Investigating the Near-Criticality of the Higgs Boson

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Based on

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio and Strumia,
JHEP **1312** *(2013) 089, arXiv:1307.3536; updated version: September 22, 2014*

Outline

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The stability bound on the Higgs mass

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Metastability scenario

Results at the Large Hadron Collider (LHC)

- Discovery of the Higgs boson at CMS and ATLAS in 2012
  with a mass $M_h = 125.15 \pm 0.24$ GeV
  [CMS Collaboration (2013, 2014); ATLAS Collaboration (2013, 2014); naive average from Giardino, Kannike, Masina, Raidal and Strumia (2014)]

- No clear evidence of new physics at the electroweak (EW) scale
  (supersymmetry (SUSY), composite Higgs, large extra dimensions, ... ?)

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The triumph of simplicity?

We do not know: still there is some room for new physics.

However, a simple Higgs doublet H with the simple potential

$$V(H) = \lambda \left(|H|^2 - \frac{v^2}{2} \right)^2$$

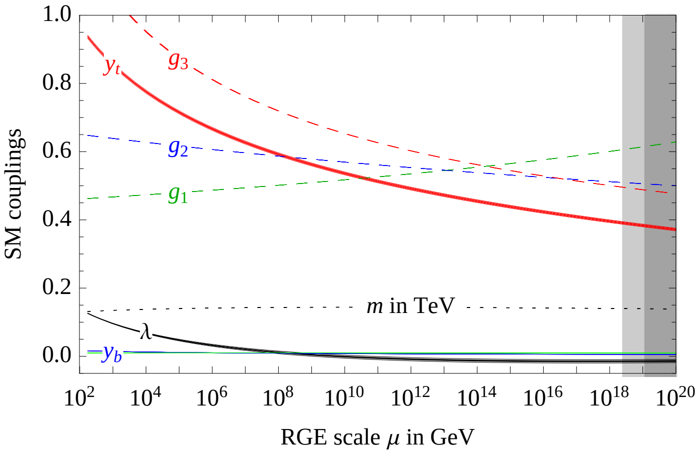
perfectly fits the data

- Measurements of G_μ provides $v = \sqrt{2}\langle |H| \rangle$ (tree level)
- and $m^2 \equiv 2\lambda v^2 = M_h^2$ (tree level) fixes the last parameter of the SM

Now, we can use the Standard Model (SM) to make predictions up to the Planck scale

Consistency: ok (up to the Planck scale)

- M_h is below the bound to push the Landau pole of λ above the Planck mass $M_{\rm Pl}$
- The Landau pole of $g_1 \equiv \sqrt{5/3} g_Y$ is at a very high energy: $\sim 10^{42}$ GeV
- The measured M_h implies that the EW vacuum expectation value (VEV) is either stable or metastable with a life-time $>$ than the age of the universe (see last part)



Solutions of the renormalization group equations (RGEs) of the most relevant SM parameters (defined in the $\overline{MS}$ scheme ...)

Still there are unsolved problems

The SM is not the final theory: apart from quantum gravity

- Dark matter
  well-motivated candidates: axion (which also solves the strong CP problem), ...
- (small) neutrino masses
  well-motivated candidates: heavy Majorana fermions, ...
- Baryon asymmetry
  Elegant solutions: Leptogenesis (possible with heavy Majorana fermions), ...

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Origin of inflation is it part of this list?

$\rightarrow$ *One possibility is that inflation is generated by the Higgs field, however, it is known that this is possible essentially only if the stability bound is not violated* [Bezrukov, Magnin, Shaposhnikov (2008, 2009); Salvio (2013)]

Introduction

The stability bound on the Higgs mass

Metastability scenario

Qualitative origin of the stability bound

$$V_{\rm eff} = V + V_1 + V_2 + \ldots$$

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2, \quad V_1(\phi) = \frac{1}{(4\pi)^2}\sum_i c_i m_i(\phi)^4\left(\ln\frac{m_i(\phi)^2}{\mu^2} + d_i\right), \quad \ldots$$

where $\phi^2 \equiv 2|H|^2$ and c_i and d_i are ~ 1 constants

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Consider the RG-improved effective potential (bare parameters $\rightarrow$ running ones) ...

$$\Longrightarrow \frac{\partial V_{\text{eff}}}{\partial \mu} = 0 \quad \text{and one is free to choose } \mu \text{ to improve perturbation theory}$$

Since at large fields, $\phi \gg v$, we have $m_i(\phi)^2 \propto \phi^2$, we choose $\mu^2 = \phi^2$, then

$$V_{\text{eff}}(\phi) = \frac{\lambda(\phi)}{4}\left(\phi^2 - v(\phi)^2\right)^2 + \dots = -\frac{m(\phi)^2}{2}\phi^2 + \lambda(\phi)\phi^4 + \dots$$

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So for $\phi \gg v$

$$V_{\text{eff}}(\phi) \simeq \frac{\lambda(\phi)}{4}\phi^4$$

- M_h contributes positively to $\lambda \to$ lower bound on M_h
- y_t contributes negatively to the running of $\lambda \to$ upper bound on M_t

Procedure to extract the stability bound

Steps of the procedure:

- $V_{\rm eff}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called *threshold corrections* or *matching conditions*) at the EW scale (e.g. at M_t) ...

Finally impose that $V_{\rm eff}$ at the EW vacuum is the absolute minimum!

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State of the art loop calculation:

- Two loop $V_{\rm eff}$ including the leading couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$ *[Martin (2002); Ford, Jack (2002)]*
- Three loop RGEs for $\{\lambda, y_t, g_3, g_2, g_1\}$ and one loop RGE for $\{y_b, y_\tau\}$... *[Mihaila, Salomon, Steinhauser (2012); Chetyrkin, Zoller (2012, 2013); Bednyakov, Pikelner, Velizhanin (March 19 and 21, 2013)]*
- Two loop values of $\{\lambda, y_t, g_3, g_2, g_1\}$ at M_t ... *[New! (2014)]*

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Previous calculations: *[...; Sher (1989); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012); ...]*

Input values of the SM observables

(used to fix the relevant parameters: $\lambda, m, y_t, g_2, g_Y$)

M_W	$=$	$80.384 \pm 0.014\ \mathrm{GeV}$	Mass of the W boson [1]
M_Z	$=$	$91.1876 \pm 0.0021\ \mathrm{GeV}$	Mass of the Z boson [2]
M_h	$=$	$125.15 \pm 0.24\ \mathrm{GeV}$	(source already quoted)
M_t	$=$	$173.34 \pm 0.76 \pm 0.3\ \mathrm{GeV}$	Mass of the top quark [3]
$V \equiv (\sqrt{2} G_\mu)^{-1/2}$	$=$	$246.21971 \pm 0.00006\ \mathrm{GeV}$	Fermi constant for μ decay [4]
$\alpha_3(M_Z)$	$=$	0.1184 ± 0.0007	$\mathrm{SU}(3)_c$ coupling (5 flavors) [5]

[1] TeVatron average: FERMILAB-TM-2532-E. LEP average: CERN-PH-EP/2006-042

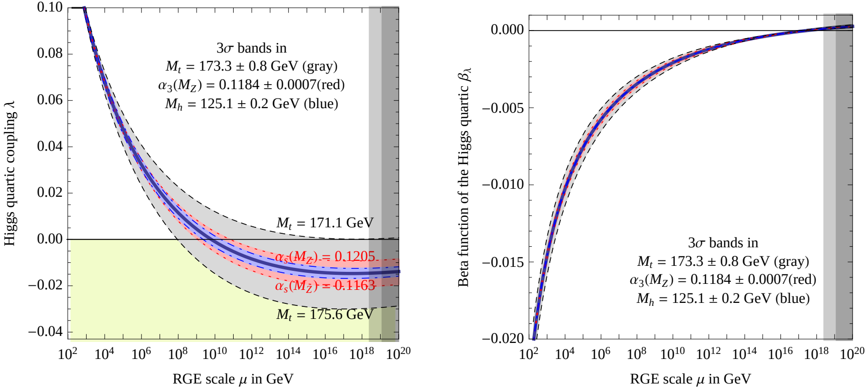
[2] 2012 Particle Data Group average, pdg.lbl.gov

[3] ATLAS, CDF, CMS, D0 Collaborations, arXiv:1403.4427. Plus an uncertainty $\mathcal{O}(\Lambda_{\rm QCD})$ because of non-perturbative effects [Alekhin, Djouadi, Moch (2013)]

[4] MuLan Collaboration, arXiv:1211.0960

[5] S. Bethke, arXiv:1210.0325

Precise running of λ and its β-function



RGE evolution of λ and its β-function varying M_t, $\alpha_3(M_Z)$, M_h by $\pm 3\sigma$.

Result for the stability bound

$$M_h > 129.6\,\mathrm{GeV} + 2.0(M_t - 173.34\,\mathrm{GeV}) - 0.5\,\mathrm{GeV}\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3\,\mathrm{GeV}$$

Combining in quadrature the experimental and theoretical uncertainties we obtain

$$M_h > (129.6 \pm 1.5)\,\mathrm{GeV}$$

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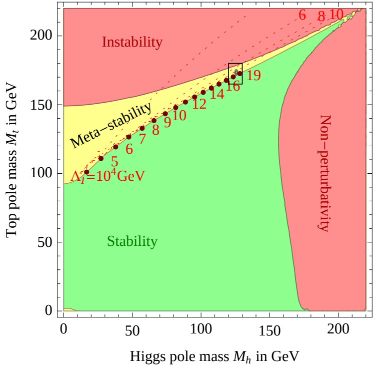
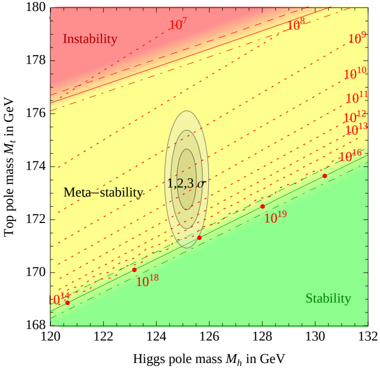
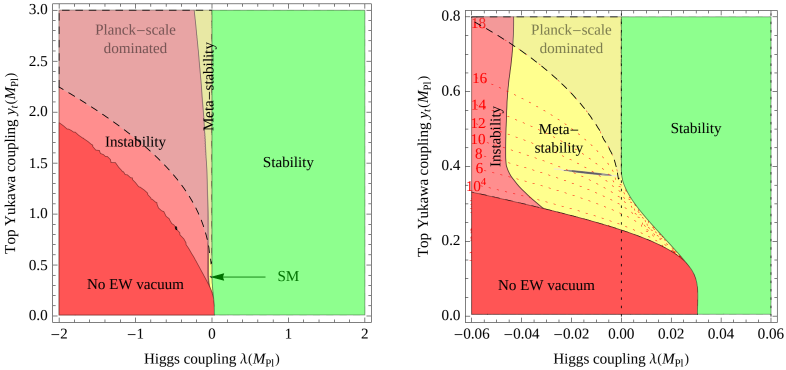




Λ_I = scale (field value) at which $V_{\rm eff}$ becomes smaller than its value at the EW scale

The SM phase diagram in terms of Planck scale couplings

$y_t(M_{\rm Pl})$ versus $\lambda(M_{\rm Pl})$



"Planck-scale dominated" corresponds to $\Lambda_I > 10^{18}$ GeV

"No EW vacuum" corresponds to a situation in which λ is negative at the EW scale

Interpretations of the near criticality

Why is $\lambda(M_{\mathrm{Pl}})$ small?

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It could be the matching with some high energy theory close to $M_{\rm Pl}$:

- High scale SUSY with $\tan\beta = 1$
  [Hall, Nomura (2009); Giudice, Strumia (2014); Cabrera, Casas, Delgado (2012); Arbey, Battaglia, Djouadi, Mahmoudi, Quevillon (2012); Ibañez, Valenzuela (2013); Hebecker, Knochel, Weigand (2013)]
- Partial $N = 2$ SUSY insuring D-flatness
  [Fox, Nelson, Weiner (2006); Benakli, Goodsell, Staub (2012)]
- An approximate Goldstone or shift symmetry
  [Hebecker, Knochel, Weigand (2012); Redi, Strumia (2012)]
- No-scale scenario *(Agravity)* together with a Z_2 symmetry: if the mirror Higgs is the field that generates $M_{\rm Pl}$, its VEV is at the Planck scale and the corresponding potential has to be nearly vanishing (to have a small cosmological constant Λ)
  [Salvio, Strumia (2014)]

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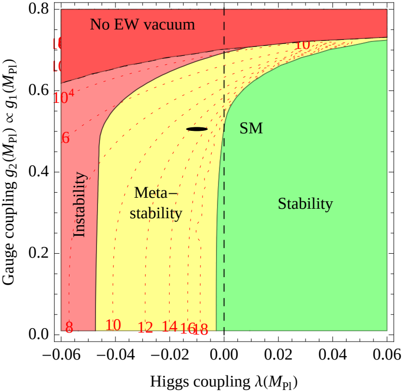
  [Salvio, Strumia (2014)]

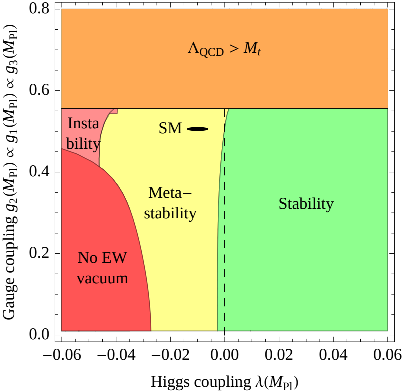
... or some property of the multiverse (not necessarily the anthropic selection)

$\lambda(M_{\rm Pl})$ small is explained if critical points in the multiverse are attractors

The SM phase diagram in terms of Planck scale couplings

Gauge coupling g_2 at $M_{\rm Pl}$ versus $\lambda(M_{\rm Pl})$

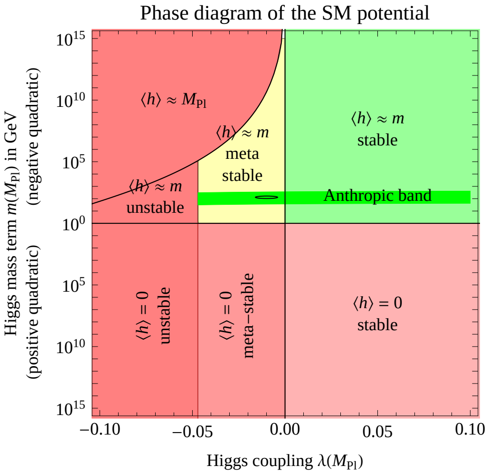




Left: $g_1(M_{\rm Pl})/g_2(M_{\rm Pl}) = 1.22$ *as in the SM, while* $y_t(M_{\rm Pl})$ *and* $g_3(M_{\rm Pl})$ *are kept to the SM value*

Right: *a common rescaling factor is applied to* g_1, g_2 *and* g_3. $y_t(M_{\rm Pl})$ *are kept to the SM value*

The SM phase diagram in terms of Higgs potential parameters



For $\lambda(M_{\mathrm{Pl}}) < 0$ there is an upper bound on m by requiring a Higgs vacuum at the EW scale. This bound is, however, much weaker than the anthropic bound of [Agrawal, Barr, Donoghue, Seckel (1997); Schellekens (2014)]

Introduction

The stability bound on the Higgs mass

Metastability scenario

If V_{eff} becomes negative much before the Planck scale

- If M_h is close to the measured central value, Higgs inflation is not possible and V_{eff} becomes negative much before M_{Pl}

- *If so, is this evidence for new physics?*
  Yes, in the sense that at least the inflaton seems to be missing in the SM

If $V_{\rm eff}$ becomes negative much before the Planck scale

- If M_h is close to the measured central value, Higgs inflation is not possible and $V_{\rm eff}$ becomes negative much before $M_{\rm Pl}$

- *If so, is this evidence for new physics?*
  Yes, in the sense that at least the inflaton seems to be missing in the SM

Rate of quantum tunnelling

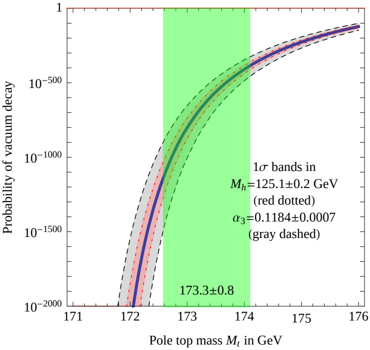
It is given by the probability of nucleating a bubble of true VEV in a spacetime volume $dV\,dt$ *[Kobzarev, Okun,Voloshin (1975); Coleman (1977); Callan, Coleman (1977)]*

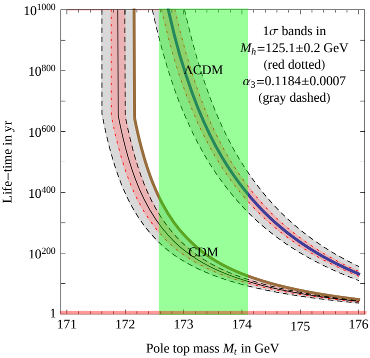
$$d\wp = dt\,dV\;\Lambda_B^4\,e^{-S(\Lambda_B)}\,.$$

$S(\Lambda_B) \equiv$ the action of the bounce of size $R = \Lambda_B^{-1}$, given by

$$S(\Lambda_B) = \frac{8\pi^2}{3|\lambda(\Lambda_B)|}$$

Vacuum life-time



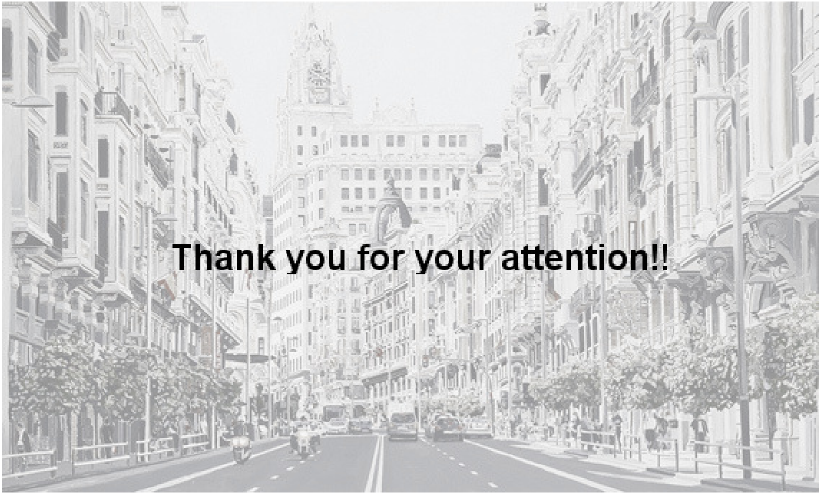


Left: *The probability that EW vacuum decay happened in our past light-cone, taking into account the expansion of the universe.*

Right: The life-time of the EW VEV, with two different assumptions for future cosmology: universes dominated by the cosmological constant (ΛCDM) or by dark matter (CDM) ...

Conclusions

- We have presented the stability bound at full next-to-next-to-leading order
- Comparing the result obtained with the experimental values of the relevant parameters we have found some tension, which we have quantified (2.8σ)
- Data vaguely indicate that the EW VEV is metastable (the life-time is $>$ than the age of the universe) and that Higgs inflation is not possible
- Absolute stability, however, is not excluded now as the measured M_h and M_t are close to the bound once the uncertainties are taken into account
- The works we have discussed call for a better determination of M_t and $\alpha_3(M_Z)$
- We appear to live very close to the boundary between stability and metastability (*near-criticality*)



Thank you for your attention!!

Extra slides

Step 1: effective potential

RG-improved tree level potential (V): classical potential with couplings replaced by the running ones

One loop (V_1): $V_{\rm eff}$ depends mainly on the top, W, Z, Higgs and Goldstone squared masses in the classical background ϕ: in the Landau gauge ... they are

$$t \equiv \frac{y_t^2\phi^2}{2}, \quad w \equiv \frac{g_2^2\phi^2}{4}, \quad z \equiv \frac{(g_2^2+3g_1^2/5)\phi^2}{4}, \quad h \equiv 3\lambda\phi^2 - m^2, \quad g \equiv \lambda\phi^2 - m^2$$

$\rightarrow (4\pi)^2 V_1$ is (in the $\overline{\rm MS}$ scheme)

$$\frac{3w^2}{2}\left(\ln\frac{w}{\mu^2}-\frac{5}{6}\right)+\frac{3z^2}{4}\left(\ln\frac{z}{\mu^2}-\frac{5}{6}\right)-3t^2\left(\ln\frac{t}{\mu^2}-\frac{3}{2}\right)+\frac{h^2}{4}\left(\ln\frac{h}{\mu^2}-\frac{3}{2}\right)+\frac{3g^2}{4}\left(\ln\frac{g}{\mu^2}-\frac{3}{2}\right)$$

In order to keep the logarithms in the effective potential small we choose

$$\mu = \phi$$

Indeed, t, w, z, h and g are $\propto \phi^2$ for $\phi \gg m$

Two loop (V_2): is very complicated, but always depend on t, w, z, h, g plus g_i

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Step 2: running couplings

For a generic coupling θ we write the RGE as

$$\frac{d\theta}{d\ln\mu^2} = \frac{\beta_\theta^{(1)}}{(4\pi)^2} + \frac{\beta_\theta^{(2)}}{(4\pi)^4} + \dots$$

They were computed before in the literature up to three loops

(very long and not very illuminating expressions at three loops)

One loop RGEs for λ, y_t^2, g_i^2 and m^2

$$\begin{aligned}
\beta_\lambda^{(1)} &= \lambda\left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) - 3y_t^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2g_1^2}{40}, \\
\beta_{y_t^2}^{(1)} &= y_t^2\left(\frac{9y_t^2}{2} - 8g_3^2 - \frac{9g_2^2}{4} - \frac{17g_1^2}{20}\right), \\
\beta_{g_1^2}^{(1)} &= \frac{41}{10}g_1^4, \quad \beta_{g_2^2}^{(1)} = -\frac{19}{6}g_2^4, \quad \beta_{g_3^2}^{(1)} = -7g_3^4, \\
\beta_{m^2}^{(1)} &= m^2\left(6\lambda + 3y_t^2 - \frac{9g_2^2}{4} - \frac{9g_1^2}{20}\right)
\end{aligned}$$

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Step 3: threshold corrections

$$
\begin{aligned}
\lambda(M_t) &= 0.12604 + 0.00206\left(\frac{M_h}{\mathrm{GeV}} - 125.15\right) - 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) \pm 0.00030_{\mathrm{th}} \\
\frac{m(M_t)}{\mathrm{GeV}} &= 131.55 + 0.94\left(\frac{M_h}{\mathrm{GeV}} - 125.15\right) + 0.17\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) \pm 0.15_{\mathrm{th}} \\
y_t(M_t) &= 0.93690 + 0.00556\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) - 0.00042\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.00050_{\mathrm{th}} \\
g_2(M_t) &= 0.64779 + 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) + 0.00011\,\frac{M_W - 80.384\,\mathrm{GeV}}{0.014\,\mathrm{GeV}} \\
g_Y(M_t) &= 0.35830 + 0.00011\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) - 0.00020\,\frac{M_W - 80.384\,\mathrm{GeV}}{0.014\,\mathrm{GeV}} \\
g_3(M_t) &= 1.1666 + 0.00314\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} - 0.00046\left(\frac{M_t}{\mathrm{GeV}} - 173.35\right)
\end{aligned}
$$

The theoretical uncertainties on the quantities are much lower than those used in previous determinations of the stability bound

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