

The Non-Linear Realization for a Composite Higgs

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work in collaboration with I. Brivio, M.B. Gavela, L. Merlo and S. Rigolin

Nature-approved QFT principles
or:
Where do (light) bosons come from?



Gauge Principle

Photon, Gluons (massless vector bosons)



Nambu-Goldstone Theorem

Pions, Kaons (massless* scalars)

Nature-approved QFT principles
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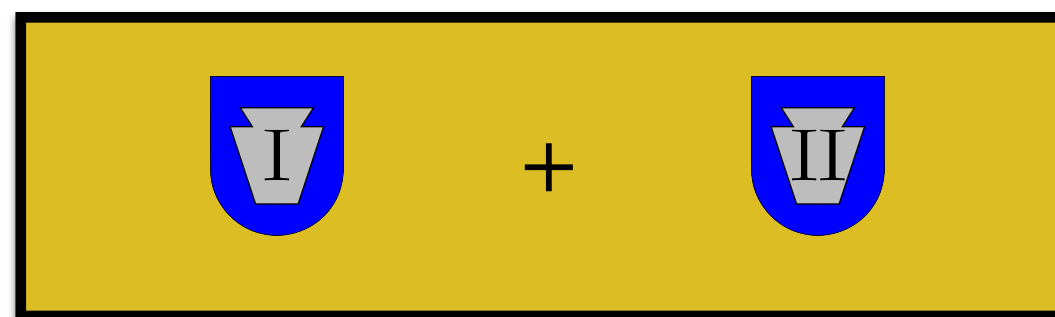
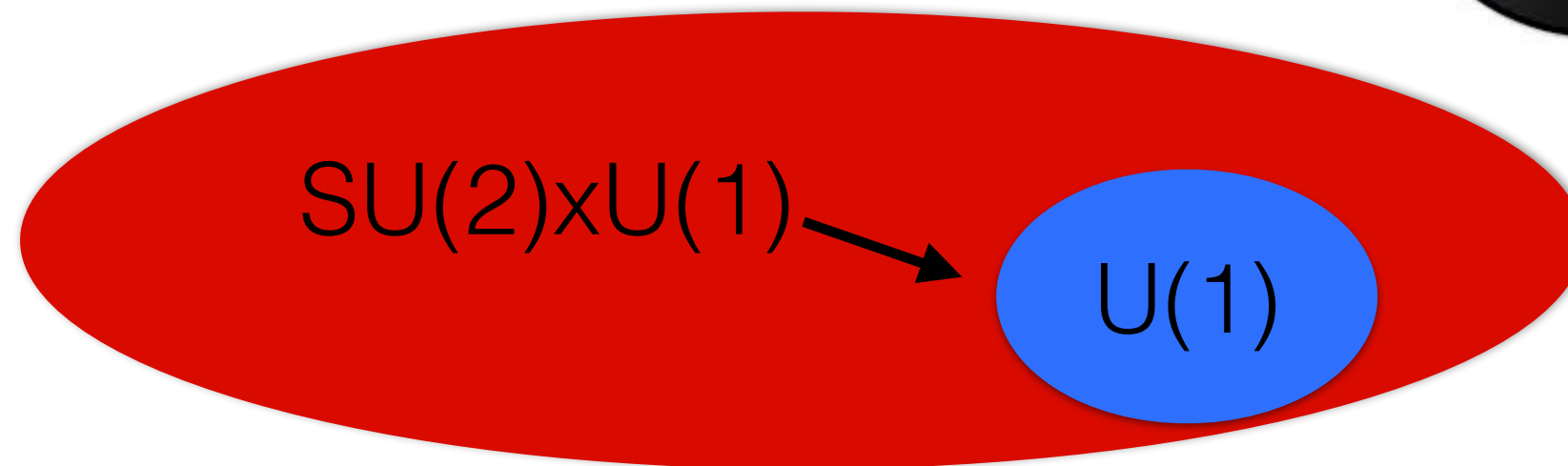
Nambu-Goldstone Theorem

Pions, Kaons (massless* scalars)

we'll stay lorentz invariant
(see Murayama's talk otherwise)

Real life example:

Electroweak Gauge Group



Massless + Massless = Massive

3 Goldstone Bosons Eaten by W,Z

Building Lagrangians for Goldstones

[à la Callan, Coleman, Wess & Zumino]

$$\mathcal{G} \rightarrow \mathcal{H}$$

$$\mathcal{G} = \mathcal{H} \oplus \mathcal{G}/\mathcal{H}$$

Generators: $\{T\}, \{X\}$

NG Bosons live in the broken group \mathcal{G}/\mathcal{H} :

$$\Omega(x) = e^{i \Xi(x)^a X_a / f}$$

and transform according to the group composition law

general element of the whole group

$$\Omega \rightarrow \mathfrak{g} \Omega = \mathfrak{b} \mathfrak{h} \Omega \mathfrak{h}^{-1} \mathfrak{h} \equiv \Omega' \mathfrak{h}'$$

transformation in the broken piece

transformation in the unbroken subgroup

$$\Omega'(x) = \mathfrak{g} \Omega(x) \mathfrak{h}^{-1}$$

Building Lagrangians for Goldstones

If, on top, the coset is symmetric, i.e.

$$\text{Grading: } \mathcal{R} : \begin{cases} T_a \rightarrow +T_a \\ X_{\hat{a}} \rightarrow -X_{\hat{a}} \end{cases}$$

$$\Sigma(x) \equiv \Omega(x)^2, \quad \text{trade omega for sigma}$$

$$\Sigma'(x) = \mathfrak{g} \Sigma(x) \mathfrak{g}_{\mathcal{R}}^{-1}$$

Lagrangian:

$$\mathcal{L} = \frac{f^2}{4} \text{Tr} (D_\mu \Sigma D^\mu \Sigma^\dagger) + \mathcal{L}^{(p^4)} + \dots$$

The expansion is
in momentum!

$$\frac{E}{f}$$

Electroweak NG bosons

$$\mathcal{G} = SU(2)_L \times U(1)_Y \quad \mathcal{H} = U(1)_{em}$$

$$\mathbf{U}(x) = e^{i\pi^a(x)\sigma_a/v}$$

$$\mathbf{U}(x) \rightarrow L \mathbf{U}(x) R^\dagger, \quad \text{custodial } SU(2) \times SU(2)$$

$$D_\mu \mathbf{U} \equiv \partial_\mu \mathbf{U} + \frac{ig}{2} W_\mu^a \sigma_a \mathbf{U} - \frac{ig'}{2} B_\mu(x) \mathbf{U} \sigma_3,$$

$$\mathbf{T}(x) \equiv \mathbf{U}(x) \sigma_3 \mathbf{U}^\dagger(x), \quad \text{Singlets } U(1)!!$$

$$\mathbf{V}_\mu(x) \equiv (\mathbf{D}_\mu \mathbf{U}(x)) \mathbf{U}^\dagger(x), \quad \text{Triplets } SU(2)$$

Electroweak NG bosons

$$\mathcal{G} = SU(2)_L \times U(1)_Y \quad \mathcal{H} = U(1)_{em}$$

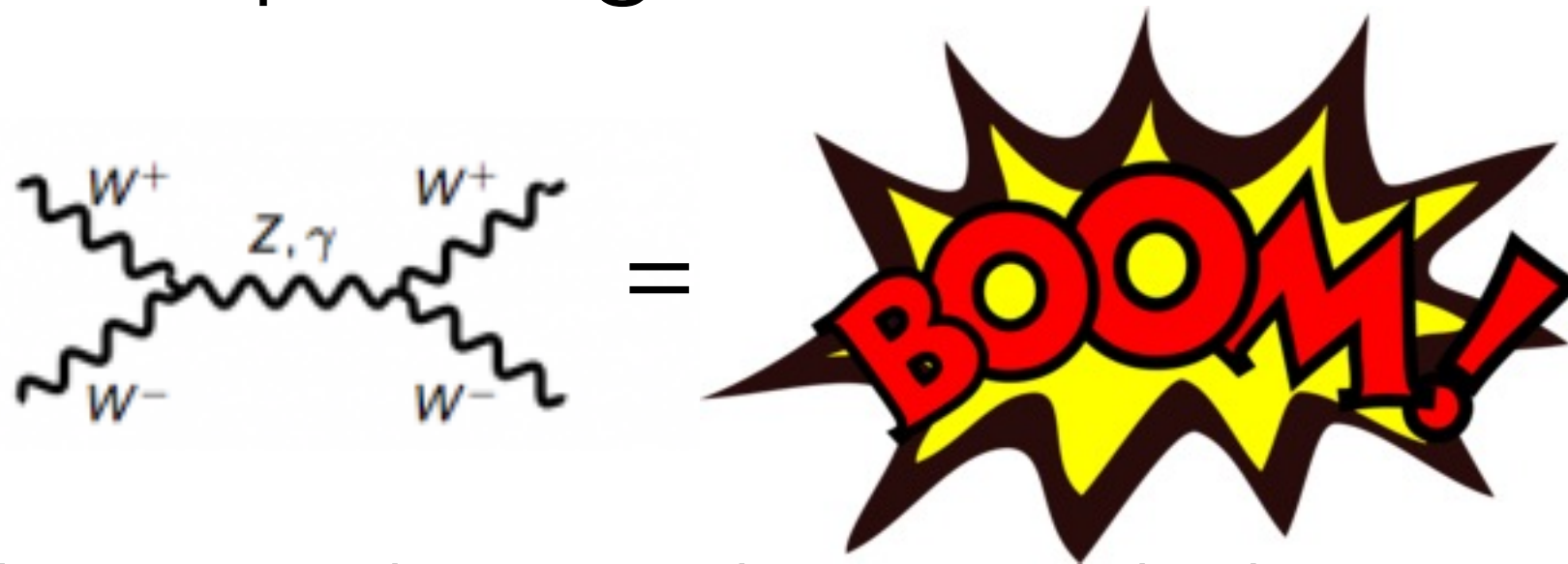
Lagrangian:

$$\begin{aligned} \mathcal{L} &= -\frac{v^2}{4} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) + c_T \frac{v^2}{4} \text{Tr}(\mathbf{V}_\mu \mathbf{T}) \text{Tr}(\mathbf{V}^\mu \mathbf{T}) + \mathcal{L}^{(p^4)} + \dots \\ &= \frac{g^2 v^2}{4} \left(W_\mu^+ W_\mu^- + \left(\frac{1}{2c_W^2} - \frac{c_T}{c_W^2} \right) Z^\mu Z_\mu \right) + \dots \end{aligned}$$

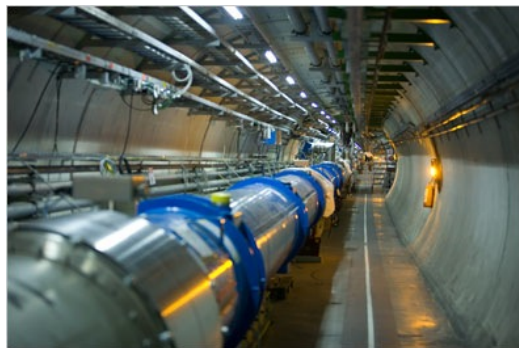
$\mathcal{L}^{(p^4)} \longrightarrow$ Appelquist Longhitano Feruglio Basis
 $(\text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu))^2$, $(\text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu))^2$...

...One Serious Problem

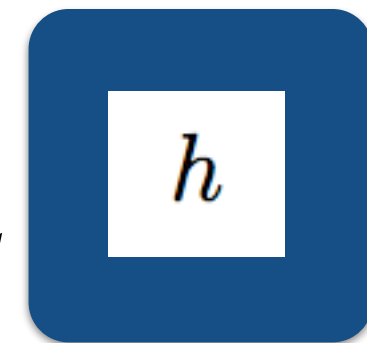
Exploding cross sections



Add some particle to paliate non-unitarity



→ maybe scalar singlet?



Chiral Higgs Lagrangian:

$$\mathcal{L} = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{v^2}{4} \text{Tr} (\mathbf{V}_\mu \mathbf{V}^\mu) \mathcal{F} \left(\frac{h}{v} \right) + \dots$$

O(p⁴): Gauge-Scalar Basis

{R.A., Gavela, Merlo, Rigolin, Yepes}

$$\mathcal{P}_W = -\frac{1}{2} \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}) \mathcal{F}_W(h)$$

$$\mathcal{P}_B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \mathcal{F}_B(h)$$

$$\mathcal{P}_1 = gg' B_{\mu\nu} \text{Tr}(\mathbf{T} \mathbf{W}^{\mu\nu})$$

$$\mathcal{P}_{14} = g \epsilon_{\mu\nu\rho\lambda} \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \text{Tr}(\mathbf{V}^\nu \mathbf{W}^{\rho\lambda})$$

$$\mathcal{P}_2 = ig' B_{\mu\nu} \text{Tr}(\mathbf{T} [\mathbf{V}^\mu, \mathbf{V}^\nu])$$

$$\mathcal{P}_{15} = \text{Tr}(\mathbf{T} \mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T} \mathcal{D}_\nu \mathbf{V}^\nu)$$

$$\mathcal{P}_3 = ig' \text{Tr}(\mathbf{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu])$$

$$\mathcal{P}_{16} = \text{Tr}([\mathbf{T}, \mathbf{V}_\nu] \mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T} \mathbf{V}^\nu)$$

$$\mathcal{P}_4 = ig' B_{\mu\nu} \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \partial^\nu (h/v)$$

$$\mathcal{P}_{17} = ig' \text{Tr}(\mathbf{T} \mathbf{W}_{\mu\nu}) \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \partial^\nu (h/v)$$

⋮

⋮

$$\dots \quad \mathcal{P}_{26} = (\text{Tr}(\mathbf{T} \mathbf{V}_\mu) \text{Tr}(\mathbf{T} \mathbf{V}_\nu))^2.$$

O(p⁴): Higgs-Derivatives Basis

$$\mathcal{P}_{\square H} = \frac{1}{v^2} (\partial_\mu \partial^\mu h)^2,$$

$$\mathcal{P}_{\Delta H} = \frac{1}{v^3} (\partial_\mu h)^2 \square h,$$

$$\mathcal{P}_{DH} = \frac{1}{v^4} ((\partial_\mu h)(\partial^\mu h))^2.$$

O(p⁴): Gauge-Scalar Basis

$$\mathcal{P}_W = -\frac{1}{2} \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}) \mathcal{F}_W(h)$$

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$$\mathcal{P}_4 = ig' B_{\mu\nu} \text{Tr}(\mathbf{T} \mathbf{V}^\mu \mathbf{V}^\nu)$$

For the pheno see Brivio's talk

{Brivio, Corbett, Éboli, Gavela, Gonzalez-Fraile, Gonzalez-Garcia, Merlo, Rigolin,}

CP odd operators

{Gavela, Gonzalez-Fraile, Gonzalez-Garcia, Merlo, Rigolin, Yepes}

Completeness check 1 loop

{Gavela, Kanshin, Machado, Saa}

NLO estimate and fermions

{Buchalla, Cata, Krause}

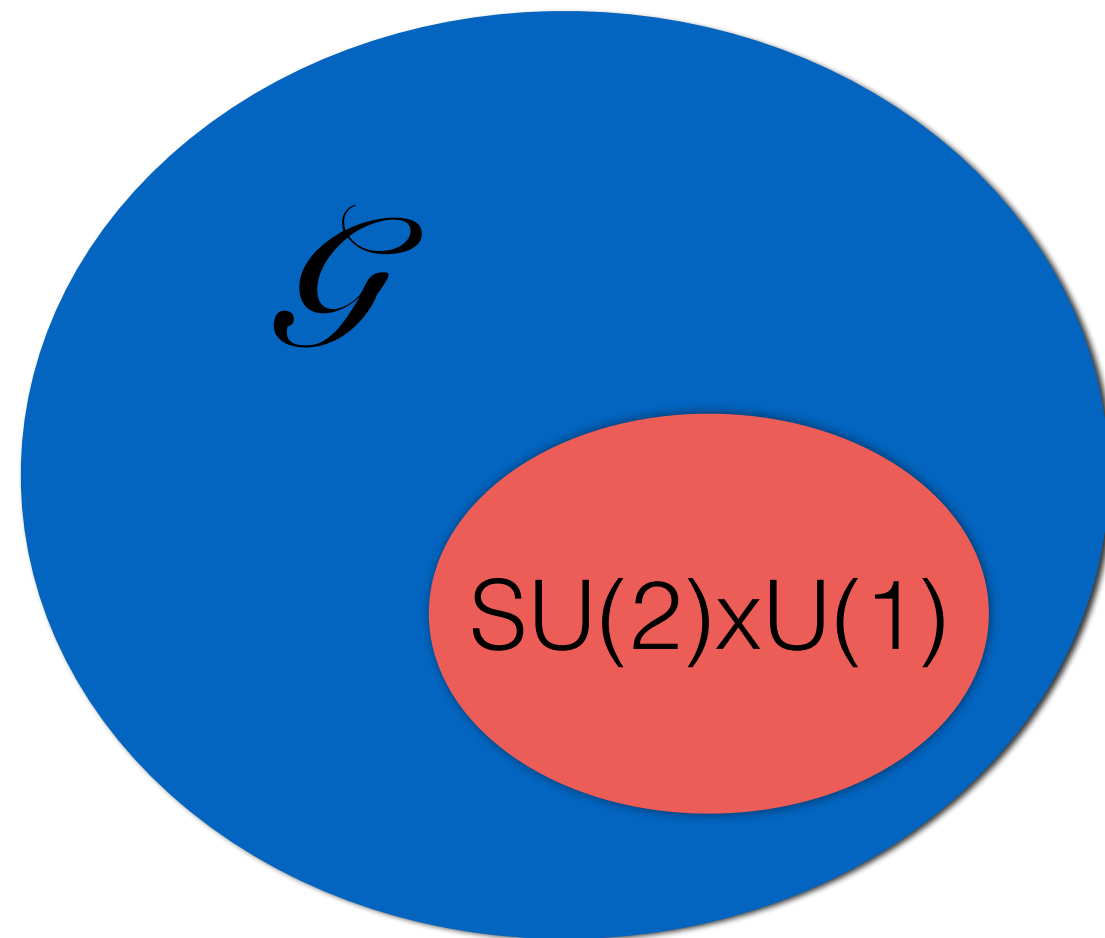
$$\mathcal{P}_{\square H} = \frac{1}{v^2} (\partial_\mu \partial^\mu h)^2,$$

$$\mathcal{P}_{\Delta H} = \frac{1}{v^3} (\partial_\mu h)^2 \square h,$$

$$\mathcal{P}_{DH} = \frac{1}{v^4} ((\partial_\mu h)(\partial^\mu h))^2.$$

But where does the Higgs singlet come from?

Use Goldstone theorem again



and assume the 4 scalar d.o.f. come in a ***SU(2)-doublet***

this scenario decouples

$$\xi = \frac{v^2}{f^2}$$

We already know how to build the Lagrangian for these models...

dim \mathcal{G}/\mathcal{H} be at least 4

$$\Sigma(x) \equiv \Omega(x)^2,$$

$$\tilde{V}_\mu = (\partial_\mu \Sigma) \Sigma^{-1},$$

in $\mathcal{L}^{(p^2)}$ only one term

$$-\frac{f^2}{4} \text{Tr} \left(\tilde{V}_\mu \tilde{V}^\mu \right),$$

in $\mathcal{L}^{(p^4)}$ ten...

$$\tilde{\mathcal{A}}_{B\Sigma} = g'^2 \text{Tr} \left(\Sigma \tilde{\mathbf{B}}_{\mu\nu} \Sigma^{-1} \tilde{\mathbf{B}}^{\mu\nu} \right),$$

$$\tilde{\mathcal{A}}_{W\Sigma} = g^2 \text{Tr} \left(\Sigma \tilde{\mathbf{W}}_{\mu\nu} \Sigma^{-1} \tilde{\mathbf{W}}^{\mu\nu} \right),$$

$$\tilde{\mathcal{A}}_1 = g g' \text{Tr} \left(\Sigma \tilde{\mathbf{B}}_{\mu\nu} \Sigma^{-1} \tilde{\mathbf{W}}^{\mu\nu} \right),$$

$$\tilde{\mathcal{A}}_2 = i g' \text{Tr} \left(\tilde{\mathbf{B}}_{\mu\nu} \left[\tilde{\mathbf{V}}^\mu, \tilde{\mathbf{V}}^\nu \right] \right),$$

$$\tilde{\mathcal{A}}_3 = i g \text{Tr} \left(\tilde{\mathbf{W}}_{\mu\nu} \left[\tilde{\mathbf{V}}^\mu, \tilde{\mathbf{V}}^\nu \right] \right),$$

$$\tilde{\mathcal{A}}_4 = \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}^\mu \right) \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}^\mu \right),$$

$$\tilde{\mathcal{A}}_5 = \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}_\nu \right) \text{Tr} \left(\tilde{\mathbf{V}}^\mu \tilde{\mathbf{V}}^\nu \right),$$

$$\tilde{\mathcal{A}}_6 = \text{Tr} \left((\mathcal{D}_\mu \tilde{\mathbf{V}}^\mu)^2 \right),$$

$$\tilde{\mathcal{A}}_7 = \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}^\mu \tilde{\mathbf{V}}_\nu \tilde{\mathbf{V}}^\nu \right),$$

$$\tilde{\mathcal{A}}_8 = \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}_\nu \tilde{\mathbf{V}}^\mu \tilde{\mathbf{V}}^\nu \right),$$

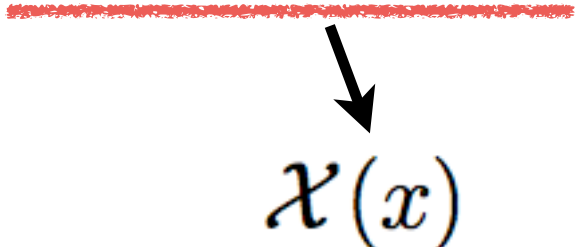
but the matter content is the same in the LE and HE bases:
we can write this Lagrangian in terms of the Chiral Lag.

Counting parameters **10 vs 31**

an example: SU(3)

$$\mathcal{G} = SU(3) \quad \mathcal{H} = SU(2)_L \times U(1)_Y$$

$$\Xi^a(x)X_a = \begin{pmatrix} 0 & 0 & \varphi_1(x) \\ 0 & 0 & \varphi_2(x) \\ \varphi_1^*(x) & \varphi_2^*(x) & 0 \end{pmatrix} = \varphi(x) \begin{pmatrix} 0 & \mathbf{U}(x)e_2 \\ (\mathbf{U}(x)e_2)^\dagger & 0 \end{pmatrix}$$


 $\mathcal{X}(x)$

$$\Sigma = e^{i\Xi^a X_a / f} = \mathbb{1} + i \sin\left(\frac{\varphi}{f}\right) \mathcal{X} + \left(\cos\left(\frac{\varphi}{f}\right) - 1\right) \mathcal{X}^2.$$

generic for a symmetric coset? $[T, X] \propto X$, $[X, X] \propto T$,

an example: SU(3)

$$\mathcal{G} = SU(3) \quad \mathcal{H} = SU(2)_L \times U(1)_Y$$

$$\begin{aligned} \mathcal{L} &= \frac{f^2}{4} \text{Tr} (D_\mu \Sigma D^\mu \Sigma^\dagger) + \mathcal{L}^{(p^4)} + \dots \\ &= \frac{1}{2} \partial_\mu h \partial^\mu h - f^2 \sin^2 \left(\frac{\varphi}{2f} \right) \text{Tr} (\mathbf{V}_\mu \mathbf{V}^\mu) \\ &\quad + \frac{f^2}{2} \sin^4 \left(\frac{\varphi}{2f} \right) \text{Tr} (\mathbf{V}_\mu \mathbf{T}) \text{Tr} (\mathbf{V}^\mu \mathbf{T}) + \mathcal{L}^{(p^4)} + \dots \end{aligned}$$

$$\frac{v^2}{4} = f^2 \sin^2 \left(\frac{\langle \varphi \rangle}{2f} \right) \quad \text{Trade } v \text{ for } \langle \varphi \rangle$$

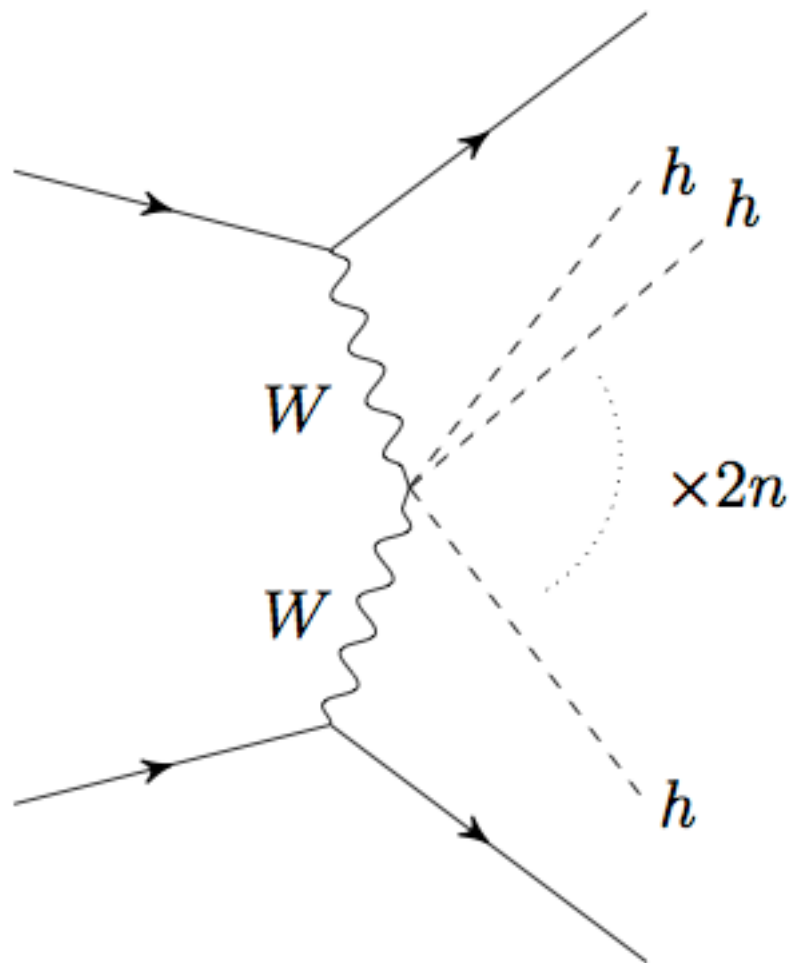
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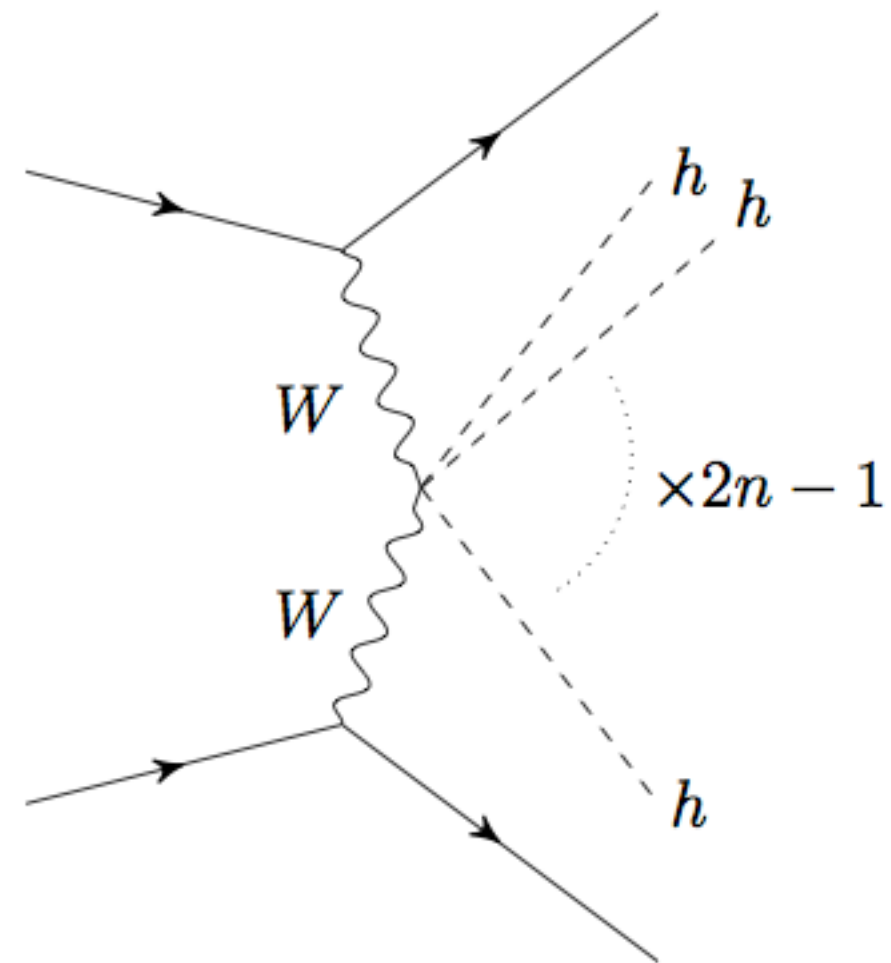
$$\begin{aligned} \mathcal{L} &= \frac{f^2}{4} \text{Tr} (D_\mu \Sigma D^\mu \Sigma^\dagger) + \mathcal{L}^{(p^4)} + \dots \\ &= \frac{1}{2} \partial_\mu h \partial^\mu h - f^2 \sin^2 \left(\frac{\varphi}{2f} \right) \text{Tr} (\mathbf{V}_\mu \mathbf{V}^\mu) \\ &\quad + \frac{f^2}{2} \sin^4 \left(\frac{\varphi}{2f} \right) \text{Tr} (\mathbf{V}_\mu \mathbf{T}) \text{Tr} (\mathbf{V}^\mu \mathbf{T}) + \mathcal{L}^{(p^4)} + \dots \\ \sin \left(\frac{\varphi}{2f} \right) &= \frac{v}{2f} \cos \left(\frac{h}{2f} \right) + \sqrt{1 - \frac{v^2}{4f^2}} \sin \left(\frac{h}{2f} \right), \end{aligned}$$

an example: SU(3)

$$\mathcal{G} = SU(3) \quad \mathcal{H} = SU(2)_L \times U(1)_Y$$



$$\sim \frac{(-1)^n}{f^{2n}} \left(\frac{v^2}{4} - \frac{f^2}{2} \right)$$



$$\sim \frac{(-1)^{n-1} v}{f^{2n-1}} \frac{1}{2} \sqrt{f^2 - \frac{v^2}{4}}$$

Unsuppressed higgs emission

an example: SU(3)

$$\mathcal{G} = SU(3) \quad \mathcal{H} = SU(2)_L \times U(1)_Y$$

$$\begin{aligned} \mathcal{L} &= \frac{f^2}{4} \text{Tr} (D_\mu \Sigma D^\mu \Sigma^\dagger) + \mathcal{L}^{(p^4)} + \dots \\ &= \frac{1}{2} \partial_\mu h \partial^\mu h - f^2 \sin^2 \left(\frac{\varphi}{2f} \right) \text{Tr} (\mathbf{V}_\mu \mathbf{V}^\mu) \\ &\quad + \frac{f^2}{2} \sin^4 \left(\frac{\varphi}{2f} \right) \text{Tr} (\mathbf{V}_\mu \mathbf{T}) \text{Tr} (\mathbf{V}^\mu \mathbf{T}) \end{aligned}$$

$$f^2 \left(\frac{\varphi^2}{4f^2} + \mathcal{O} \left(\frac{\varphi^2}{4f^2} \right) \right)$$

$$\left[H = U \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \right]$$

$$D^\mu H^\dagger D_\mu H$$

SM

an example: SU(3)

$$\mathcal{G} = SU(3) \quad \mathcal{H} = SU(2)_L \times U(1)_Y$$

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rho parameter

$$\alpha_{\text{em}} \Delta T = \frac{\xi}{4} \implies f \gtrsim 2 \text{ TeV}.$$

Other (Custodial-preserving) models

Georgi Kaplan
Little Higgs

$$\mathcal{G} = SU(5) \quad \mathcal{H} = SO(5)$$

(Get rid of 10 NGB)

Minimal
Composite
Higgs

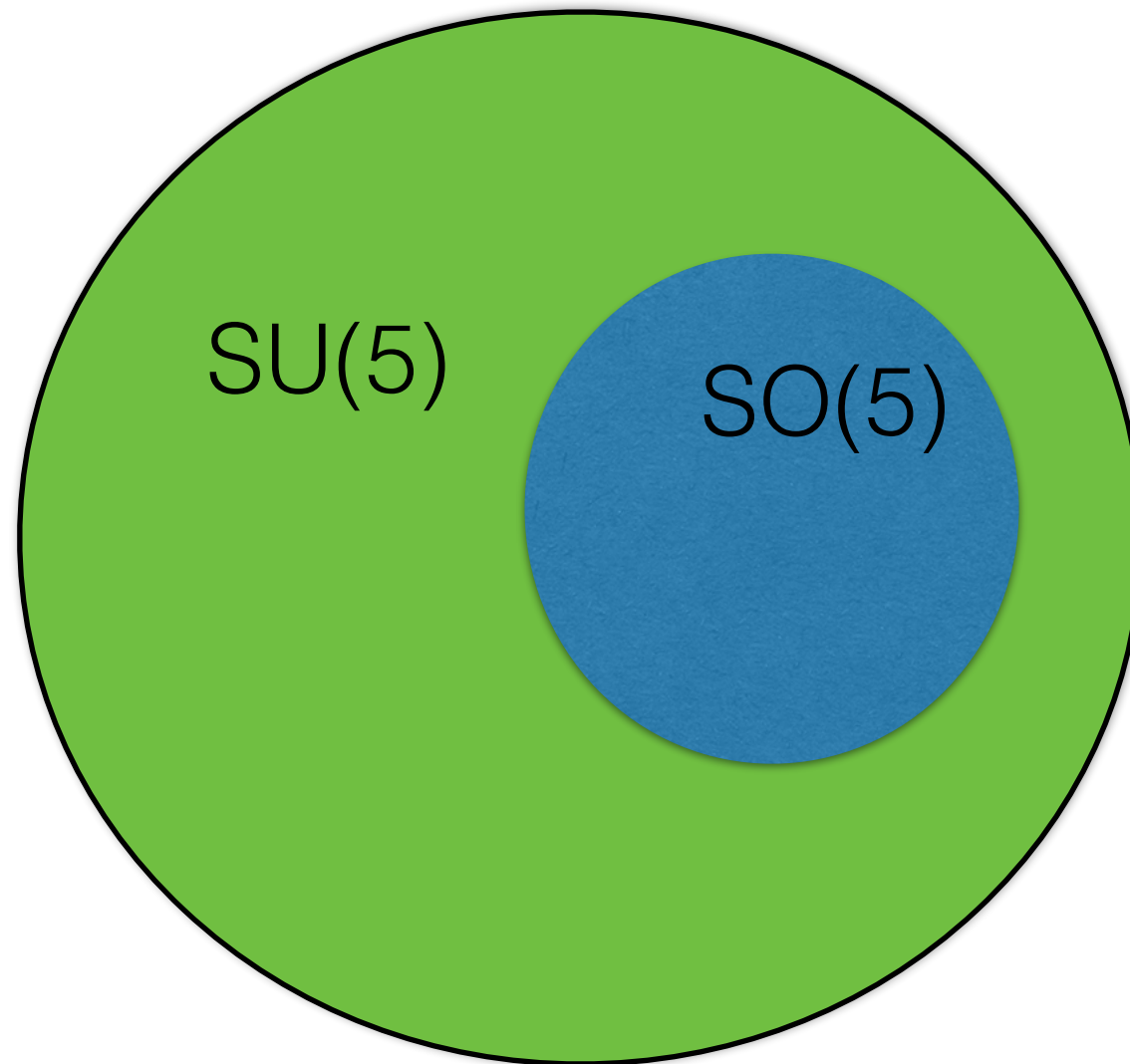
$$\mathcal{G} = SO(5) \quad \mathcal{H} = SO(4)$$

[Pomarol, Agashe, Contino]

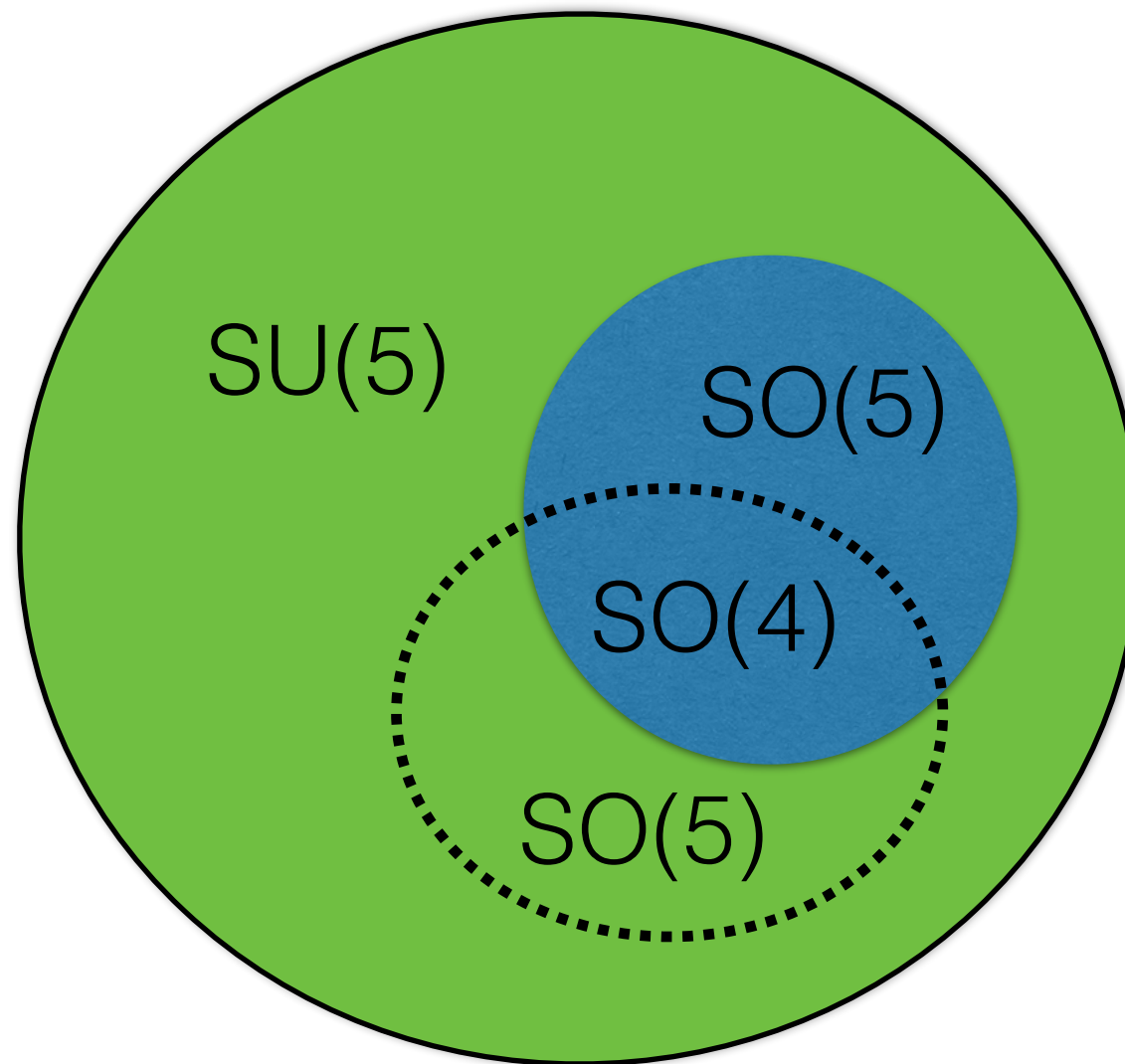
Model Table

$c_i \mathcal{F}_i(h)$	$SU(5)/SO(5)$ $SO(5)/SO(4)$	$SU(3)/SU(2) \times U(1)$
$c_1 \mathcal{F}_1(h)$	$\tilde{c}_1 \sin^2 \frac{\varphi}{2f}$	$\frac{\tilde{c}_1}{4} \sin^2 \frac{\varphi}{f}$
$c_2 \mathcal{F}_2(h)$	$\tilde{c}_2 \sin^2 \frac{\varphi}{2f}$	$\frac{\tilde{c}_2}{4} \sin^2 \frac{\varphi}{f}$
$c_3 \mathcal{F}_3(h)$	$2\tilde{c}_3 \sin^2 \frac{\varphi}{2f}$	$\frac{\tilde{c}_3}{2} \sin^2 \frac{\varphi}{f}$
$c_4 \mathcal{F}_4(h)$	$\tilde{c}_2 \sqrt{\xi} \sin \frac{\varphi}{f}$	$\frac{\tilde{c}_2}{2} \sqrt{\xi} \sin \frac{2\varphi}{f}$
$c_5 \mathcal{F}_5(h)$	$-2\tilde{c}_3 \sqrt{\xi} \sin \frac{\varphi}{f}$	$-2\tilde{c}_3 \sqrt{\xi} \sin \frac{\varphi}{f}$
$c_6 \mathcal{F}_6(h)$	$16\tilde{c}_4 \sin^4 \frac{\varphi}{2f} - \frac{1}{2}\tilde{c}_6 \sin^2 \frac{\varphi}{f}$	$8(2\tilde{c}_4 + \tilde{c}_7) \sin^4 \frac{\varphi}{2f} - \frac{1}{2}\tilde{c}_6 \sin^2 \frac{\varphi}{f}$
$c_7 \mathcal{F}_7(h)$	$-2\tilde{c}_6 \sqrt{\xi} \sin \frac{\varphi}{f}$	$-2\tilde{c}_6 \sqrt{\xi} \sin \frac{\varphi}{f}$
\vdots	\vdots	\vdots

"SU(5)" and "SO(5)" equivalence



"SU(5)" and "SO(5)" equivalence



Choose 4 generators in $\mathfrak{g}/\mathfrak{h}$, commute them and commute the commutators

if its a doublet...

Linear

$$(\mathbf{D}_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (\mathbf{D}_\nu \Phi) =$$

$$B_{\mu\nu} \text{Tr}(\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \frac{(v+h)^2}{4} + B_{\mu\nu} \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \frac{(v+h)^2}{4}$$

or Non-Linear

$$i g' \text{Tr} \left(\tilde{\mathbf{B}}_{\mu\nu} \left[\tilde{\mathbf{V}}^\mu, \tilde{\mathbf{V}}^\nu \right] \right) =$$

$$B_{\mu\nu} \text{Tr}(\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \sin^2 \left[\frac{\varphi}{2f} \right] + B_{\mu\nu} \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \sin^2 \left[\frac{\varphi}{2f} \right]$$

same relative factor for any number of Higgs legs

Summary

- The Higgs Chiral Lagrangian is the less prejudiced scenario (I know)
- If the Higgs Doublet is a set of pNG bosons correlations appear that tell apart this class of theories
- Do all such models have a universal functional dependence in the Higgs singlet?