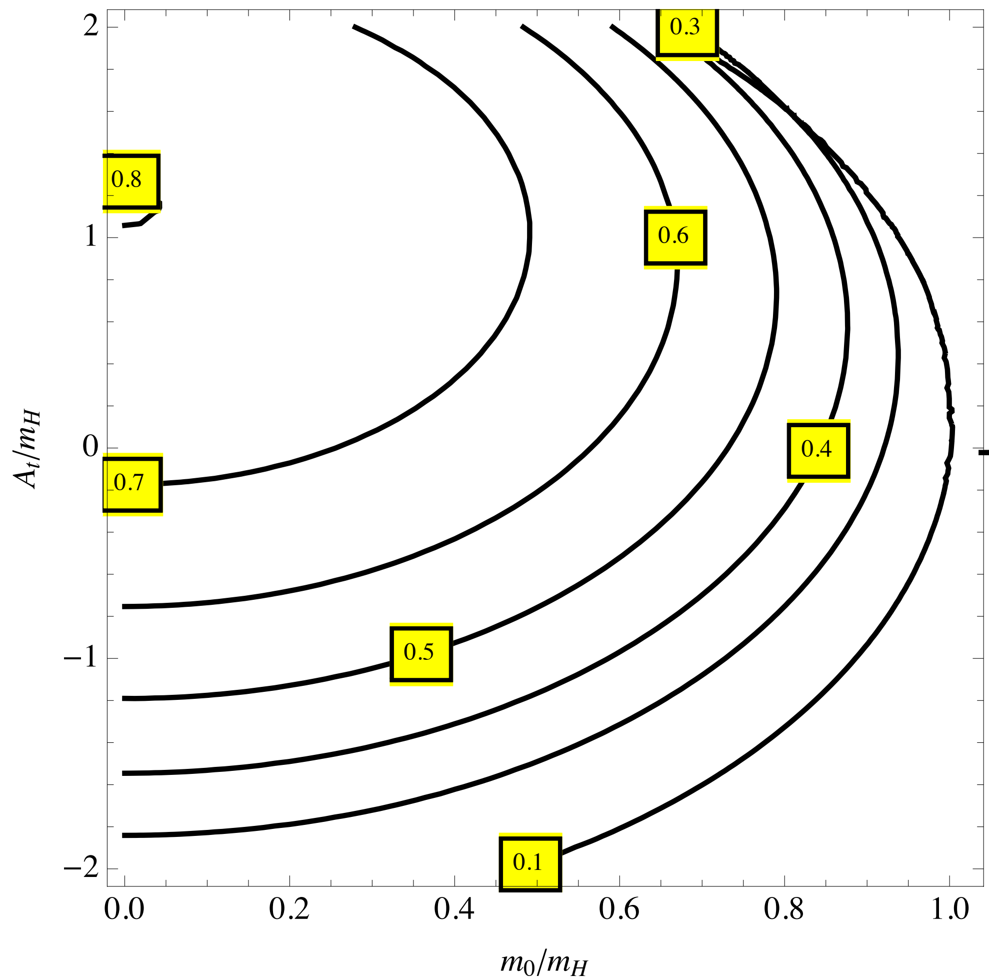


NUHM

$$M_Q = M_U = m_0 \quad M_a = m_{1/2} \quad M_{H_U} = M_{H_D} = M_H$$

M=10^{16} GeV

Contour lines of $m_{1/2}/m_H$



Original FP

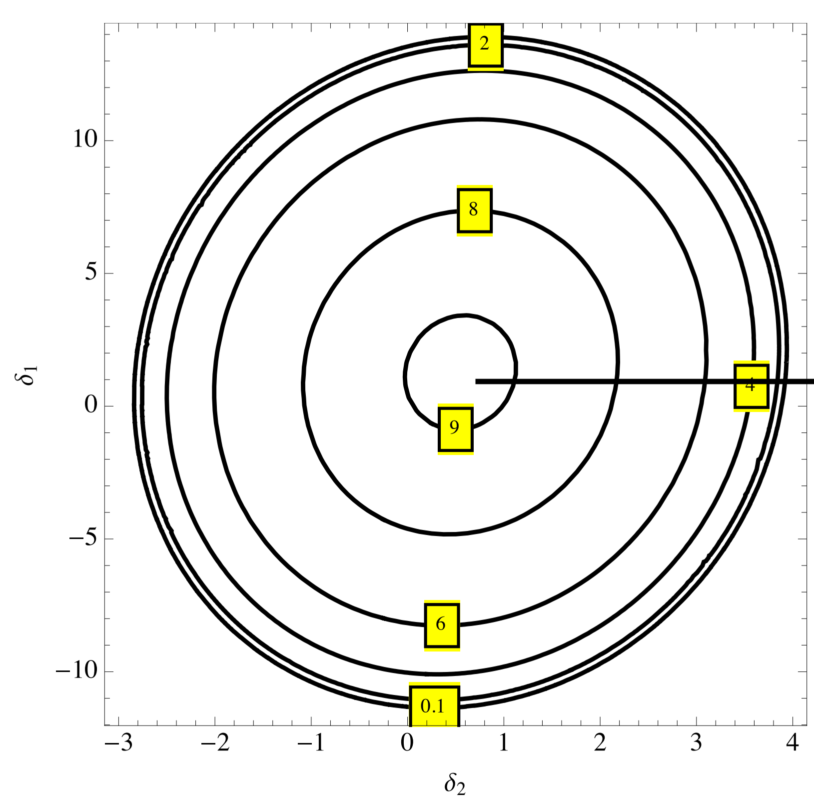
Non-universal gaugino masses:

M=10^{16} GeV
δ_3=1

$$M_Q = M_U = M_{H_U} = m_0 \quad M_a = \delta_a m_{1/2}$$

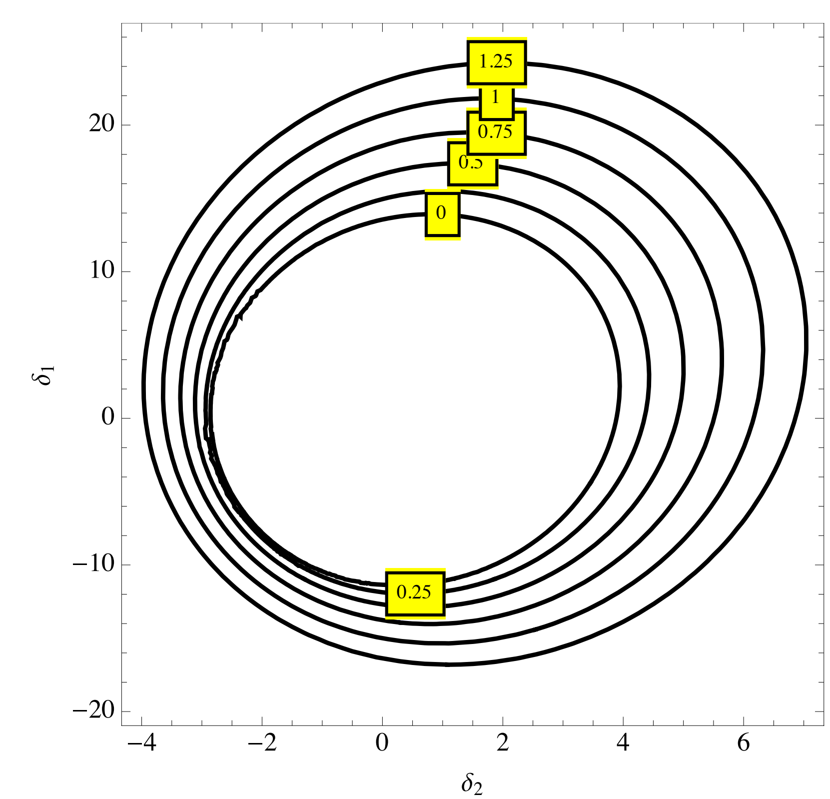
Contour lines
of $m_0/m_{1/2}$

A_t=0



Original FP

A_t=-2.5 m_0



- Standard gauge mediation does not work, it is a very predictive theory:

$$\Lambda_G = \frac{NF}{4\pi M} \quad \Lambda_S = \frac{\sqrt{N}F}{4\pi M}$$

$$m_Q^2 = 2\left(\frac{4}{3}\alpha_3^2 + \frac{3}{4}\alpha_2^2 + \frac{1}{60}\alpha_1^2\right)\Lambda_S^2$$

$$m_U^2 = 2\left(\frac{4}{3}\alpha_3^2 + \frac{4}{15}\alpha_1^2\right)\Lambda_S^2$$

$$m_{H_U}^2 = 2\left(\frac{3}{4}\alpha_2^2 + \frac{3}{20}\alpha_1^2\right)\Lambda_S^2$$

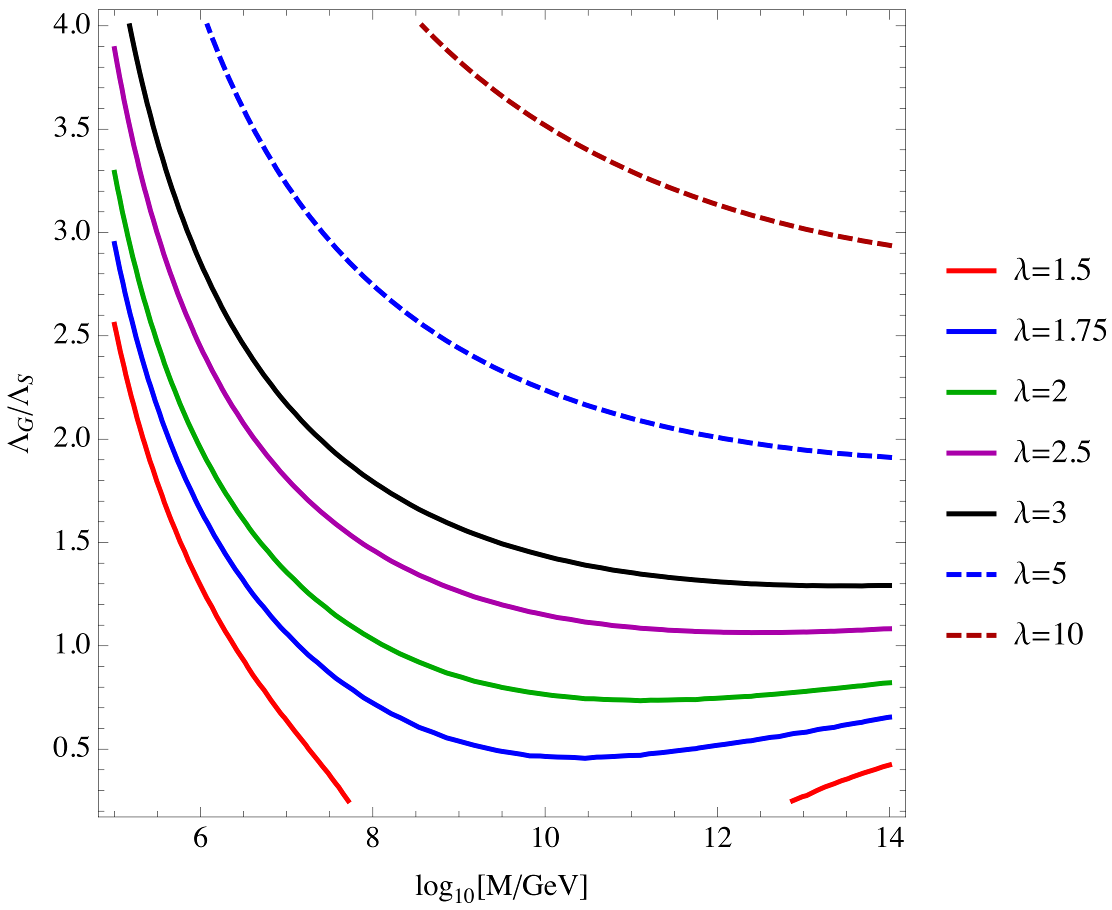
$$M_a = \alpha_a \Lambda_G, \quad A_t = 0$$

- It has only one ratio as free parameter plus M.

- One can modify the contribution to the soft mass of the Higgses by direct coupling to the messengers:

$$M^2_{H_U} = (1+\lambda) M^2_L$$

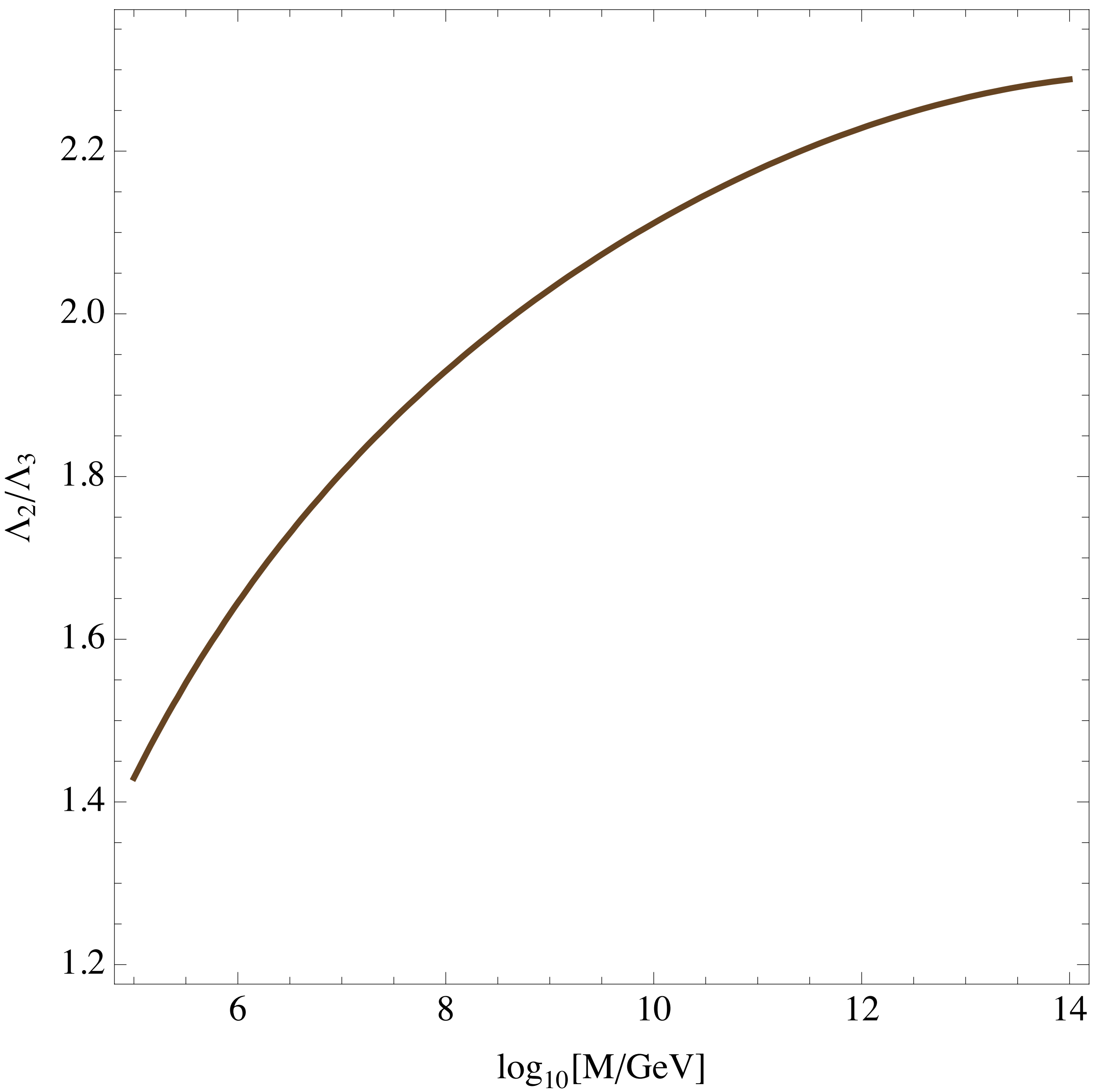
Λ_G/Λ_S labels the number of messengers



- Another possible modification includes different F-terms for color and EW interactions.

$$
\begin{aligned}
m_Q^2 &= 2\left[\frac{4}{3}\alpha_3^2\Lambda_3^2 + \frac{3}{4}\alpha_2^2\Lambda_2^2 + \frac{1}{60}\alpha_1^2\left(\frac{2}{5}\Lambda_3^2 + \frac{3}{5}\Lambda_2^2\right)\right] \\
m_U^2 &= 2\left[\frac{4}{3}\alpha_3^2\Lambda_3^2 + \frac{4}{15}\alpha_1^2\left(\frac{2}{5}\Lambda_3^2 + \frac{3}{5}\Lambda_2^2\right)\right] \\
m_{H_U}^2 &= 2\left[\frac{3}{4}\alpha_2^2\Lambda_2^2 + \frac{3}{20}\alpha_1^2\left(\frac{2}{5}\Lambda_3^2 + \frac{3}{5}\Lambda_2^2\right)\right] \\
M_1 &= \alpha_1\left(\frac{2}{5}\Lambda_3 + \frac{3}{5}\Lambda_2\right) \\
M_2 &= \alpha_2\Lambda_2, \quad M_3 = \alpha_3\Lambda_3
\end{aligned}
$$

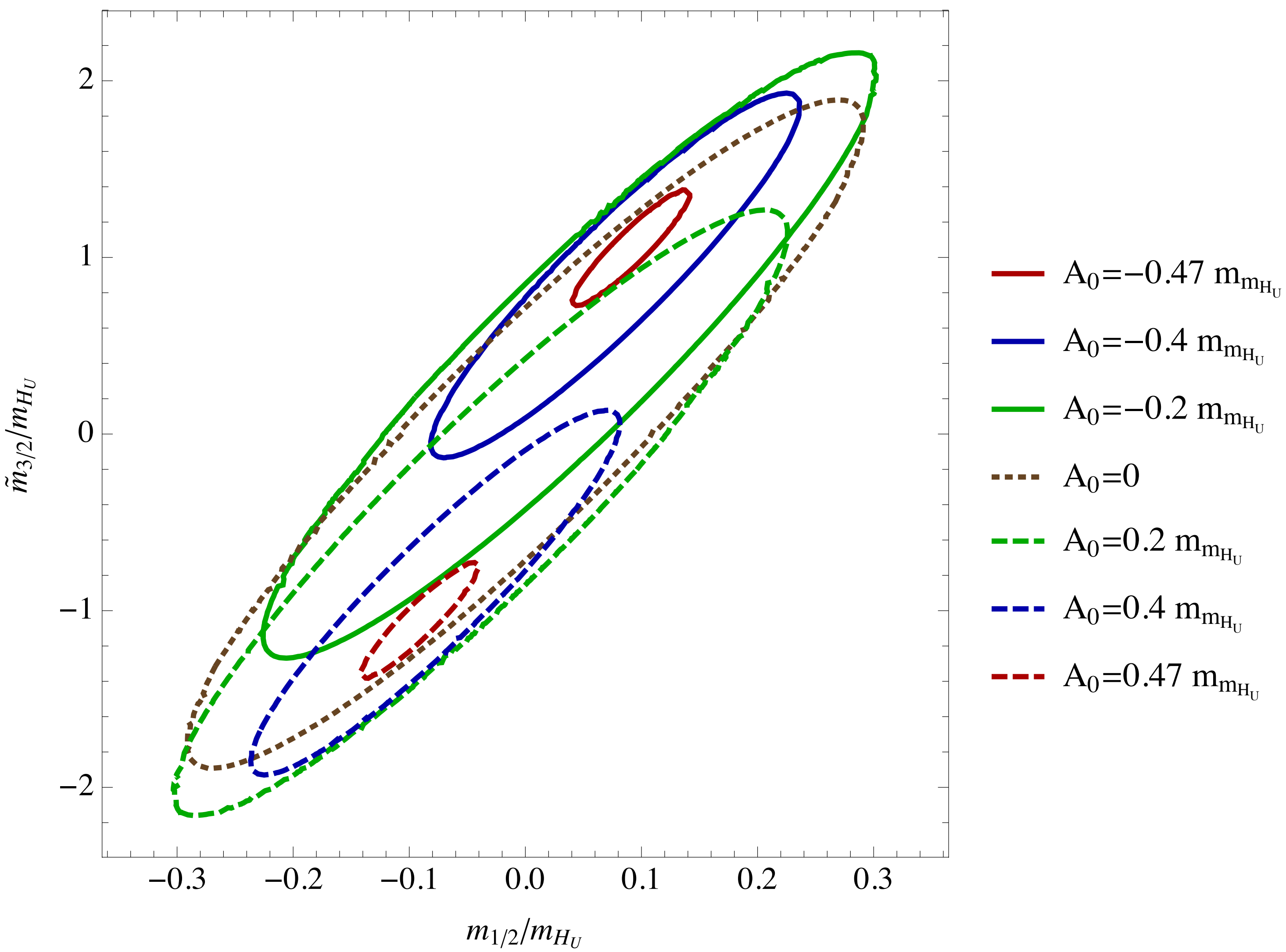
Λ_3/Λ_2 labels the relative contributions



- The final model we study is mirage-mediation:

$$
\begin{aligned}
m_{H_U}^2 &= m_0^2 + \left[3\alpha_t\left(6\alpha_t - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1\right) - \frac{3}{2}\alpha_2^2 b_2 - \frac{3}{10}\alpha_1^2 b_1\right]\widetilde{m}_{3/2}^2 \\
m_Q^2 &= m_0^2 + \left[\alpha_t\left(6\alpha_t - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1\right) - \frac{8}{3}\alpha_3^2 b_3 - \frac{3}{2}\alpha_2^2 b_2 - \frac{1}{30}\alpha_1^2 b_1\right]\widetilde{m}_{3/2}^2 \\
m_U^2 &= m_0^2 + \left[2\alpha_t\left(6\alpha_t - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1\right) - \frac{8}{3}\alpha_3^2 b_3 - \frac{8}{15}\alpha_1^2 b_1\right]\widetilde{m}_{3/2}^2 \\
A_t &= A_0 - \left(6\alpha_t - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1\right)\widetilde{m}_{3/2} \\
M_a &= m_{1/2} + \alpha_a b_a \widetilde{m}_{3/2}
\end{aligned}
$$

$$\tilde{m}_{3/2} = m_{3/2}/4\pi$$



2
1
0
−1
−2
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
$\tilde{m}_{3/2}/m_{H_U}$
$m_{1/2}/m_{H_U}$
$A_0=-0.47\ m_{m_{H_U}}$
$A_0=-0.4\ m_{m_{H_U}}$
$A_0=-0.2\ m_{m_{H_U}}$
$A_0=0$
$A_0=0.2\ m_{m_{H_U}}$
$A_0=0.4\ m_{m_{H_U}}$
$A_0=0.47\ m_{m_{H_U}}$

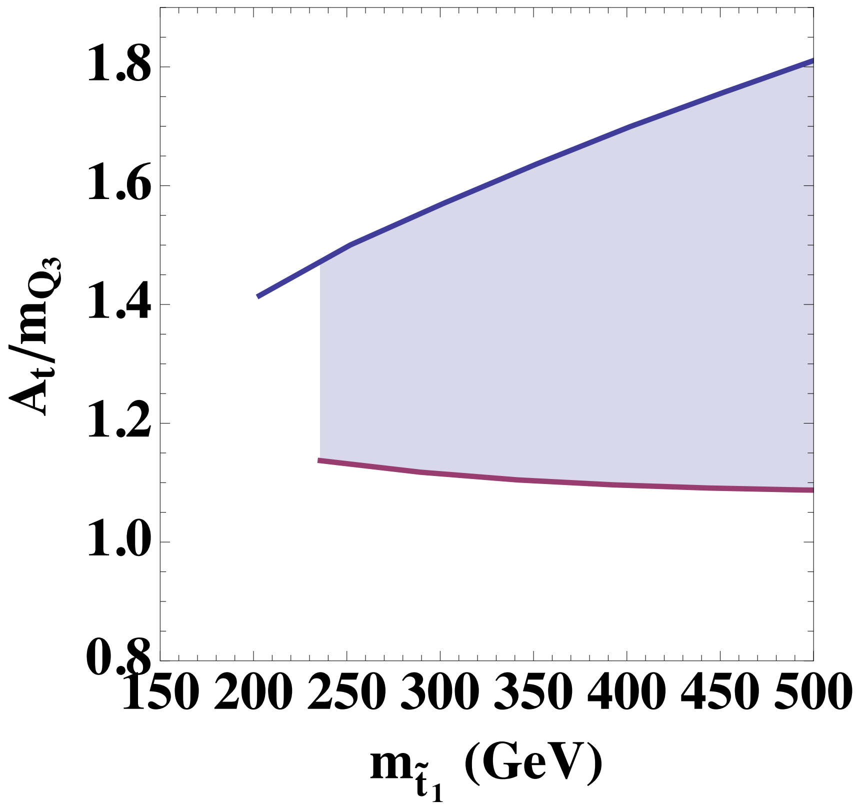
$M=10^{16}$ GeV

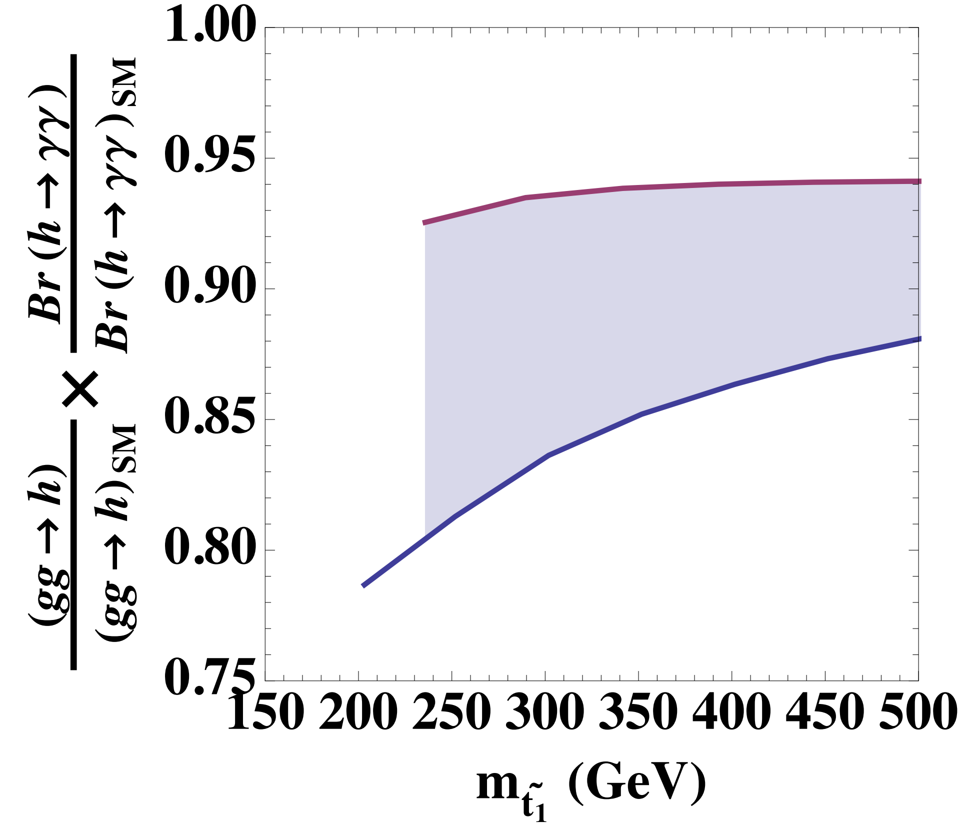
The LSS

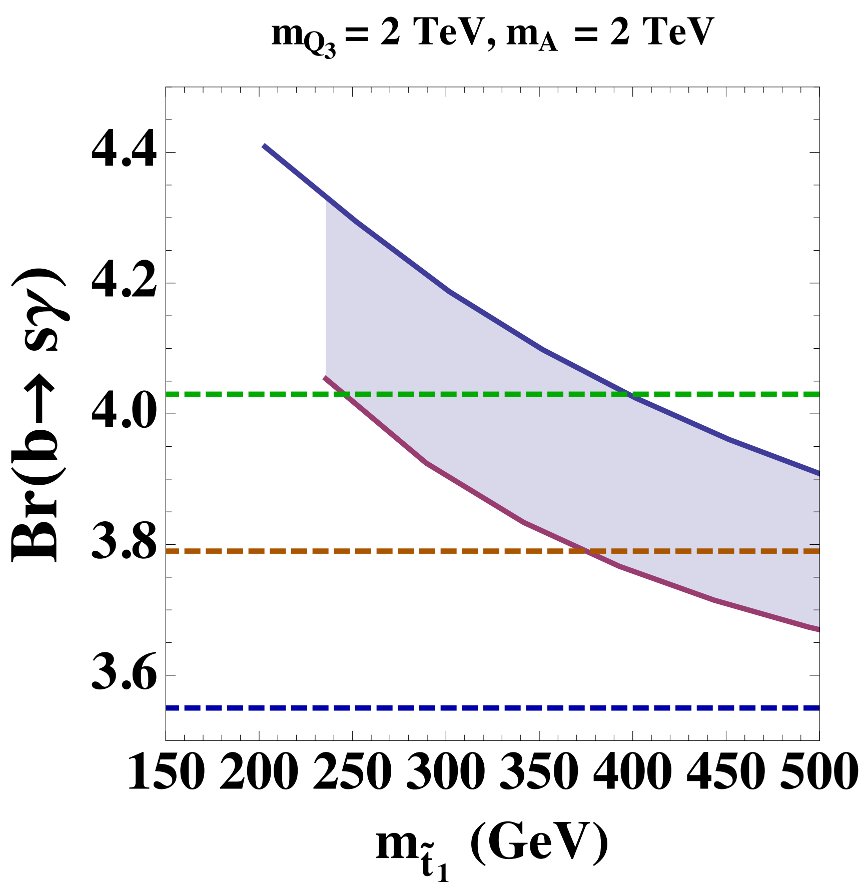
- One scenario that has deserved some attention is the one where at least one stop is quite light ~200 GeV.

- In other for this scenario to be viable one has to check the Higgs and flavor phenomenology.

- Before accommodating a light stop into the focus point I am going to show the constrains coming from the LHC and B-physics.







Parameters:

$1 < A_t < 1.8\ M_Q$

M_Q=2 TeV

M_A=2 TeV

tan β=8

‘Double’ focus point

- Using the generic solution for any soft mass:

$$
\begin{aligned}
m_Q^2(\mathcal{Q}) &= m_Q^2 + d_Q \left\{ \eta_{Q_L}[\mathcal{Q},\mathcal{M}](m_{Q_L}^2 + m_{U_R}^2 + m_{H_U}^2) \right. \\
&+ \sum_a \left[\eta_a[\mathcal{Q},\mathcal{M}] - 2\left(c_{H_U}^a - \frac{c_Q^a}{d_Q} \right) F_a[\mathcal{Q},\mathcal{M}] \right] M_a^2 \\
&+ \left. \sum_{a \neq b} \eta_{ab}[\mathcal{Q},\mathcal{M}] M_a M_b + \sum_a \eta_{aA}[\mathcal{Q},\mathcal{M}] M_a A_t + \eta_A[\mathcal{Q},\mathcal{M}] A_t^2 \right\}
\end{aligned}
$$

$$
F_a[\mathcal{Q},\mathcal{M}] = \frac{1}{b_a} \frac{\alpha_a^2(\mathcal{M}) - \alpha_a^2(\mathcal{Q})}{\alpha_a^2(\mathcal{M})} = \frac{\alpha_a(\mathcal{Q})}{2\pi} \log(\mathcal{M}/\mathcal{Q}) \left(2 - \frac{b_a \alpha_a(\mathcal{Q})}{2\pi} \log(\mathcal{M}/\mathcal{Q}) \right)
$$

- We can impose the ‘double focus’ point solution:

$$
m_{H_U}^2(\mathcal{Q}_0) \simeq m_{U_R}^2(\mathcal{Q}_0) \simeq 0
$$

- Imposing the two conditions leads to the following value of the mass for the squark doublet:

$$m^2_{Q_L}(\mathcal{Q}_0) = m^2_{Q_L} - \frac{1}{2} m^2_{U_R} + \sum_a d_a M_a^2 F_a[\mathcal{Q}_0, \mathcal{M}]$$

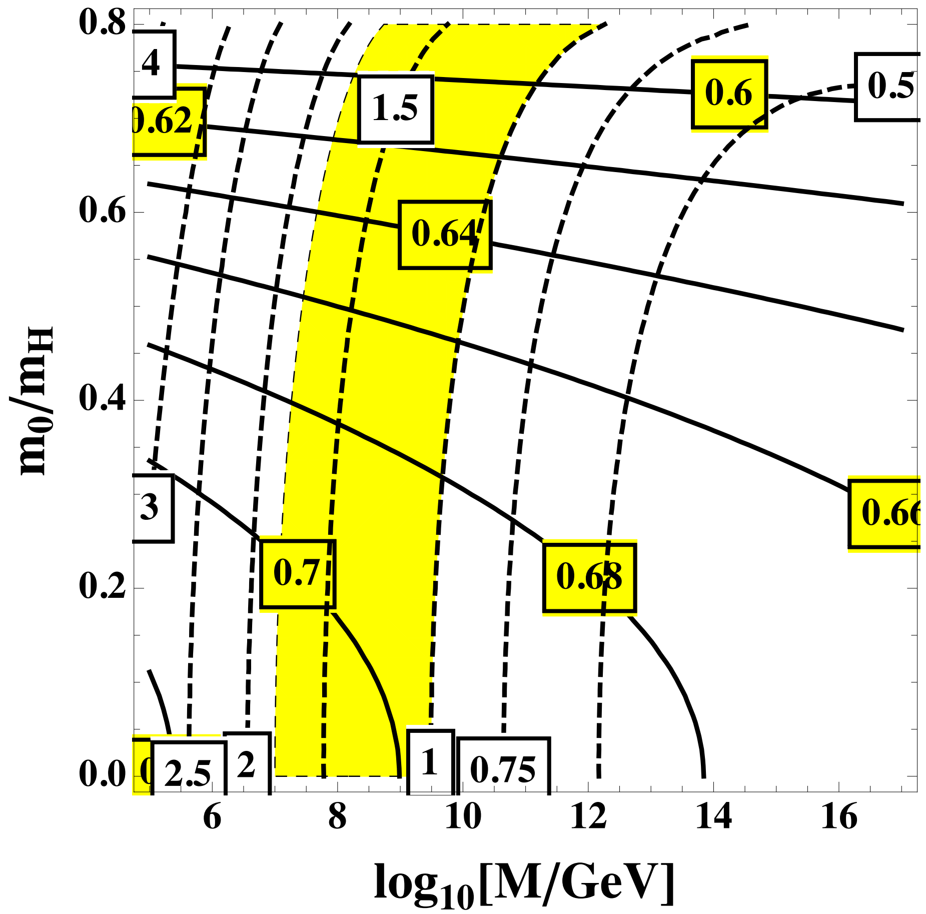
- Gauginos and A_t can be also calculated as:

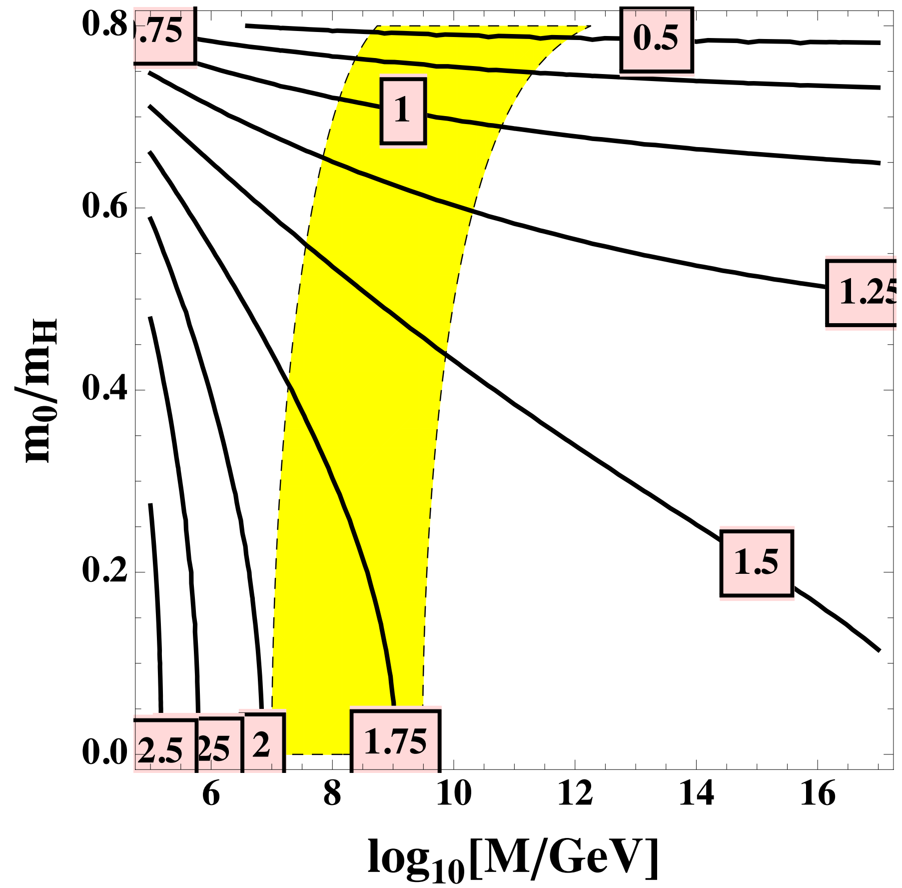
$$M_a(\mathcal{Q}_0) = \frac{\alpha_a(\mathcal{Q}_0)}{\alpha_a(\mathcal{M})} M_a$$

$$A_t(\mathcal{Q}_0) = \sum_a \gamma_a(\mathcal{Q}_0, \mathcal{M}) M_a(\mathcal{M}) + \gamma_A(\mathcal{Q}_0, \mathcal{M}) A_t(\mathcal{M})$$

y(M)=Log(M/GeV)

$$\begin{aligned}
\gamma_1(\mathcal{Q}_0, \mathcal{M}) &= 0.0149 - 0.0054\, y(\mathcal{M}) + 0.0001\, y^2(\mathcal{M}) \\
\gamma_2(\mathcal{Q}_0, \mathcal{M}) &= 0.0924 - 0.0336\, y(\mathcal{M}) + 0.0008\, y^2(\mathcal{M}) \\
\gamma_3(\mathcal{Q}_0, \mathcal{M}) &= 0.3979 - 0.1418\, y(\mathcal{M}) + 0.0021\, y^2(\mathcal{M}) \\
\gamma_A(\mathcal{Q}_0, \mathcal{M}) &= 1.2576 - 0.1058\, y(\mathcal{M}) + 0.0030\, y^2(\mathcal{M})
\end{aligned}$$





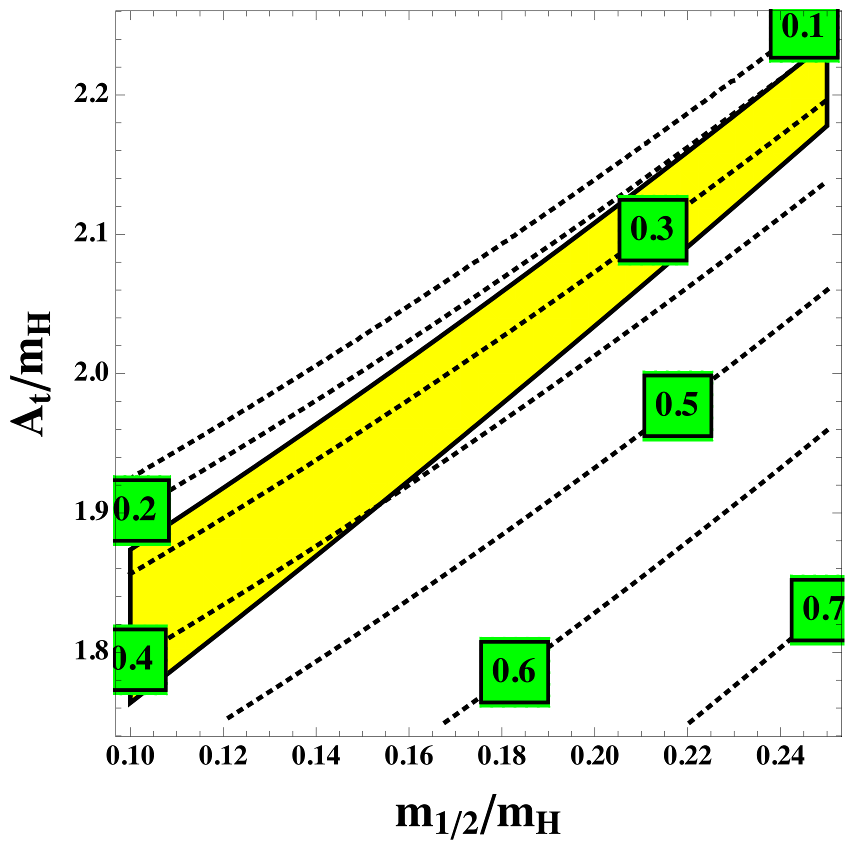
M_Q/M_H (A_t/M_Q)

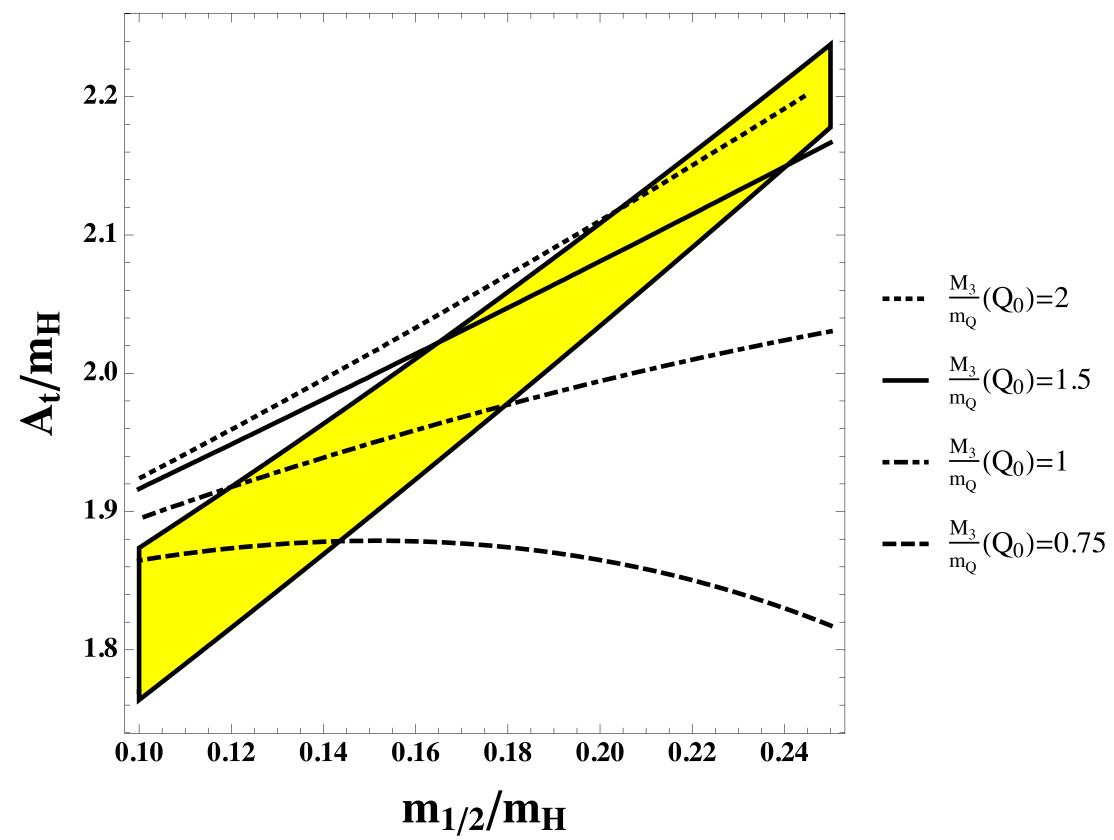
M_3/M_Q

- NUHM boundary conditions.

$\frac{m_0}{m_H}$	$\frac{A_t}{m_H}$	$\frac{m_{1/2}}{m_H}$	$\frac{m_{Q_L}(Q_0)}{m_H}$	$\frac{m_{U_R}(Q_0)}{m_{Q_L}(Q_0)}$	$\frac{A_t(Q_0)}{m_{Q_L}(Q_0)}$	$\frac{M_3(Q_0)}{m_{Q_L}(Q_0)}$	$\frac{M_2(Q_0)}{m_{Q_L}(Q_0)}$	$\frac{M_1(Q_0)}{m_{Q_L}(Q_0)}$
0.6	2.35	0.52	0.65	0.22	1.03	1.27	0.74	0.59

$M=10^{10}$ GeV





M_Q/M_H (A_t/M_Q)

M_3/M_Q

- Non-universal scalar boundary conditions.

$\frac{m_{1/2}}{m_H}$	$\frac{A_t}{m_H}$	$\frac{m_{Q_L}}{m_H}$	$\frac{m_{U_R}}{m_H}$	$\frac{m_{Q_L}(\mathcal{Q}_0)}{m_H}$	$\frac{m_{U_R}(\mathcal{Q}_0)}{m_{Q_L}(\mathcal{Q}_0)}$	$\frac{A_t(\mathcal{Q}_0)}{m_{Q_L}(\mathcal{Q}_0)}$	$\frac{M_3(\mathcal{Q}_0)}{m_{Q_L}(\mathcal{Q}_0)}$	$\frac{M_2(\mathcal{Q}_0)}{m_{Q_L}(\mathcal{Q}_0)}$	$\frac{M_1(\mathcal{Q}_0)}{m_{Q_L}(\mathcal{Q}_0)}$
0.13	1.85	0.66	0.79	0.40	0.18	1.05	0.70	0.28	0.15

$M=10^{16}$ GeV

- Since in most of these scenarios M_1 is greater than the mass of the stop the LSP will be higgsino-like.

- It can not be a viable thermal DM, it annihilates to efficiently.

- The stop will primarily decay into chargino-bottom and the chargino will decay into neutralino and soft leptons therefore the signal will be like a sbottom. (b's+MET)

- In general as long as the splitting between the stop and the LSP is few tens of GeV there is no bound.

When is EWSB compatible with the focus point?

- I have analyzed the different situations when, due to RGE evolution, the soft mass of the Higgs will be ‘zero’.

- This corresponds to a situation where EWSB is triggered by a negative mass in one of the directions.

- Is this always possible? Can we draw any conclusions on how EWSB is achieved?

- Lets start with the quadratic potential of the MSSM:

$$V_2 = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2(H_1 \cdot H_2 + h.c.)$$

- We can identify two physical degrees of freedom:
  - The SM-tachyonic state
  - The Heavy Higgs

$$\begin{aligned} H &= \cos\beta H_1 - \sin\beta \widetilde{H}_2 \\ \mathcal{H} &= \sin\beta H_1 + \cos\beta \widetilde{H}_2 \ . \end{aligned}$$

- Bellow the mass of all SUSY particles (Q_0) we match to:

$$V_{\rm SM} = -m^2(\mathcal{Q}_0)|H|^2 + \frac{\lambda(\mathcal{Q}_0)}{2}|H|^4 + \cdots$$

- λ has as boundary condition:

$$\Delta^{(0)}\lambda = \frac{1}{4}(g^2 + g'^2)c_{2\beta}^2$$

$$16\pi^2\Delta^{(1)}\lambda = 6y_t^4 s_\beta^4 X_t^2\left(1 - \frac{X_t^2}{12}\right) - \frac{1}{2}y_b^4 s_\beta^4(\mu/\mathcal{Q}_0)^2 + \frac{3}{4}y_t^2 s_\beta^2(g^2 + g'^2)X_t^2 c_{2\beta}$$
$$+ \left(\frac{1}{6}c_{2\beta}^2 - \frac{3}{4}\right)g^4 - \frac{1}{2}g^2 g'^2 - \frac{1}{4}g'^4 - \frac{1}{16}(g^2 + g'^2)^2 s_{4\beta}^2$$

$$(16\pi^2)^2\Delta^{(2)}\lambda = 16y_t^4 s_\beta^4 g_3^2\left(-2X_t + \frac{1}{3}X_t^3 - \frac{1}{12}X_t^4\right) + \mathcal{O}(h_t^6 s_\beta^4,\, g^4,\, g^2 g'^2,\, g'^4)$$

- And at the pole mass of the Higgs we will have:

$$m^2(m_H) = \frac{1}{2}m_H^2 \qquad m_H^2 = 2\lambda(m_H)v^2$$

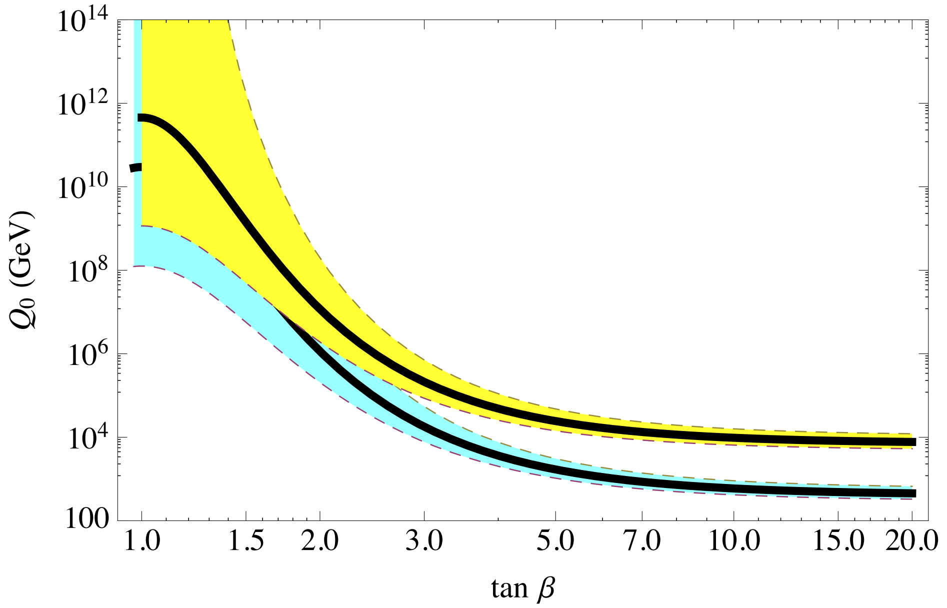
- The relation between the parameters of the MSSM Higgs potential and the SM one can be written as:

$$m_2^2(\mathcal{Q}_0) = \frac{m_{\mathcal{H}}^2(\mathcal{Q}_0) - m^2(\mathcal{Q}_0)\tan^2\beta}{\tan^2\beta + 1}$$

- We will calculate the value of Q_0 from the known mass of the Higgs and suppose that all SUSY particles and the second Higgs are degenerate.

- We will use the previous formula to calculate the sign of soft mass of H_2.

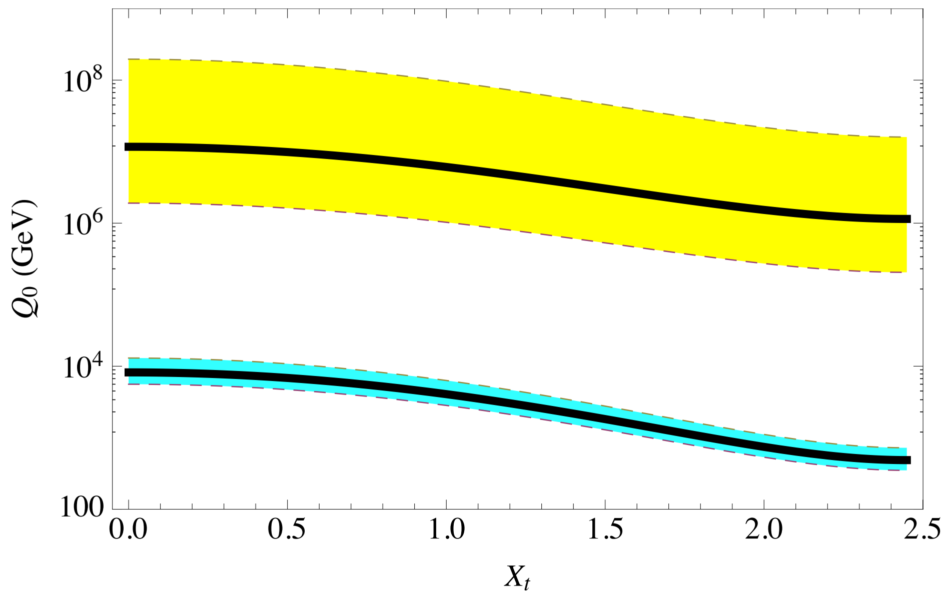
- The value of Q_0 is determined by the physical mass of the Higgs 125 GeV:



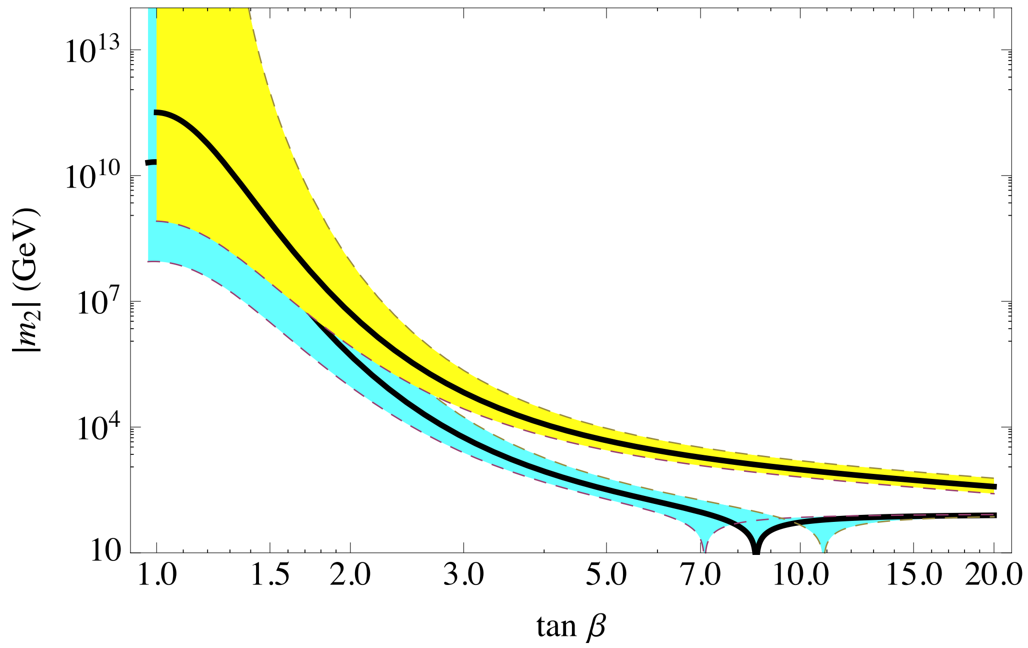
Errors are due to M_{top} and α_s

X_t=0 & X_t max

tan β=2 & 15



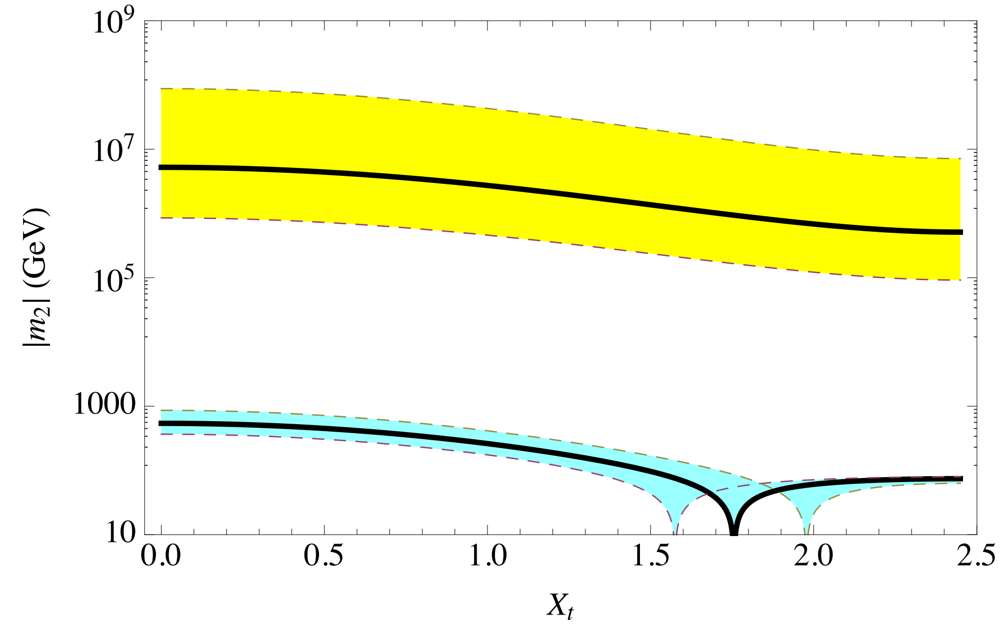
- We can then calculate the value of m_2:



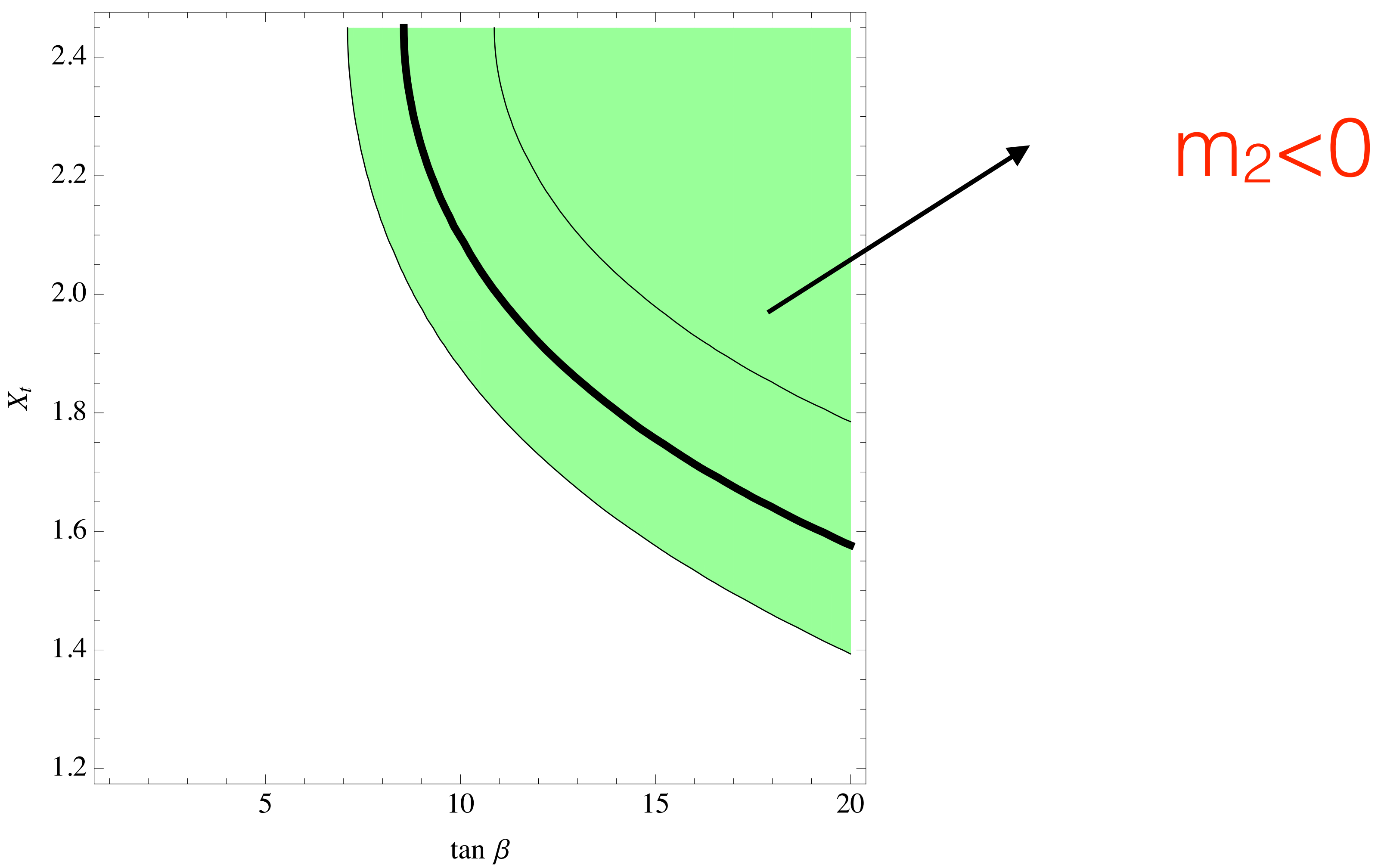
Errors are due to M_{top} and α_s

tan β=2 & 15

X_t=0 & X_t max



- We can summarize the results in this plot:



Conclusions

- The focus point scenario could have a reduce fine-tuning if….
- The boundary conditions necessary come from some UV scenario
- I have analyzed the different boundary conditions and SUSY breaking scales where it is possible.
- In SUGRA inspired models you have plenty of room.

- There are solutions with NUHM and also with non-universal gaugino masses.

- In GMSB you need to deviate from the minimal set-up.

- AMSB also has a focus point solution.

- There are even situations more restrictive where one can realize the LSS with a focus point solution.

- In general these LSS scenarios require the LSP to be Higgsino.
- In the last part of the talk I have studied for which part of the parameter space one needs a tachyonic mass for the H_u to trigger EWSB.
- In the simplified model where everything decouples at the same scale this kind of braking only occurs for $\tan\beta>7$.