Natural Supersymmetry in Warped Space

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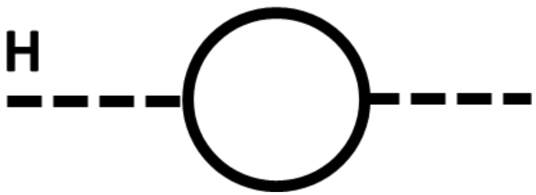
B. Heidenreich and YN, arXiv:1407.5095[hep-ph]

Questions in Higgs Mechanism

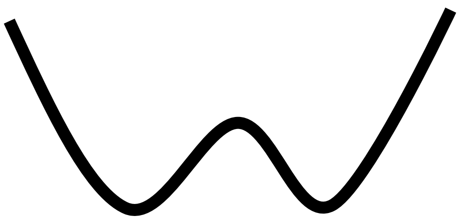
Naturalness



Quantum correction destabilizes the EW scale.

Dynamics of EWSB

Higgs potential is ad hoc.



Why ?

Yukawa hierarchies

$$m_u \sim 3\ \mathrm{MeV} \quad \ll \quad m_t \sim 173\ \mathrm{GeV}$$

Yukawa couplings have to be hierarchical.

We need BSM physics !

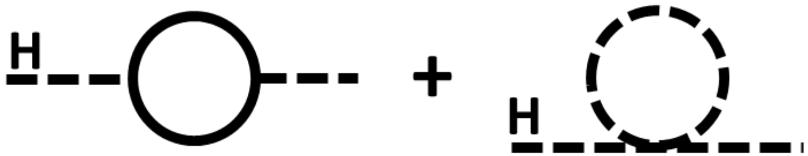
Supersymmetry

SUSY is a nice framework to address several questions !

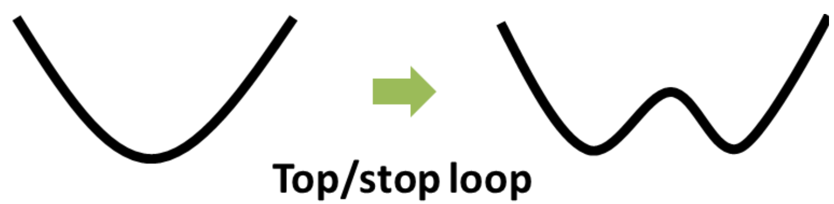
Naturalness

Quadratic divergence is cancelled.

γ, Z, … (spin 1)	⟷	gaugino (spin 1/2)
quark, …	⟷	squark, …
Higgs	⟷	Higgsino



Radiative EWSB



SUSY breaking can drive EWSB radiatively.

Problems of SUSY

Little hierarchy problem

$$m_{\rm 1st,\,2nd} > 5 \times 10^4\,{\rm TeV}$$ → **Fine-tuning !**

M. Bona, the UK Flavour Workshop (2013)

Yukawa hierarchies

SUSY itself does not address Yukawa hierarchies.

Light Higgs

125 GeV Higgs is heavy for SUSY.

Stop mass bound

$$m_{\tilde{t}} > 650\,{\rm GeV}$$ **Tuning**

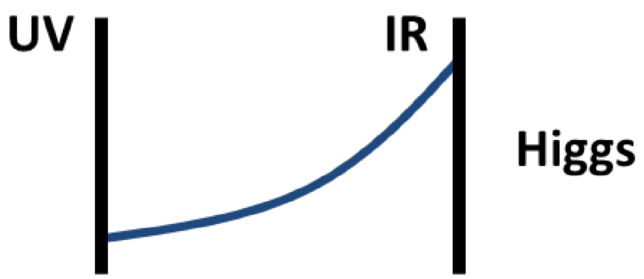
To solve these questions, …

SUSY + Compositeness

is good.

AdS/CFT

Composite Higgs ⟷



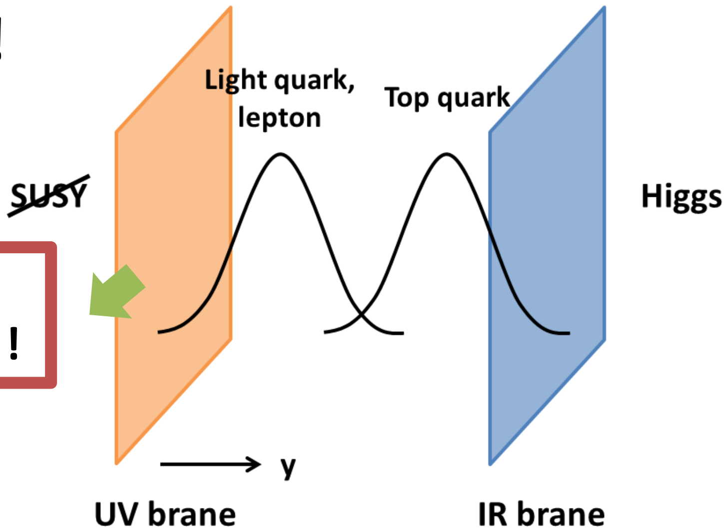
Warped Natural SUSY

Gherghetta, Pomarol (2003) , Sundrum (2009) , Larsen, Nomura, Roberts (2012) ...

Combine SUSY and Randall-Sundrum !

Quadratic divergence is cutoff at the IR scale (~ 10 TeV).

Yukawa hierarchies !



For naturalness, we only need ...

Light stops, gauginos and Higgsinos.

The other squarks and sleptons are heavy.

Little hierarchy problem is solved !

This pattern of SUSY breaking is naturally realized.

Accidental SUSY

Even with large ~~SUSY~~ on UV brane, IR brane SUSY is maintained.

SUSY Yang-Mills coupled to SCFT **Sundrum (2009)**

$$\mathcal{L}(\mu) = -\frac{1}{4}F_{\mu\nu}^{a\ 2} + \bar{\lambda}D.\sigma\lambda + \mathcal{L}_{SCFT} + g A_\mu^a J_a^\mu + \tilde{g}\lambda_a \Psi_J^a - \frac{1}{2}g_D^2 D^a D^a$$

$$\frac{d1/g^2}{d\ln\mu} = \frac{d1/\tilde{g}^2}{d\ln\mu} = \frac{d1/g_D^2}{d\ln\mu} \equiv -b_{CFT} \sim \frac{\mathcal{O}(N_{CFT})}{16\pi^2}$$

➡ **IR free !**

$$\frac{\Delta\tilde{g}^2}{g^2} \equiv \frac{\tilde{g}^2 - g^2}{g^2} \approx \frac{g^2}{g_0^2}\frac{\Delta\tilde{g}_0^2}{g_0^2}$$

$$\frac{\Delta g_D^2}{g^2} \equiv \frac{g_D^2 - g^2}{g^2} \approx \frac{g^2}{g_0^2}\frac{\Delta g_{D0}^2}{g_0^2}$$

Focusing effect in the IR

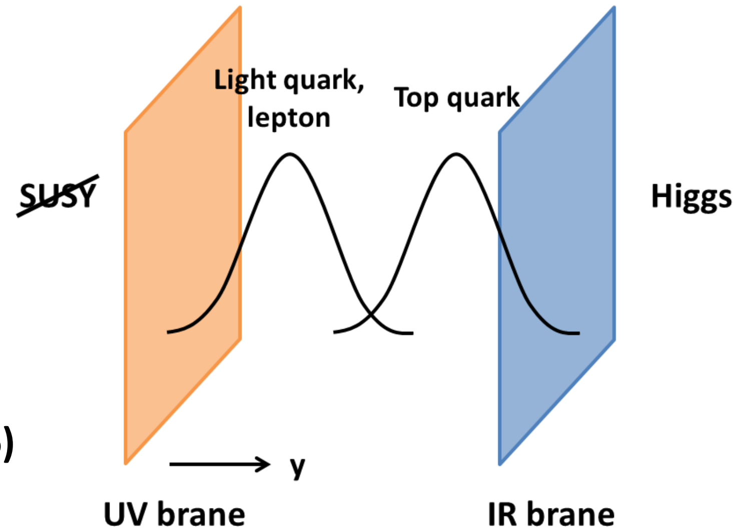
Warped Natural SUSY

Gherghetta, Pomarol (2003) , Sundrum (2009) , Larsen, Nomura, Roberts (2012) ...

EW breaking is driven by top/stop loop.

125 GeV Higgs is naturally obtained in λ SUSY.

Barbieri, Hall, Nomura, Rychkov (2006)



$$W = \lambda S H_u H_d \qquad \lambda > 0.7$$

(without encountering Landau pole)

Let's pursue a fully realistic model !

SUSY RS Model

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \quad (0 \le |y| \le \pi R)$$

Randall, Sundrum (1999)

$$S^1/Z_2$$

Extended SUSY in the bulk ➡ ***N = 1 SUSY on the branes***

Compactification scale : $k' \equiv ke^{-k\pi R} = \mathcal{O}(10)\ \mathrm{TeV} \qquad kR \sim 10$

SM gauge fields in the bulk

Wavefunction profile of the zero mode is flat.

$$A_\mu(x,y) \simeq \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)}(x)$$

SUSY RS Model

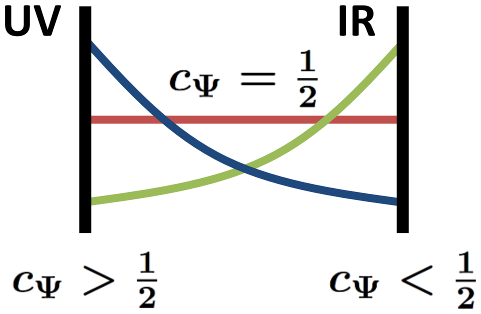
Matter (hyper-)multiplets in the bulk

$$S_\Psi = \int d^5x \Big\{ e^{-2k|y|} \int d^4\theta \left(\Psi^\dagger \Psi + \Psi^c \Psi^{c\dagger}\right) + e^{-3k|y|} \int d^2\theta\, \Psi^c \left[\partial_y - \left(\frac{3}{2} - c_\Psi\right) k\epsilon(y)\right] \Psi + \text{h.c.} \Big\}$$

Bulk mass parameter

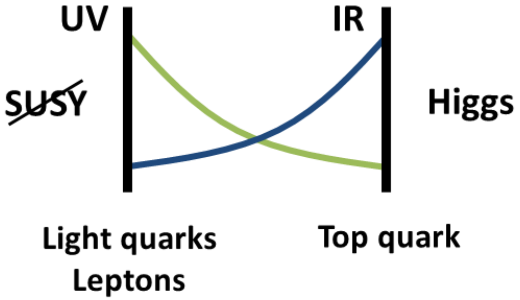
Wavefunction profile of the zero mode

$$\Psi(x,y) \simeq \frac{e^{-(c_\Psi - \frac{3}{2})k|y|}}{\sqrt{\frac{1}{(c_\Psi - \frac{1}{2})k}\left(1 - e^{-2\pi k R (c_\Psi - \frac{1}{2})}\right)}} \Psi^{(0)}(x)$$



Yukawa hierarchy

Yukawa coupling on IR brane



$$S_{\text{Yukawa}} = \int d^5x\, \delta(y-\pi R)\, e^{-3\pi kR} \Big\{ \int d^2\theta \left(\tilde{y}_u^{ij} H_u Q_i \bar{u}_j + \tilde{y}_d^{ij} H_d Q_i \bar{d}_j + \tilde{y}_\nu^{ij} H_u L_i \bar{\nu}_j + \tilde{y}_e^{ij} H_d L_i \bar{e}_j \right) + \text{h.c.} \Big\}$$

$$\Rightarrow \quad y_u^{ij} = \tilde{y}_u^{ij} k\, \zeta_{Q_i} \zeta_{\bar{u}_j}, \quad y_d^{ij} = \tilde{y}_d^{ij} k\, \zeta_{Q_i} \zeta_{\bar{d}_j}, \quad y_\nu^{ij} = \tilde{y}_\nu^{ij} k\, \zeta_{L_i} \zeta_{\bar{\nu}_j}, \quad y_e^{ij} = \tilde{y}_e^{ij} k\, \zeta_{L_i} \zeta_{\bar{e}_j}$$

$$\zeta_\Psi \simeq \begin{cases} \sqrt{c_\Psi - \frac{1}{2}}\; e^{-(c_\Psi - \frac{1}{2})\pi kR} & (c_\Psi \gg 1/2) \\ \frac{1}{\sqrt{2\pi kR}} & (c_\Psi \sim 1/2) \\ \sqrt{\frac{1}{2} - c_\Psi} & (c_\Psi \ll 1/2) \end{cases}$$

Light quarks , Leptons ($c_\Psi \gg 1/2$)

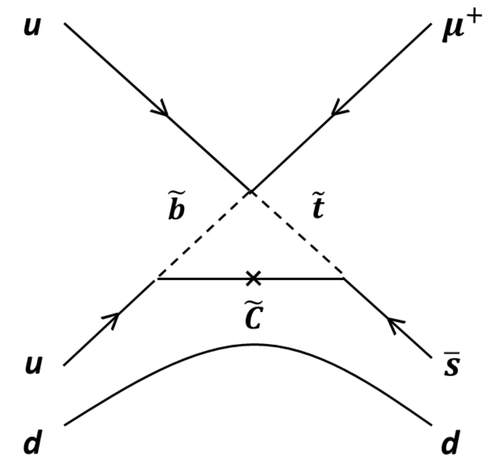
Top quark ($c_\Psi \ll 1/2$)

Proton Decay

Even if we impose R-parity as usual, …

$$W_{\rm IR} \sim \frac{1}{\Lambda_{\rm IR}} QQQL$$ × (wavefunction factors)

O(10) TeV !



Rapid proton decay …

RPV is natural in SUSY RS !

Z_3 lepton number symmetry

$$L \to e^{2\pi i/3}\, L, \qquad \bar{\nu} \to e^{-2\pi i/3}\, \bar{\nu}, \qquad \bar{e} \to e^{-2\pi i/3}\, \bar{e}$$

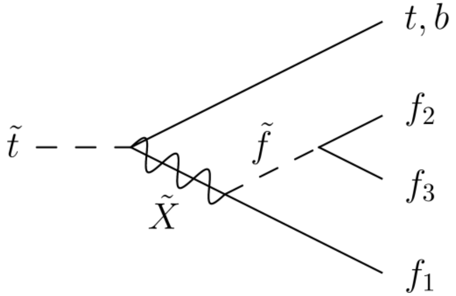
Anomaly free ➡ **Three generations !**

R-parity Violation

LSPs can decay promptly and evade searches based on missing transverse energy !

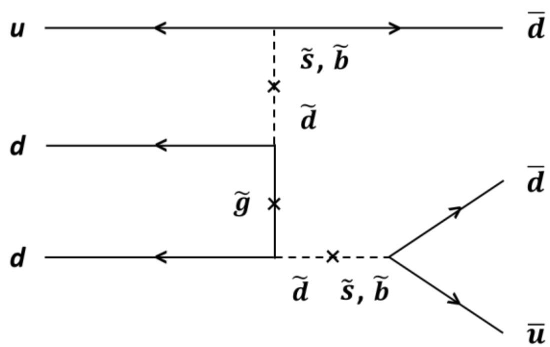
BNV couplings

$$W_{\rm BNV} = \frac{1}{2}\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k$$

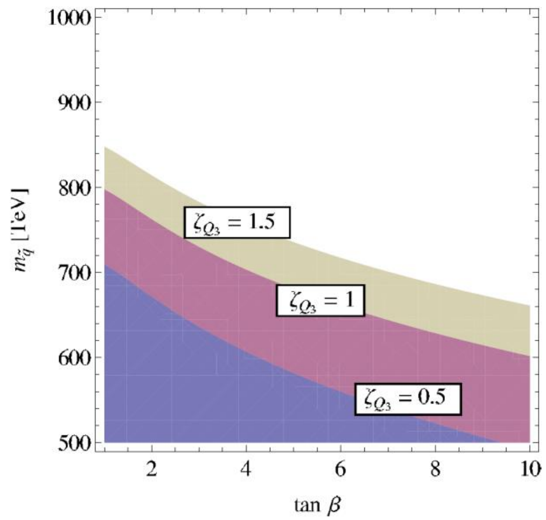


Stop and sbottom $m_{\tilde{b},\,\tilde{t}} \gtrsim 100\,\mathrm{GeV}$

Constraints from ΔB = 2 processes are satisfied !



n - $\bar{n}$ **oscillations**



U(1) D-term Problem

Strassler (2003)

Sundrum (2009)

Heavy scalars ➡ $\mathcal{L}_{\rm FI} \sim \int d^4\theta \, \frac{m^2_{\tilde{q},\tilde{l}}}{16\pi^2} \, g_Y V_Y$ **near UV brane**

➡ **Large Higgs soft masses !**

The gauge field couples marginally to the CFT current J^μ_Y

($\Delta = 3$)

➡ **Scaling dimension of** D_Y **:** $\Delta = 2$

➡ **SCFT admits a relevant deformation :** $\Delta\mathcal{L} = M_D^2 \, D_Y$

Conformal phase breaks down at M_D

TeV Unification

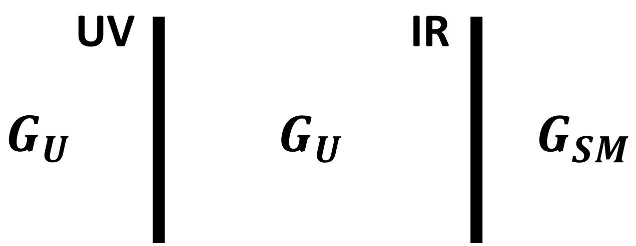
Extend the SM gauge group to forbid the relevant deformation

- **Semi-simple group (SU(5) , …)**
- **Left-right symmetry under which the U(1) D-term transforms nontrivially**

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times \boxed{U(1)_{B-L}}$$

The unified group is broken on IR brane by boundary conditions

cf. The Higgsless model of EWSB



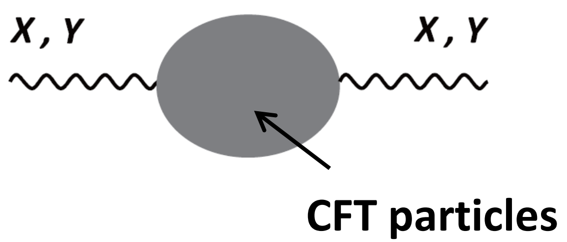
Extra gauge fields → Dirichlet boundary condition on IR brane

ND Gauge Field

Extra gauge fields with ND boundary conditions satisfy

$$\frac{J_0(m/k)}{Y_0(m/k)} = \frac{J_1(m/k')}{Y_1(m/k')} \quad \Rightarrow \quad \boxed{m_0 \simeq \sqrt{\frac{2}{\pi k R}}\, k'}$$

$$\mathcal{L}_{SU(5)} \sim -\frac{1}{4}\left\{\frac{1}{g_{\rm UV}^2} + \frac{N_{\rm CFT}}{16\pi^2}\log\left(\frac{M_{\rm pl}}{\Lambda_{\rm IR}}\right)\right\}\sum_{a\ \text{(All)}} (F_{\mu\nu}^a)^2 + \frac{1}{2}\frac{N_{\rm CFT}}{16\pi^2}\Lambda_{\rm IR}^2 \sum_{\alpha\ \text{(Broken)}} (A_\mu^\alpha)^2$$



$$\Rightarrow \quad m_0^2 \sim \frac{\Lambda_{\rm IR}^2}{\log\left(M_{\rm pl}/\Lambda_{\rm IR}\right)} \qquad \log\left(M_{\rm pl}/\Lambda_{\rm IR}\right) \simeq \pi k R$$

$k' = \mathcal{O}(10)\ \mathrm{TeV}$ ➡ **ND gauge fields may be discovered at LHC !**

The SU(5) Model

Coupling unification can be realized by IR brane-localized kinetic term

UV | IR

SU(5) | SU(5) | G_{SM}

$$S_{\rm IR} = \int d^5x\, \delta(y-\pi R) \left\{ \frac{1}{4\tilde{g}_a^2} \int d^2\theta\, {\rm Tr}\, W^{a\alpha} W^a{}_\alpha + {\rm h.c.} \right\} \quad a = 3, 2, 1$$

Split multiplets for quarks and leptons : $\mathbf{10_Q},\ \mathbf{10_{\bar{u},\bar{e}}}$ **&** $\bar{\mathbf{5}}_{\bar{\mathbf{d}}},\ \bar{\mathbf{5}}_{\mathbf{L}}$

Z_3 lepton number symmetry $\quad \omega_3 \equiv e^{2\pi i/3}$

$$\mathbf{10}_Q \to \omega_3 \mathbf{10}_Q, \quad \mathbf{10}_{\bar{u},\bar{e}} \to \omega_3^{-1} \mathbf{10}_{\bar{u},\bar{e}}, \quad \bar{\mathbf{5}}_L \to \omega_3 \bar{\mathbf{5}}_L, \quad \bar{\mathbf{5}}_{\bar{d}} \to \omega_3^{-1} \bar{\mathbf{5}}_{\bar{d}}$$

Extra fields in split multiplets : $Q', \bar{u}', \bar{d}', L', \bar{e}'$

Light Exotics

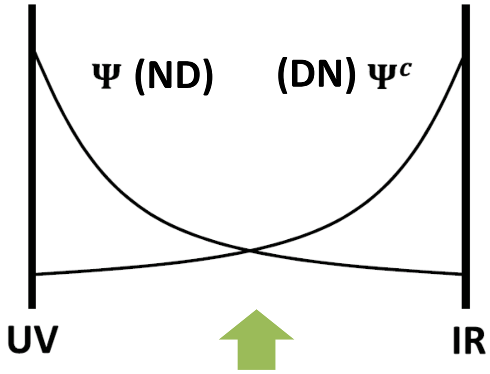
Obtain sizable masses of exotics by ND boundary conditions ?

→ No …

Exotics satisfy $$\frac{J_{c-1/2}(m/k)}{Y_{c-1/2}(m/k)} = \frac{J_{c+1/2}(m/k')}{Y_{c+1/2}(m/k')}$$

For $c \gg 1/2$

$$m \simeq 2\sqrt{c+\frac{1}{2}}\,\zeta\, k'$$



Exponentially small overlap !!

Light exotics always appear …

$$M_{Q'_1} \sim 2\zeta_{\bar{u}_1} k' \ll M_Z$$

The SU(5) model is excluded …

Split Couplings without Exotics

A way to avoid light exotics in split multiplets

$$\left[\begin{array}{l}\Psi_A = (A, B') \\ \Psi_B = (A', B)\end{array}\right.$$

Introduce a new multiplet on UV brane :

$$\bar{\Psi}_{\rm UV} = (\bar{A}_{\rm UV}, \bar{B}_{\rm UV})$$

A mass term : $M_{\rm UV}\bar{\Psi}_{\rm UV}(s_\theta \Psi_A - c_\theta \Psi_B)$

➡ **A light multiplet :** $\hat{\Psi} = (\hat{A}, \hat{B}) = c_\theta \Psi_A + s_\theta \Psi_B$

Yukawa couplings on IR brane : $\mathcal{L}_{\rm IR} = A\mathcal{O}_A + B\mathcal{O}_B + \ldots$

➡ $$\mathcal{L}_{\rm eff} = y_A \hat{A}\mathcal{O}_A + y_B \hat{B}\mathcal{O}_B + \ldots$$

$$\left[\begin{array}{l}y_A \sim c_\theta \zeta_A \\ y_B \sim s_\theta \zeta_B\end{array}\right.$$

The Left-Right Model

The symmetry is broken on IR brane : $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$

W', Z' may be discovered at LHC via $W' \to \ell\nu$ **,** $Z' \to \ell^+\ell^-$

Split multiplets :

$$\mathcal{Q}_Q = (Q, U'') \qquad \mathcal{Q}_{\bar{u}} = (Q', U_{\bar{u}}) \qquad \mathcal{Q}_{\bar{d}} = (Q'', U_{\bar{d}})$$

$U_{\bar{u}} \nearrow (\bar{u}, \bar{d}')$

Introduce new multiplets on UV brane to avoid light exotics : $\bar{\mathcal{Q}}_{1,2}$

Yukawa couplings on IR brane : $W_{\text{Yukawa}} = Q\bar{u}H_u + Q\bar{d}H_d$

Summary

To pursue a fully realistic SUSY RS model ...

Proton decay problem

Lepton number symmetry

RPV

Light stop

Viable pattern of RPV is naturally derived !

U(1) D-term problem

TeV unification !

ND gauge fields may be discovered at LHC !

Thank you.

Backup

Yukawa hierarchy

Wavefunction factors

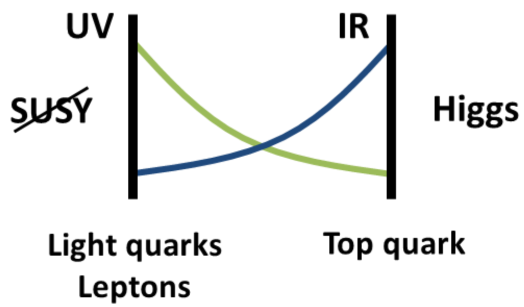
$$m_{u_i} \simeq \zeta_{Q_i}\zeta_{\bar{u}_i} v \sin\beta, \qquad m_{d_i} \simeq \zeta_{Q_i}\zeta_{\bar{d}_i} v \cos\beta$$

$$|\,(V_{\rm CKM})_{ij}\,| \simeq \frac{\zeta_{Q_j}}{\zeta_{Q_i}} \qquad \text{for} \quad j \leq i$$

$$|\,(V_{\rm CKM})_{21}\,| \simeq \lambda, \qquad |\,(V_{\rm CKM})_{32}\,| \simeq \lambda^2, \qquad |\,(V_{\rm CKM})_{31}\,| \simeq \lambda^3 \qquad \lambda \sim 0.2$$

$$\left[\begin{array}{lll}
\zeta_{Q_1} \simeq \lambda^3\zeta_{Q_3}, & \zeta_{Q_2} \simeq \lambda^2\zeta_{Q_3}, & \\
\zeta_{\bar{u}_1} \simeq \dfrac{m_u}{\lambda^3\zeta_{Q_3} v \sin\beta}, & \zeta_{\bar{u}_2} \simeq \dfrac{m_c}{\lambda^2\zeta_{Q_3} v \sin\beta}, & \zeta_{\bar{u}_3} \simeq \dfrac{m_t}{\zeta_{Q_3} v \sin\beta}, \\
\zeta_{\bar{d}_1} \simeq \dfrac{m_d}{\lambda^3\zeta_{Q_3} v \cos\beta}, & \zeta_{\bar{d}_2} \simeq \dfrac{m_s}{\lambda^2\zeta_{Q_3} v \cos\beta}, & \zeta_{\bar{d}_3} \simeq \dfrac{m_b}{\zeta_{Q_3} v \cos\beta}
\end{array}\right.$$

R-parity violation



<u>RPV coupling on IR brane</u>

$$S_{\mathrm{RPV,\,IR}} = \int d^5x\, \delta(y-\pi R)\, e^{-3k\pi R} \left(\int d^2\theta\, \frac{1}{2} \tilde{\lambda}^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k + \mathrm{h.c.} \right)$$

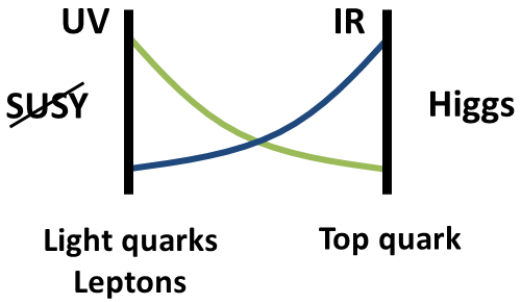
$$\Rightarrow \quad W^{4D}_{\mathrm{RPV,\,IR}} = \frac{1}{2} \lambda^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \qquad \lambda^{ijk} = \tilde{\lambda}^{ijk} k^{3/2} \zeta_{\bar{u}_i} \zeta_{\bar{d}_j} \zeta_{\bar{d}_k}$$

Coupling size is proportional to wavefunction factors !

	sb	bd	ds
u	8×10^{-6}	2×10^{-6}	1×10^{-6}
c	7×10^{-4}	2×10^{-4}	1×10^{-4}
t	3×10^{-3}	1×10^{-3}	6×10^{-4}

$\tan\beta = 3$ and $\zeta_{Q_3} = 1$

R-parity violation



RPV coupling on UV brane

$$S_{\text{RPV, UV}} = \int d^5x\, \delta(y) \left(\int d^2\theta\, \frac{1}{2} \tilde{\lambda}'^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k + \text{h.c.} \right)$$

$$\Rightarrow \quad W^{4D}_{\text{RPV, UV}} = \frac{1}{2} \lambda'^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \qquad \lambda'^{ijk} = \tilde{\lambda}'^{ijk} k^{3/2} \eta_{\bar{u}_i} \eta_{\bar{d}_j} \eta_{\bar{d}_k}$$

$$\eta_\Psi \simeq \begin{cases} \sqrt{c_\Psi - \frac{1}{2}} \simeq \sqrt{\frac{1}{\pi k R} \log \zeta_\Psi^{-1}} & (c_\Psi \gg 1/2) \\ \frac{1}{\sqrt{2\pi k R}} \simeq \zeta_\Psi & (c_\Psi \sim 1/2) \\ \sqrt{\frac{1}{2} - c_\Psi}\; e^{-(\frac{1}{2} - c_\Psi)\pi k R} \simeq \zeta_\Psi e^{-\pi k R \zeta_\Psi^2} & (c_\Psi \ll 1/2) \end{cases}$$

	sb	bd	ds
u	0.04	0.05	0.05
c	0.02	0.03	0.03
t	5×10^{-16}	6×10^{-16}	6×10^{-16}

$$\tan\beta = 3 \text{ and } \zeta_{Q_3} = 1$$

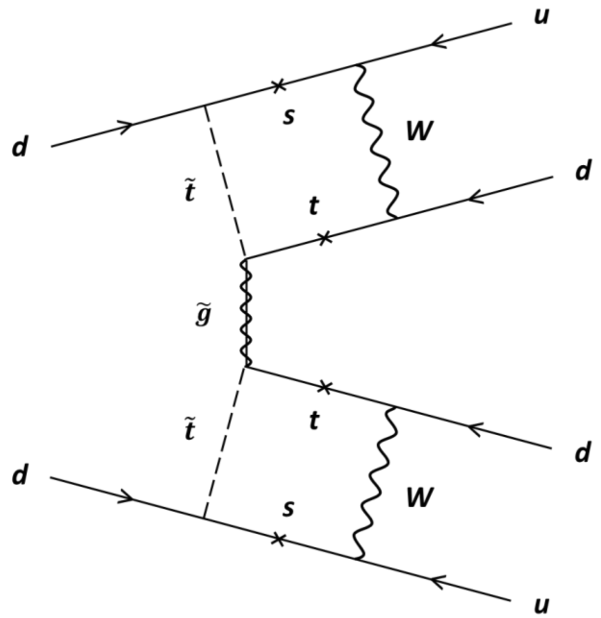
R-parity violation

n - $\bar{n}$ oscillations (RPV on IR brane)

Constraint : $\tau_{n-\bar{n}} \geq 2.44 \times 10^{8}\,\mathrm{s}$

If the scalars of light quarks are very heavy , …

The leading diagram must involve only light superpartners.



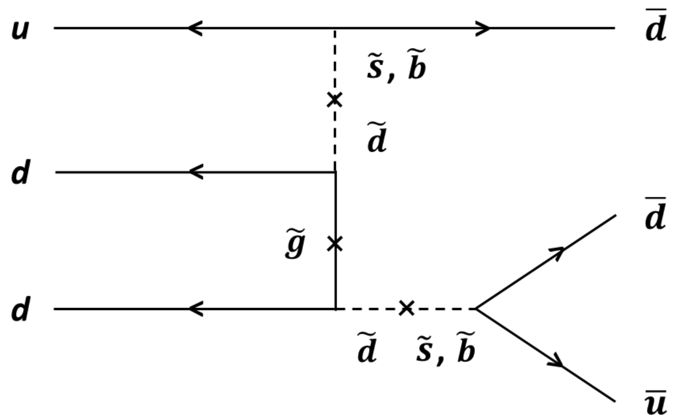
$$\tau_{n-\bar{n}} \sim (3 \times 10^{10}\,\mathrm{s}) \left(\frac{\lambda_{tds}}{6 \times 10^{-4}}\right)^{-2} \left(\frac{m_{\tilde{g}}}{1.2\,\mathrm{TeV}}\right) \left(\frac{m_{\tilde{t}}}{300\,\mathrm{GeV}}\right)^{4}$$

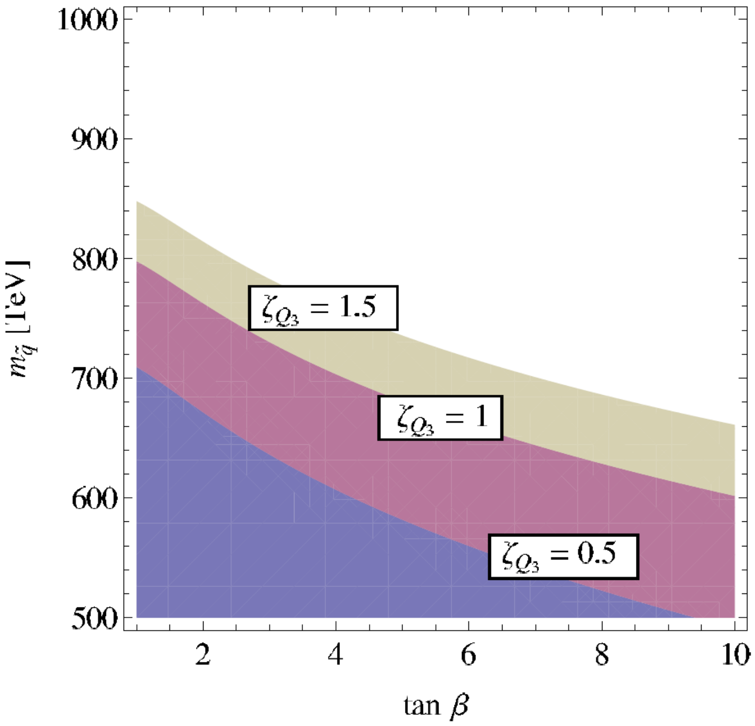
The bound is easily satisfied !

R-parity violation

n - $\bar{n}$ oscillations (RPV on UV brane)

Sizable coupling for light quarks.





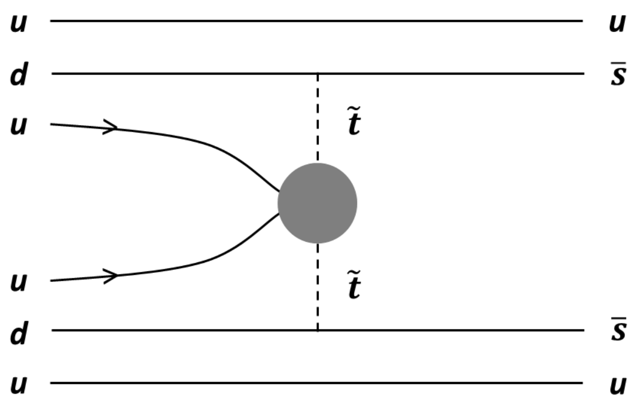
$$\mathcal{M}_{n-\bar{n}} \sim 4\pi\alpha_3 \, \lambda'^{\,2}_{uds} \, \tilde{\Lambda} \left(\frac{\tilde{\Lambda}}{m_{\tilde{g}}}\right) \left(\frac{\tilde{\Lambda}^2}{m_{\tilde{q}}^2}\right)^2$$

$$\tilde{\Lambda} \sim \Lambda_{\rm QCD} \sim 250\,{\rm MeV}$$

The bound is weaker than the FCNC bound.

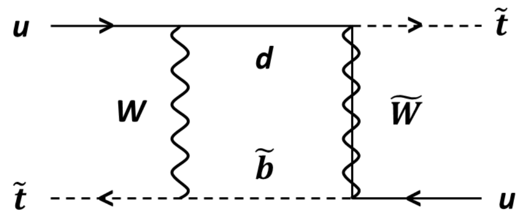
R-parity violation

<u>**Dinucleon decay (RPV on IR brane)**</u>



Constraint : $\tau_{pp\to K^+K^+} \geq 1.7\times 10^{32}\,\text{yrs}$

If the scalars of light quarks
are very heavy , …

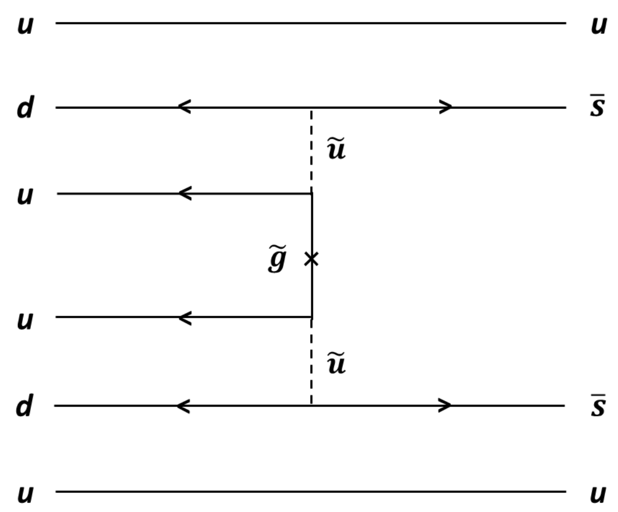


The leading diagram must involve only light superpartners.

$$\tau_{pp\to K^+K^+} \sim (4\times 10^{39}\,\text{yrs})\left(\frac{\lambda_{tds}}{6\times 10^{-4}}\right)^{-4}\left(\frac{m_{\tilde{W}}}{600\,\text{GeV}}\right)^{2}\left(\frac{m_{\tilde{t},\tilde{b}}}{300\,\text{GeV}}\right)^{12}$$

R-parity violation

Dinucleon decay (RPV on UV brane)



Scalars of light quarks are very heavy,

but sizable coupling for light quarks.

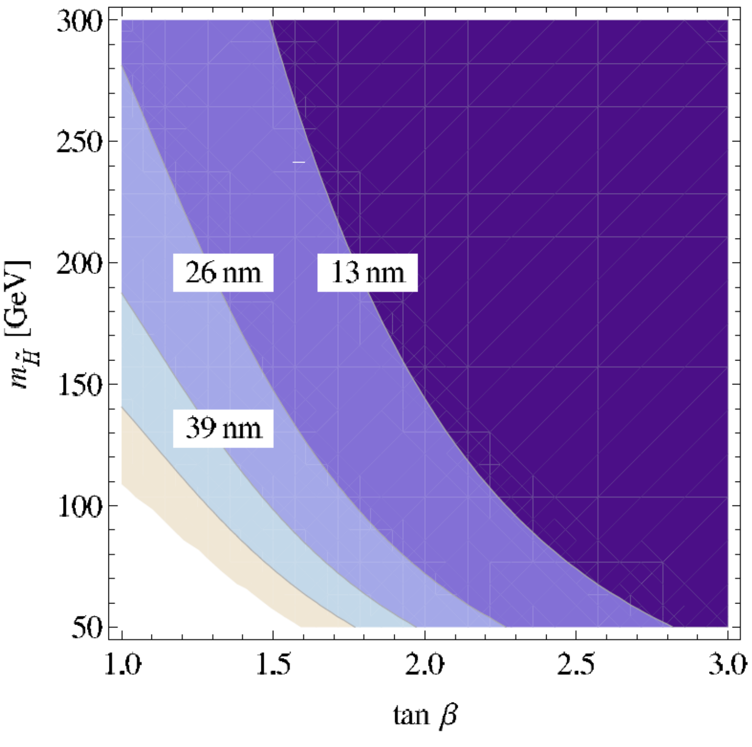
For $\tan\beta = 3$ and $\zeta_{Q_3} = 1$

$$\tau_{pp\to K^+K^+} \sim (5\times 10^{35}\,\text{yrs})\left(\frac{\lambda'_{uds}}{0.05}\right)^{-4}\left(\frac{m_{\tilde{g}}}{1.2\,\text{TeV}}\right)^{2}\left(\frac{m_{\tilde{q}}}{1000\,\text{TeV}}\right)^{8}$$

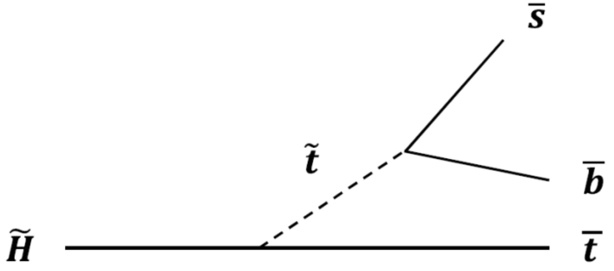
Constraints from ΔB = 2 processes are satisfied !

R-parity violation

LSP decay (Constraint from displaced vertex)



Higgsino LSP



$$\Gamma_{\tilde{H}} \sim \frac{m_{\tilde{H}}}{128\pi^3} |\lambda_{tsb}|^2$$

If LSP is lighter than top quark, decay length is still short.

U(1) D-term problem

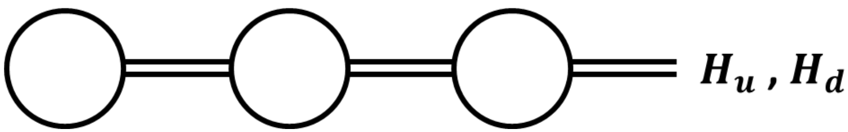
Strassler (2003)

Sundrum (2009)

Heavy scalars ➡ $$\mathcal{L}_{\mathrm{FI}} \sim \int d^4\theta \, \xi \, V_1$$

➡ **Large Higgs soft masses !**

Three-site model



	$U(1)_1$	$U(1)_2$	$U(1)_3$
Σ_1	1	-1	0
$\bar{\Sigma}_1$	-1	1	0
Σ_2	0	1	-1
$\bar{\Sigma}_2$	0	-1	1
H_u	0	0	$1/2$
H_d	0	0	$-1/2$

$$W \sim X_1 \left(\Sigma_1 \bar{\Sigma}_1 - v_1^2\right) + X_2 \left(\Sigma_2 \bar{\Sigma}_2 - v_2^2\right)$$

➡ $$|\Sigma_1|^2 - |\bar{\Sigma}_1|^2 \sim |\Sigma_2|^2 - |\bar{\Sigma}_2|^2 \sim \frac{\xi}{g_Y}$$