

Analytics for jet substructure

Gavin Salam (CERN) based on work with Dasgupta, Fregoso & Marzani

Physics challenges in the face of LHC-14 Madrid, September 2014

LHC reach v. lumi



http://cern.ch/collider-reach with A. Weiler

Boosted searches



Most obvious way of detecting a boosted decay is through the mass of the jet





But jet mass is **poor** in practice: e.g., narrow W resonance

highly smeared by QCD radiation

(mainly underlying event/ pileup)

cf. calculations by Rubin '10



apologies for omitted taggers, arguable links, etc.



To fully understand "Boost" you want to study all possible signal (W/Z/H/top/...) and QCD jets.

But you need to start somewhere. We chose the QCD jets because:

(a) they have the richest structure.

(b) once you know understand the QCD jets, the route for understanding signal jets becomes clear too.

> arXiv:1307.0007 Dasgupta, Fregoso, Marzani & GPS +Dasgupta, Fregoso, Marzani & Powling, 1307.0013

study 3 taggers/groomers

Cannot possibly study all tools These 3 are widely used

Trimming



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Pruning



study 3 taggers/groomers

Cannot possibly study all tools These 3 are widely used

Trimming





Mass-drop tagger (MDT, aka BDRS)



The key variables

For phenomenology

Jet mass: m

[as compared to W/Z/H or top mass] For QCD calculations



[R is jet opening angle – or radius]

Because p is invariant under boosts along jet direction













But only for a limited range of masses

What might we want to find out?



And maybe you can make better taggers

Key calculations related to plain jet mass

- Catani, Turnock, Trentadue & Webber, '91: heavy-jet mass in e+e-
- Dasgupta & GPS, '01: hemisphere jet mass in e+e⁻ (and DIS)
 (→ non-global logs)
- Appleby & Seymour, '02; Delenda, Appleby, Dasgupta & Banfi '06: impact of jet boundary (→ clustering logs)
- Gehrmann, Gehrmann de Ridder, Glover '08; Weinzierl '08
 Chien & Schwartz '10: heavy-jet mass in e+e- to higher accuracy
- Rubin '10: filtering for jet masses
- Li, Li & Yuan '12, Dasgupta, Khelifa-Kerfa, Marzani & Spannowsky '12, Chien & Schwartz '12, Jouttenus, Stewart, Tackmann, Waalewijn '13: jet masses at hadron colliders
- Hatta & Ueda '13: non-global logs beyond large-N_C limit
- Forshaw, Seymour et al '06-'12, Catani, de Florian & Rodrigo '12: factorization breaking terms (aka super-leading logs)

Jet masses are hard! Will tagging/grooming make them impossible?



Take all particles in a jet of radius **R** and recluster them into subjets with a jet definition with radius

$R_{sub} < R$

The subjets that satisfy the condition

 $p_t^{(subjet)} > \mathbf{Z_{cut}} p_t^{(jet)}$

are kept and merged to form the trimmed jet.

Trimming Krohn, Thaler & Wang '09

two parameters: R_{sub} and z_{cut}

Use z_{cut} because signals (bkgds) tend to have large (small) z_{cut}



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Our approximations

- •ρ«1 logs of p get resummed
- pretend $R \ll 1$

• $Z_{cut} \ll 1$, but (log z_{cut}) not large

> These approximations are not as "wild" as they might sound.

> > They can also be relaxed.

But our aim for now is to understand the taggers — we



Leading Order — 2-body kinematic plane

At O(α_s), a quark jet emits a gluon. We study this as a function of the gluon momentum fraction z and the quark-gluon opening angle θ



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jet mass

$$\rho = z(1-z)\theta^2$$

matrix element			
<u> </u>	$\frac{\alpha_s C_F}{\pi} \frac{d\theta^2}{\theta^2}$	$\frac{d^2}{dz} = \frac{dz}{dz}$	
emission probability ~ constant in log θ – log z plane			
θ _{qg} 0.1	R _s	ub	1
both particle	s in 1 subjet	2 subjets $\rho = 0.200$	
		trimmed away	0.1 ² cut 0.01
		unnineu away	z



$$\rho = z(1-z)\theta^2$$



matrix element $\alpha_s C_F \ d\theta^2 \ dz$ $\pi \frac{\theta^2}{\theta^2}$ emission probability ~ constant in $\log \theta - \log z$ plane θ_{qg} 0.1 R_{sub} both particles in 1 subjet 2 subjets $\rho = 0.200$ 0.1 z_{cut}

Z

0.01

trimmed away



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Trimming at LO in α_s



$$\frac{\rho}{\sigma} \frac{d\sigma^{(\text{trim,LO})}}{d\rho} =$$

$$\frac{\alpha_s C_F}{\pi} \left(\ln \frac{r^2}{\rho} - \frac{3}{4} \right)$$

$$\frac{\alpha_s C_F}{\pi} \left(\ln \frac{1}{z_{\rm cut}} - \frac{3}{4} \right)$$

$$\frac{\alpha_s C_F}{\pi} \left(\ln \frac{1}{\rho} - \frac{3}{4} \right)$$

continue with all-order resummation of terms $\alpha_s^n \ln^m \rho$

Inputs

QCD pattern of multiple soft/collinear emission

Analysis of taggers' behaviour for 1, 2, 3, ... n, emissions Establish which simplifying approximations to use for tagger & matrix elements

\rightarrow all orders in α_s

Output

approx. formula for tagger's mass distribution for $\rho \ll 1$ $\frac{\rho}{\sigma}\frac{d\sigma}{d\rho} =$ ∞ $\sum c_{nm} \alpha_s^n \ln^m \rho$ n=1

keeping only terms with largest powers of ln ρ , e.g. m = 2n, 2n-1
Trimming

$$\rho^{\text{trim}}(k_1, k_2, \dots k_n) \simeq \sum_{i}^{n} \rho^{\text{trim}}(k_i)$$
$$\sim \max_{i} \{\rho^{\text{trim}}(k_i)\}$$

Trimmed jet reduces (~) to sum of trimmed emissions



Matrix element

$$\sum_{n} \frac{1}{n!} \prod_{i}^{n} \frac{d\theta_{i}^{2}}{\theta_{i}^{2}} \frac{dz_{i}}{z_{i}} \frac{\alpha_{s}(\theta_{i} z_{i} p_{t}^{\text{jet}}) C_{F}}{\pi}$$

can use QED-like independent emissions, as if gluons don't split

+ virtual corrections, essentially from unitarity

















Full resummation also needs treatment of running coupling

What logs, what accuracy?

Express accuracy for " $cumulative dist^{n}$ " $\Sigma(\rho)$:

$$\Sigma(\rho) = \int_0^{\rho} d\rho' \frac{1}{\sigma} \frac{d\sigma}{d\rho'}$$

Use shorthand $L = \log 1/\rho$

Trimming's **leading logs** (LL, in Σ) are: $\alpha_s L^2, \ \alpha_s^2 L^4, \ \dots \ \text{I.e.} \ \alpha_s^n L^{2n}$

Just like the jet mass

We also have **next-to-leading logs** (NLL): $\alpha_s^n L^{2n-1}$

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Could we do better? Yes: NLL in In Σ :

 $\ln \Sigma: \alpha_s^n L^{n+1} \text{ and } \alpha_s^n L^n$

Trimmed mass is like plain jet mass (with $R \rightarrow R_{sub}$), and this accuracy involves **non-global logs**, **clustering logs**

Trimming: MC v. analytics



Non-trivial agreement! (also for dependence on parameters)

Trimming: MC v. analytics



Non-trivial agreement! (also for dependence on parameters)

For a jet clustered with C/A:

- 1. undo last clustering step to break jet (mass m) into two subjets with $m_1 > m_2$
- 2. If significant mass-drop ($m_1 < \mu m$) and subjet energy-sharing not too asymmetric $\min(p_{t1}^2, p_{t2}^2)\Delta R_{12}^2 < y_{\rm cut}m^2$

jet is tagged.

3. Otherwise discard subjet 2, and go to step 1 with jet \rightarrow subjet 1.

Mass-Drop Tagger

Butterworth, Davison, Rubin & GPS '08

two parameters: µ and y_{cut} (~ z_{cut})



decluster &













Jet is always split to give two subjets, and so y_{cut} $(\sim z_{cut})$ is always applied.



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Analytic Jet Substructure





What MDT does wrong beyond LO:

Follows a soft branch (p₂+p₃ < y_{cut} p_{jet}) with "accidental" small mass, when the "right" answer was that the (massless) hard branch had no substructure

Subjet is soft, but has more substructure than hard subjet

MDT's leading logs (LL, in Σ) are:

$$\alpha_s L, \alpha_s^2 L^3, \dots$$
 I.e. $\alpha_s^n L^{2n-1}$

quite complicated to evaluate

A simple fix: **"modified"** Mass Drop Tagger: When recursing, **follow branch with larger (m²+p_t²)** (rather than the one with larger m)



Modification has almost no phenomenological impact, but big analytical consequences...

modified Mass Drop Tagger

At most "single logs", at all orders, i.e. $\alpha_s L, \, \alpha_s^2 L^2, \, \dots \, \text{I.e.} \, \alpha_s^n L^n$

Logs exclusively collinear – much simpler than jet mass

- no non-global logs
- no clustering logs
- no super-leading (factorization-breaking) logs First time anything like this has been seen

Fairly simple formulae; e.g. [fixed-coupling]

$$\Sigma^{(\mathrm{mMDT})}(\rho) = \exp\left[-\frac{\alpha_s C_F}{\pi} \left(\ln\frac{y_{\mathrm{cut}}}{\rho}\ln\frac{1}{y_{\mathrm{cut}}} - \frac{3}{4}\ln\frac{1}{\rho} + \frac{1}{2}\ln^2\frac{1}{y_{\mathrm{cut}}}\right)\right]$$

mMDT MC v. Resummation





[mMDT is closest we have to a scale-invariant tagger, though exact behaviour depends on q/g fractions]

mMDT MC v. Resummation

gluon jets



[mMDT is closest we have to a scale-invariant tagger, though exact behaviour depends on q/g fractions]

mMDT resummation v. fixed order



Because we only have single logs, fixed-order is valid over a broader than usual range of scales

(helped by fortuitous cancellation between running coupling and single-log Sudakov)

NLO from NLOJet++

mMDT: comparing many showers



mMDT: comparing many showers



Issue found in Pythia 6 pt-ordered shower \rightarrow promptly identified and fixed by Pythia authors!

mMDT: comparing many showers



Issue found in Pythia 6 p_t -ordered shower \rightarrow promptly identified and fixed by Pythia authors!

Analytic Jet Substructure

comparing to data



Since LO quite close to full resummation, you can try comparing LO directly to data.

Remarkable agreement! [see backup for non-pert effects]

Dasgupta, Siodmok & Powling, in prep.

Looking beyond

Soft Drop

mMDT has a single-logarithmic (pure collinear) distribution that's free of non-global logs

A generalisation is **Soft Drop**

Uncluster C/A jets as with mMDT, but stop only if

$$\frac{\min(p_{t1}, p_{t2})}{p_t} > z_{\text{cut}} \left(\frac{\Delta R_{12}}{R}\right)^{\beta}$$

For $\beta > 0$, get double-log distⁿ without NG logs [mMDT corresponds to $\beta = 1$]

Performance for finding signals (S/ \sqrt{B})

At high pt, substantial gains from new Y-pruning (probably just indicative of potential for doing better)



At low pt (moderate m/pt), all taggers quite similar

Combining variables



Experiments often combine [trimming/pruning/MDT/etc.] with a shape cut, typically N-subjettiness, $\tau_{21} = \tau_2 / \tau_1$

Next: understand τ₂₁. **Qu.:** apply before or after MMDT?

Prelim. answer: take τ₂ from full jet, τ₁ from mMDT jet

Work in progress, Dasgupta, GPS, Soyez & Sarem-Schunk



Work in progress, Dasgupta, GPS, Soyez & Sarem-Schunk



- Taggers may be quite simple to write, but potentially involved to understand.
- Contrast this with pt cuts for standard jet analyses (mostly) simple
- Still, many taggers/groomers are within calculational reach.
- Calculations help put the field on solid ground & potentially open road to new, better tools
Summary table

	highest logs	transition(s)	Sudakov peak	NGLS	-
plain mass	$\alpha_s^n L^{2n}$		$L \simeq 1/\sqrt{\bar{\alpha}_s}$	yes	
trimming	$\alpha_s^n L^{2n}$	$z_{ m cut},r^2 z_{ m cut}$	$L \simeq 1/\sqrt{\bar{\alpha}_s} - 2\ln r$	yes	-
pruning	$lpha_s^n L^{2n}$	$z_{ m cut},z_{ m cut}^2$	$L \simeq 2.3 / \sqrt{\bar{\alpha}_s}$	yes	
MDT	$\alpha_s^n L^{2n-1}$	$y_{\mathrm{cut}}, \frac{1}{4}y_{\mathrm{cut}}^2, y_{\mathrm{cut}}^3$		yes	
Y-pruning	$\alpha_s^n L^{2n-1}$	$z_{ m cut}$	(Sudakov tail)	yes	
mMDT	$lpha_s^n L^n$	$y_{ m cut}$	—	no	
					-

Special: only single logarithms ($L = \ln \rho$) \rightarrow more accurately calculable Special: better exploits signal/bkgd differences

EXTRAS

SIGNAL

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3



Zbb BACKGROUND

Cluster event, C/A, R=1.2

Butterworth, Davison, Rubin & GPS '08

arbitrary norm₆₇

SIGNAL

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3



Zbb BACKGROUND

Fill it in, \rightarrow show jets more clearly

Butterworth, Davison, Rubin & GPS '08

arbitrary norm₆₈



p_t [GeV]

90[.]

80

70

60

50

40

30

20

10

0

5

Hardest jet, pt=246.211 m=150.465

-2



Consider hardest jet, m = 150 GeV

Butterworth, Davison, Rubin & GPS '08

arbitrary norm₆₉

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3



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arbitrary norm₇₃

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3





arbitrary norm₇₄

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What about non-perturbative effects?

[on 3 TeV jets?!]

Hadronisation effects



Nearly all taggers have large hadronisation effects: 15 - 60%for m = 30 - 100 GeV

Hadronisation effects



Exception is (m)MDT.

In some cases just few % effect.

m-dependence of hadronisation even understood analytically!



Underlying Event (UE)



Underlying event impact much reduced relative to jet mass

Almost zero for mMDT (this depends on jet pt)

MDT phenomenology ATLAS measurment of the jet mass with MDT [JHEP 1205 (2012)] Hadronization + MPI effects Plain Mass ATLAS MDT



red line – parton level blue line – hadron level



mMDT – not very sensitive to hadronization!

Dasgupta, Siodmok & Powling, in prep.

Examples of NLO checks



Dasgupta, Fregoso, Marzani & Powling, 1307.0013

mMDT: impact of μ and of filtering



µ parameter basically irrelevant
(simpler tagger discards it)

filtering leaves results unchanged (up to and incl. NNLL)

What about cuts on shapes/radiation

E.g. cuts on N-subjettiness, tight mass drop, etc.?

- These cuts are nearly always for a jet whose mass is somehow groomed. All the structure from the grooming persists.
- So tagging & shape must probably be calculated together



Take a jet and define $R_{prune} = m / p_t$ Recluster with k_t or C/A alg. At each i+j clustering step, if $p_{ti} \text{ Or } p_{tj} < \mathbf{Z}_{cut} p_{t(i+j)asdf}$ $\Delta R_{ij} > R_{prune}$ discard softer prong. Acts similarly to filtering, but with dynamic subjet radius

Pruning Ellis, Vermillion & Walsh '09

one (main) parameter: *z*_{cut}

we'll study variant with C/A reclustering



Dynamical choice of R_{prune} means that two prongs are always separated by $> R_{prune}$. So, unlike trimming, *z*_{cut} always applied.



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pruning beyond 1st order: consider multiple emissions



What pruning sometimes does

Chooses R_{prune} based on a soft p₃ (dominates total jet mass), and leads to a single narrow subjet whose mass is also dominated by a soft emission (p₂, within R_{prune} of p₁, so not pruned away).

Sets pruning radius, but gets pruned away → "wrong" pruning radius → makes this ~ trimming

A simple fix: "Y" pruning

Require at least one successful merging with $\Delta R > R_{prune}$ and $z > z_{cut}$ — forces 2-pronged ("Y") configurations





"Y" pruning ~ an isolation cut on radiation around the tagged object exploits W/Z/H colour singlet

What logs, what accuracy?

At leading order pruning (= Y-pruning): **no double logs**! $\alpha_s L$, but no $\alpha_s L^2$

Full Pruning's leading logs (LL, in Σ) are:

$$\alpha_s L, \alpha_s^2 L^4, \dots$$
 I.e. $\alpha_s^n L^{2n}$

we also have NLL

Y-Pruning's leading logs (LL, in Σ) are:

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 I.e. $\alpha_s^n L^{2n-1}$

we also have NLL

Could we do better? Yes: NLL in In Σ , but involves **non-global logs**, **clustering logs**

Analytic Jet Substructure

quark jets



Non-trivial agreement! (also for dependence on parameters)