Dark Matter at the ILC

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Work done in Collaboration with: Conley, Wienemann

Huck, Krämer, Schmeier, Tattersall

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Outline

1) Measuring a very light neutralino LSP mass at the ILC

2) Effective DM-operator analysis at the ILC

I) Measuring a very light neutralino LSP mass at the ILC

Text book knowledge:

LHC: If they exist, all SUSY Particles must be very heavy.

Every SUSY Particle?

No, not EVERY SUSY particle!

The lightest Neutralino can still be massless!

$\mathcal{M}_{\tilde{\chi}^0} = 0$ very much allowed

Heinemeyer, Kittel, Langenfeld, Weiglein, D: EPJC

Neutralino Search at LEP

- Chargino Search: $M_{\tilde{\chi}_1^\pm} > 94\,\text{GeV} \quad \Rightarrow \quad |\mu|,\ M_2 \gtrsim 100\,\text{GeV}$

- SUSY GUT: $M_1 = \frac{5}{3}\tan^2\theta_w\, M_2$

- Insert bound on M_2: $\quad \Rightarrow \quad M_1 \gtrsim 50\,\text{GeV}$

- Insert into Neutralino Mass matrix: $\quad \Rightarrow \quad \boxed{M_{\tilde{\chi}_1^0} \gtrsim 46\,\text{GeV}}$

- Now drop GUT Assumption

Massless Neutralino

- For massless neutralino: $\det(\mathcal{M}_{\chi^0})=0$

$$\Rightarrow M_1 = \frac{M_2 M_Z^2 \sin(2\beta) s_W^2}{\mu M_2 - M_Z^2 \sin(2\beta) c_W^2}$$

- Estimate

$$M_1 \approx \frac{M_Z^2 \sin(2\beta) s_W^2}{\mu} \approx 2.5\ \text{GeV} \left(\frac{10}{\tan\beta}\right)\left(\frac{150\ \text{GeV}}{\mu}\right)$$

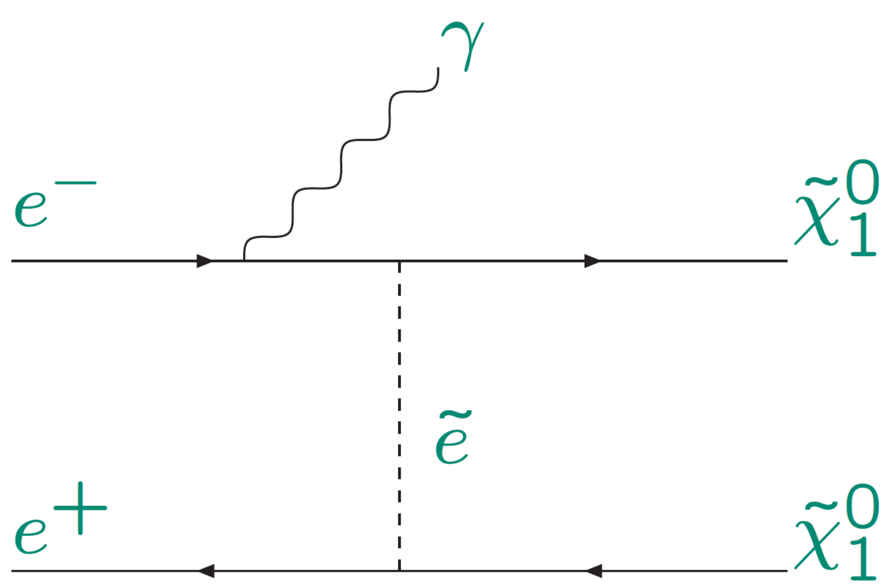
- Some fine-tuning between M_1 and M_2 required

- Very light neutralino is dominantly BINO

- For complex M_1, μ solutions do not always exist

Radiative Neutralino Prod. $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$

O. Kittel, U. Langenfeld, HD



+ 2 more diagrams

- Standard Model background: $e^+e^- \to \nu\bar{\nu}\gamma$ & eeγ J. List et al
- Opal 1999 observed: 138 events (stat. error: 11.7 events)
- SM-Theory: 141.1 ± 1.1 events, $\longrightarrow$ No discrepancy

Production of $\tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ at e^+e^--Colliders. $M_{\tilde{e}} = 150$ GeV

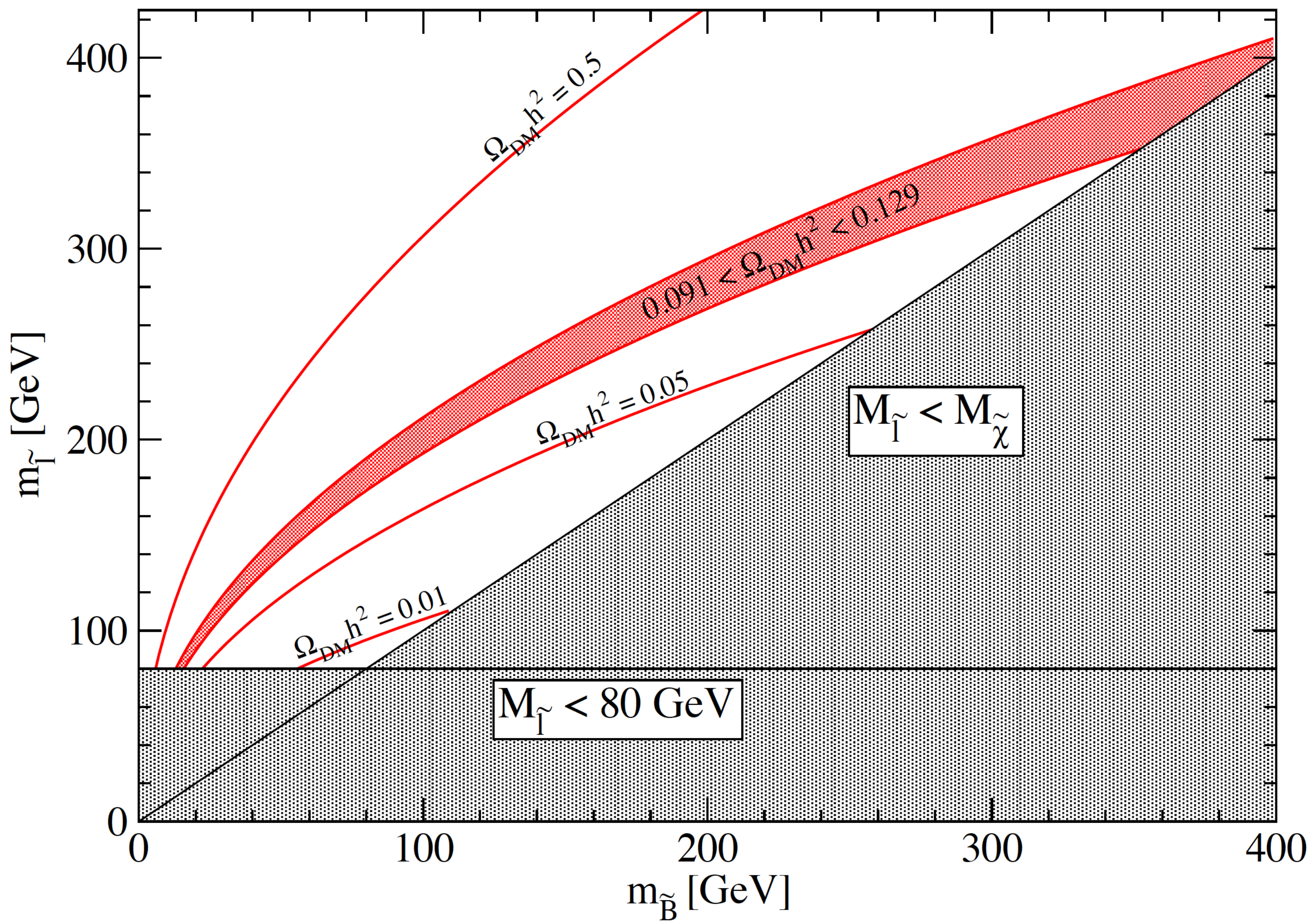
Exp.	Int. Lum.(pb^{-1})	Energy	Cross-sect.(fb)	Events
LEP	6.65	130	5.87	0.04
	5.96	136	6.14	0.04
	9.89	161	7.11	0.07
	10.28	172	7.44	0.08
	54.5	183	7.72	0.42
	75.	200	8.05	0.60
KEK-B	7×10^5	10.5	6.74×10^{-2}	47
BaBar	3.9×10^5	10.5	6.74×10^{-2}	26
ILC	3×10^5	500	6.19	1857

- Large hadronic background at B-factories; $S/\sqrt{B} < 0.1$; $S/B < 0.01$
- Only chance is at ILC with polarized beams
- Note: there is also a background from $e^+e^- \to \tilde{\nu}\tilde{\nu}\gamma$

HD, O. Kittel, U. Langenfeld

Dark Matter Heinemeyer, Kittel, Langenfeld, Weiglein, HD

- Hot Dark Matter: $m_{\tilde{\chi}_1^0} \leq 0.07$ eV
- Cold Dark Matter, estimate (pre Higgs): $m_{\tilde{\chi}_1^0} > 13$ GeV



- post Higgs: $m_{\tilde{\chi}_1^0} > 30$ GeV Scopel, Fornengo, Bottino
- Arbey, Battaglia, Mahmoudi: $M_{\chi^0} \approx 10$ GeV allowed if $M_{\tilde{b}} \approx 15$ GeV

Question: LSP Mass

- Assume we discover SUSY at the LHC?
- Can we determine the LSP mass at the LHC?
- Yes, if it is heavy, larger than about 50 GeV
- How about a (very) light neutralino?
    - Note: if we measure: $\mathcal{M}_{\tilde{\chi}^0}$ < 25 GeV

      have excluded CMSSM neutralino DM!
- Must go to ILC

Measuring a light χ_1^0 Mass at the ILC

Conley, Wienemann, HD

- $e^+e^- \to \tilde{e}^-\tilde{e}^+ \to e^+e^-\chi_1^0\chi_1^0$

- $E_+ = \text{max[Energy}(e^-)]$, $\quad E_- = \text{min[Energy}(e^-)]$

- $M_{\tilde{e}} = \sqrt{s}\dfrac{\sqrt{E_+E_-}}{E_+ + E_-}$; $\quad M_{\chi_1^0} = M_{\tilde{e}}\sqrt{1 - \dfrac{E_+E_-}{\sqrt{s}/2}}$

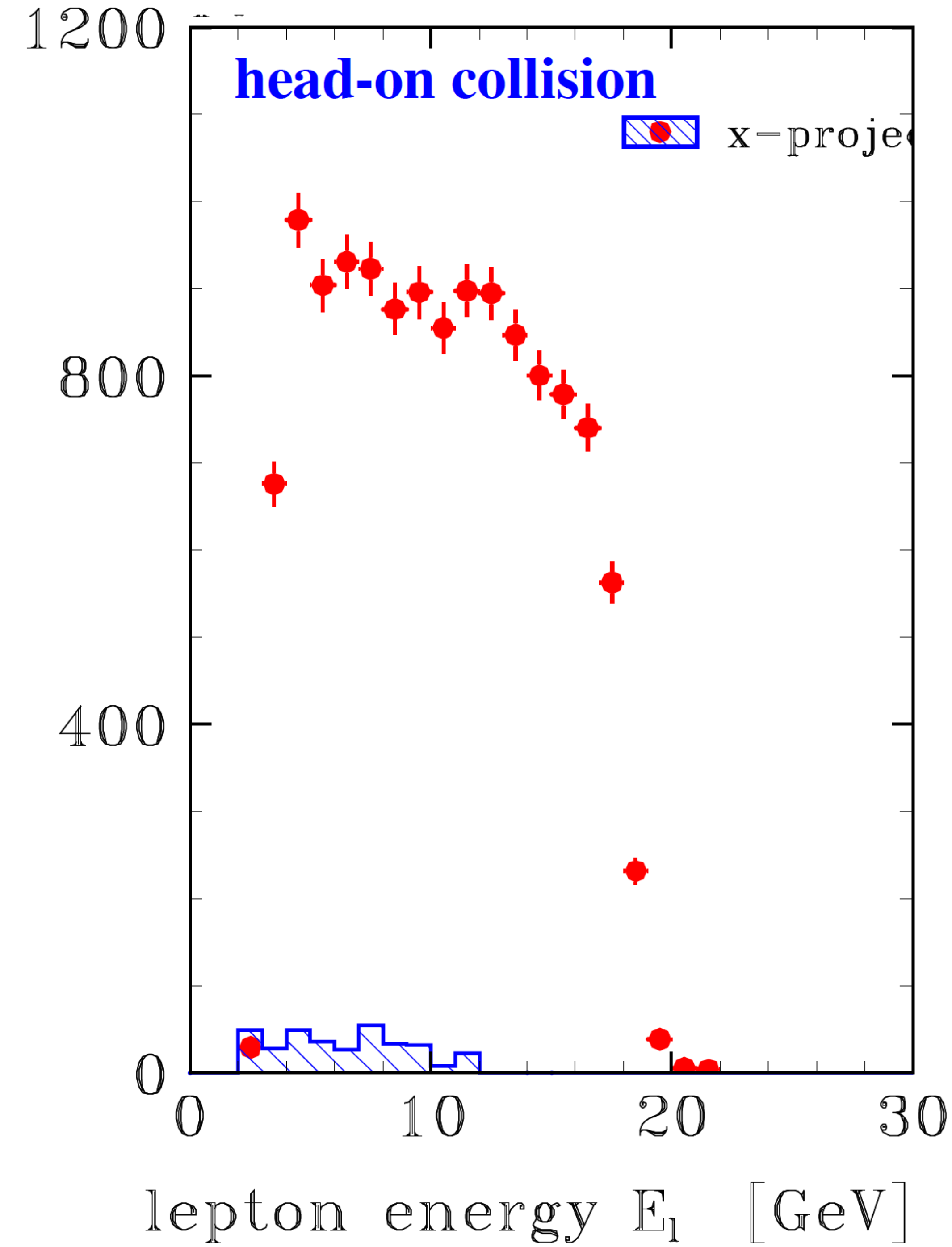
- $\dfrac{\delta M_{\chi_1^0}}{\delta E_\pm} = \dfrac{\delta M_{\tilde{e}}}{\delta E_\pm}\dfrac{M_{\chi_1^0}}{M_{\tilde{e}}} - \dfrac{M_{\tilde{e}}^2}{M_{\chi_1^0}\sqrt{s}} \quad \Longrightarrow \quad \delta M_{\chi_1^0} \simeq \dfrac{M_{\tilde{e}}^2}{M_{\chi_1^0}\sqrt{s}}\sqrt{\delta E_+^2 + \delta E_-^2}$

- Can not measure an LSP mass below about 10 GeV

- Background: just introduced via efficiency

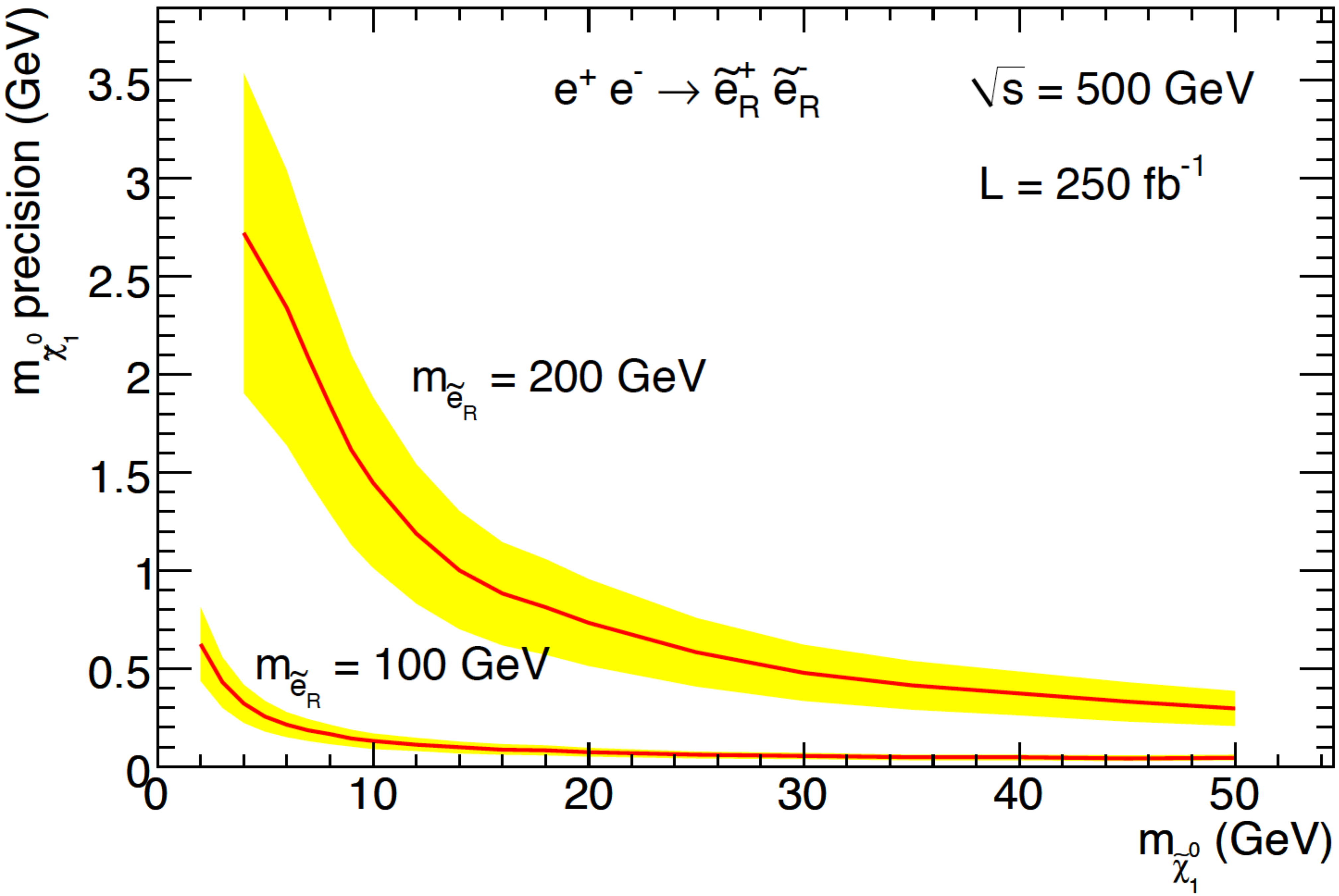
H.U. Martyn

- Energy distribution: box
- Edges smeared out by experimental effects



- This example: $e^+_L e^-_R \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R \rightarrow \mu^+ \tilde{\chi}^0_1 \; \mu^- \tilde{\chi}^0_1$

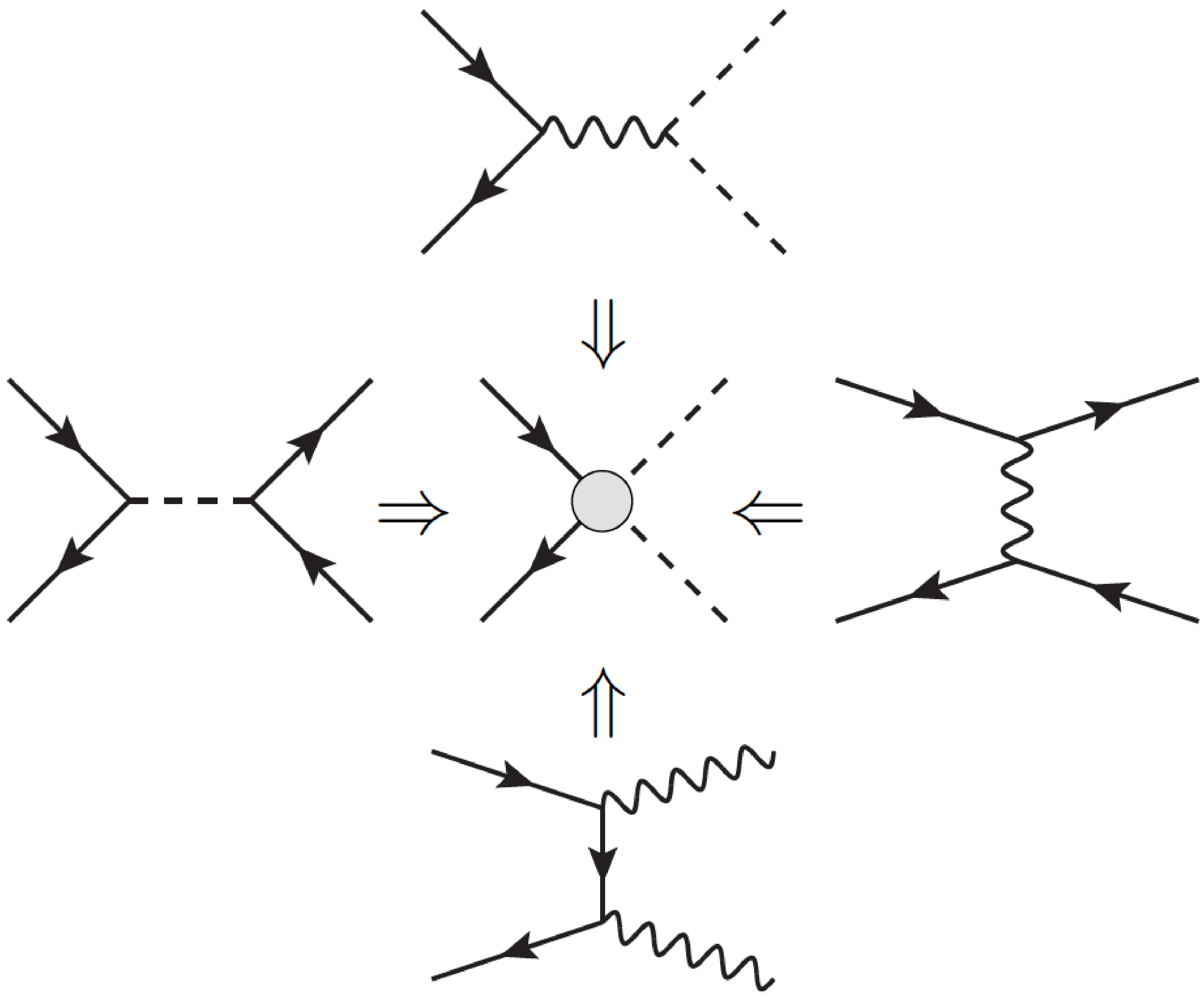
Conley, Wienemann, HD



- Yellow band: 30% uncertainty related to our simplified MC
- Beam polarization: (e-,e+)=(+80%,-60%)
- **Currently not possible to measure mass below 10 GeV!**

2) Effective DM-operator analysis at the ILC

Daniel Schmeier, Jamie Tattersall, HD



2) Effective DM-operator analysis at the ILC

Daniel Schmeier, Jamie Tattersall, HD

Some Previous Work

- Search for Monojets at Tevatron

  Bai, Fox, Harnik arXiv:1005.3797

- LEP Analysis with Monophotons

  Fox, Harnik, Kopp, Tsai; arXiv:1103.0240

- LHC Analyses with Monophotons/Monojets

  Goodman, Ibe, Rajaraman, Shepherd, Tait, Yu; arXiv:1008.1783,
  CMS Collaboration; arXiv:1206.5663,
  ATLAS Collaboration; ATLAS-CONF-2012-084

- ILC monophotons in nonrelativistic approximation

  Birkedal, Matchev, Perelstein; arXiv:hep-ph/0403004
  Bartels, Berggren, List; arXiv:1206.6639

Our Work

- Extensive list of possible effective, relativistic operators
- ILC monophoton analysis of these, including detector–effects

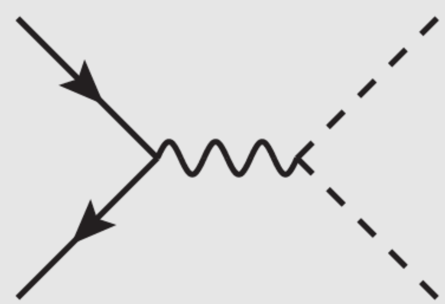
- Relate to underlying UV theory

Effective Approach

- DM interacts pairwise with SM fermions by single mediating particle

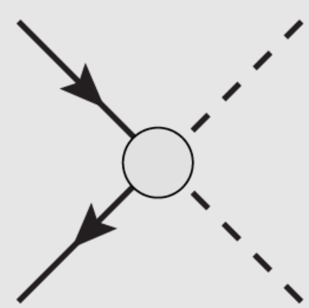
- For very heavy mediators obtain effective coupling $G_{\rm eff} = \frac{g_\chi g_\psi}{M_\Omega^2}$

Example: Scalar DM, Vector Mediator ("SV")



$$-\mathcal{L}_{\rm UV} = g_\psi \bar\psi \gamma^\mu \psi Z_\mu + g_\chi Z_\mu \chi^\dagger \overleftrightarrow{\partial_\mu} \chi + \frac{1}{2} M_\Omega^2 Z^\mu Z_\mu$$

$$\overset{s \ll M_\Omega^2}{\Longrightarrow} -\mathcal{L}_{\rm eff} = \frac{g_\chi g_\psi}{M_\Omega^2} \chi^\dagger \overleftrightarrow{\partial_\mu} \chi \bar\psi \gamma^\mu \psi$$



- Eff. field theory: can go beyond previous non-relativistic approach

Some Assumptions

- DM particles interact with SM only via heavy mediator
- DM is colorless, SU(2) singlet with no hypercharge
- Obtain eff. theory by integrating out mediator, but do not consider UV complete theory
- Two scenarios:
    1. All SM particles couple with same strength
    2. Coupling proportional to mass: “Yukawa coupling”
- No resonances or co-annihilation in relic density comp.
- DM & mediator: Scalar, Dirac-Ferm. Maj.-Ferm. Vector

Outline

- Define models
- Choose benchmarks (too many models!)
- Consider ILC constraints: $e^+e^- \to \chi\chi + \gamma$ Mono photons

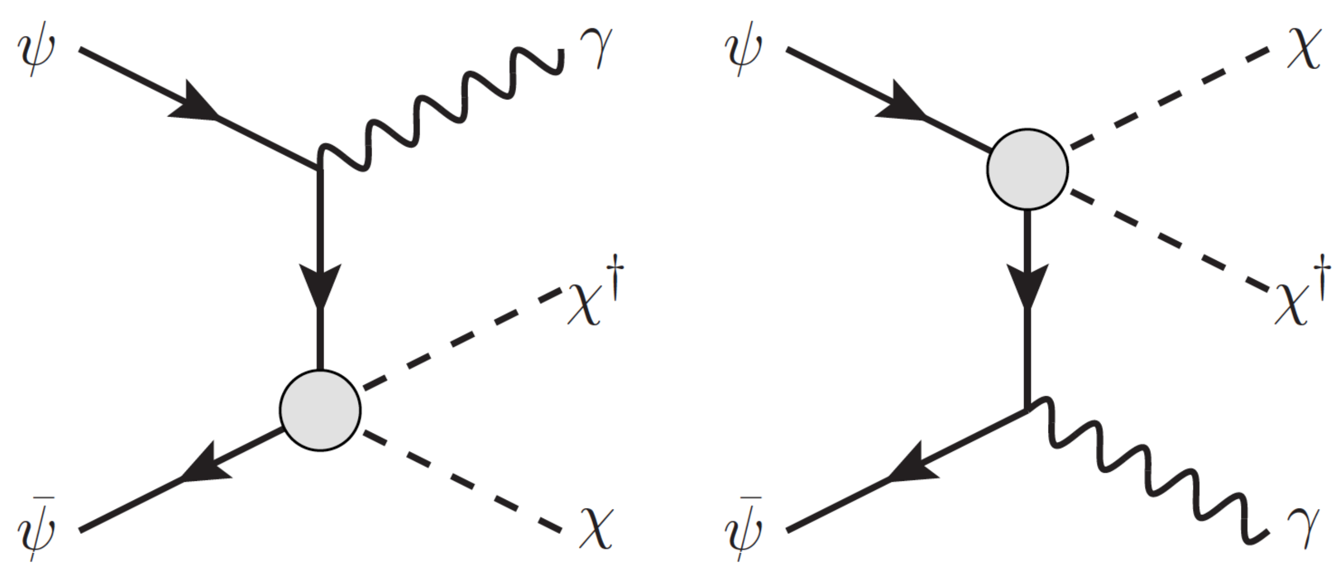
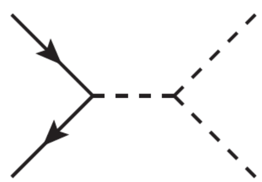
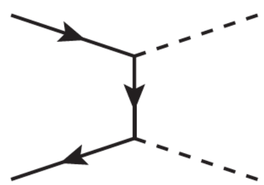
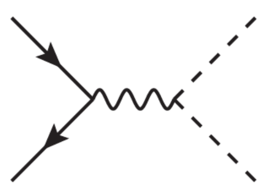
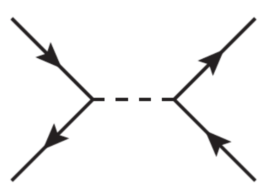


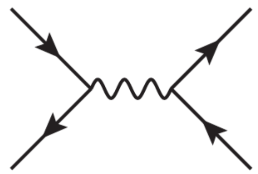
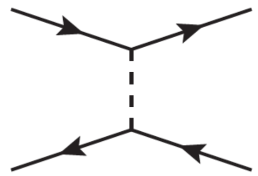
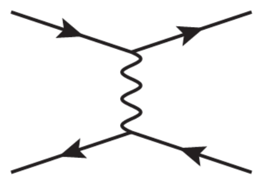
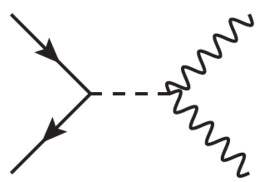
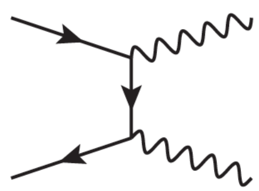
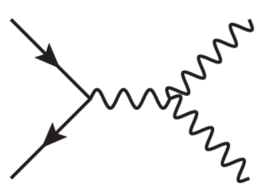
FIG. 1. Diagrams for radiative pair production of dark matter. Terms in which the heavy mediator can emit a photon are neglected.

- Xenon direct detection, ie need relic density
- Pamela indirect detection (SI, SD)

The models —>

TABLE I. List of interaction vertices for S(calar), F(ermion) and V(ector) dark matter, χ, before and after integrating out the heav mediator scalar field ϕ, spinor field η or vector field Z^μ with mass M_Ω. ψ denotes the standard model fermion. $\partial X^{\mu\nu} \equiv \partial^\mu X^\nu -$ $\partial^\nu X^\mu$. Fermionic tS and tV models denote cases where the mediator is exchanged in the t-channel. Note that all Lagrangians are hermitian by construction.

DM	Med.	Diagram	$-\mathcal{L}_{\rm UV}$ $-\mathcal{L}_{\rm eff}$
S	S		$g_\chi \chi^\dagger \chi \phi + \bar{\psi}(g_s + i g_p \gamma^5)\psi \phi$ $\frac{g_\chi}{M_\Omega^2} \chi^\dagger \chi \bar{\psi}(g_s + i g_p \gamma^5)\psi$
S	F		$\bar{\eta}(g_s + g_p \gamma^5)\psi \chi + \bar{\psi}(g_s - g_p \gamma^5)\eta \chi^\dagger$ $\frac{1}{M_\Omega}[(g_s^2 - g_p^2)\bar{\psi}\psi \chi^\dagger \chi + \frac{i}{M_\Omega}\chi^\dagger \bar{\psi}(g_s^2 + g_p^2 - 2 g_s g_p \gamma^5)\gamma^\mu \partial_\mu(\psi \chi)]$
S	V		$g_\chi(\chi^\dagger \partial_\mu \chi - \chi \partial_\mu \chi^\dagger) Z^\mu + \bar{\psi}\gamma^\mu(g_l P_L + g_r P_R)\psi Z_\mu$ $\frac{g_\chi}{M_\Omega^2} \bar{\psi}\gamma^\mu(g_l P_L + g_r P_R)\psi(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger)$
F	S		$\bar{\chi}(g_{s1} + g_{p1}\gamma^5)\chi \phi + \bar{\psi}(g_{s2} + g_{p2}\gamma^5)\psi \phi$ $\frac{1}{M_\Omega^2} \bar{\chi}(g_{s1} + i g_{p1}\gamma^5)\chi \bar{\psi}(g_{s2} + i g_{p2}\gamma^5)\psi$

F	V		$\bar{\psi}\gamma^{\mu}(g_{l1}P_L + g_{r1}P_R)\psi Z_{\mu} + \bar{\chi}\gamma^{\mu}(g_{l2}P_L + g_{r2}P_R)\chi Z_{\mu}$ $\frac{1}{M_{\Omega}^2}\bar{\psi}\gamma^{\mu}(g_{l1}P_L + g_{r1}P_R)\psi\bar{\chi}\gamma_{\mu}(g_{l2}P_L + g_{r2}P_R)\chi$
F	tS		$\bar{\chi}(g_lP_L + g_rP_R)\psi\phi + \bar{\psi}(g_lP_R + g_rP_L)\chi\phi$ $\frac{1}{M_{\Omega}^2}\bar{\psi}(g_lP_R + g_rP_L)\chi\bar{\chi}(g_lP_L + g_rP_R)\psi$
F	tV		$\bar{\psi}\gamma^{\mu}(g_lP_L + g_rP_R)\chi Z_{\mu} + \bar{\chi}\gamma^{\mu}(g_lP_L + g_rP_R)\psi Z_{\mu}$ $\frac{1}{M_{\Omega}^2}\bar{\psi}\gamma^{\mu}(g_lP_L + g_rP_R)\chi\bar{\chi}\gamma_{\mu}(g_lP_L + g_rP_R)\psi$
V	S		$-g_{\chi}\chi^{\mu}\chi_{\mu}\phi + \bar{\psi}(g_s + ig_p\gamma^5)\psi\phi$ $-\frac{g_{\chi}}{M_{\Omega}^2}\chi^{\mu}\chi_{\mu}\bar{\psi}(g_s + ig_p\gamma^5)\psi$
V	F		$-\bar{\eta}\gamma^{\mu}(g_lP_L + g_rP_R)\chi_{\mu} + \bar{\psi}\gamma^{\mu}(g_lP_L + g_rP_R)\eta\chi^{\dagger}_{\mu}$ $\frac{1}{M_{\Omega}}[g_lg_r\bar{\psi}\gamma^{\nu}\gamma^{\rho}\psi\chi^{\dagger}_{\nu}\chi_{\rho} + \frac{i}{M_{\Omega}}\chi^{\dagger}_{\nu}\bar{\psi}\gamma^{\nu}\gamma^{\mu}\gamma^{\rho}(g_l^2P_L + g_r^2P_R)\partial_{\mu}(\psi\chi_{\rho})]$
V	V		$ig_{\chi}[Z_{\mu}\chi^{\dagger}_{\nu}\partial\chi^{\mu\nu} + Z_{\mu}\chi_{\nu}\partial\chi^{\mu\nu} + \chi^{\dagger}_{\mu}\chi_{\nu}\partial Z^{\mu\nu}] + \bar{\psi}\gamma_{\mu}(g_lP_L + g_rP_R)\psi Z^{\mu}$ $\frac{ig_{\chi}}{M_{\Omega}^2}\bar{\psi}\gamma^{\mu}(g_lP_L + g_rP_R)\psi[\chi^{\nu}\partial\chi^{\dagger}_{\mu\nu} - \chi^{\dagger,\nu}\partial\chi_{\mu\nu} + \partial^{\nu}(\chi^{\dagger}_{\nu}\chi_{\mu} - \chi^{\dagger}_{\mu}\chi_{\nu})]$

Benchmark Models

Operators	Definition	Name
SS, VS, FS,	$g_p = 0$	scalar
FtS, FtSr:	$g_s = 0$	pseudoscalar
SF, SFr:	$g_p = 0, M_\Omega = 1$ TeV	scalar_low
	$g_p = 0, M_\Omega = 10$ TeV	scalar_high
	$g_s = 0, M_\Omega = 1$ TeV	pseudoscalar_low
	$g_s = 0, M_\Omega = 10$ TeV	pseudoscalar_high
SV, FV, FtV,	$g_l = g_r$	vector
FtVr, VV:	$g_l = -g_r$	axialvector
	$g_l = 0$	right–handed
VF, VFr:	$g_l = g_r, M_\Omega = 1$ TeV	vector_low
	$g_l = -g_r, M_\Omega = 10$ TeV	vector_high
	$g_l = g_r, M_\Omega = 1$ TeV	axialvector_low
	$g_l = -g_r, M_\Omega = 10$ TeV	axialvector_high
FVr :	$g_l = 0$	right–handed

TABLE II. Benchmark models with specific values for the coupling constants shown in Table I.

ILC Study

Detector effects based on ILD simulation study by C. Bartels, J. List und M. Berggren: arXiv:1206.6639v1 [hep-ex]

Background Processes

- $\nu\bar{\nu}\gamma(\gamma)$: Mainly left–chiral due to W–exchange
- $e^+e^-\gamma$: No polarisation dependence. Small efficiency (two undetected leptons) but high cross section (purely QED)

Simulation of Signal and $\nu\bar{\nu}\gamma(\gamma)$

- CalcHEP with ISR + Beamstrahlung for $M_\chi \in [1\text{GeV}, 490\text{GeV}]$
- ΔE and ϵ based on ILC Letter of Intent and 1206.6639

$e^+e^-\gamma$–background

- Detector Effects crucial → Results from 1206.6639

Backgrounds

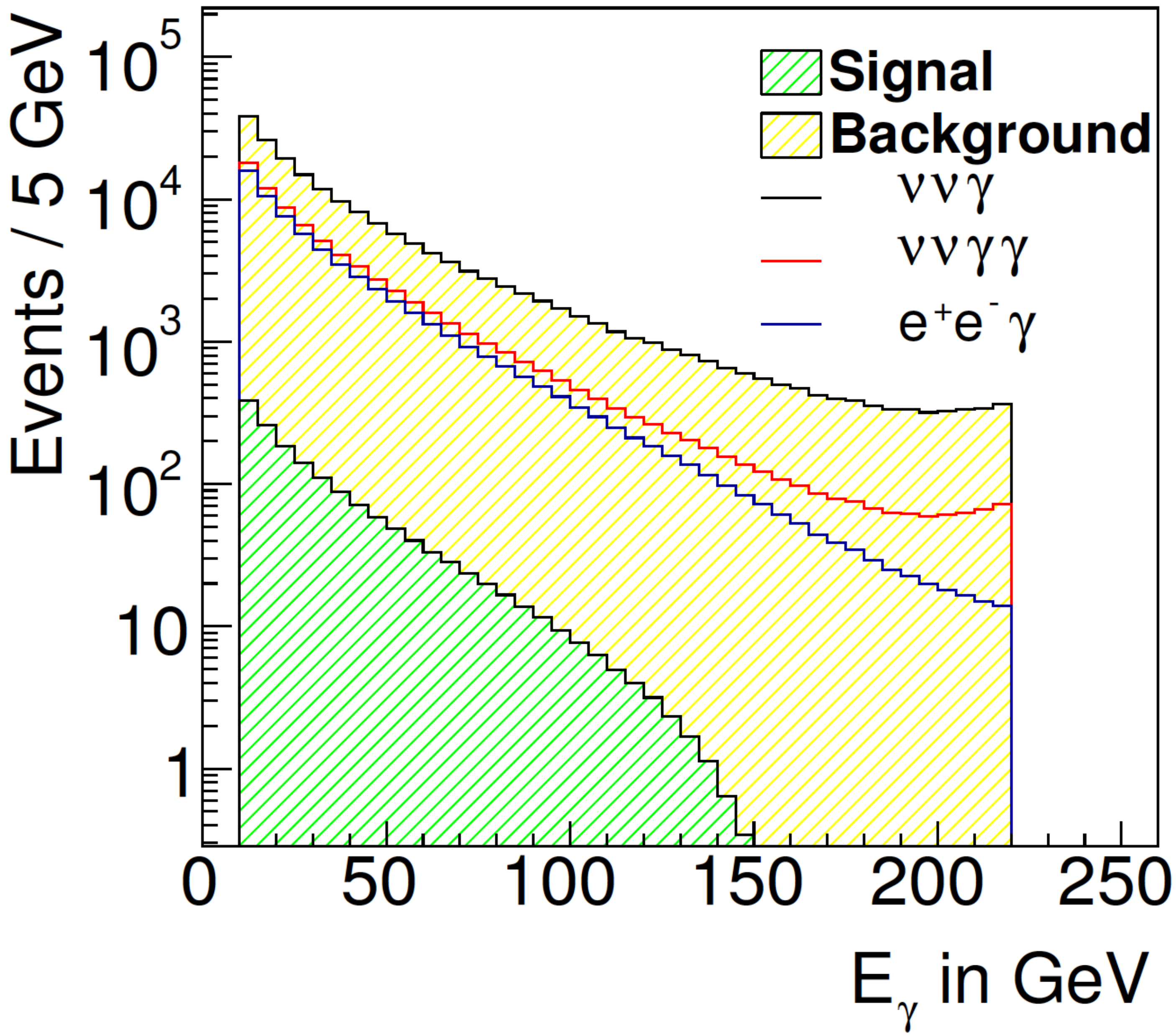


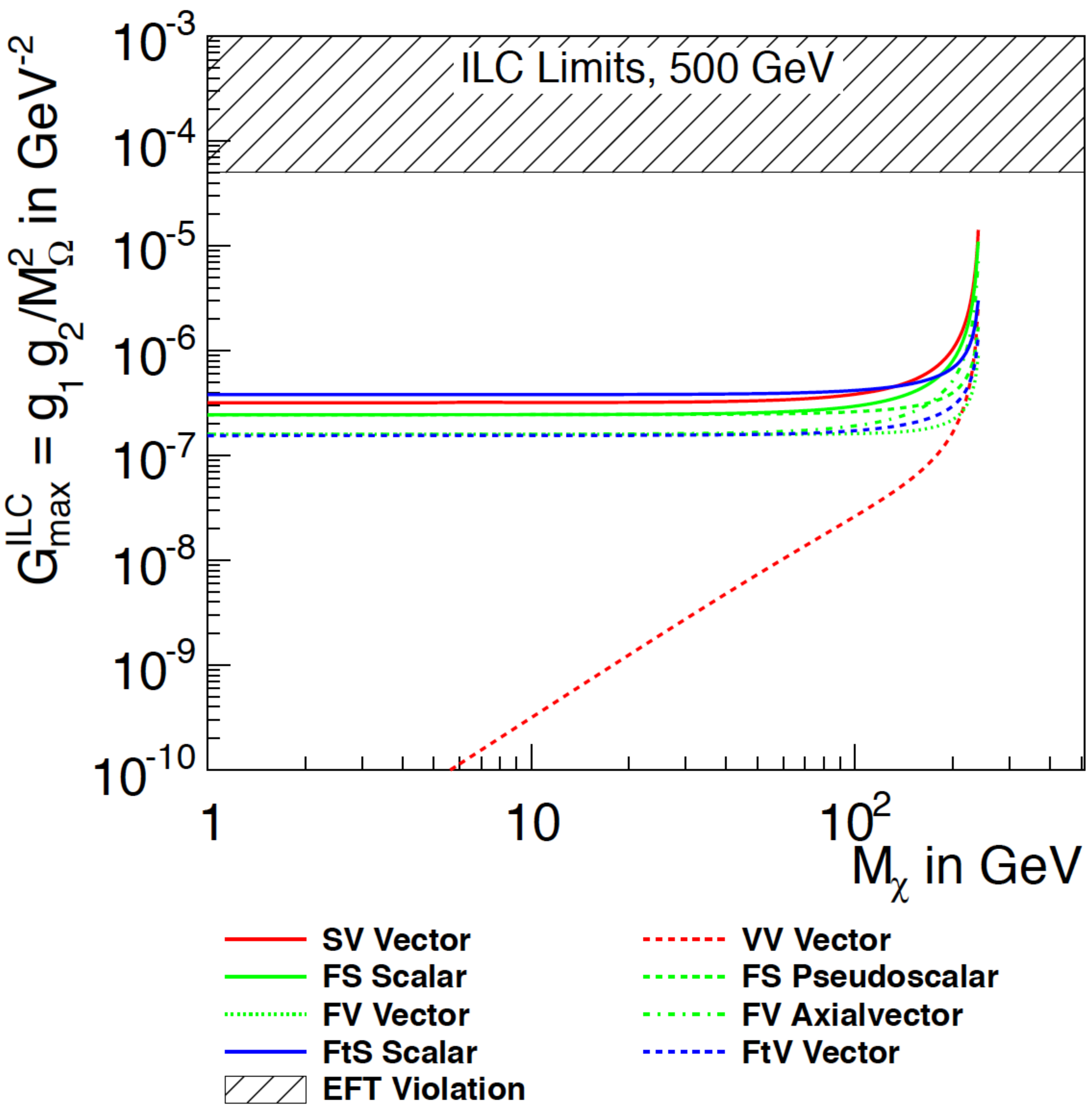
FIG. 4. Photon energy distributions of the most dominant background contributions (stacked) compared to an example signal (FS Scalar, $M_\chi = 150$ GeV) with a total cross section of 100 fb. All spectra are taken after selection for an unpolarised initial state.

Model	$\frac{d\sigma}{dx\, d\cos\theta}$
SS	$\frac{\hat\beta F_{x\theta}}{32\pi M_\Omega^4} G_{s+p} g_\chi^2 C_s$
SF	$\frac{\hat\beta F_{x\theta}}{32\pi M_\Omega^2}\left[G_{s-p}^2 C_s + \frac{\hat\beta^2 \hat s}{12 M_\Omega^2}\boldsymbol{V_{x\theta}}\left[(g_s+g_p)^4 C_R + (g_s-g_p)^4 C_L\right] + \boldsymbol{A}_{\mathrm{SF}}\right]$
SFr	$\frac{\hat\beta}{16\pi M_\Omega^2}\left[F_{x\theta} G_{s-p}^2 C_s + \boldsymbol{A}_{\mathrm{SFr}}\right]$
SV	$\frac{\hat s \hat\beta^3 F_{x\theta}}{96\pi M_\Omega^4}\boldsymbol{V_{x\theta}}\left[g_l^2 C_L + g_r^2 C_R\right] g_\chi^2$
FS	$\frac{\hat s \hat\beta F_{x\theta}}{16\pi M_\Omega^4} G_{s+p} C_s \left[g_s^2 \hat\beta^2 + g_p^2\right]$
FV	$\frac{\hat\beta F_{x\theta}}{48\pi M_\Omega^4}\boldsymbol{V_{x\theta}}\left[G_{l+r}\hat s \hat\beta^2 + 3\left(g_l+g_r\right)^2 M_\chi^2\right]\left[g_l^2 C_L + g_r^2 C_R\right]$
FVr	$\frac{\hat s \hat\beta^3 F_{x\theta}}{48\pi M_\Omega^4}\boldsymbol{V_{x\theta}}\left(g_l - g_r\right)^2\left[g_l^2 C_L + g_r^2 C_R\right]$
FtS	$\frac{F_{x\theta}\hat\beta}{48\pi M_\Omega^4} G_{s+p}^2\left[\boldsymbol{V_{x\theta}}(\hat s - M_\chi^2) + \boldsymbol{A}_{\mathrm{FtS}}\right]$
FtSr	$\frac{\hat\beta F_{x\theta}}{192\pi M_\Omega^4} G_{s+p}^2\left[3(\hat s - 2M_\chi^2) C_P + \boldsymbol{V_{x\theta}} 2(\hat s - 4M_\chi^2) C_V\right]$
FtV	$\frac{\hat\beta F_{x\theta}}{48\pi M_\Omega^4}\left[6 G_{lr}^2 C_s(\hat s - 2M_\chi^2) + (\hat s - M_\chi^2)\boldsymbol{V_{x\theta}}(g_l^4 C_L + g_r^4 C_R)\right]$
FtVr	$\frac{\hat\beta F_{x\theta}}{48\pi M_\Omega^4}\left[12 G_{lr}^2 C_s(\hat s - 2M_\chi^2) + (\hat s - 4M_\chi^2)\boldsymbol{V_{x\theta}}(g_l^4 C_L + g_r^4 C_R)\right]$

TABLE III. Analytical differential cross sections for the process $e^+e^- \to \chi\chi\gamma$ in the various effective models. Terms in bold do not appear in the Weizsäcker–Williams approach and are given in Appendix C 1 where we also define all used abbreviations. Models with a suffix ‘r’ correspond to the case of real particles. Cross sections for SSr, FSr and VSr are twice as large as in the complex case while SV and VV vanish completely for real particles.

ILC Results

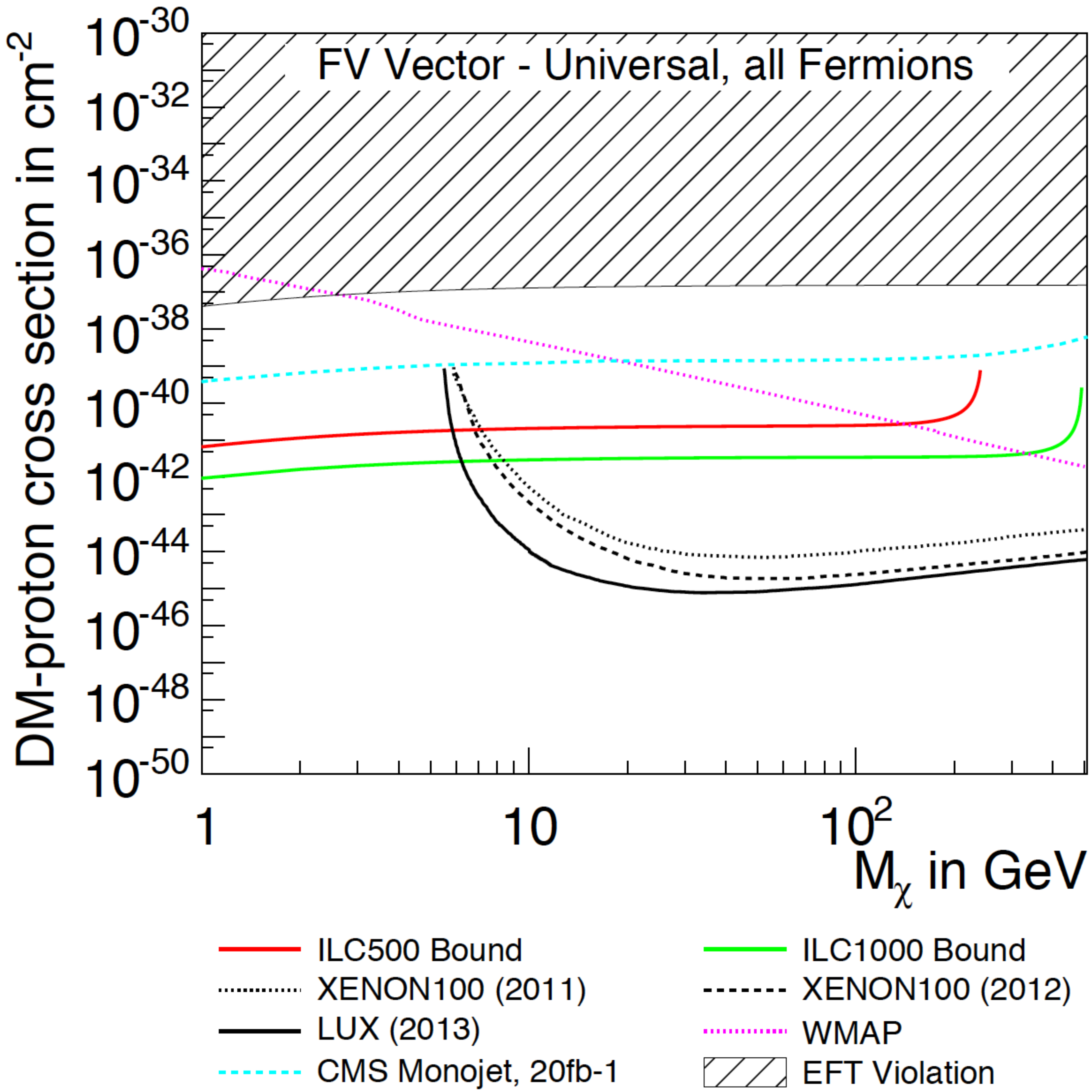
- We show results for $\mathcal{L} = 500\text{fb}^{-1}$, $\sqrt{s} = 500\text{GeV}$, $\Delta P/P = 0.01$
- Polarisation with largest $S/\Delta B$ ($P^{+} = \pm 0.3$ can be better than 0.6)



Remarks

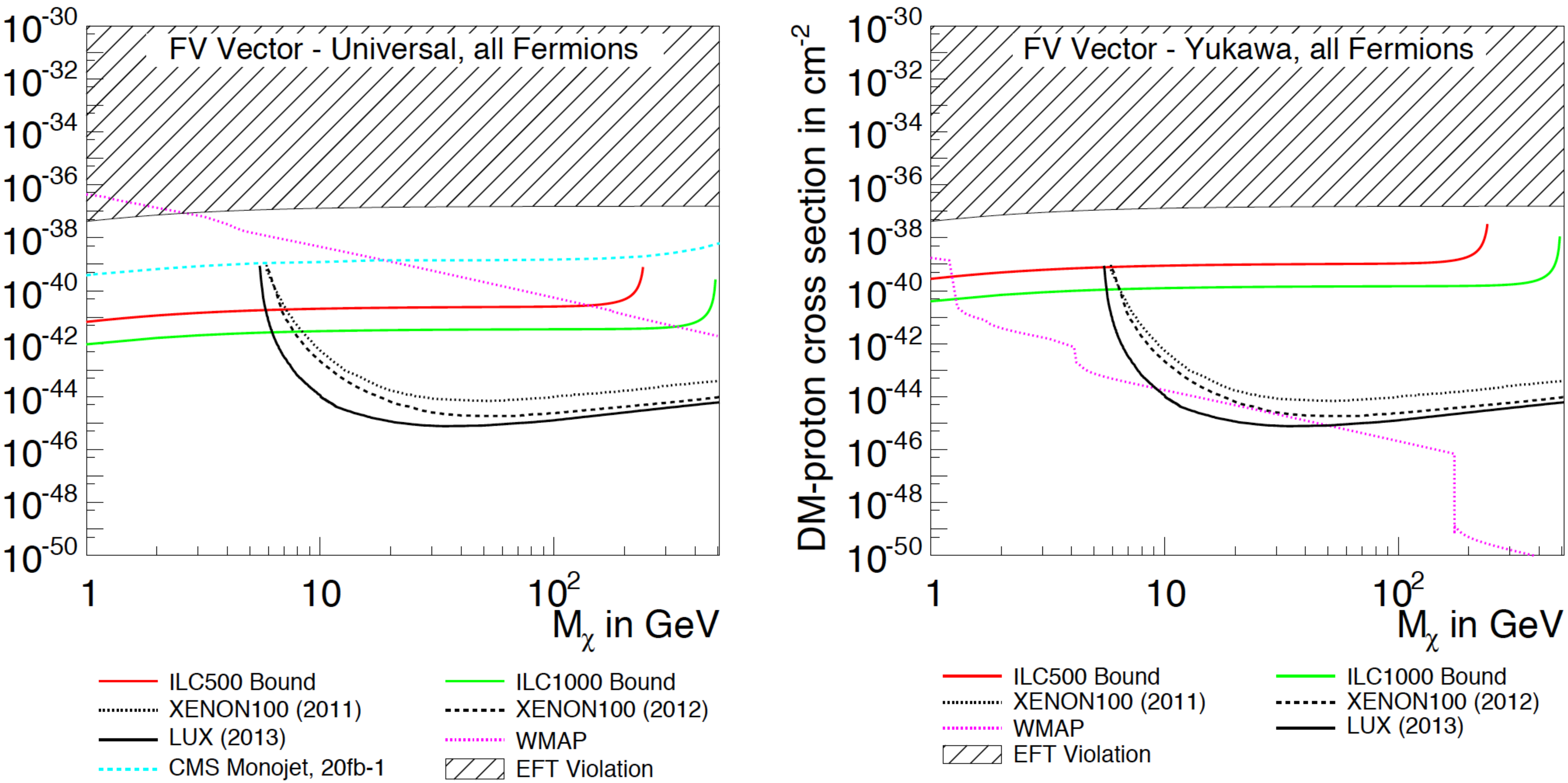
- Exclusion limits are given to 90% CL
- $G = 10^{-7}\text{GeV}^{-2}$ corresponds to about $\sigma = 0.3$ fb
- Models with vector WIMPs have theoretical problems ($\sigma \propto s/M_\chi^4$)

Comparison Results: Spin Independent

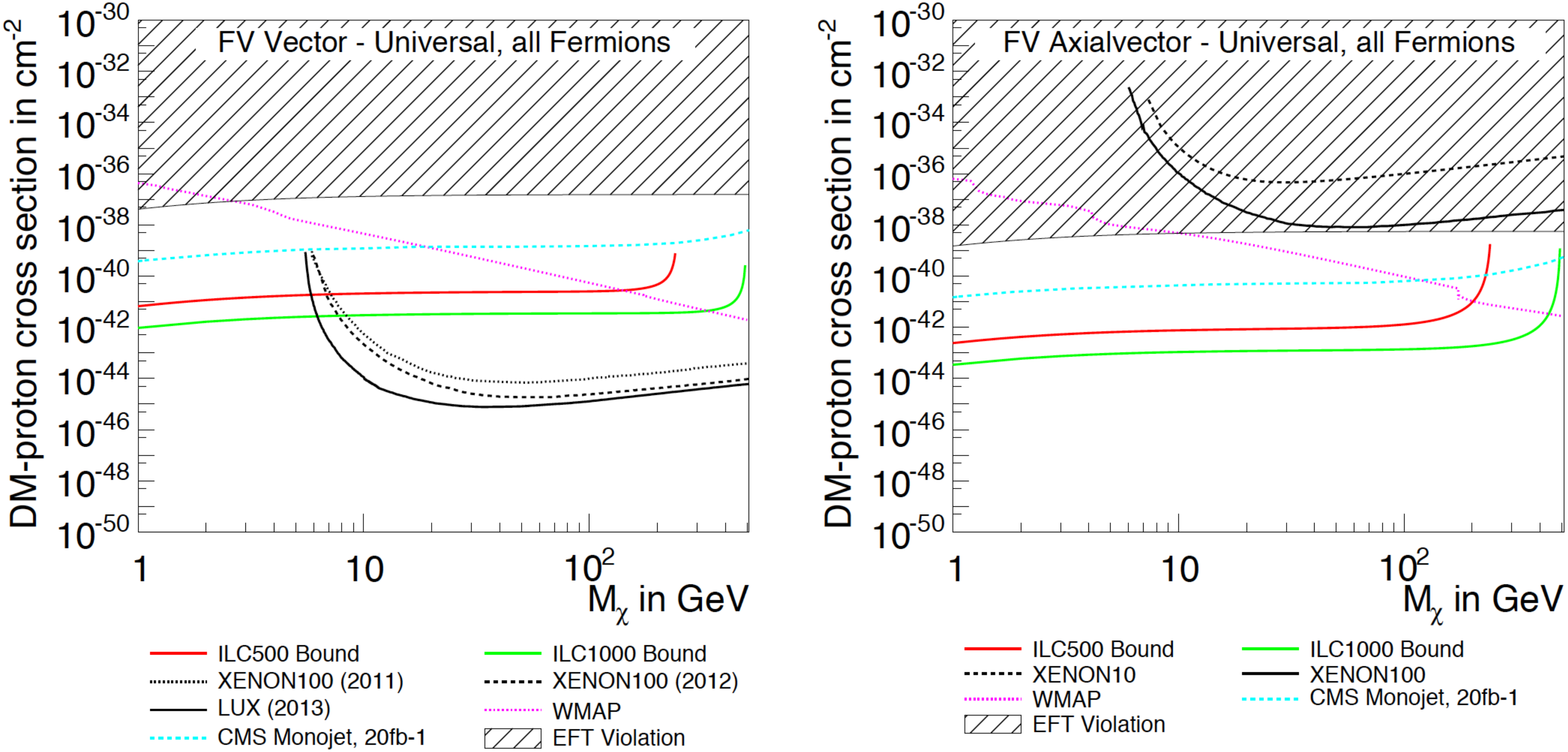


- ILC good for light DM

Comparison Results: Universal vs Yukawa

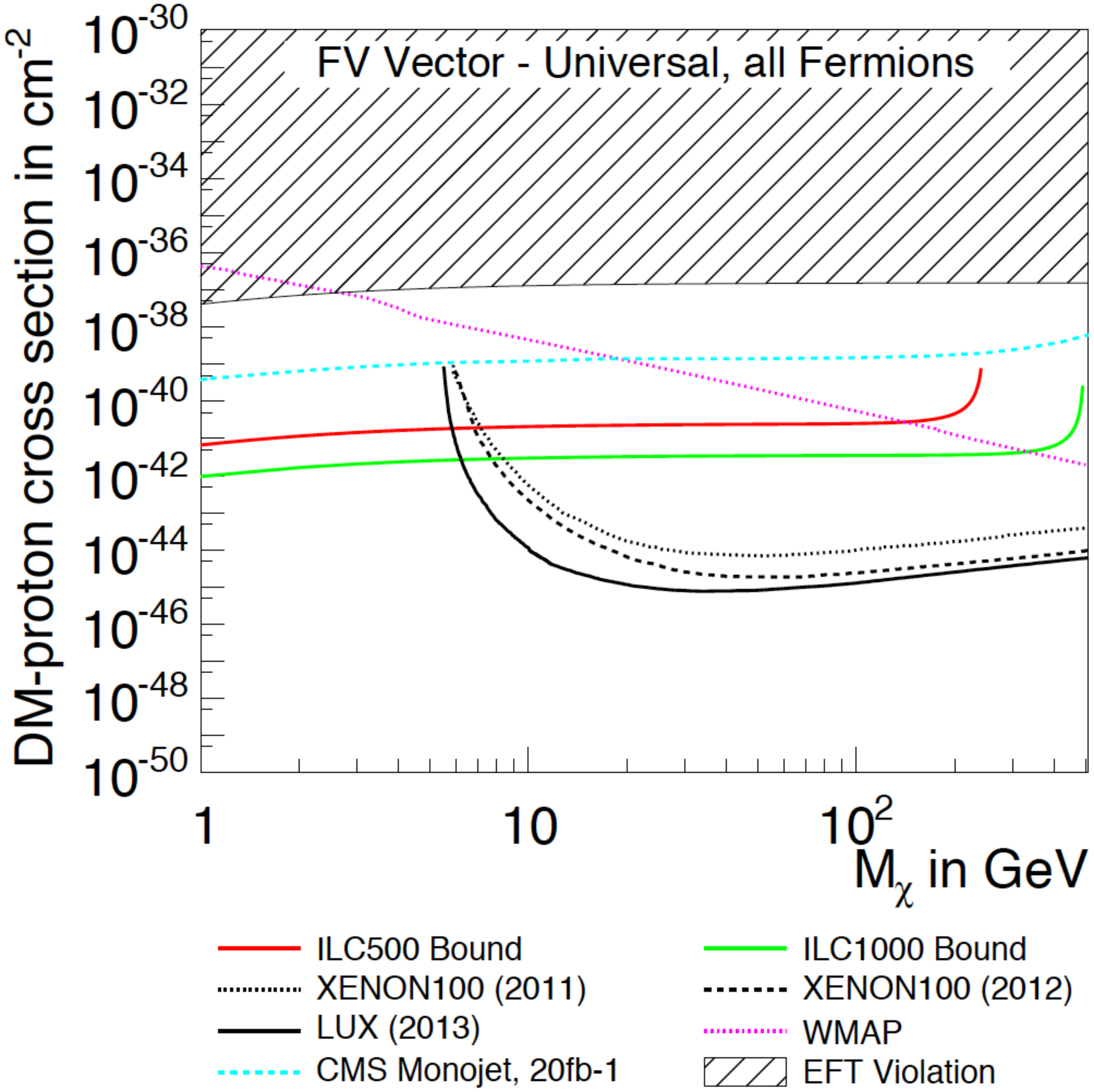


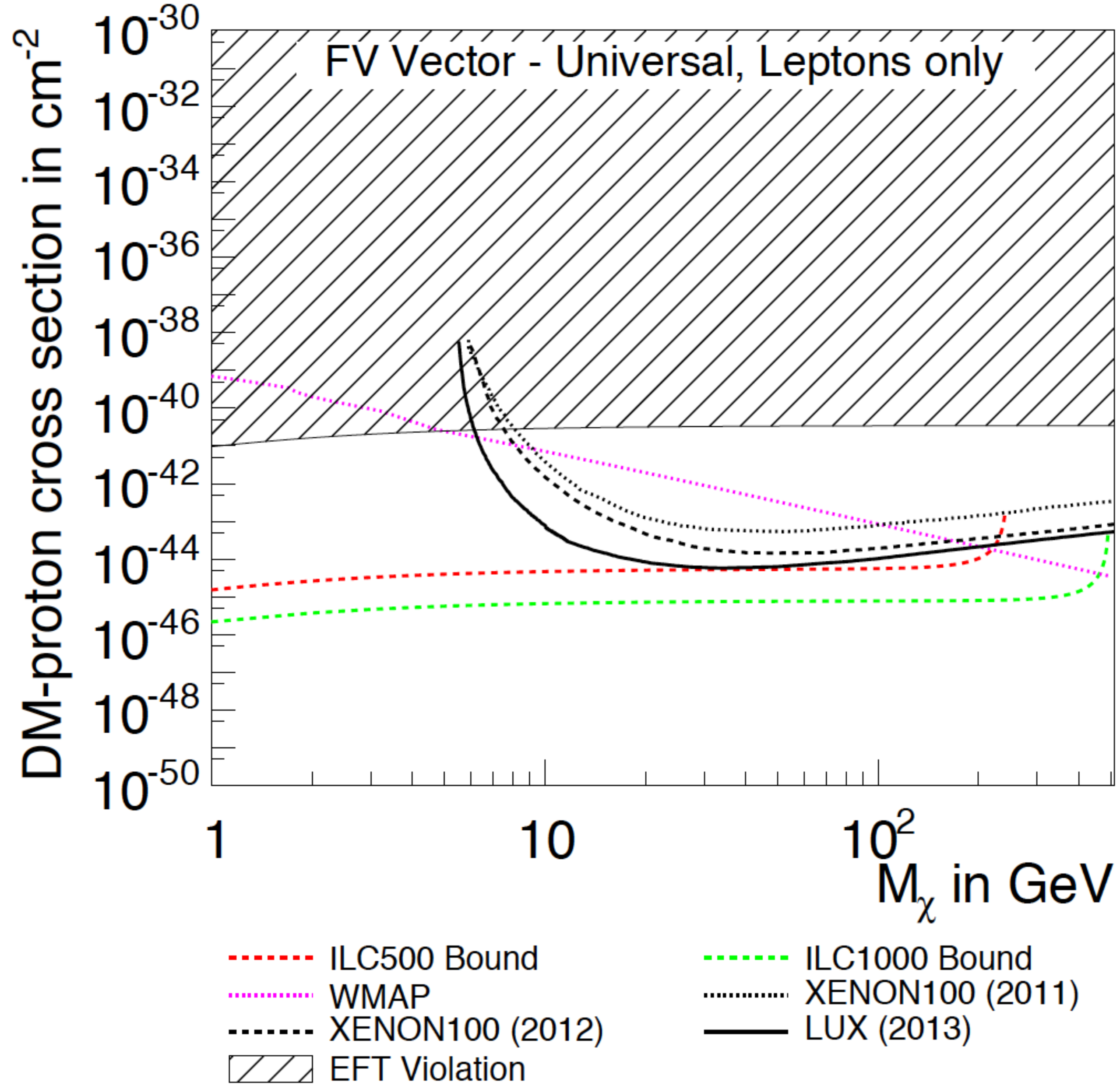
Comparison Results: Spin Indep. vs Spin Dep.



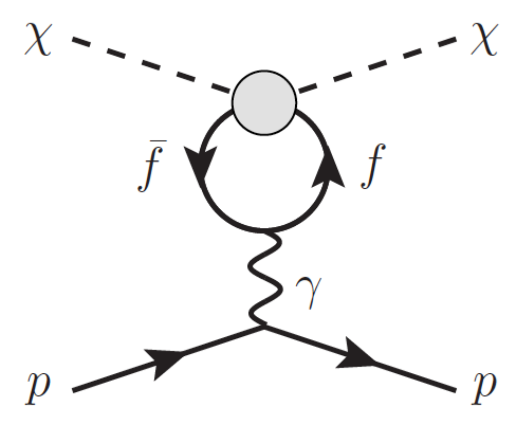
- Spin dependent case, collider is strongest

Fermions vs Leptons only

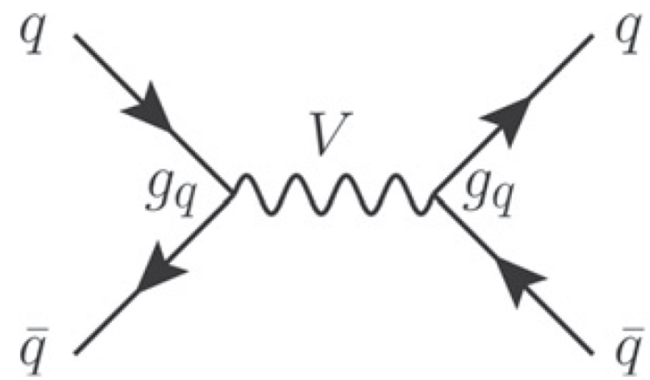




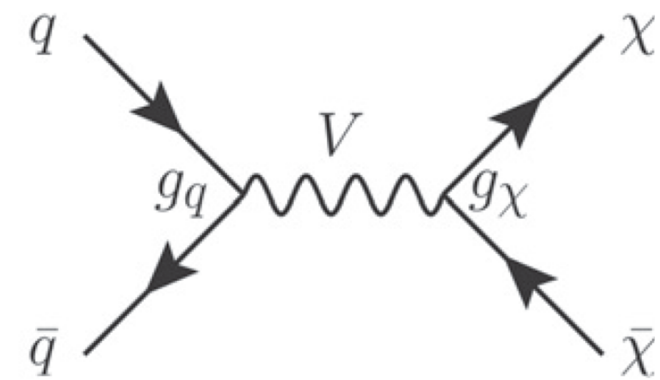
- DM coupling to nucleons loop-suppressed



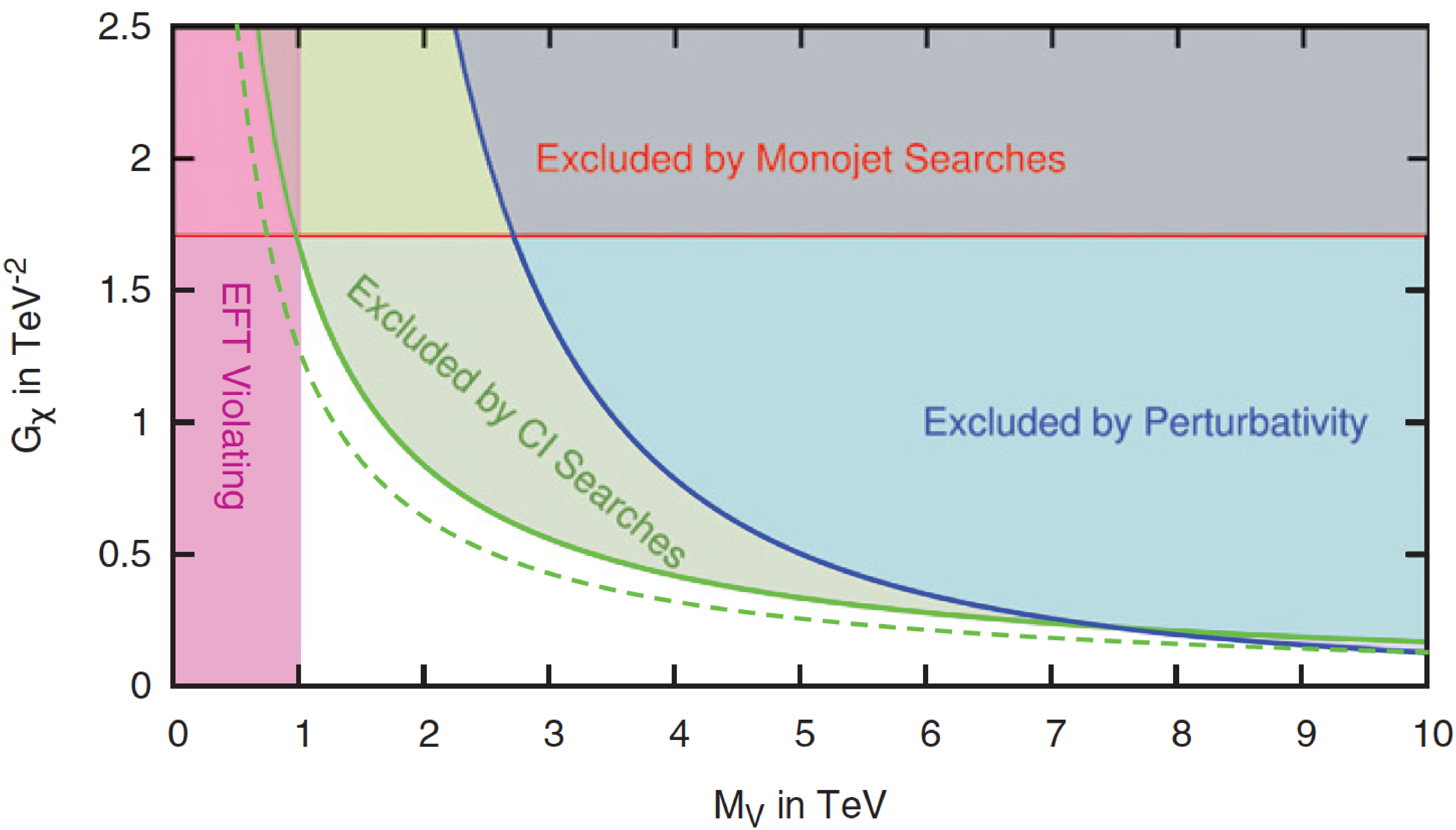
Contact Interactions @ LHC



(a) Elastic quark scattering (plus a corresponding t-channel contribution).



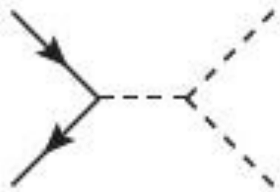
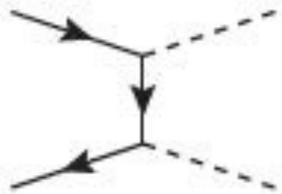
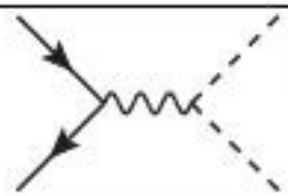
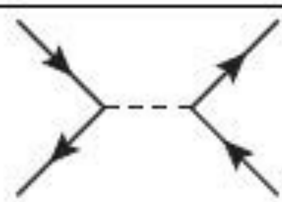
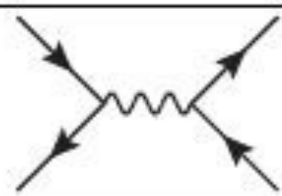
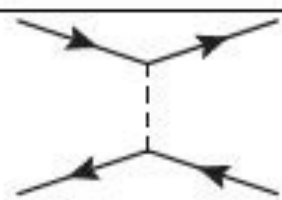
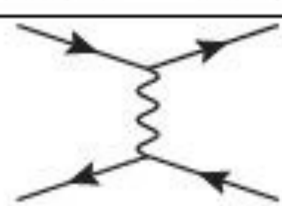
(b) Pair production of χ.



Summary

- Measuring light neutralino mass at ILC
  - Very light neutralino is allowed
  - Measure mass via: $e^+e^- \to \tilde{e}^+\tilde{e}^- \to e^+e^-\chi_1^0\chi_1^0$
  - Very accurate at high χ_1^0 mass
  - Impossible below about 5-10 GeV
- Eff. Operator DM Analysis at ILC
  - Eff. models describe int. with just 2 parameters: M_χ and $G_{\rm eff}$
  - ILC can look for pair produced WIMPs via mono photons
  - Polraisation can help reduce (some) background
  - LHC limits can be improved by orders of magnitude

Backups

DM	Med.	Diagram	$-\mathscr{L}_{\mathrm{UV}}$ $-\mathscr{L}_{\mathrm{eff}}$
S	S		$g_\chi \chi^\dagger \chi \phi + \bar{\psi}(g_s + i g_p \gamma^5)\psi\phi$ $\frac{g_\chi}{M_\Omega^2}\chi^\dagger\chi\bar{\psi}(g_s + i g_p\gamma^5)\psi$
S	F		$\bar{\eta}(g_s + g_p\gamma^5)\psi\chi + \bar{\psi}(g_s - g_p\gamma^5)\eta\chi^\dagger$ $\frac{1}{M_\Omega}\left[(g_s^2 - g_p^2)\bar{\psi}\psi\chi^\dagger\chi + \frac{i}{M_\Omega}\chi^\dagger\bar{\psi}\left(g_s^2 + g_p^2 - 2g_s g_p\gamma^5\right)\gamma^\mu\partial_\mu\left(\psi\chi\right)\right]$
S	V		$g_\chi(\chi^\dagger\partial_\mu\chi - \chi\partial_\mu\chi^\dagger)Z^\mu + \bar{\psi}\gamma^\mu(g_l P_L + g_r P_R)\psi Z_\mu$ $\frac{g_\chi}{M_\Omega^2}\bar{\psi}\gamma^\mu\left(g_l P_L + g_r P_R\right)\psi\left(\phi^\dagger\partial_\mu\phi - \phi\partial_\mu\phi^\dagger\right)$
F	S		$\bar{\chi}\left(g_s + g_p\gamma^5\right)\chi\phi + \bar{\psi}\left(g_s + g_p\gamma^5\right)\psi\phi$ $\frac{1}{M_\Omega^2}\bar{\chi}\left(g_s + i g_p\gamma^5\right)\chi\bar{\psi}\left(g_s + i g_p\gamma^5\right)\psi$
F	V		$\bar{\psi}\gamma^\mu(g_l P_L + g_r P_R)\psi Z_\mu + \bar{\chi}\gamma^\mu(g_l P_L + g_r P_R)\chi Z_\mu$ $\frac{1}{M_\Omega^2}\bar{\psi}\gamma^\mu\left(g_l P_L + g_r P_R\right)\psi\;\bar{\chi}\gamma_\mu\left(g_l P_L + g_r P_R\right)\chi$
F	tS		$\bar{\chi}\left(g_s + g_p\gamma^5\right)\psi\phi + \bar{\psi}\left(g_s + g_p\gamma^5\right)\chi\phi$ $\frac{1}{M_\Omega^2}\bar{\psi}\left(g_s - g_p\gamma^5\right)\chi\bar{\chi}\left(g_s + g_p\gamma^5\right)\psi$
F	tV		$\bar{\psi}\gamma^\mu(g_l P_L + g_r P_R)\chi Z_\mu + \bar{\chi}\gamma^\mu(g_l P_L + g_r P_R)\psi Z_\mu$ $\frac{1}{M_\Omega^2}\bar{\psi}\gamma^\mu\left(g_l P_L + g_r P_R\right)\chi\bar{\chi}\gamma_\mu\left(g_l P_L + g_r P_R\right)\psi$

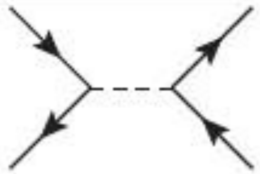
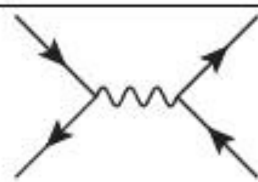
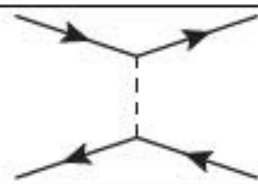
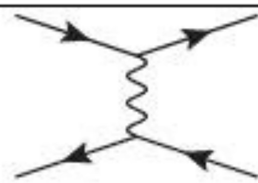
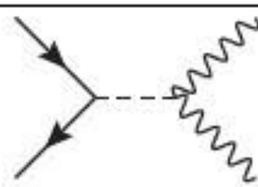
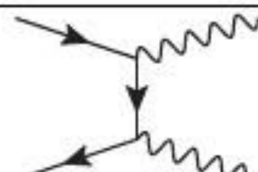
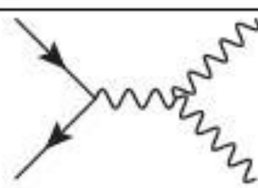
F	S		$\bar\chi\left(g_s+g_p\gamma^5\right)\chi\phi+\bar\psi\left(g_s+g_p\gamma^5\right)\psi\phi$ $\frac{1}{M_\Omega^2}\bar\chi\left(g_s+ig_p\gamma^5\right)\chi\bar\psi\left(g_s+ig_p\gamma^5\right)\psi$
F	V		$\bar\psi\gamma^\mu(g_lP_L+g_rP_R)\psi Z_\mu+\bar\chi\gamma^\mu(g_lP_L+g_rP_R)\chi Z_\mu$ $\frac{1}{M_\Omega^2}\bar\psi\gamma^\mu\left(g_lP_L+g_rP_R\right)\psi\;\bar\chi\gamma_\mu\left(g_lP_L+g_rP_R\right)\chi$
F	tS		$\bar\chi\left(g_s+g_p\gamma^5\right)\psi\phi+\bar\psi\left(g_s+g_p\gamma^5\right)\chi\phi$ $\frac{1}{M_\Omega^2}\bar\psi\left(g_s-g_p\gamma^5\right)\chi\bar\chi\left(g_s+g_p\gamma^5\right)\psi$
F	tV		$\bar\psi\gamma^\mu(g_lP_L+g_rP_R)\chi Z_\mu+\bar\chi\gamma^\mu(g_lP_L+g_rP_R)\psi Z_\mu$ $\frac{1}{M_\Omega^2}\bar\psi\gamma^\mu\left(g_lP_L+g_rP_R\right)\chi\bar\chi\gamma_\mu\left(g_lP_L+g_rP_R\right)\psi$
V	S		$-\chi^\mu\chi_\mu\phi+\bar\psi(g_s+ig_p\gamma^5)\psi\phi$ $-\frac{g_\chi}{M_\Omega^2}\chi^\mu\chi_\mu\bar\psi\left(g_s+ig_p\gamma^5\right)\psi$
V	F		$-\bar\eta\gamma^\mu(g_lP_L+g_rP_R)\chi_\mu+\bar\psi\gamma^\mu(g_lP_L+g_rP_R)\eta\chi^\dagger_\mu$ $\frac{1}{M_\Omega}\left[g_lg_r\bar\psi\gamma^\nu\gamma^\rho\psi\;\chi^\dagger_\nu\chi_\rho+\frac{i}{M_\Omega}\chi^\dagger_\nu\bar\psi\gamma^\nu\gamma^\mu\gamma^\rho\left(g_l^2P_L+g_r^2P_R\right)\partial_\mu\left(\psi\chi_\rho\right)+\right]$
V	V		$ig_\chi\left[Z_\mu\chi^\dagger_\nu\partial\chi^{\mu\nu}+Z_\mu\chi_\nu\partial\chi^{\mu\nu}+\chi^\dagger_\mu\chi_\nu\partial Z^{\mu\nu}\right]+\bar\psi\gamma_\mu(g_lP_L+g_rP_R)\psi$ $\frac{ig_\chi}{M_\Omega^2}\bar\psi\gamma^\mu\left(g_lP_L+g_rP_R\right)\psi\left[\chi^\nu\partial\chi^\dagger_{\mu\nu}-\chi^{\dagger,\nu}\partial\chi_{\mu\nu}+\partial^\nu\left(\chi^\dagger_\nu\chi_\mu-\chi^\dagger_\mu\chi_\nu\right)\right]$

TABLE I. List of interaction vertices for S(calar), F(ermion) and V(ector) dark matter, χ, before and after integrating out the heavy mediator scalar field ϕ, spinor field η or vector field Z^μ with mass M_Ω. ψ denotes the Standard Model fermion. $\partial X^{\mu\nu}\equiv\partial^\mu X^\nu-\partial^\nu X^\mu$. tS and tV denote cases where the mediator is exchanged in the t-channel.

VS	$\frac{\hat{\beta}F_{x\theta}}{128\pi M_\chi^4 M_\Omega^4} G_{s+p} g_\chi^2 C_s (12M_\chi^4 - 4M_\chi^2\hat{s} + \hat{s}^2)$
VF	$\frac{\hat{\beta}F_{x\theta}}{3840\pi M_\chi^4 M_\Omega^2} \left[40G_{lr}^2 C_s (7M_\chi^4 - 2M_\chi^2\hat{s} + \hat{s}^2) + \frac{1}{M_\Omega^2}\left(g_l^4 C_L + g_r^4 C_R\right)\left(40M_\chi^6 - 22M_\chi^4\hat{s} + 56M_\chi^2\hat{s}^2 + 3\hat{s}^3\right) + \boldsymbol{A}_{\mathrm{VF}}\right]$
VFr	$\frac{\hat{\beta}F_{x\theta}}{3840\pi M_\chi^4 M_\Omega^2} \left[60G_{lr}^2 C_s (12M_\chi^4 - 4M_\chi^2\hat{s} + \hat{s}^2) + \frac{1}{M_\Omega^2}\left(g_l^4 C_L + g_r^4 C_R\right)\left(320M_\chi^6 - 104^4\hat{s} + 32M_\chi^2\hat{s}^2 + \hat{s}^3\right) + \boldsymbol{A}_{\mathrm{VFr}}\right]$
VV	$\frac{\hat{s}\hat{\beta}^3 F_{x\theta}\boldsymbol{V}_{x\theta}}{3840\pi M_\chi^4 M_\Omega^4} \left[g_l^2 C_L + g_r^2 C_R\right] g_\chi^2 (M_\chi^4 + 20M_\chi^2\hat{s} + \hat{s}^2)$

TABLE III. Analytical differential cross sections for the process $e^+e^- \to \chi\chi\gamma$ in the various effective models. Terms in bold do not appear in the Weizsäcker–Williams approach and are given in Appendix C 1 where we also define all used abbreviations. Models with a suffix ‘r’ correspond to the case of real particles. Cross sections for SSr, FSr and VSr are twice as large as in the complex case while SV and VV vanish completely for real particles.

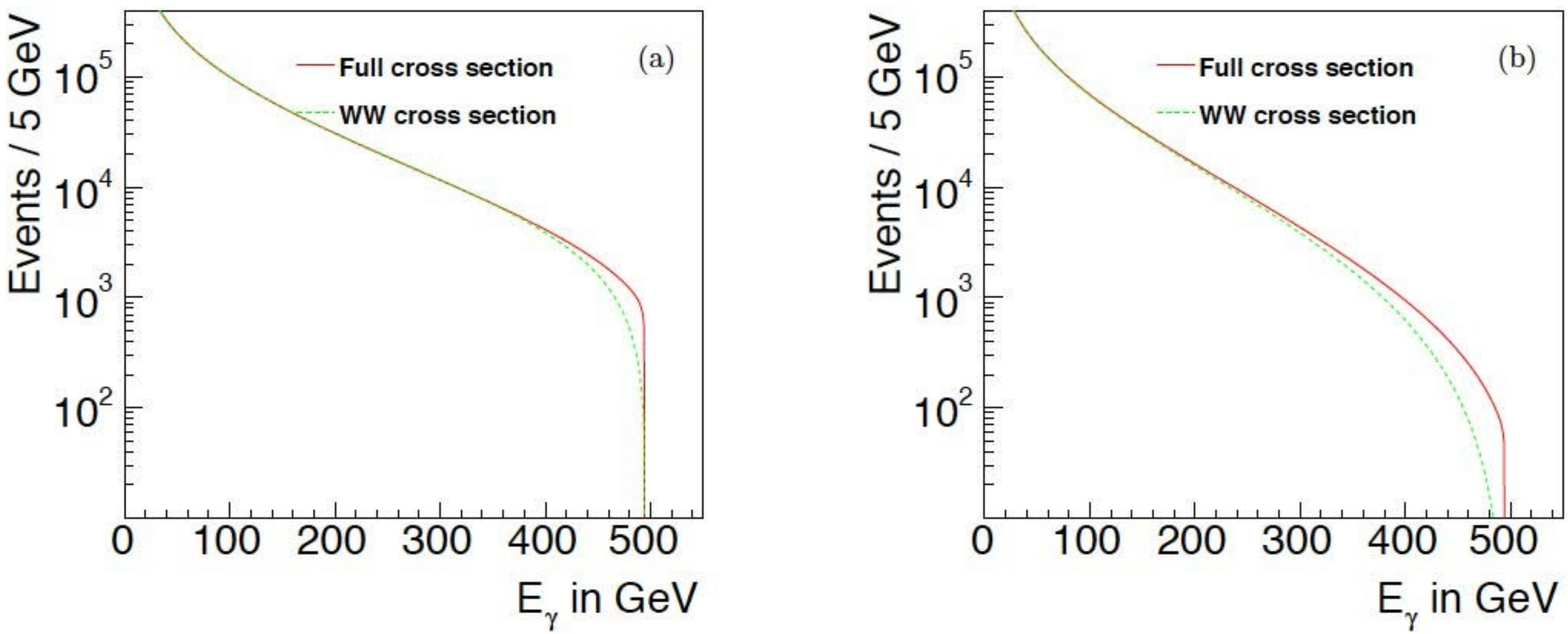


FIG. 2. Comparison of tree level photon energy distributions in the WW–approximation and the analytical solution for $M_\chi = 50$ GeV, $|\cos\theta_\gamma|_{\max} = 0.98$ and $\sqrt{s} = 1$ TeV. (a) SV, (b) FtS.