RGE of $d = 6$ Operators in SM EFT

Elizabeth Jenkins

Department of Physics
University of California, San Diego

Madrid HEFT2014, September 29, 2014

Papers

- C. Grojean, E.E. Jenkins, A.V. Manohar and M. Trott, "Renormalization Group Scaling of Higgs Operators and $h \to \gamma\gamma$ Decay," JHEP 1304:016 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, "On Gauge Invariance and Minimal Coupling," JHEP 1309:063 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, "Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and λ Dependence," JHEP 1310:087 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, "Naive Dimensional Analysis Counting of Gauge Theory Amplitudes and Anomalous Dimensions," Phys. Lett. B726 (2013) 697-702.
- E.E. Jenkins, A.V. Manohar and M. Trott, "Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence," JHEP 1401:035 (2014).
- R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, "Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology," JHEP 1404:159 (2014).
- R. Alonso, E.E. Jenkins and A.V. Manohar, "Holomorphy without Supersymmetry in the Standard Model Effective Field Theory," arXiV:1409.0868 [hep-ph].

Generalization of the SM to SM EFT

- Discovery of Higgs boson h with mass 126 GeV at LHC and no other experimental signal of new physics up to energies $\Lambda \sim 1$ TeV implies that SM gives good description of nature up to high energies Λ.
- If no new particles below Λ, it is possible to study effect of arbitrary new physics at high energy Λ on electroweak scale observables by generalizing SM theory to SM EFT containing Lagrangian terms with $d > 4$. Low-energy fields of SM EFT are SM fields.

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^{d-4}} \sum_i C_i O_i^{(d)}$$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^{2}} \sum_i C_i O_i^{(6)} + \cdots$$

Generalization of the SM to SM EFT

- Leading higher dimension operators are $d = 6$.
- Assuming B and L conservation, there are 59 independent dimension-six operators which form complete basis of $d = 6$ operators.
- 59 operators divided into eight operator classes.

$$1 : X^3 \quad 2 : H^6 \quad 3 : H^4 D^2 \quad 4 : X^2 H^2 \quad 5 : \psi^2 H^3$$

$$6 : \psi^2 XH \quad 7 : \psi^2 H^2 D \quad 8 : \psi^4$$

$$X = G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \qquad \psi = q, l, u, d, e$$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

Dimension Six Operators

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

Dimension Six Operators

$8 : (\bar{L}L)(\bar{L}L)$		$8 : (\bar{R}R)(\bar{R}R)$		$8 : (\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8 : (\bar{L}R)(\bar{R}L) + \text{h.c.}$		$8 : (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

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Dimension Six Operators

$$1 : X^3 \quad 2 : H^6 \quad 3 : H^4 D^2 \quad 4 : X^2 H^2 \quad 5 : \psi^2 H^3$$
$$6 : \psi^2 XH \quad 7 : \psi^2 H^2 D \quad 8 : \psi^4$$

- Operators in classes 1,2,3,4 are flavor singlet
- Operators in classes 5,6,7 have two flavor indices p, r
- Operators in class 8 have four flavor indices p, r, s, t

$n_g = 1, 2, 3 \Rightarrow$ number of $d = 6$ operator coefficients is large!

Counting of Coefficients

Class	N_{op}	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_g(9n_g-7)$	1	30
8 : $(\bar{L}L)(\bar{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$\frac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
8 : $(\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
8 : $(\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
8 : $(\bar{L}R)(\bar{R}L)$	1	n_g^4	1	81	n_g^4	1	81
8 : $(\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac{1}{8}(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

Table : Number of CP-even and CP-odd coefficients in $\mathcal{L}^{(6)}$ for n_g flavors. The total number of coefficients is $(107n_g^4+2n_g^3+135n_g^2+60)/4$, which is 76 for $n_g=1$ and 2499 for $n_g=3$.

SM EFT

- Higher dimensional operators generated at scale $\mu = \Lambda$ by integrating out new physics particles. Analysis model independent.

$$\mathcal{L}^{(6)} = \sum_i C_i(\Lambda) O_i^{(6)}(\Lambda) = \sum_i C_i(m_h) O_i^{(6)}(m_h)$$

- Matrix elements of operators taken at $\mu = m_h$ (electroweak scale) for electroweak observables.
- RG evolution of dimension-six operator coefficients produces interesting mixing. Need to do RG evolution for accurate measurements of $h \to \gamma\gamma$, $h \to Z\gamma$, $gg \to h$.
- We have computed full one-loop anomalous dimension matrix γ_{ij} for dimension-six operators!

Structure of Anomalous Dimension Matrix

- Rescale Operators according to Naive Dimensional Analysis: $X \to gX$, and chirality flip operators scaled by Yukawa coupling y: $\psi^2 H^3 \to y\psi^2 H^3$, $\psi^2 XH \to gy\psi^2 XH$
- Anomalous dimension entries

$$\hat{\gamma} \propto \left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda} \left(\frac{y^2}{16\pi^2}\right)^{n_y} \left(\frac{g^2}{16\pi^2}\right)^{n_g}, \qquad N = n_\lambda + n_y + n_g$$

$$N = 1 + w_i - w_j$$

w	operators
2	H^6
1	$H^4 D^2,\ y\psi^2 H^3,\ \psi^2 H^2 D,\ \psi^4$
0	$g^2 X^2 H^2,\ gy\psi^2 XH$
-1	$g^3 X^3$

	$\{2\}$	$\{3,5,7,8\}$	$\{4,6\}$	$\{1\}$
$\{2\}$	1	2	3	4
$\{3,5,7,8\}$	0	1	2	3
$\{4,6\}$	-1	0	1	2
$\{1\}$	-2	-1	0	1

Anomalous Dimension Matrix

	H^6	H^4D^2	$y\psi^2H^3$	ψ^2H^2D	ψ^4	$g^2X^2H^2$	$gy\psi^2XH$	g^3X^3
Class	2	3	5	7	8	4	6	1
NDA Weight	2	1	1	1	1	0	0	-1
H^6	λ, y^2, g^2	$\lambda^2, \lambda g^2, g^4$	$\lambda y^2, y^4$	$\lambda y^2, \lambda g^2, \cancel{y^4}$	0	$\lambda g^4, g^6$	0	$\cancel{\lambda g^6}$
H^4D^2	0	λ, y^2, g^2	$\cancel{y^2}$	y^2, g^2	0	$\cancel{g^4}$	$\cancel{y^2 g^2}$	$\cancel{g^6}$
$y\psi^2H^3$	0	λ, y^2, g^2	λ, y^2, g^2	λ, y^2, g^2	λ, y^2	g^4	$\cancel{g^2\lambda}, g^4, g^2y^2$	$\cancel{g^6}$
ψ^2H^2D	0	g^2, y^2	$\cancel{y^2}$	$g^2, \cancel{\lambda}, y^2$	g^2, y^2	$\cancel{g^4}$	$\cancel{g^2 y^2}$	$\cancel{g^6}$
ψ^4	0	0	0	g^2, y^2	g^2, y^2	0	g^2y^2	$\cancel{g^6}$
$g^2X^2H^2$	0	$\cancel{1}$	0	$\cancel{1}$	0	λ, y^2, g^2	y^2	g^4
$gy\psi^2XH$	0	0	$\cancel{1}$	$\cancel{1}$	1	g^2	g^2, y^2	g^4
g^3X^3	0	0	0	0	0	$\cancel{1}$	0	g^2

Table : Form of the one-loop anomalous dimension matrix $\hat{\gamma}_{ij}$ for dimension-six operators $\hat{Q}_i$ rescaled according to naive dimensional analysis. The operators are ordered by NDA weight, rather than by operator class. The possible entries allowed by the one-loop Feynman graphs are shown. The cross-hatched entries vanish.

RGE

- RG running of SM parameters due to $d = 6$ operators. All corrections calculated.
- RG running of $d = 6$ operators. All corrections calculated. Formulae are many pages.
- RG mixing of $d = 6$ operators gives nontrivial implications for flavor structure of BSM physics.
- $d = 6$ operators affect interpretation of SM parameters at tree level.

Holomorphy

- Observation: 1-loop anomalous dimension matrix respects holomorphy to a large extent.

Recall $d = 6$ Operator Classes

$$1 : X^3 \quad 2 : H^6 \quad 3 : H^4 D^2 \quad 4 : X^2 H^2 \quad 5 : \psi^2 H^3$$

$$6 : \psi^2 XH \quad 7 : \psi^2 H^2 D \quad 8 : \psi^4$$

Divide $d = 6$ Operators into Holomorphic, Antiholomorphic and Non-Holomorphic Operators

$$X^{\pm}_{\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} \mp i\widetilde{X}_{\mu\nu}\right), \qquad \widetilde{X}^{\pm}_{\mu\nu} = \pm i X^{\pm}_{\mu\nu},$$

$$\widetilde{X}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} X^{\alpha\beta}/2 \qquad \widetilde{\widetilde{X}}_{\mu\nu} = -X_{\mu\nu}$$

$$X \to X^{\pm}$$

$$\psi \to L, R$$

Holomorphy

Definition

The holomorphic part of the Lagrangian, $\mathcal{L}_{\mathfrak{h}}$, is the Lagrangian constructed from the fields X^+, R, $\overline{L}$, but none of their hermitian conjugates.

Holomorphy

$$6 : \psi^2 XH + \text{h.c.}$$

$$\frac{i}{2}\epsilon^{\alpha\beta\mu\nu}\sigma_{\mu\nu}P_R = -\sigma^{\alpha\beta}P_R$$

$$Q_{RX} = \left(\overline{L}\sigma^{\mu\nu}R\right) X^{\mu\nu}\ H = \left(\overline{L}\sigma^{\mu\nu}R\right) X^{+\mu\nu}\ H,$$
$$Q_{RX}^{\dagger} = \left(\overline{R}\sigma^{\mu\nu}L\right) X^{\mu\nu}\ H = \left(\overline{R}\sigma^{\mu\nu}L\right) X^{-\mu\nu}\ H,$$

$$\mathcal{L}^{d=6} \supset C_{RX}Q_{RX} + C_{RX}^{*}Q_{RX}^{\dagger}$$

$$6 : Q_{RX},\ Q_{RX}^{\dagger}\ \rightarrow\ Q_{RX},\ Q_{RX}^{\dagger}$$

Holomorphy

$$1 : X^3$$

$$Q_X = f^{ABC} X_\mu^{A\nu} X_\nu^{B\rho} X_\rho^{C\mu},$$
$$Q_{\widetilde{X}} = f^{ABC} \widetilde{X}_\mu^{A\nu} X_\nu^{B\rho} X_\rho^{C\mu},$$

$$Q_{X,+} \equiv \frac{1}{2}\left(Q_X - iQ_{\widetilde{X}}\right) = f^{ABC} X_\mu^{+A\nu} X_\nu^{+B\rho} X_\rho^{+C\mu}$$
$$Q_{X,-} \equiv \frac{1}{2}\left(Q_X + iQ_{\widetilde{X}}\right) = f^{ABC} X_\mu^{-A\nu} X_\nu^{-B\rho} X_\rho^{-C\mu}$$

$$\mathcal{L}^{d=6} \supset C_X Q_X + C_{\widetilde{X}} Q_{\widetilde{X}} = C_{X,+} Q_{X,+} + C_{X,-} Q_{X,-}$$
$$C_{X,\pm} \equiv \left(C_X \pm iC_{\widetilde{X}}\right)$$

$$1 : Q_X,\ Q_{\widetilde{X}} \ \rightarrow\ Q_{X,+},\ Q_{X,-}$$

Holomorphy

$$4 : X^2 H^2$$

$$Q_{HX} = X_{\mu\nu} X^{\mu\nu} \; H^\dagger H,$$
$$Q_{H\widetilde{X}} = X_{\mu\nu} \widetilde{X}^{\mu\nu} \; H^\dagger H,$$

$$Q_{HX,+} \equiv {X^+}^2 \; H^\dagger H = \frac{1}{4} \left(X - i\widetilde{X} \right)^2 \; H^\dagger H$$
$$Q_{HX,-} \equiv {X^-}^2 \; H^\dagger H = \frac{1}{4} \left(X + i\widetilde{X} \right)^2 \; H^\dagger H$$

$$\mathcal{L}^{d=6} \supset C_{HX} Q_{HX} + C_{H\widetilde{X}} Q_{H\widetilde{X}} = C_{HX,+} Q_{HX,+} + C_{HX,-} Q_{HX,-}$$
$$C_{HX,\pm} \equiv \left(C_{HX} \pm i C_{H\widetilde{X}} \right)$$

$$4 : Q_{HX}, \; Q_{H\widetilde{X}} \;\to\; Q_{HX,+}, \; Q_{HX,-}$$

Holomorphy

$$8 : \psi^4 = (\overline{L}R)\,(\overline{L}R)\,,(\overline{R}L)\,(\overline{R}L)$$

$$Q_{LRLR'} = \left(\overline{L}^j R\right)\epsilon_{jk}\left(\overline{L}^k R'\right),$$

$$Q^\dagger_{LRLR'}$$

$$\mathcal{L}^{d=6} \supset C_{LRLR'}Q_{LRLR'} + C^*_{LRLR'}Q^\dagger_{LRLR'}$$

$$8 : Q_{LRLR'},\ Q^\dagger_{LRLR'} \ \rightarrow\ Q_{LRLR'},\ Q^\dagger_{LRLR'}$$

Holomorphic + *Antiholomorphic*

Holomorphy

$$8 : \psi^4 = \left(\overline{L}R\right)\left(\overline{R}L\right)$$

$$Q_{ledq} = \left(\overline{l}^j e\right)\left(\overline{d} q_j\right) ,$$

$$Q^\dagger_{ledq}$$

$$\mathcal{L}^{d=6} \supset C_{ledq} Q_{ledq} + C^*_{ledq} Q^\dagger_{ledq}$$

$$8 : Q_{ledq},\ Q^\dagger_{ledq} \ \rightarrow\ Q_{ledq},\ Q^\dagger_{ledq}$$

Non – holomorphic

Holomorphy

$$8 : \psi^4 = JJ = (\overline{L}\gamma^\mu L)(\overline{L}\gamma_\mu L),\ (\overline{R}\gamma^\mu R)(\overline{R}\gamma_\mu R),\ (\overline{L}\gamma^\mu L)(\overline{R}\gamma^\mu R)$$

$$2 : H^6 = \left(H^\dagger H\right)^3$$

$$3 : H^4 D^2 = \left(H^\dagger H\right)\Box\left(H^\dagger H\right),\ \left(H^\dagger D_\mu H\right)^*\left(H^\dagger D_\mu H\right)$$

$$\mathrm{Self-Hermitian}$$

$$\mathit{Non-holomorphic}$$

Holomorphy

$$5 : \psi^2 H^3 + \text{h.c.}$$

$$Q_{RH} = \left(H^\dagger H\right) (\overline{L}R)\, H,$$
$$Q_{RH}^\dagger = \left(H^\dagger H\right) (\overline{R}L)\, H^\dagger$$

$$\mathcal{L}^{d=6} \supset C_{RH} Q_{RH} + C_{RH}^* Q_{RH}^\dagger$$

Non – holomorphic

Holomorphy

$$7 : \psi^2 H^2 D$$

$$Q_{HL} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right) \left(\overline{L}\gamma^\mu L\right),$$
$$Q_{HR} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right) \left(\overline{R}\gamma^\mu R\right)$$

$$Q_{Hud} = i \left(\widetilde{H}^\dagger D_\mu H\right) \left(\overline{u}\gamma^\mu d\right),$$
$$Q^\dagger_{Hud} = i \left(\widetilde{H}^\dagger D_\mu H\right) \left(\overline{d}\gamma^\mu u\right)$$

$$\mathrm{Self-Hermitian\ (except\ Q_{Hud})}$$

$$\mathit{Non-holomorphic}$$

Holomorphy

$$\mathcal{L}^{d=6} = \mathcal{L}_{\mathfrak{h}} + \mathcal{L}_{\overline{\mathfrak{h}}} + \mathcal{L}_{\mathfrak{n}} = C_{\mathfrak{h}} Q_{\mathfrak{h}} + C_{\overline{\mathfrak{h}}} Q_{\overline{\mathfrak{h}}} + C_{\mathfrak{n}} Q_{\mathfrak{n}}$$

$$Q_{\mathfrak{h}} \subset \left\{ X^{+3},\ X^{+2} H^2,\ (\overline{L}\sigma^{\mu\nu} R)\ X^+ H,\ (\overline{L}R)(\overline{L}R) \right\}$$

$$Q_{\overline{\mathfrak{h}}} \subset \left\{ X^{-3},\ X^{-2} H^2,\ (\overline{R}\sigma^{\mu\nu} L)\ X^- H,\ (\overline{R}L)(\overline{R}L) \right\}$$

$$Q_{\mathfrak{n}} \subset \left\{ H^6,\ H^4 D^2,\ \psi^2 H^3,\ \psi^2 H^2 D,\ (\overline{L}R)(\overline{R}L),\ JJ \right\}$$

Definition

The holomorphic part of the Lagrangian, $\mathcal{L}_{\mathfrak{h}}$, is the Lagrangian constructed from the fields X^+, R, $\overline{L}$, but none of their hermitian conjugates.

Holomorphy

$$\dot{C}_i \equiv 16\pi^2 \mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_i = \sum_{j=\mathfrak{h},\bar{\mathfrak{h}},\mathfrak{n}} \gamma_{ij} C_j \,, \qquad i = \mathfrak{h}, \bar{\mathfrak{h}}, \mathfrak{n}$$

$$\begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}} & \gamma_{\mathfrak{h}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{h}\mathfrak{n}} \\ \gamma_{\bar{\mathfrak{h}}\mathfrak{h}} & \gamma_{\bar{\mathfrak{h}}\bar{\mathfrak{h}}} & \gamma_{\bar{\mathfrak{h}}\mathfrak{n}} \\ \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix} = \begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}} & 0 & 0 \\ 0 & \gamma^*_{\mathfrak{h}\mathfrak{h}} & 0 \\ \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma^*_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix}$$

$$\begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}} & \gamma_{\mathfrak{h}\mathfrak{n}} \\ \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix} = \begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}} & 0 \\ \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix}$$

$$\dot{C}_{\mathfrak{h}} = \sum_{j=\mathfrak{h}} \gamma_{\mathfrak{h}j} C_j$$

Holomorphy

	$(X^+)^3$	$(X^+)^2H^2$	ψ^2X^+H	$(\bar{L}R)(\bar{L}R)$	$(\bar{L}R)(\bar{R}L)$	JJ	ψ^2H^3	H^6	H^4D^2	ψ^2H^2D
$(X^+)^3$	$\mathfrak{h}$	$\to 0$	0	0	0	0	0	0	0	0
$(X^+)^2H^2$	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}$	0	0	$\nexists$	0	0	$\to 0$	$\to 0$
ψ^2X^+H	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}_F$	$\to 0$	$\to 0$	$\to 0$	0	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{L}R)$	$\to 0$	$\nexists$	$\mathfrak{h}_F$	$\mathfrak{h}_F$	$Y_u^\dagger Y_{e,d}^\dagger$	$Y_u^\dagger Y_{e,d}^\dagger$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{R}L)$	$\to 0$	$\nexists$	$\to 0$	$Y_uY_d,\ Y_u^\dagger Y_e^\dagger$	$\mathfrak{h}_F$	$*$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
JJ	$\to 0$	$\nexists$	$\to 0$	$Y_uY_{e,d}$	$*$	$*$	$\nexists$	$\nexists$	$\nexists$	$*$
ψ^2H^3	$\to 0$	$Y_{u,d,e}^\dagger$	$\mathfrak{h}$	$\mathfrak{h}$	$*$	$*$	$*$	$\nexists$	$*$	$*$
H^6	$\to 0$	$*$	$\nexists$	$\nexists$	$\nexists$	$\nexists$	$*$	$*$	$*$	$*$
H^4D^2	$\to 0$	$\to 0$	$\to 0$	$\nexists$	$\nexists$	$\nexists$	$\to 0$	$\nexists$	$*$	$*$
ψ^2H^2D	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$*$	$\to 0$	$\nexists$	$*$	$*$

Table : Form of the one-loop anomalous dimension matrix for $d = 6$ operators. Y is a Yukawa coupling. The first 4 rows and columns involve holomorphic operators (and their conjugates), and the rest involve non-holomorphic operators. The RGE for the rows can depend on the C of each column, or their conjugates. Entries which must vanish by NDA are denoted by 0, those for which there is no one-loop diagram (after taking equations of motion into account) are denoted by $\nexists$, and those which vanish by explicit computation are denoted by $\to 0$. Entries with $\mathfrak{h}$ are non-zero, and satisfy holomorphy, i.e. they depend on C but not C^*. Entries with $\mathfrak{h}_F$ satisfy holomorphy because anti-holomorphic contributions are forbidden by NDA and flavor symmetry. Entries with $*$ are non-zero, but not holomorphic. The red entries violate holomorphy.

Summary

- Complete RGE of dimension-six operators of SM EFT computed for first time.
- Contribution of dimension-six operators to running of SM parameters calculated for first time.
- RG evolution of dimension-six operators important for Higgs boson production $gg \to h$ and decay $h \to \gamma\gamma$ and $h \to Z\gamma$, which occur at one loop in SM.
- Significant constraints on flavor structure of SM EFT. Test of MFV hypothesis.
- Approximate holomorphy of 1-loop anomalous dimension matrix of dimension-six operators.