Open Problems in High Precision Event Generators

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State of the art

- Merged tree level matrix elements interfaced to Parton Shower generators (ME-PS) available with a high degree of automation. Widely used by the experimental collaboration
- NLO calculation interfaced to shower generators (NLO+PS) available with a high degree of automation also for complex processes ($2 \to 3$, $2 \to 4$). Widely used by experimental collaborations
- Merged NLO+PS generators available
- First NNLO+PS generators appearing

"Merging" refers to generators for processes with an increasing number of associated jet.

	LO+PS H	LO+PS HJ	LO+PS HJJ	Merged
Inclusive	$\mathcal{O}(\alpha_s^2)$ (LO)	NO	NO	$\mathcal{O}(\alpha_s^2)$
1 jet	$\mathcal{O}(\alpha_s^3)$ approximate	$\mathcal{O}(\alpha_s^3)$ (LO)	NO	$\mathcal{O}(\alpha_s^3)$
2 jet	...	$\mathcal{O}(\alpha_s^4)$ approximate	$\mathcal{O}(\alpha_s^4)$ (LO)	$\mathcal{O}(\alpha_s^4)$
3 jet	...	...	$\mathcal{O}(\alpha_s^5)$ approximate	appr.
4 jet	...	...	...	

	NLO-PS H	NLO-PS HJ	NLO-PS HJJ
Inclusive	$\mathcal{O}(\alpha_s^2+\alpha_s^3)$ (NLO)	NO	NO
1 jet	$\mathcal{O}(\alpha_s^3)$ (LO)	$\mathcal{O}(\alpha_s^3+\alpha_s^4)$ (NLO)	NO
2 jet	approximate $\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^4)$ (i.e. LO)	$\mathcal{O}(\alpha_s^4+\alpha_s^5)$ (NLO)
3 jet		approximate $\mathcal{O}(\alpha_s^5)$	$\mathcal{O}(\alpha_s^5)$ (i.e. LO)
4 jet			approximate $\mathcal{O}(\alpha_s^6)$

	H	H+HJ merging (1)	H+HJ+HJJ merging (2)	NNLO-PS H
Incl.	$\mathcal{O}(\alpha_s^2+\alpha_s^3)$ (NLO)	$\mathcal{O}(\alpha_s^2+\alpha_s^3)$ (NLO)	$\mathcal{O}(\alpha_s^2+\alpha_s^3)$ (NLO)	$\mathcal{O}(\alpha_s^2+\alpha_s^3+\alpha_s^4)$ (NNLO)
1 jet	$\mathcal{O}(\alpha_s^3)$ (LO)	$\mathcal{O}(\alpha_s^3+\alpha_s^4)$ (NLO)	$\mathcal{O}(\alpha_s^3+\alpha_s^4)$ (NLO)	$\mathcal{O}(\alpha_s^3+\alpha_s^4)$ (NLO)
2 jet	approximate $\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^4)$ (LO)	$\mathcal{O}(\alpha_s^4+\alpha_s^5)$ (NLO)	$\mathcal{O}(\alpha_s^4)$ (LO)
3 jet		approximate $\mathcal{O}(\alpha_s^5)$	$\mathcal{O}(\alpha_s^5)$ (LO)	approximate $\mathcal{O}(\alpha_s^5)$
4 jet			approximate $\mathcal{O}(\alpha_s^5)$	

ME-PS:
AlpGen (Mangano,Moretti,Piccinini,Pittau,Polosa)
Sherpa (Gleisberg, Höche,Krauss,Schonherr,Schumann,Siegert,Winter)
MadGraph (Alwall,Herquet,Maltoni,Mattelaer,Stelzer)

`NLO-PS`:
`(a)MC@NLO` (Frixione,Webber,Frederix,Hirschi,Maltoni,Pittau,Torielli ...)
`POWHEG-BOX` (Alioli,Oleari,Re,Hamilton,Zanderighi,P.N. + ...)
`POWHEL` (Garzelli, Kardos, Papadopoulos, Trocsanyi + ...)
`Sherpa` (`POWHEG` and `MC@NLO` variants, Höche,Krauss,Schonherr,Siegert)
Herwig++ (`POWHEG` and `MC@NLO` variants, Plätzer, Gieseke ...)
New proposal: `VINCIA` (Giele et al, 2013), `GENEVA` (Alioli et al,),
CKKW-L extensions (Lönnblad, Prestel, 2013)

All generators are automated to a certain extent; noticeably:
`aMC@NLO` FULLY automated: "`MG5_aMC> generate p p > e+ ve [QCD]`"
generates NLO+PS W^+ events in hadronic collisions ...

Open Problems

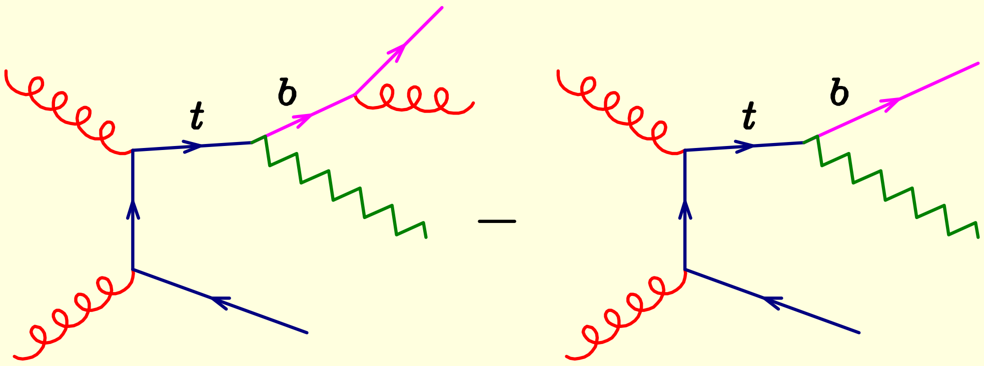
Not all is done. Most important open problems (in my opinion ...):

- Processes with intermediate resonances
  Problems arise in case of coloured-resonance mediated processes, also in the narrow width approximation. More problems arise if we wish to include non-resonant effects.

- NLO+PS merging (and NNLOPS).
  Although general procedures are advocated for NLO+PS merging that may work in practice, whether or not they achieve the desired accuracy is at times a matter of definitions.

Problems with resonances: NLO calculations

Standard schemes for NLO calculation fail in the narrow resonance limit.
Example: FKS in $t\bar{t}$ production



FKS subtraction term kinematics does not preserve the bgW mass.
(b direction preserved; $W\bar{b}$ recoiling system boosted along b direction and b momentum set to conserve 4-momentum)
Thus: when bgW is on shell, the counterterm is off-shell, spoiling IR cancellation in the narrow width approximation. The same happens with CS dipoles (W four momentum preserved.)

Message #1:
Current NLO subtraction schemes fail in the narrow width limit

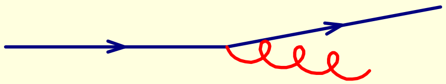
As long as we have finite width, current schemes converge given an unlimited amount of CPU time, i.e. brute force solutions are sometimes possible.
NLO calculations of $W^+W^-b\bar{b}$ have been performed by Bevilacqua etal, 2011, and Denner etal, 2012, in the 5-flavour scheme, and Frederix, 2014, massive b.

If only resonant graphs are included: radiative corrections for production and decays are distinct, and can be separated.
One can use the different subtraction schemes in production and decays;

If non resonant contributions, and interference between non-resonant and resonant graphs are included, more work is needed.

Problems with resonances: PS

Key problem: momentum reshuffling.



Collinear splitting conservers momentum only in the strict collinear limit. Shower Monte Carlo enforce exact momentum conservation by "Momentum reshuffling" (i.e. adjust the momenta by subleading corrections to enforce momentum conservation).

For example (Herwig): If a Final State particle undergoes splitting, and its 3-momentum is kept fixed to balance the 3-momenta of all other FS particles, its energy becomes larger. In order to restore energy conservation, all 3-momenta are rescaled down by a common factor.

If we have a radiating resonance decay, this procedure does not conserve the resonance mass. Hence: in this case, Herwig does momentum reshuffling maintaining the resonance 4-momentum fixed, by rescaling the momenta of the resonance decay products in the resonance rest frame.

Problems with resonances: NLO+PS

POWHEG example:

$$\bar{B}\exp\left[-\int\frac{R}{B}\,\mathrm{d}\Phi_{\mathrm{rad}}\right]\frac{R}{B}\,\mathrm{d}\Phi_{\mathrm{rad}}$$

Here R contains the radiation, and B is the underlying Born kinematics.
The standard POWHEG underlying Born mapping does not preserve resonance virtuality: if R is on shell, B is off shell, R/B LARGE!

More quantitatively: consider for example $t \to bW$; b splits into a bg with mass m^2. The bW mass in the counterterm differs from the original top virtuality by an amount m^2/E_b. So, we expect that

The b jet mass profile is distorted when $m^2_{\mathrm{jet}}/E_b \approx \Gamma_{\mathrm{top}}$.

Message #2:
Current NLO+PS schemes fail in the narrow width limit

Brute force solution: Full $W^+W^-b\bar{b}$ production in `POWHEL` has been implemented (Kardos,Garzelli,Trocsanyi 2014), using the standard `POWHEG BOX` mapping. The euristic argument given above would imply unphysical features of jet structure when $m_{\rm jet} \approx \sqrt{\Gamma E} \approx 8\,{\rm GeV}$. Studies in this direction are being pursued.

NLO+PS with radiating resonances in `narrow width limit`

`POWHEG-BOX-V2` can deal with radiation in resonance decays in the zero-width limit in a fully general way. In order to implement a process one must:

- Specify the resonance and its decay products in the user provided subprocess list. For example:
  ```
  realfl:[ 0, 0,  6, -6, 24,-24,-11, 12, 13,-14,  5, -5, 0]
  realrs:[ 0, 0,  0,  0,  3,  4,  5,  5,  6,  6,  3,  4, 3]
  ```
  represents a real graph for $gg \to (t \to (W \to \bar{e}\nu)\, b\, g)(\bar{t} \to (W^- \to \mu\bar{\nu}))$.
- Virtual corrections should include virtual corrections to resonance decays.
- Real correction should yield separately the radiation from the hard interaction (if the radiated parton does not belong to a resonance), and the radiation from each decaying resonance, depending upon `realrs[n]`

Implementation in $t\bar{t}$ production

(Campbell,Ellis,Re,P.N.)

Matrix elements from Campbell,Ellis,2012.

Narrow resonance decay machinery from `POWHEG BOX V2`.

Further problems:

- Finite width effects
- Multiplicative vs. additive corrections to resonance decays.

Finite width effects

In certain applications (for example, t mass measurement from end-points) the finite width of the top may have an effect. We implement it in the following way:

- We generate the Born phase space with finite width for the t and $\bar{t}$
- We project the Born finite width phase space onto a zero width phase space with a top mass equal to the average top virtuality. The projection conserves the total CM partonic momentum.
- Matrix elements are computed with the projected phase space.
- Real emission events are projected backward into real emission events initiated by the original (off-shell top) kinematics.
- The final cross section is reweighted either with
  - $\rightarrow$ The exact Born matrix elements for the production of the given final state after decay (from MadGraph), (including interference effects) divided by the projected on-shell matrix element
  - $\rightarrow$ The Breit Wigner shapes of the t and $\bar{t}$

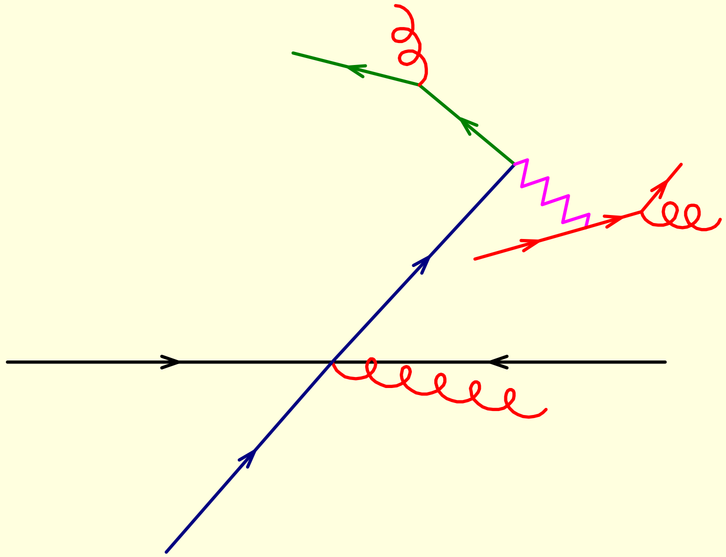
Multiplicative vs. additive corrections

In `POWHEG` only the hardest emission is generated. All remaining emissions are committed to the Shower generator.

In $t\bar{t}$ production we can have: a production emission, an emission from either t or $\bar{t}$, and an emission for each W decay, if applicable, for a total of 5 emissions. In `POWHEG` each emission is tried, and only the hardest one is kept. All the remaining one have to be generated by the shower.

Undesirable feature: the emission in hard production is more likely to have large transverse momentum. Emissions from decay become thus rare, and most of the time they will be handled by the shower.
It would of course be more desirable to have an emission from the hard production, plus one for each decaying resonance.

Present solution: keep hard radiation and the emissions from all decaying resonances, and merged them into a single radiation phase space with several radiated partons, up to one for each resonance.



From a given underlying Born configuration (take away the radiated gluons), in the longitudinal rest frame of the $t\bar{t}$ system:
ISR: transverse boost of the whole $t\bar{t}$ system
t radiation in decay: W boosted along its momentum in t rest frame
W radiation in decay: either q or $\bar{q}$ direction preserved. In order to combine all of them:

- Start from Born phase space, including W radiation in decay if present.
- If t radiation in decay is present, add the corresponding gluon, and replace the b and W momentum (boosting the W system along its momentum in the t rest frame)
- If ISR is present, perform transverse boost of $t\bar{t}$ system and add the gluon

This prescription guarantees that

- The hardest radiation is generated with full NLO accuracy, and the subsequent ones are at least accurate in the collinear limit
- Rotationally invariant shape observables for the resonance decays are all NLO accurate.

However: further radiation from the shower must be vetoed at different scales for radiation in production (scalup) and for each radiating resonance.

Standard LHIUP (Les Houches Interface for User Processes) allows only for a single scale for vetoing radiation ... Extensions are needed!

Some plots

We have generated LH events for leptonic top decays (e^+, μ^-), with radiation in decays not included (LO Dec) and included (NLO Dec).

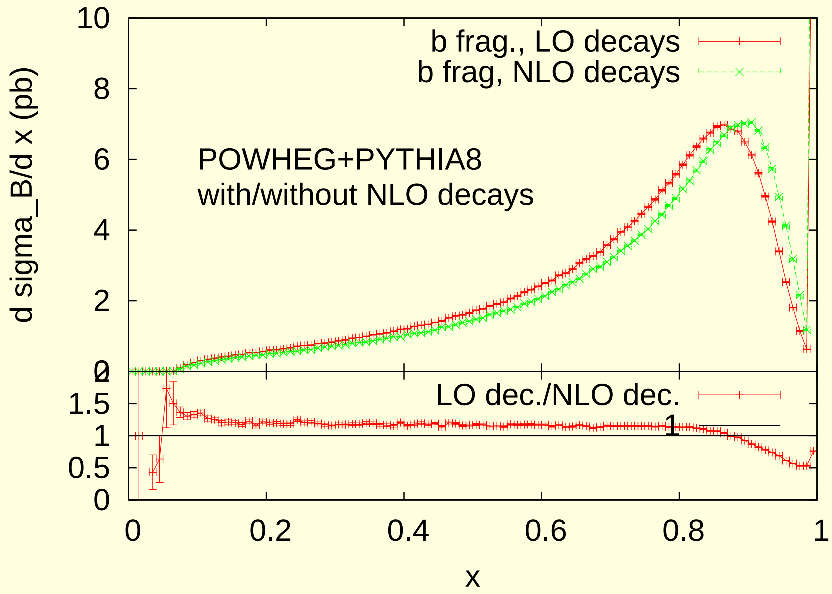
The events of the LO Dec sample were fed to `Pythia8`, with no further action. Pythia8 takes care of adding radiation in top decays.
The events of the NLO Dec sample were fed to `Pythia8`. Care was taken to compute the transverse momentum of radiation in top decays (in the top rest frame) and instruct `Pythia8` to veto radiation in resonance decays (using `canSetResonanceScale` and `scaleResonance` in `UserHooks` class)

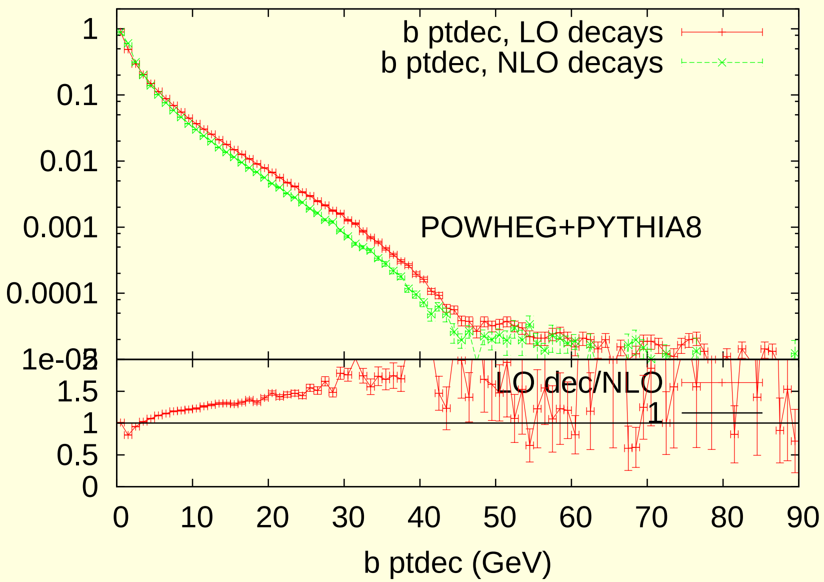
In the following plots:

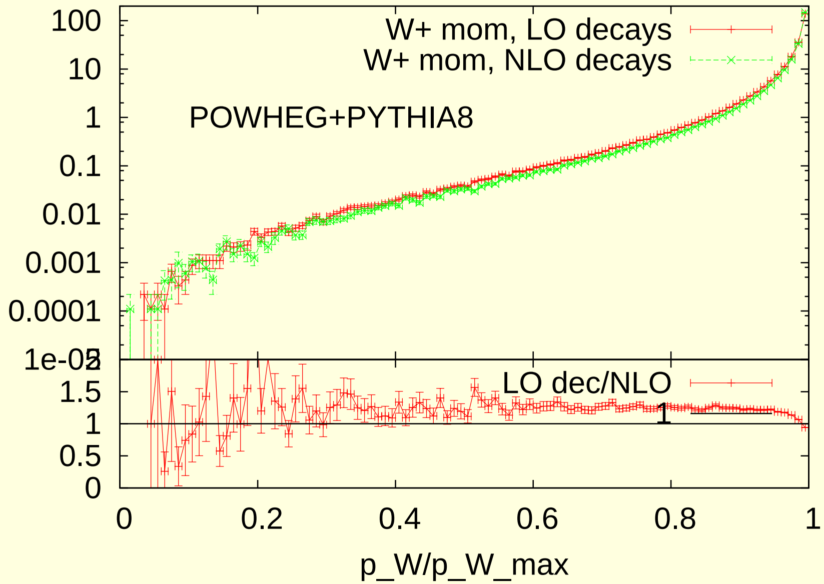
- b stands for the MC truth b from t decay
- W^+ stands for the MC truth W^+
- l^+ stands for fermion coming from $t \to W^+ \to l^+$ (MC truth)
- $b\,\mathrm{jet}$ stands for anti-kt jet ($R = 0.5$) containing b

b fragmentation properties in t decays

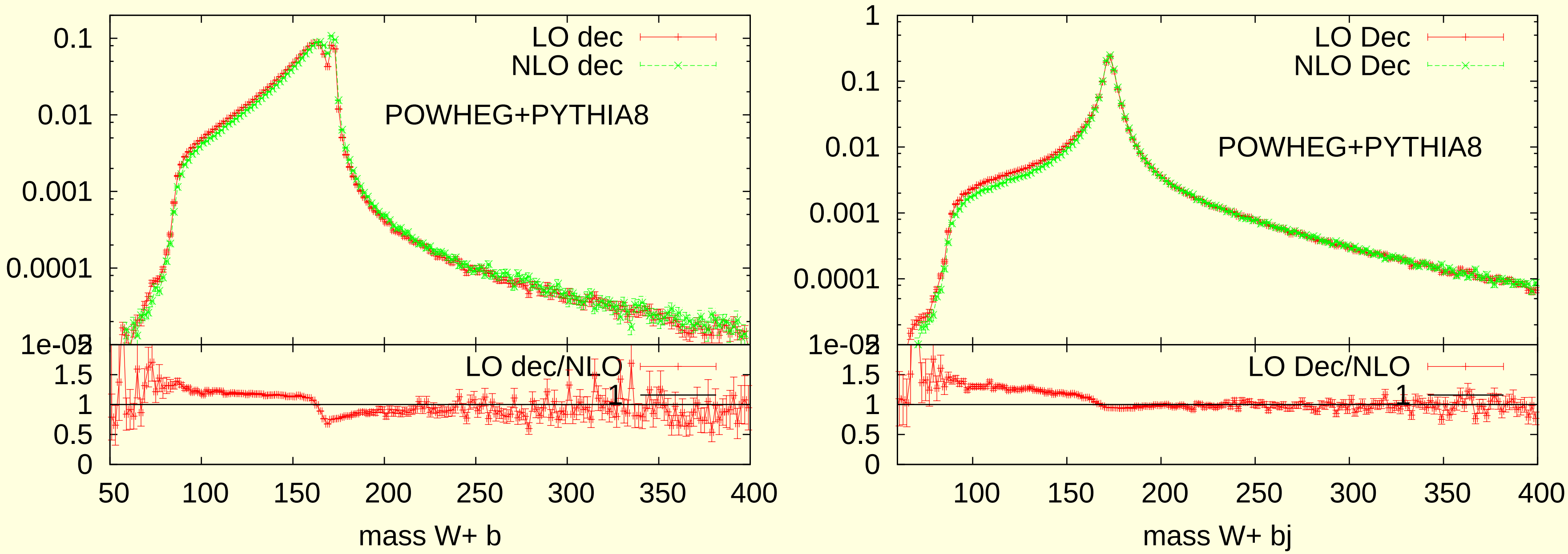
Observables computed in t rest frame.
b stands for hardest b flavoured hadron



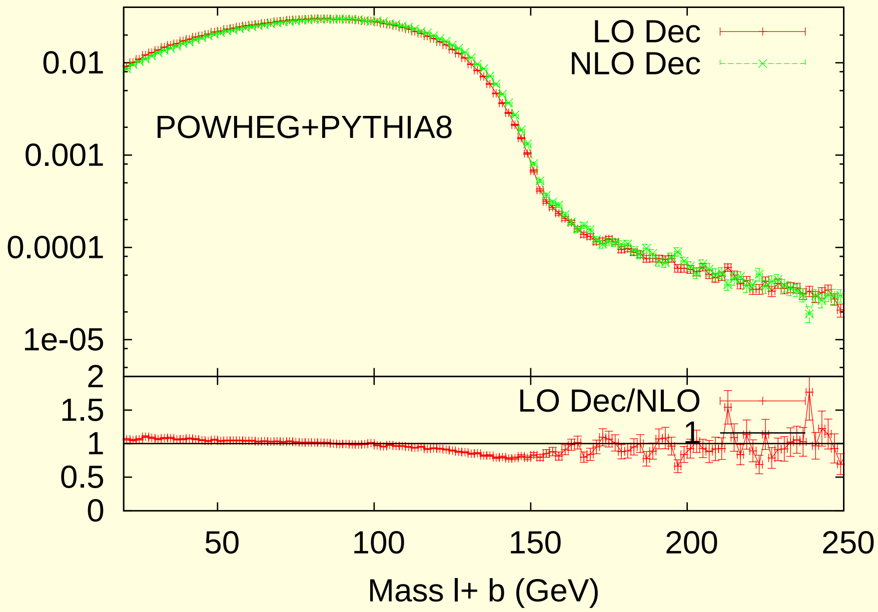


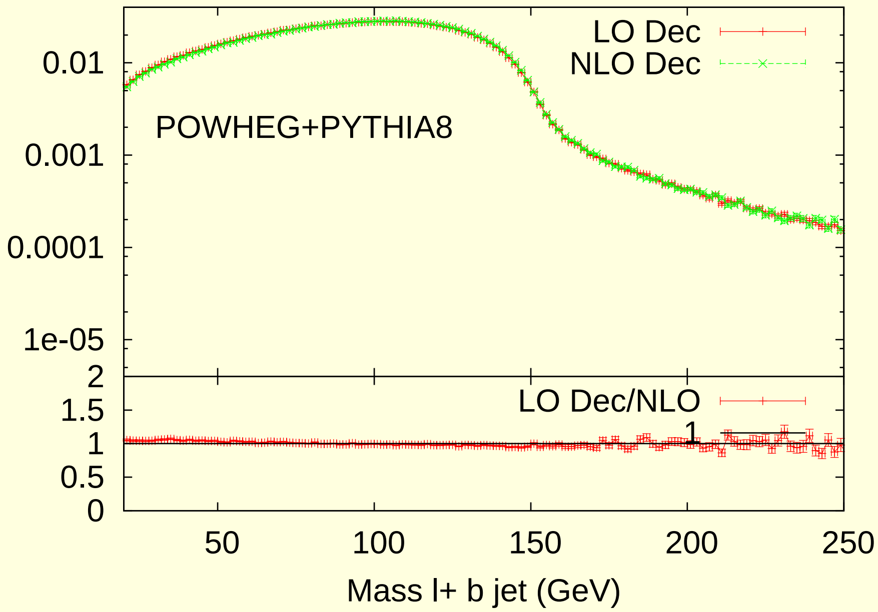


t mass (pseudo) observables



Notice small peak in $W^{+}b$ plot, due to $x=1$ peak in b fragmentation function. Sensibly different shapes around the top peak.





Effect of different fragmentation behaviour shows up in M_{l+b}, but not in $M_{l+b\,\mathrm{jet}}$.

Radiating resonances: conclusions

- NLO accurate simulation of processes with radiating resonances is an open problem in several respects
- A reliable method for NLO subtractions when resonances are present is needed
- No problems at NLO in the narrow width limit
- Several (solvable?) problems with NLO+PS in the narrow width limit:
  - Inclusion of finite width effects
  - Multiplicative versus Additive NLO corrections to decays
  - Insufficient LHIUP: time to review the standard ...
- For NLO+PS including interference in decays, further studies are needed ...

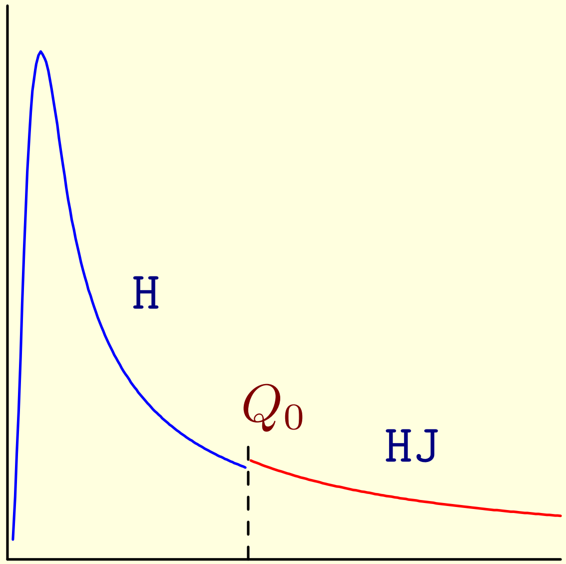
NLO+PS merging: the problem

Focus upon Higgs production; call H and HJ the NLO+PS generators for Higgs and Higgs + 1 jet production. Accuracy:

	NLO-PS `H`	NLO-PS `HJ`	Matched
Inclusive	$\mathcal{O}(\alpha_s^2+\alpha_s^3)$ (NLO)	NO	$\mathcal{O}(\alpha_s^2+\alpha_s^3)$ (NLO)
1 jet	$\mathcal{O}(\alpha_s^3)$ (LO)	$\mathcal{O}(\alpha_s^3+\alpha_s^4)$ (NLO)	$\mathcal{O}(\alpha_s^3+\alpha_s^4)$ (NLO)
2 jet	approximate $\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^4)$ (i.e. LO)	$\mathcal{O}(\alpha_s^4)$ (i.e. LO)
3 jet	...	approximate $\mathcal{O}(\alpha_s^5)$	approximate $\mathcal{O}(\alpha_s^5)$
4 jet	...	...	...

Naive approach:

- Start with the H and HJ generator
- Introduce a separation scale: $\Lambda_{\rm QCD} \ll Q_0 \ll M_H$
- Use H for $p_T^H < Q_0$
- Use HJ for $p_T^H > Q_0$



NLO accuracy requires that the sum of the areas below the blue and red lines yield an $\mathcal{O}(\alpha_s^3)$ accurate result. Is it so? We would like Q_0 as small as possible ...

The "blue" integral has roughly the structure

$$\sigma_0(1+C\alpha_s)\exp\underbrace{[L(\alpha_s L)^n, (\alpha_s L)^n, \alpha_s(\alpha_s L)^n, \ldots]}_{LL \qquad NLL \qquad NNLL}$$

where $\alpha_s = \alpha_s(M_{\rm H})$, $L = \log M_{\rm H}^2/Q_0^2$. In NLO+PS generators, it is guaranteed that for $Q_0 \approx M_H$ the formula is α_s^3 accurate, up to corrections of order α_s^4.
If Q_0 is near the Sudakov peak (i.e. $\alpha_s L^2 \approx 1$), the first NNLL term yields a correction factor: $1+\alpha_s^2 L \approx 1+\alpha_s^{1.5}$.

So: unless the NLO+PS Sudakov form factor is accurate at NNLL, the "blue" contribution misses unknown terms of relative order $\alpha_s^{1.5}$.
(NLO accuracy requires unknown terms of relative order α_s^2).

So: we can't take Q_0 at the Sudakov peak. What is the value of Q_0 such that full NLO accuracy is achieved?

We must have $\alpha_s^2 L \approx \alpha_s^2$, i.e. $Q_0 \approx M_H$!!

The "green" integral has the structure

$$\alpha_s^2(\alpha_s L^2, \alpha_s L, \alpha_s^2 L^4, \alpha_s^2 L^3, \alpha_s^2 L^2, \alpha_s^2 L) + Q_0 \, \text{suppressed terms}$$

We must collect all terms with any power of L into a Sudakov form factor, if we want this to hold up to terms of relative order α_s^2 near the Sudakov region.

Can we live with less precision? can we tolerate $\alpha_s^{1.5}$ instead of α_s^2?

Well: keep in mind that it is $\alpha_s^{1.5}$ at best (i.e. only if Sudakov is NLL accurate).

For the purpose of this talk, I define "strict" NLO accuracy to require α_s^2 leftovers.

How to deal with the problem

- Practical approach: close your eyes and do it! Check that Q^0 dependency is small.
- Correct normalization of region below Q^0 so that NLO accuracy for the total inclusive cross section is enforced (UNLOPS method)
- Use highly accurate Sudakov form factors (Geneva)
- `MiNLO` method

Current merging approaches:

- SHERPA, [Hoeche, Krauss, Schonherr, Siegert, arXiv:1207.5030], traditional merging with matching scales.

- FxFx, [Frederix, Frixione, arXiv:1209.6215], traditional merging with matching scales; check matching scale dependence a posteriori.

- UNLOPS, [Platzer, arXiv:1211.5467], [Lönnblad, Prestel, arXiv:1211.7278], force unitarity by subtracting appropriate terms (UNLOPS method).

- GENEVA, [Alioli, Bauer, Berggren, Hornig, Tackmann, Vermilion, Walsh, Zuberi, arXiv:1211.7049], increase precision in LL resummation to reach formal accurate matching

Matching using MiNLO

MiNLO (Multiscale-improved NLO) is a method for adding to an NLO amplitude Sudakov form factors and coupling rescaling without spoiling NLO accuracy.
In other words, it is a method to apply CKKW to an NLO calculation.
The "green line" integrand for `HJ` yields (for small Higgs p_T)

$$\sigma_0 \frac{1}{p_T^2}(\alpha_s L, \alpha_s, \alpha_s^2 L^3, \alpha_s^2 L^2, \alpha_s^2 L, \alpha_s^2) + \text{finite reminder}$$

With MiNLO it becomes

$$\sigma_0 \frac{1}{p_T^2}(\alpha_s, \alpha_s L, \alpha_s^2 L, \alpha_s^2) \exp S_{\text{MiNLO}}(f(x_1, p_T) Q_0, M_H)$$

i.e., higher logs are absorbed in the Sudakov form factor.

On the other hand, NNLL analytic resummation would yield (schematically) a perfect differential

$$\frac{d\sigma^{\rm NNLL}}{dy\,dp_T^2} = \sigma_0 \frac{d}{dq_T^2}\{f(x_1,p_T)f(x_2,p_T)\exp S(M_{\rm H},p_T)\}$$

NNLL accuracy implies that

$$\int \frac{d\sigma}{dy\,dp_T^2}\,dp_T^2$$

is NLO accurate. On the other hand:

$$\frac{d\sigma^{\rm NNLL}}{dy\,dp_T^2} = \sigma_0 \frac{1}{p_T^2}\left[\alpha_s, \alpha_s L, \alpha_s^2 L, \alpha_s^2, \ldots\right]\exp S(M_{\rm H},p_T)$$

where ... stands for missing higher order terms, having at most one power of L.

The integral of each $\alpha_s^n L^m/p_T^2$ term is of order $\alpha_s^{n-\frac{m+1}{2}}$.
Comparing this result with the MiNLO one, it is easy to show that the only missing term needed to render the MiNLO result NLO accurate upon full integration, is a B_2 term in the MiNLO Sudakov form factor.

More precisely, the effect of a B_2 term is

$$\sigma_0 \frac{1}{q_T^2}[\alpha_S, \alpha_S^2, \alpha_S L, \alpha_S^2 L] \times \exp \mathcal{S} \times \left\{ \exp\left[-\int_{q_T^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2} B_2 \alpha_S^2 \right] - 1 \right\}$$

$$\Longrightarrow \frac{1}{q_T^2}[\alpha_S^3 L^2] \times \exp \mathcal{S} \Longrightarrow [\alpha_s(Q^2)]^{3-\frac{2+1}{2}} = [\alpha_s(Q^2)]^{1.5}$$

In case of $H/W/Z + 1\,\mathrm{jet}$, it is in fact possible to modify the MiNLO Sudakov form factor by carefully including the B_2 term in such a way that integrating over the radiated jet we achieve NLO accuracy for inclusive $H/W/Z$ distributions. (Hamilton,Oleari,Zanderighi,P.N. 2012)

NNLO+PS generators

Given an `H-HJ` merged generator (accurate at order α_s^3 for fully inclusive quantities, and at order α_s^4 for Higgs plus one jet observables), it is easy to prove that NNLO accuracy can be achieved as follows:

- Generate events with the NLO+PS merged generator
- Reweight the event cross section as a function of $y_{\rm H}$ with the factor

$$\frac{\frac{d\sigma^{\rm NNLO}}{d\,y_{\rm H}}}{\frac{d\sigma^{\tt H-HJ}}{d\,y_{\rm H}}}$$

so that the Higgs rapidity distribution becomes NNLO accurate

In order for the proof to work, it is essential that

$$\frac{\dfrac{d\sigma^{\mathrm{NNLO}}}{d\,y_{\mathrm{H}}}}{\dfrac{d\sigma^{\mathtt{H-HJ}}}{d\,y_{\mathrm{H}}}} = \frac{\sigma^{(0)}_{\mathrm{NNLO}} + \alpha_s \sigma^{(1)}_{\mathrm{NNLO}} + \alpha_s^2 \sigma^{(2)}_{\mathrm{NNLO}}}{\sigma^{(0)}_{\mathtt{H-HJ}} + \alpha_s \sigma^{(1)}_{\mathtt{H-HJ}}} = 1 + \mathcal{O}(\alpha_s^2)$$

i.e. that $d\sigma_{\mathtt{H-HJ}}/dy_{\mathrm{H}}$ is NLO accurate.

If the ratio was $1 + \mathcal{O}(\alpha_s)$, distributions like the Higgs transverse momentum, that have the expansion:

$$\frac{d\sigma}{dp_t} = \alpha_s^3 \frac{d\sigma^{(3)}}{dp_t} + \alpha_s^4 \frac{d\sigma^{(3)}}{dp_t}$$

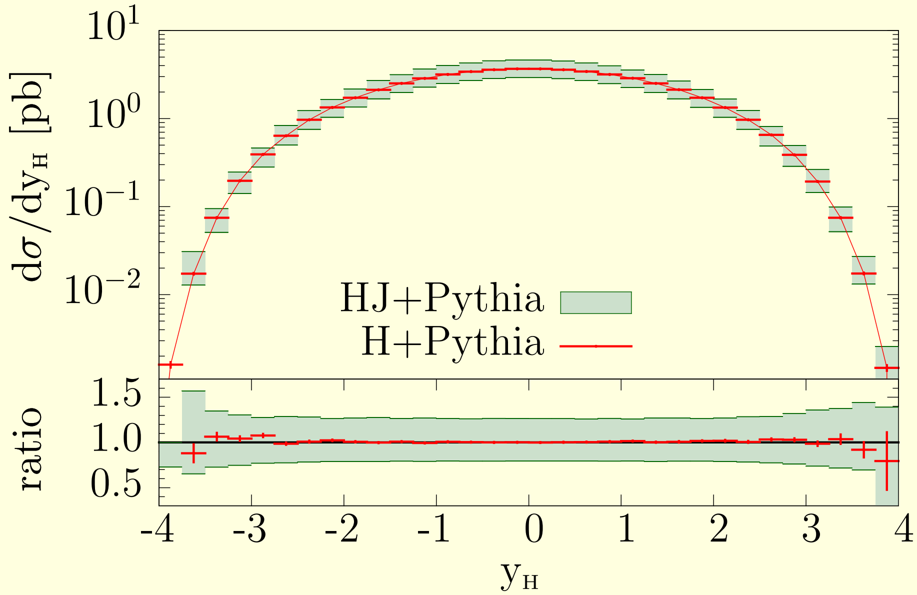
so that by reweighting:

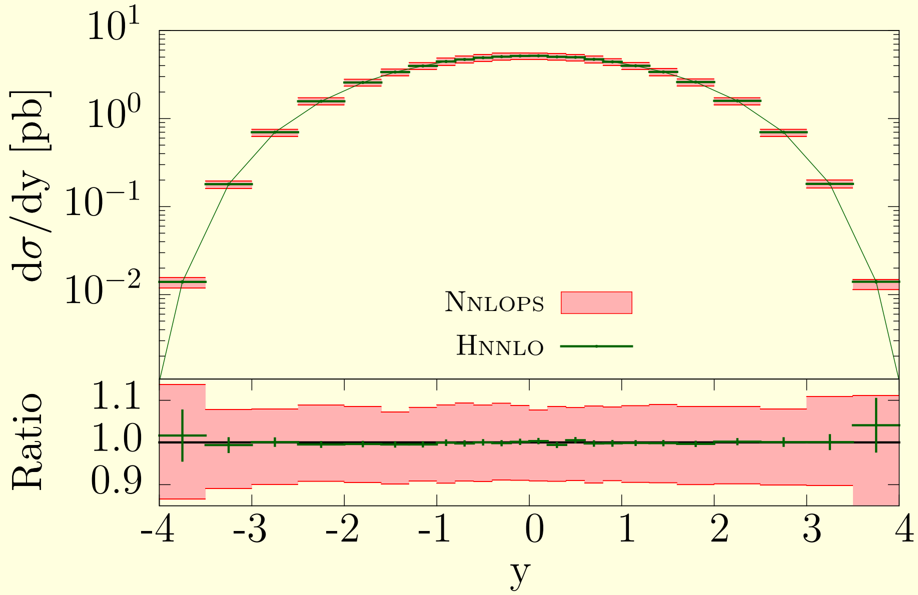
$$(1 + \mathcal{O}(\alpha_s)) \times \frac{d\sigma}{dp_t} = \alpha_s^3 \frac{d\sigma^{(3)}}{dp_t} + \alpha_s^4 \frac{d\sigma^{(3)}}{dp_t} + \underbrace{\alpha_s^3 \frac{d\sigma^{(3)}}{dp_t} \times \mathcal{O}(\alpha_s)}_{\mathcal{O}(\alpha_s^4)}$$

we get spurious terms of order α_s^4, spoiling its α_s^4 accuracy.

First result on Higgs production at NNLO+PS:

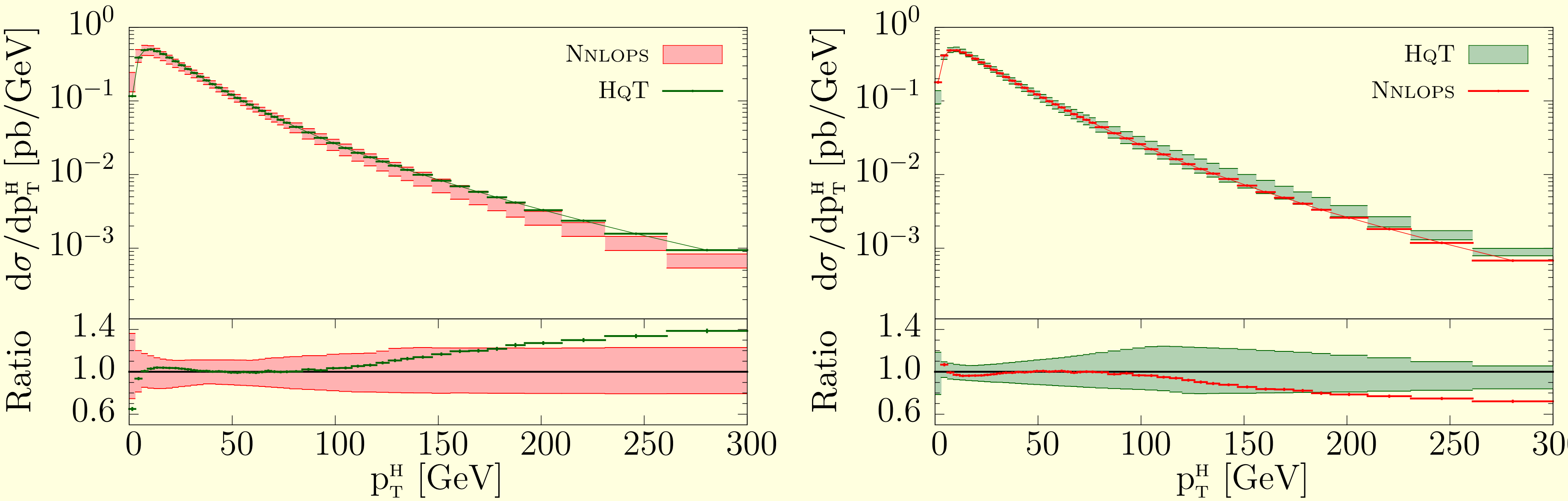
Zanderighi,Hamilton,Re,P.N. Aug. 2013, reweighting `MiNLO` generator from Zanderighi,Hamilton,Oleari,P.N. 2012





Accuracy: (left) `H-HJ` MiNLO: ∼30%, (right) NNLO+PS:∼10%

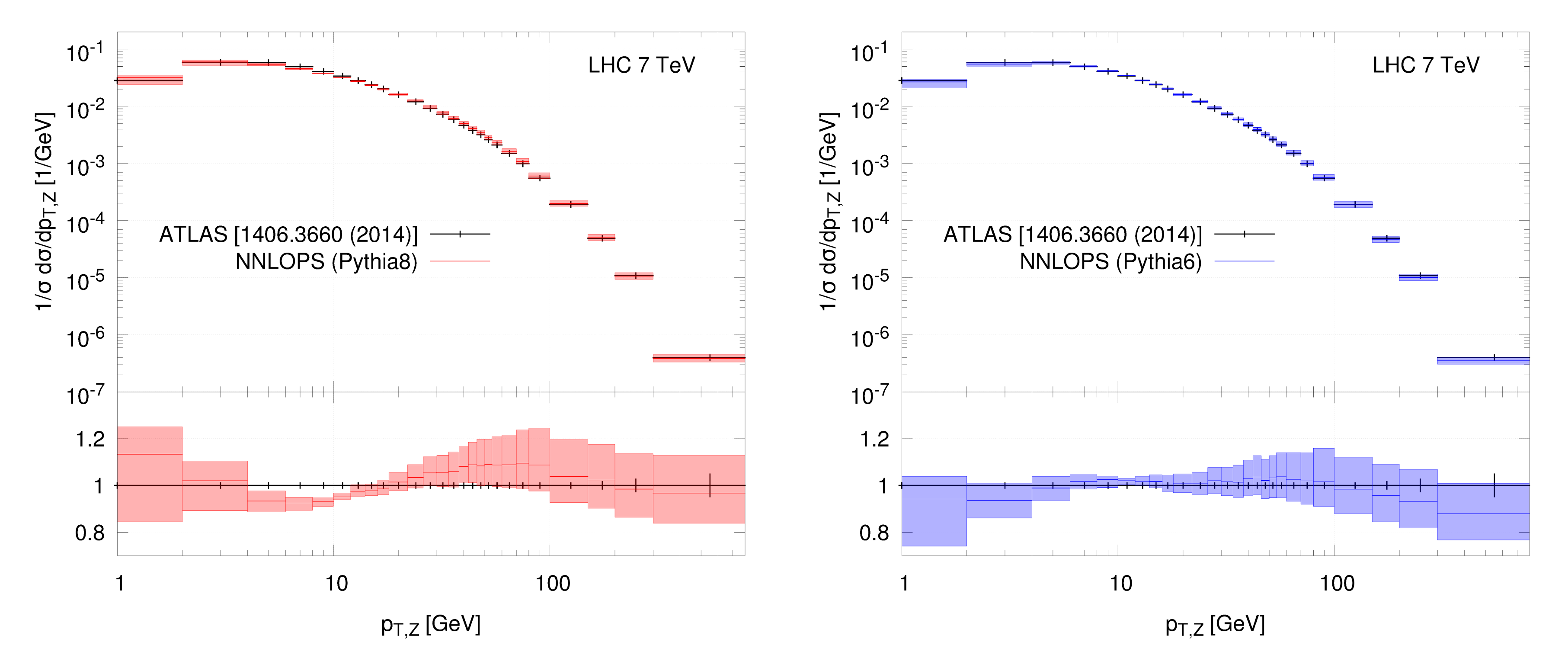
Higgs transverse momentum comparison to HqT



- HqT: dedicated program for NNLO+NNLL calculation of $\mathrm{d}\sigma^{\mathrm{H}}/\mathrm{d}p_{\mathrm{T}}$, Bozzi,Catani,De Florian,Ferrera,Grazzini,Tommasini
- Good agreement at small/moderate p_{T}
- Large p_{T}: it will be interesting to compare to $H+1j$ NNLO calculation by Boughezal,Caola,Melnikov,Petriello,Schulze Feb. 2013
- Approach valid in all production processes of colourless massive systems

Results on Drell-Yan NNLO+PS using the MiNLO Method:

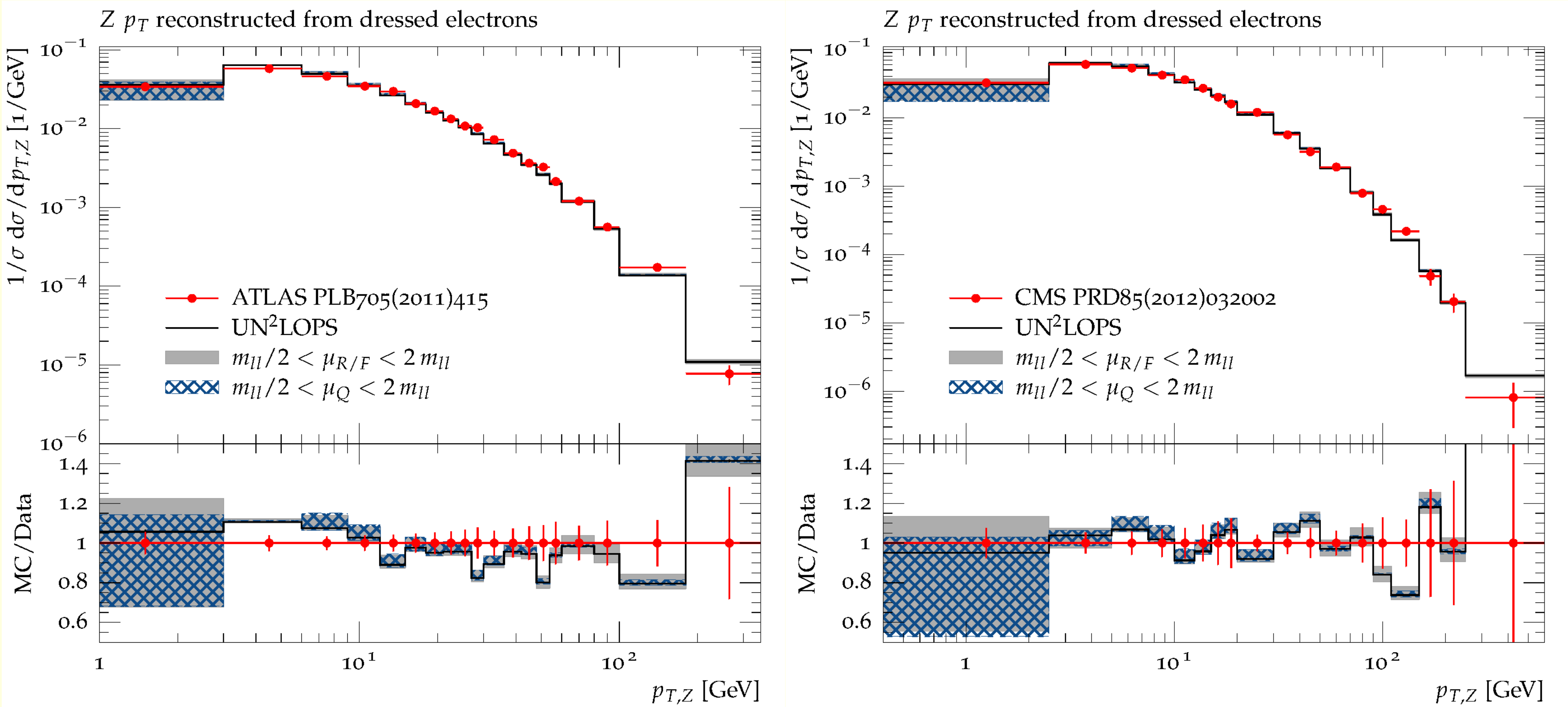
Karlberg,Re,Zanderighi,2014:



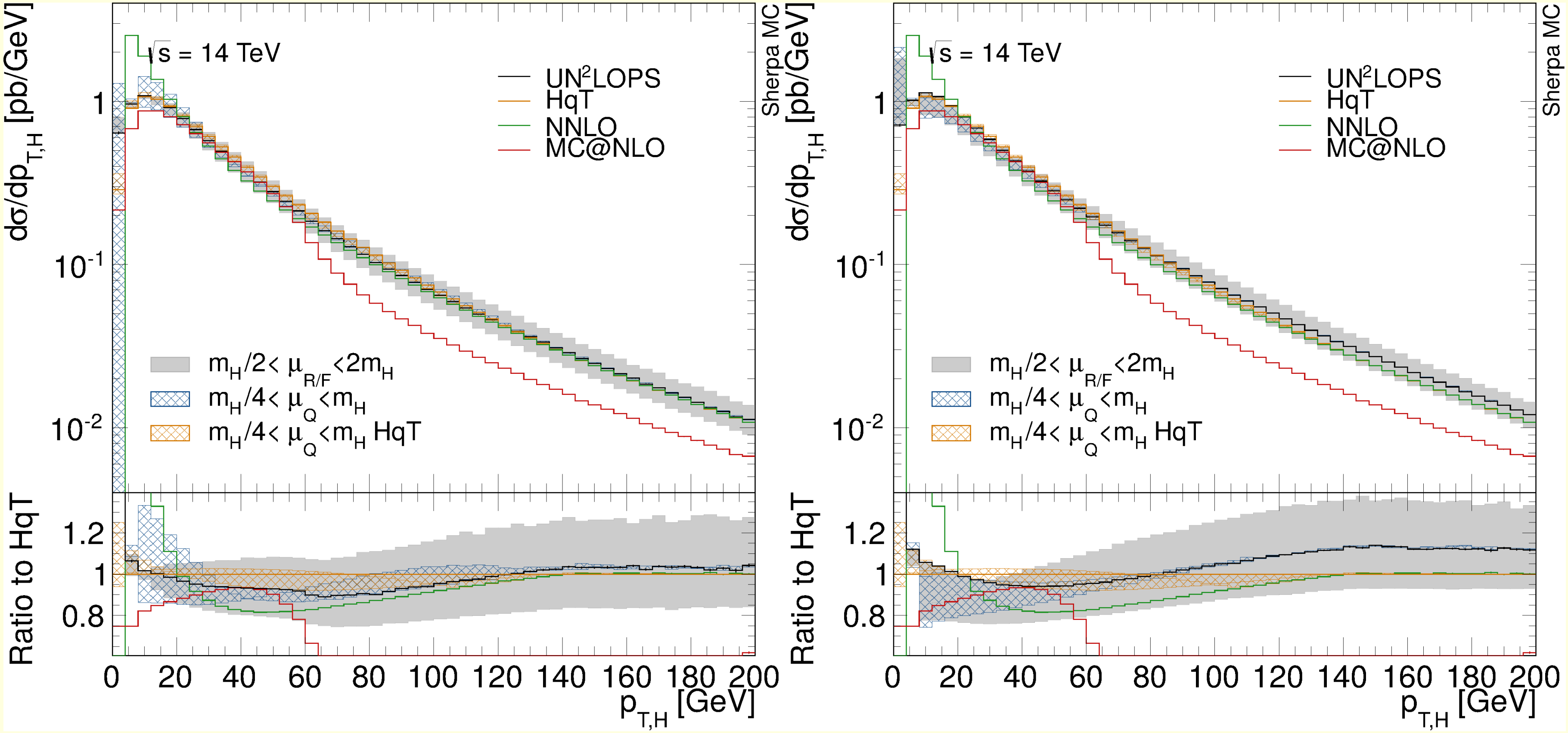
Monte Carlo tunes still play an important role.

Other methods:

NNLO+PS for Drell Yan pair production, Höche,Ye Li,Prestel, May 2014, based upon the UNLOPS merging method by Lönnblad, Prestel, 2012:



UNLOPS for Higgs



Höche,Li,Prestel,2014

UNLOPS method: NLO matching

$$\underbrace{\left\{\int \mathrm{d}\Phi_{\mathrm{B}}\,\bar{B}^{t_c}(\Phi_{\mathrm{B}})+\int_{t_c}\mathrm{d}\Phi[1-\Pi_0(t,Q)]\,R(\Phi)\right\}}_{H\,\text{Born kinematics, no further radiation}}+\underbrace{\int_{t_c}\mathrm{d}\Phi[\Pi_0(t,Q)]\,R(\Phi)}_{H+1\,\text{parton kinematics}\,+\,\text{shower}}$$

where t_c is a cutoff scale for showering, and

$$\bar{B}^{t_c}(\Phi_{\mathrm{B}})=B(\Phi_{\mathrm{B}})+V(\Phi_{\mathrm{B}})+\int^{t_c}\mathrm{d}\Phi_{\mathrm{rad}}\,R$$

In inclusive cross sections: terms multiplied by Π_0 cancel, NLO result recovered. This scheme is pushed to NNLO.
Notice: no Sudakov suppression on the term in curly bracket

In essence, in `UNLOPS` NLO and NNLO accuracy is restored by contributions that are added in the no-radiation bin. In this it differs from `MC@NLO` and `POWHEG` methods, where the no-radiation bin is Sudakov suppressed (i.e. suppressed by more than any power of α_s).

Conclusions

- The MiNLO approach looks very promising, but at present is satisfies the "strict" NLO requirement only for production of colour neutral systems (H, W/Z, HZ, HW, etc.), up to 1 jet merging.
- The Geneva approach seems to satisfy the "strict" requirement; needs more field testing (in my opinion) ...
- Traditional matching (`Sherpa`, `FxFx`) fully general (i.e. can be applied to any process), but relax the requirement of "strict" NLO accuracy, independent upon the matching scale
- `UNLOPS` forces "strict" NLO accuracy, but relaxes requirements about the description of the Sudakov region.
- Geneal problem: LO, NLO NNLO accuracy for inclusive observables with LL, NLL, NNLL accuracy in reduced phase space regions.

Backup

NNLO+PS: main claim

(Zanderighi, Hamilton, Oleari, P.N. 2013) From 1st-level NLO+PS merging, NNLO accuracy can be reached by reweighting.

Here we prove this in the example of Higgs production (proof easely extended to the general case.

Begin with the following (trivial) theorem:

A parton level Higgs boson production generator that is accurate at $\mathcal{O}(\alpha_s^4)$ for all IR safe observables that vanish with the maximum transverse momenta of all light partons, and that also reaches accuracy for the $\mathcal{O}(\alpha_s^4)$ inclusive Higgs rapidity distribution, achieves the same level of precision for all IR safe observables, i.e. it is fully NNLO accurate.

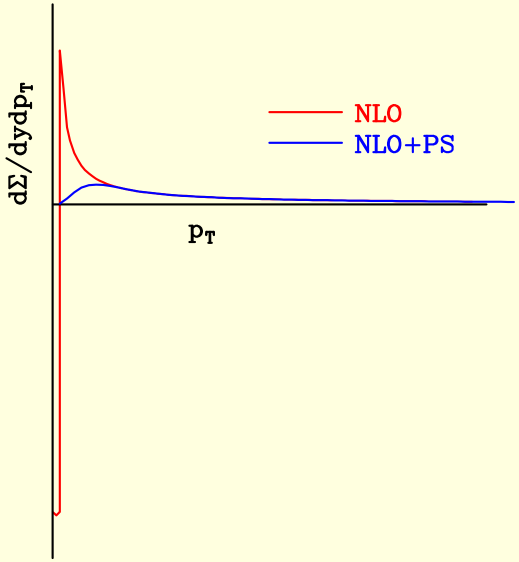
NLO accuracy by reweighting

Proof: $F(\Phi)$ is an IR safe observables; $F(y_H)$ is its "Born level" value

$$\langle F\rangle = \int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} F(\Phi) = \underbrace{\int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} \left(F(\Phi) - F(y_\Phi)\right)}_{\text{accurate at } \mathcal{O}(\alpha_s^4) \text{ by 1st hypothesis}} + \underbrace{\int \mathrm{d}y \frac{\mathrm{d}\sigma}{\mathrm{d}y} F(y)}_{\mathcal{O}(\alpha_s^4) \text{ by 2nd hypothesis}}$$

Does the 1st hypothesis apply also to an NLO-PS generator?

The difference with respect to a parton level generator is that the soft and collinear singularities are spread out over the Sudakov regions. For the Higgs:



NLO result: divergent distribution at low $p_{\rm T}$;
Negative divergent spike at $p_{\rm T}=0$, so that

$$\int \frac{d\sigma^{\rm NLO}}{dy\,dp_{\rm T}}\,{\rm d}p_{\rm T} = \frac{d\sigma^{\rm NLO}}{dy}$$

NLO+PS result: smooth Sudakov shape at small $p_{\rm T}$, all positive, with

$$\int \frac{d\sigma^{\rm NLO+PS}}{dy\,dp_{\rm T}}\,{\rm d}p_{\rm T} = \frac{d\sigma^{\rm NLO}}{dy}$$

(The proof of the 1st hypothesis for a NLO+PS generator can be carried out by expanding the Sudakov form factors in terms of a normalized "+" distribution plus higher order terms)

NNLO generator for Higgs production

(Hamilton,Re,Zanderighi,P.N. 2013)
Variant reweighting schemes

$$\begin{aligned} \mathrm{d}\sigma &= \mathrm{d}\sigma_A + \mathrm{d}\sigma_B \\ \mathrm{d}\sigma_A &= \mathrm{d}\sigma \times h(p_T) \\ \mathrm{d}\sigma_B &= \mathrm{d}\sigma \times (1 - h(p_T)) \end{aligned}$$

with

$$h(p_T) = \frac{(\beta m_{\mathrm{H}})^\gamma}{(\beta m_{\mathrm{H}})^\gamma + p_T^\gamma},$$

and reweight by

$$W(y, p_T) = h(p_T) \times \frac{\int \mathrm{d}\sigma_A^{\mathrm{NNLO}} \delta(y - y(\Phi))}{\int \mathrm{d}\sigma_A^{\mathtt{MiNLO}} \delta(y - y(\Phi))} + (1 - h(p_T)),$$

that yields

$$\int \mathrm{d}\sigma^{\mathtt{MiNLO}} \delta(y - y(\Phi)) \, W(y, p_T) = \left(\frac{\mathrm{d}\sigma_A}{\mathrm{d}y}\right)^{\mathrm{NNLO}} + \left(\frac{\mathrm{d}\sigma_B}{\mathrm{d}y}\right)^{\mathtt{MiNLO}}$$

We have adopted a further variant that has the advantage of yielding exactly the NNLO rapidity distribution:

$$W(y, p_T) = h(p_T) \times \frac{\int (\mathrm{d}\sigma^{\mathrm{NNLO}} - \mathrm{d}\sigma_B^{\mathtt{MiNLO}})\delta(y - y(\Phi))}{\int \mathrm{d}\sigma_A^{\mathtt{MiNLO}}\delta(y - y(\Phi))} + (1 - h(p_T)).$$

Numerically one needs to compute and store the (one-parameter) functions of y that appear in the fraction. After that one generates events normally, and reweights them by the W factor.

The NNLO cross section is computed with HNNLO (Grazzini, 2008)

Uncertainties

Uncertainties are estimated by the 7 point scale variation

$$(K_R, K_F) = (0.5, 0.5), (1, 0.5), (0.5, 1), (1, 1), (2, 1), (1, 2), (2, 2)$$

that is performed independently in the NNLO calculation and in the `MiNLO` one.

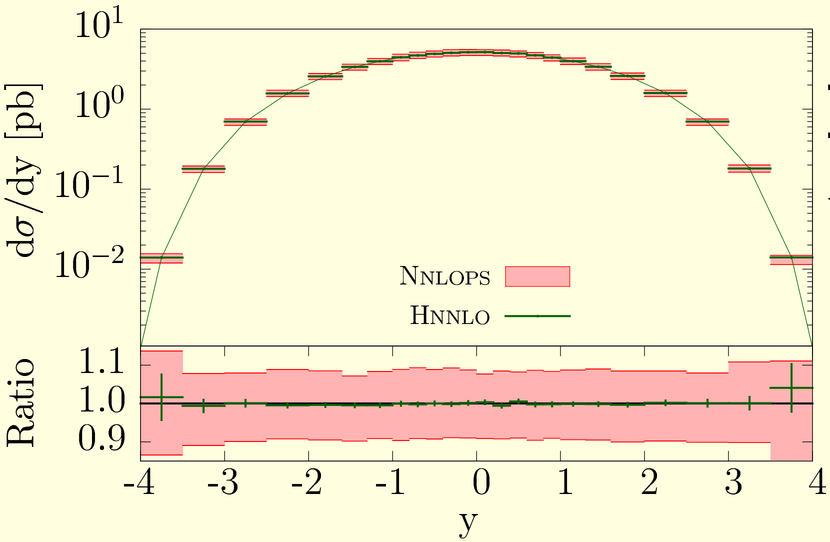
In other words, we assume conservatively that scale uncertainties in the NNLO and in the `MiNLO` results are uncorrelated.

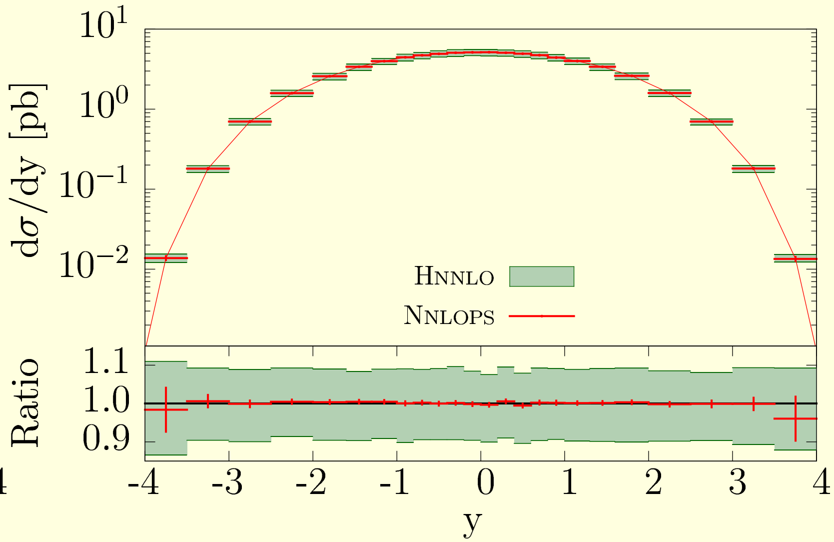
The value of p_T in the h function has been taken as the transverse momentum of the hardest jet.

Central scale for HNNLO: $m_H/2$, in slight tension with the `MiNLO` choice.

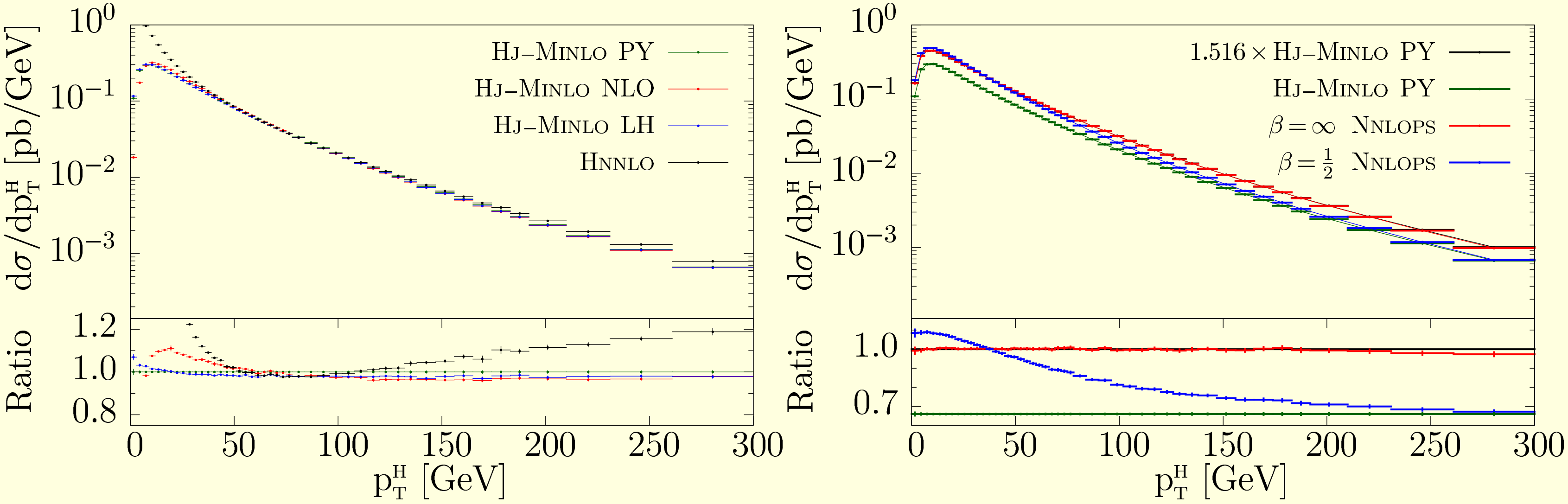
We use $\gamma = 2$ in the h function, and consider the range $0.5 < \beta < \infty$.

Results



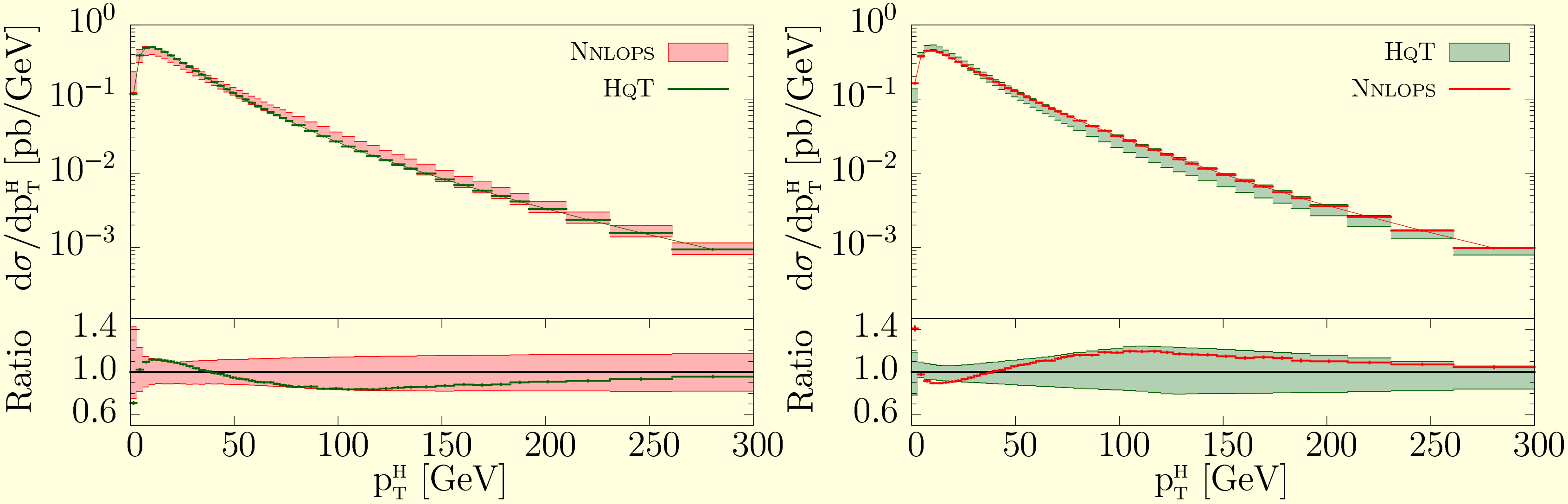


By construction, the rapidity distribution is exactly the same in NNLO-PS and in fixed order

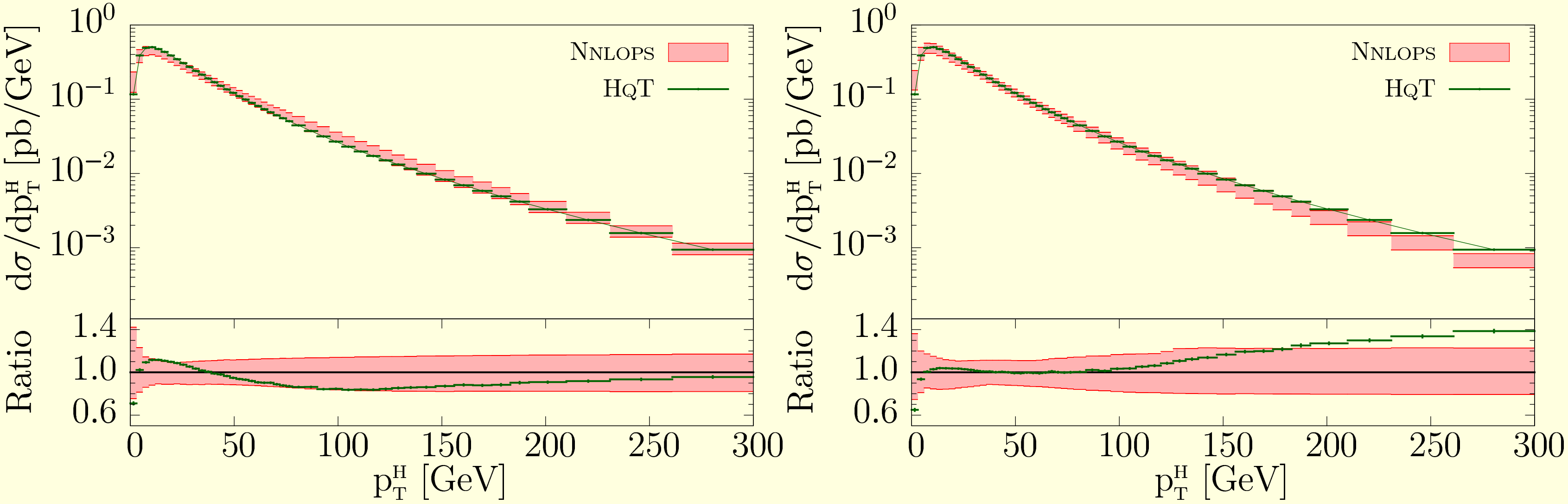


Left:
Comparison of the high p_T distribution with HNNLO, using M_H as scales

Right:
Effect of β variation

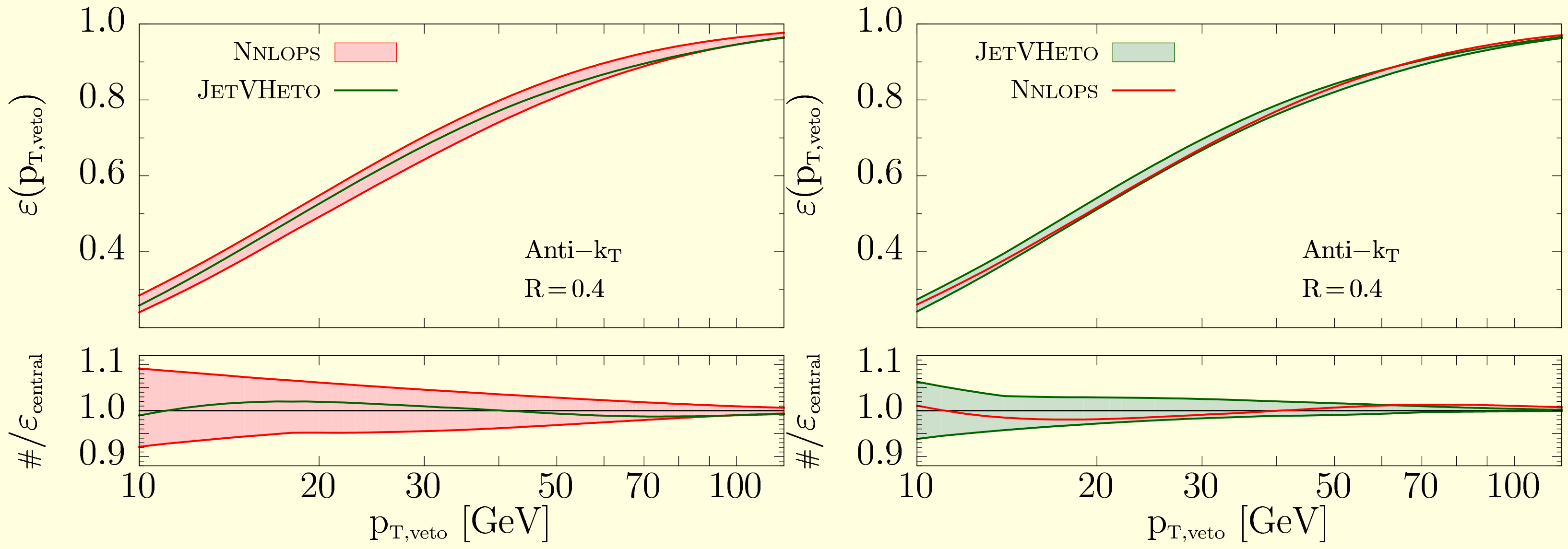


HqT and NNLO-PS error bands comparable



p_T spectrum with error bands, $\beta = \infty$ (left), $\beta = 1/2$ (right)

Choice of β analogous to the choice of the resummation scale in `HqT`.
$\beta = 1/2$ corresponds to $Q_{\mathrm{res}} = M_H/2$.



- JetVHeto: NNLL resummed, $\mu_R = \mu_F = m_H/2$, 7pts band (Banfi,Monni,Salam,Zanderighi, 2012)
- Fair agreement