AN ASYMPTOTIC SOLUTION OF LARGE-N QCD, AND OF N=1 SUSY YM

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Solving QCD at large-N is a long standing difficult problem, actually a dream ...

An easier problem is to solve it only asymptotically in the UV

In a sense we already have an asymptotic solution:
It is standard perturbation theory

But solving the large-N theory, even only asymptotically, is much more interesting:

This solution would replace QCD as a theory of gluons and quarks, that is strongly coupled in the infrared in perturbation theory, with a theory of glueballs and mesons that is weakly coupled at all scales

We have found an asymptotic solution of massless QCD at large-N in a sense specified later, by a new purely field-theoretical method, based on fundamental principles, that we call the Asymptotically-Free Bootstrap

It expresses uniquely 2 and 3 point correlators of any spin (explicitly for lower spins) in terms of glueball and meson propagators, in such a way that the result is asymptotic in the UV to RG-improved perturbation theory

It extends to certain primitive r-point correlators and S-matrix amplitudes to all 1/N orders

To say it in a nutshell, the asymptotically free bootstrap is a version of the conformal bootstrap, that is based on the

OPE + conformal symmetry

suitably modified to take into account

OPE + asymptotic freedom in the UV

as opposed to the conformal symmetry,
but combined with the

Kallen-Lehmann representation
in the large-N limit of confining asymptotically free theories

Of course the asymptotically free bootstrap leads to much weaker results than the conformal bootstrap

In fact, the asymptotically-free bootstrap constructs asymptotic universality classes of massive local gauge-invariant correlators in the UV, for large-N confining asymptotically-free gauge theories with no mass scale in perturbation theory, admitting a large-N limit of 't Hooft type

(for example also large-N n=1 SUSY YM)
but it does not provide spectral information

To get spectral information we must lift the asymptotic solution to an actual solution. We will discuss ideas in this direction, based on a completely new type of topological strings, in the second part of the talk

Implications of the asymptotic solution

First and foremost, an asymptotic solution of this kind is a guide to find out an actual solution by other methods, either field theoretical or string theoretical

Besides, it gives us a rigorous asymptotic estimate for the two-point (scalar) correlator that controls the mass gap in large-N Yang-Mills, as we will see

Moreover, it provides an easy way to understand how good or bad approximate solutions proposed in the past are and to check forthcoming proposed exact solutions (easy because based only on fundamental principles of field theory)

Yet, the most fundamental consequence of the asymptotically-free bootstrap is the explicit structure of the asymptotic S-matrix in certain sectors

This puts the strongest constraints on any (string ?) solution for the S-matrix of large-N QCD and of n=1 SUSY YM

so explicit, and so strong constraints, that we conjecture that they determine uniquely the large-N QCD S-matrix on the string side

as we will see in the second part of the talk

What makes possible the Asymptotically-Free Bootstrap is a recently-proved Asymptotic Theorem for large-N two-point correlators

M.B. Glueball and meson propagators of any spin in large-N QCD
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The large-N limit of (massless) SU(N) QCD:

$$Z = \int \delta A \delta \bar{\psi} \delta \psi \, e^{-\frac{N}{2g^2} \int \sum_{\alpha\beta} Tr\left(F^2_{\alpha\beta}\right) + i \sum_f \bar{\psi}_f \gamma^\alpha D_\alpha \psi_f \, d^4x}$$

(G. 't Hooft 1974)

The following remarkable simplifications occur

For example, in the pure glue sector:

$$< \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n) >_{conn} \sim N^{2-n}$$

thus at the leading 1/N order:

$$< \frac{1}{N}\sum_{\alpha\beta} trF^2_{\alpha\beta}(x_1)...\frac{1}{N}\sum_{\alpha\beta} trF^2_{\alpha\beta}(x_k) >=$$

$$< \frac{1}{N}\sum_{\alpha\beta} trF^2_{\alpha\beta}(x_1) > ... < \frac{1}{N}\sum_{\alpha\beta} trF^2_{\alpha\beta}(x_k) >$$

At next to leading 1/N order, because of the vanishing of the interaction associated to 3 and multi-point correlators,

two-point correlators, assuming confinement, are an infinite sum of free fields satisfying the the Kallen-Lehmann representation (A. Migdal, 1977):

$$\int \langle \mathcal{O}^{(s)}(x)\mathcal{O}^{(s)}(0)\rangle_{conn} e^{-ip\cdot x} d^4x = \sum_{n=1}^{\infty} P^{(s)}\left(\frac{p_\alpha}{m_n^{(s)}}\right) \frac{|<0|\mathcal{O}^{(s)}(0)|p,n,s>'|^2}{p^2+m_n^{(s)2}}$$

$$<0|\mathcal{O}^{(s)}(0)|p,n,s,j> = e_j^{(s)}\left(\frac{p_\alpha}{m}\right) <0|\mathcal{O}^{(s)}(0)|p,n,s>'$$

$$\sum_j e_j^{(s)}\left(\frac{p_\alpha}{m}\right)\overline{e_j^{(s)}\left(\frac{p_\alpha}{m}\right)} = P^{(s)}\left(\frac{p_\alpha}{m}\right)$$

But the large-N correlators have to match perturbation theory for large momentum, Migdal (1977), Polyakov book "Gauge Fields and Strings" (1987), actually RG-improved perturbation theory

What is the large momentum behavior of two-point correlators of any integer spin s in pure Yang-Mills, in QCD with massless quarks, in n=1 SUSY YM or in any asymptotically free gauge theory massless in perturbation theory ?

$$\int \langle \mathcal{O}^{(s)}(x)\mathcal{O}^{(s)}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x = ?$$

For example:

$$\mathcal{O}^{(s)} = Tr(F^2_{\alpha\beta}), \bar{\psi}\gamma^{\alpha}\psi, T_{\alpha\beta}, \cdots$$

The answer is simple but not completely trivial, as we will see momentarily. We have found it by standard methods:

Perturbation Theory +

Asymptotic Freedom +

Renormalization Group +

Some non-trivial subtlety . . .

$$\frac{\gamma_0}{\beta_0} \neq 0, 1$$

$$\int \langle \mathcal{O}^{(s)}(x) \mathcal{O}^{(s)}(0) \rangle_{conn} \, e^{-ip\cdot x} d^4x$$

$$\sim P^{(s)}\left(\frac{p_\alpha}{p}\right) p^{2D-4} \left[\frac{1}{\beta_0 \log(\frac{p^2}{\Lambda^2_{QCD}})} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log(\frac{p^2}{\Lambda^2_{QCD}})}{\log(\frac{p^2}{\Lambda^2_{QCD}})} + O\left(\frac{1}{\log(\frac{p^2}{\Lambda^2_{QCD}})}\right)\right)\right]^{\frac{\gamma_0}{\beta_0}-1}$$

up to a polynomial in momentum, i.e. a contact term, i.e. a distribution supported at x=0 in coordinate space (this is the first subtlety) that must be subtracted;

$P^{(s)}(\frac{p_\alpha}{p})$ is the projector obtained substituting $m^2 = -p^2$ in the massive projector of spin s $P^{(s)}(\frac{p_\alpha}{m})$ (this is the second subtlety)

For conserved currents:

$$\frac{\gamma_0}{\beta_0} = 0$$

$$\int \langle \mathcal{O}^{(s)}(x) \mathcal{O}^{(s)}(0) \rangle_{conn} \, e^{-ip\cdot x} d^4x$$

$$\sim P^{(s)}(\frac{p_\alpha}{p}) \, p^{2D-4} \log(\frac{p^2}{\Lambda_{QCD}^2}) \left(1 + O(\frac{\log\log(\frac{p^2}{\Lambda_{QCD}^2})}{\log(\frac{p^2}{\Lambda_{QCD}^2})}) \right)$$

up to contact terms. $P^{(s)}(\frac{p_\alpha}{p})$ is the projector obtained substituting $m^2 = -p^2$ in the massive projector of spin s $P^{(s)}(\frac{p_\alpha}{m})$

Definitions:

$$\gamma_{\mathcal{O}^{(s)}}(g) = -\frac{\partial \log Z^{(s)}}{\log \mu} = -\gamma_0 g^2 + \cdots$$

$$\beta(g) = \frac{\partial g}{\partial \log \mu} = -\beta_0 g^3 - \beta_1 g^5 + \cdots$$

Therefore, at the leading large-N order it must hold:

$$\sum_{n=1}^{\infty} P^{(s)}\big(\frac{p_\alpha}{m_n^{(s)}}\big)\,\frac{|<0|\mathcal{O}^{(s)}(0)|p,n,s>'|^2}{p^2+m_n^{(s)2}}$$

$$\sim P^{(s)}\big(\frac{p_\alpha}{p}\big)\,p^{2D-4}\left[\frac{1}{\beta_0\log(\frac{p^2}{\Lambda^2_{QCD}})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{p^2}{\Lambda^2_{QCD}})}{\log(\frac{p^2}{\Lambda^2_{QCD}})}+O\big(\frac{1}{\log(\frac{p^2}{\Lambda^2_{QCD}})}\big)\right)\right]^{\frac{\gamma_0}{\beta_0}-1}$$

up to contact terms

Fundamental question:

Which are the constraints on the residues and the poles that follow from this asymptotic equality ?

Oddly, neither Migdal, despite he is the father of the subject, nor other people found out any answer, with the exception of

J. Mondejar, A. Pineda [hep-th/0704.1417]
[hep-th/0803.3625]

who worked out some particular cases

In the case of Migdal because he followed a path different from his own fundamental idea, for deep reasons discussed later

The answer to the fundamental question, after 38 years,

is the following Asymptotic Theorem (M.B.):

$$\int \langle \mathcal{O}^{(s)}(x)\mathcal{O}^{(s)}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x \sim \sum_{n=1}^{\infty} P^{(s)}\big(\frac{p_\alpha}{m_n^{(s)}}\big)\frac{m_n^{(s)2D-4} Z_n^{(s)2}\rho_s^{-1}(m_n^{(s)2})}{p^2+m_n^{(s)2}}$$

$$= P^{(s)}\big(\frac{p_\alpha}{p}\big)\, p^{2D-4}\sum_{n=1}^{\infty}\frac{Z_n^{(s)2}\rho_s^{-1}(m_n^{(s)2})}{p^2+m_n^{(s)2}} + \cdots$$

$$\sim P^{(s)}\big(\frac{p_\alpha}{p}\big)\, p^{2D-4}\int_{m_1^{(s)2}}^{\infty}\frac{Z^{(s)2}(m)}{p^2+m^2}dm^2 + \cdots$$

$$\sim P^{(s)}\big(\frac{p_\alpha}{p}\big)\, p^{2D-4}\left[\frac{1}{\beta_0\log(\frac{p^2}{\Lambda^2_{QCD}})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{p^2}{\Lambda^2_{QCD}})}{\log(\frac{p^2}{\Lambda^2_{QCD}})}+O(\frac{1}{\log(\frac{p^2}{\Lambda^2_{QCD}})})\right)\right]^{\frac{\gamma_0}{\beta_0}-1}$$

$$\sum_{n=1}^{\infty} f(m_n^{(s)2}) \sim \int_1^{\infty} f(m_n^{(s)2})dn = \int_{m_1^{(s)2}}^{\infty} f(m^2)\rho_s(m^2)dm^2$$

$$Z_n^{(s)} \equiv Z^{(s)}(m_n^{(s)}) = \exp\int_{g(\mu)}^{g(m_n^{(s)})}\frac{\gamma_{\mathcal{O}^{(s)}}(g)}{\beta(g)}dg$$

$$Z_n^{(s)2} \sim \left[\frac{1}{\beta_0\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}}{\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}}+O(\frac{1}{\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}})\right)\right]^{\frac{\gamma_0}{\beta_0}}$$

$$\langle \mathcal{O}^{(s)}(x)\mathcal{O}^{(s)}(0)\rangle_{conn} \sim \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int P^{(s)}\left(\frac{p_\alpha}{m_n^{(s)}}\right) \frac{m_n^{(s)2D-4} Z_n^{(s)2} \rho_s^{-1}(m_n^{(s)2})}{p^2 + m_n^{(s)2}} e^{ip\cdot x} d^4p$$

Spin 1:

$$\int \langle \mathcal{O}^{(1)}_{\alpha}(x)\mathcal{O}^{(1)}_{\beta}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x$$

$$\sim \sum_{n=1}^{\infty} (\delta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_n^{(1)2}}) \frac{m_n^{(1)2D-4} Z_n^{(1)2} \rho_1^{-1}(m_n^{(1)2})}{p^2 + m_n^{(1)2}}$$

$$\sim p^{2D-4}(\delta_{\alpha\beta} - \frac{p_\alpha p_\beta}{p^2}) \sum_{n=1}^{\infty} \frac{Z_n^{(1)2} \rho_1^{-1}(m_n^{(1)2})}{p^2 + m_n^{(1)2}} + \cdots$$

Spin 2:

$$\int \langle \mathcal{O}^{(2)}_{\alpha\beta}(x)\mathcal{O}^{(2)}_{\gamma\delta}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x$$

$$\sim \sum_{n=1}^{\infty} \left[\frac{1}{2}\eta_{\alpha\gamma}(m_n^{(2)})\eta_{\beta\delta}(m_n^{(2)}) + \frac{1}{2}\eta_{\beta\gamma}(m_n^{(2)})\eta_{\alpha\delta}(m_n^{(2)}) - \frac{1}{3}\eta_{\alpha\beta}(m_n^{(2)})\eta_{\gamma\delta}(m_n^{(2)})\right] \frac{m_n^{(2)2D-4} Z_n^{(2)2} \rho_2^{-1}(m_n^{(2)2})}{p^2 + m_n^{(2)2}}$$

$$\sim p^{2D-4}\left[\frac{1}{2}\eta_{\alpha\gamma}(p)\eta_{\beta\delta}(p) + \frac{1}{2}\eta_{\beta\gamma}(p)\eta_{\alpha\delta}(p) - \frac{1}{3}\eta_{\alpha\beta}(p)\eta_{\gamma\delta}(p)\right] \sum_{n=1}^{\infty} \frac{Z_n^{(2)2} \rho_2^{-1}(m_n^{(2)2})}{p^2 + m_n^{(2)2}} + \cdots$$

$$\eta_{\alpha\beta}(m) = \delta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2} \qquad \eta_{\alpha\beta}(p) = \delta_{\alpha\beta} - \frac{p_\alpha p_\beta}{p^2}$$

Proof of the RG estimate in the coordinate representation using the fact that the operator O is multiplicatively renormalizable in the coordinate representation, because contact terms do not occur for x away from 0

$$\langle \mathcal{O}_D(x)\mathcal{O}_D(0)\rangle_{conn} \sim C_0(x^2)$$

$$\left(x_\alpha \frac{\partial}{\partial x_\alpha} + \beta(g)\frac{\partial}{\partial g} + 2(D + \gamma_{\mathcal{O}_D}(g))\right) C_0(x^2) = 0$$

$$C_0(x^2) = \frac{1}{x^{2D}}\, \mathcal{G}_0(g(x))\, Z^2_{\mathcal{O}_D}(x\mu, g(\mu))$$

$$\mathcal{G}(g(x)) \sim 1 + O(g^2(x))$$

$$C_0(x^2) \sim \tfrac{1}{x^{2D}}\, g(x)^{\frac{2\gamma_0(\mathcal{O}_D)}{\beta_0}}$$

$$\sim \frac{1}{x^{2D}} \left(\frac{1}{\beta_0 \log(\frac{z_0^2}{x^2\Lambda^2_{QCD}})} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log(\frac{z_0^2}{x^2\Lambda^2_{QCD}})}{\log(\frac{z_0^2}{x^2\Lambda^2_{QCD}})}\right)\right)^{\frac{\gamma_0}{\beta_0}}$$

Euler-MacLaurin formula, in order to extract the large-momentum asymptotics (Migdal, decades ago ...)

$$\sum_{k=k_1}^{\infty} G_k(p) = \int_{k_1}^{\infty} G_k(p) dk - \sum_{j=1}^{\infty} \frac{B_j}{j!} \left[\partial_k^{j-1} G_k(p) \right]_{k=k_1}$$

$$< O(x)O(0) >_{conn} = \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{R_n m_n^{2D-4} \rho^{-1}(m_n^2)}{p^2 + m_n^2} e^{ip\cdot x} d^4p$$

$$= \frac{1}{4\pi^2 x^2} \sum_{n=1}^{\infty} R_n m_n^{2D-4} \rho^{-1}(m_n^2) \sqrt{x^2 m_n^2} K_1(\sqrt{x^2 m_n^2})$$

$$\sim \frac{1}{4\pi^2 x^2} \int_1^{\infty} R_n m_n^{2D-4} \rho^{-1}(m_n^2) \sqrt{x^2 m_n^2} K_1(\sqrt{x^2 m_n^2}) dn$$

$$= \frac{1}{4\pi^2 x^2} \int_{m_1^2}^{\infty} R(m) m^{2D-4} \sqrt{x^2 m^2} K_1(\sqrt{x^2 m^2}) dm^2$$

$$= \frac{1}{4\pi^2 x^2} \int_{m_1^2 x^2}^{\infty} R(\frac{z}{x})(\frac{z^2}{x^2})^{D-2} z K_1(z) \frac{dz^2}{x^2}$$

$$= \frac{1}{4\pi^2 (x^2)^D} \int_{m_1^2 x^2}^{\infty} R(\frac{z}{x}) z^{2D-3} K_1(z) dz^2$$

$$z^2 = x^2 m^2$$

Proof of the Asymptotic theorem in the coordinate representation

$$\int_{m_1^2x^2}^{\infty} R(\frac{z}{x}) z^{2D-3} K_1(z) dz^2 \sim \left(\frac{1}{\beta_0 \log(\frac{1}{x^2\Lambda_{QCD}^2})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{1}{x^2\Lambda_{QCD}^2})}{\log(\frac{1}{x^2\Lambda_{QCD}^2})}\right)\right)^{\frac{\gamma_0}{\beta_0}}$$

$$\int_{m_1^2x^2}^{\infty} R(\frac{z}{x}) z^{2D-3} K_1(z) dz^2$$

$$\sim R(\frac{z_0}{x}) \int_0^{\infty} z^{2D-3} K_1(z) dz^2 \sim \left(\frac{1}{\beta_0 \log(\frac{1}{x^2\Lambda_{QCD}^2})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{1}{x^2\Lambda_{QCD}^2})}{\log(\frac{1}{x^2\Lambda_{QCD}^2})}\right)\right)^{\frac{\gamma_0}{\beta_0}}$$

$$R(\frac{z}{x}) \sim \left(\frac{1}{\beta_0 \log(\frac{z^2}{x^2\Lambda_{QCD}^2})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{z^2}{x^2\Lambda_{QCD}^2})}{\log(\frac{z^2}{x^2\Lambda_{QCD}^2})}\right)\right)^{\frac{\gamma_0}{\beta_0}}$$

$$\frac{1}{\log(\frac{z^2}{x^2\Lambda_{QCD}^2})} = \frac{1}{\log(\frac{z_0^2}{x^2\Lambda_{QCD}^2})}\left(1+\frac{\log(\frac{z^2}{z_0^2})}{\log(\frac{z_0^2}{x^2\Lambda_{QCD}^2})}\right)^{-1}$$

$$R(\frac{z_0}{x}) \sim Z_{\mathcal{O}}^2(x\mu, g(\mu))$$

Proof of the Asymptotic Theorem in momentum representation

$$\int \langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x = \sum_{n=1}^{\infty} \frac{R_n m_n^{2D-4} \rho^{-1}(m_n^2)}{p^2 + m_n^2}$$

$$m_n^{2D-4} = (-p^2 + p^2 + m_n^2)^{D-2}$$

$$\int \langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x = (-p^2)^{D-2} \sum_{n=1}^{\infty} \frac{R_n \rho^{-1}(m_n^2)}{p^2 + m_n^2} + \cdots$$

$$\sum_{n=1}^{\infty} P^{(s)}\Big(\frac{p_\alpha}{m_n^{(s)}}\Big)\frac{m_n^{(s)2D-4} Z_n^{(s)2}\rho_s^{-1}(m_n^{(s)2})}{p^2+m_n^{(s)2}}$$

$$= P^{(s)}\Big(\frac{p_\alpha}{p}\Big)\, p^{2D-4}\sum_{n=1}^{\infty}\frac{Z_n^{(s)2}\rho_s^{-1}(m_n^{(s)2})}{p^2+m_n^{(s)2}} + \cdots$$

$$m_n^{(s)2D-4} P^{(s)}\Big(\frac{p_\alpha}{m_n^{(s)}}\Big)$$

$$m_n^{2d} \to p^{2d}; -p^{2d}$$

$$-p^2 \to m_n^2$$

$$P^{(s)}\Big(\frac{p_\alpha}{m_n}\Big) \to P^{(s)}\Big(\frac{p_\alpha}{p}\Big)$$

Euler-McLaurin formula:

$$\sum_{k=k_1}^{\infty} G_k(p) = \int_{k_1}^{\infty} G_k(p) dk - \sum_{j=1}^{\infty} \frac{B_j}{j!} \left[\partial_k^{j-1} G_k(p) \right]_{k=k_1}$$

$$\sum_{n=1}^{\infty} \frac{R_n \rho^{-1}(m_n^2)}{p^2 + m_n^2}$$

$$\sim \int_1^{\infty} \frac{R_n \rho^{-1}(m_n^2)}{p^2 + m_n^2} dn$$

$$= \int_{m_1^2}^{\infty} \frac{R(m) \rho^{-1}(m^2)}{p^2 + m^2} \rho(m^2) dm^2$$

$$= \int_{m_1^2}^{\infty} \frac{R(m)}{p^2 + m^2} dm^2$$

$$\nu = \frac{p^2}{\Lambda_{QCD}^2}; k = \frac{m^2}{\Lambda_{QCD}^2}; K = \frac{\Lambda^2}{\Lambda_{QCD}^2}$$

$$\int_{m_1^2}^{\Lambda^2} \frac{R(m)}{p^2+m^2} dm^2 = Z^2_{\mathcal{O}}(p)\mathcal{G}_0(g(p))$$

$$\nu = \frac{p^2}{\Lambda^2_{QCD}}; k = \frac{m^2}{\Lambda^2_{QCD}}; K = \frac{\Lambda^2}{\Lambda^2_{QCD}}$$

$$\int_{k_1}^{K} \frac{R(\sqrt{k})}{\nu+k} dk = Z^2_{\mathcal{O}}(\sqrt{\nu})\mathcal{G}_0(g(\sqrt{\nu}))$$

$$\int_{k_1}^{K} \frac{R(\sqrt{k})}{\nu+k} dk = \left(\frac{1}{\beta_0 \log \nu}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log\log \nu}{\log \nu}\right)\right)^{\frac{\gamma_0}{\beta_0}-1}$$

This is an integral equation of Fredholm type, for which a solution exists if and only if it is unique:

$$R(\sqrt{k}) \sim Z^2(\sqrt{k}) \sim \left(\frac{1}{\beta_0 \log \frac{k}{c}}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log\log \frac{k}{c}}{\log \frac{k}{c}}\right)\right)^{\frac{\gamma_0}{\beta_0}}$$

$$\gamma' = \frac{\gamma_0}{\beta_0}$$

$$\beta_0^{-\gamma'} \int_1^\infty \left(\frac{1}{\log(\frac{k}{c})} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log(\frac{k}{c})}{\log(\frac{k}{c})} \right) \right)^{\gamma'} \frac{dk}{k+\nu}$$

$$\sim \frac{1}{\gamma'-1} \beta_0^{-\gamma'} \left(\log \frac{1+\nu}{c} \right)^{-\gamma'+1} - \frac{\beta_1}{\beta_0^2} \beta_0^{-\gamma'} \left(\log(\frac{1+\nu}{c}) \right)^{-\gamma'} \log\log(\frac{1+\nu}{c})$$

$$= \frac{\beta_0^{-\gamma'}}{\gamma'-1} \left(\log \frac{1+\nu}{c} \right)^{-\gamma'+1} \left[1 - \frac{\beta_1(\gamma'-1)}{\beta_0^2} \left(\log(\frac{1+\nu}{c}) \right)^{-1} \log\log(\frac{1+\nu}{c}) \right]$$

$$\sim \frac{1}{\beta_0(\gamma'-1)} \left(\beta_0 \log \frac{1+\nu}{c} \right)^{-\gamma'+1} \left[1 - \frac{\beta_1}{\beta_0^2} \left(\log(\frac{1+\nu}{c}) \right)^{-1} \log\log(\frac{1+\nu}{c}) \right]^{\gamma'-1}$$

$$\sim \left(\frac{1}{\beta_0 \log \nu} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log \nu}{\log \nu} \right) \right)^{\gamma'-1}$$

It agrees with naive RG estimate in momentum representation, assuming the operator O to be multiplicatively renormalizable, that is technically false

$$\int \langle \mathcal{O}_D(x)\mathcal{O}_D(0)\rangle_{conn} e^{-ip\cdot x} d^4x \sim C_0(p^2)$$

$$\left(p_\alpha \frac{\partial}{\partial p_\alpha} - \beta(g)\frac{\partial}{\partial g} - 2(D-2+\gamma_{\mathcal{O}_D}(g))\right) C_0(p^2) = 0$$

$$C_0(p^2) = p^{2D-4}\,\mathcal{G}_0(g(p))\, Z^2_{\mathcal{O}_D}(\frac{p}{\mu}, g(\mu))$$

$$\mathcal{G}(g(p)) \sim \log\frac{p^2}{\Lambda^2_{QCD}} \sim \frac{1}{g^2(p)}$$

$$\int p^{2D-4}\log\frac{p^2}{\mu^2} e^{ipx} d^4p \sim \frac{1}{x^{2D}}$$

$$C_0(p^2) \sim p^{2D-4} g(p)^{\frac{2\gamma_0(\mathcal{O}_D)}{\beta_0}-2}$$

$$\sim p^{2D-4}\left[\frac{1}{\beta_0\log(\frac{p^2}{\Lambda^2_{QCD}})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{p^2}{\Lambda^2_{QCD}})}{\log(\frac{p^2}{\Lambda^2_{QCD}})}+O(\frac{1}{\log(\frac{p^2}{\Lambda^2_{QCD}})})\right)\right]^{\frac{\gamma_0}{\beta_0}-1}$$

Why Migdal did not look for something similar to the asymptotic theorem ?

Migdal starting point was that the OPE looks seemingly incompatible with (his own) Kallen-Lehmann representation, since terms involving Bernoulli numbers, that he argued should match the OPE, cannot get logarithmic corrections implied in general by anomalous dimensions !

$$\sum_{k=k_1}^{\infty} G_k(p) = \int_{k_1}^{\infty} G_k(p)dk - \sum_{j=1}^{\infty} \frac{B_j}{j!} \left[\partial_k^{j-1} G_k(p)\right]_{k=k_1}$$

In fact, Migdal way out, imposing that there are no power-like corrections in the OPE, was dispersion relations and ratios of Bessel functions in 1977 !

as AdS/Gauge Theory correspondence found 20 years later !

Migdal (1977)

Polchinski-Strassler (Hard Wall) (2001)

$$\int < trF^2(x)trF^2(0) >_{conn} e^{-ip\cdot x}d^4x \sim \quad p^4\left[2\frac{K_1(\frac{p}{\mu})}{I_1(\frac{p}{\mu})} - \log p\right] \sim -p^4\left[\log p + O(e^{-2\frac{p}{\mu}})\right]$$

Yet Migdal objection to his own Kallen-Lehmann representation is in fact void for the first few most relevant coefficient functions in the OPE, i.e. asymptotically ...

$$G^{(2)}(x) = \langle \mathcal{O}_D(x)\mathcal{O}_D(0)\rangle_{conn} = C_{2D}(x) + \sum_{D_1>0} C_{2D-D_1}(x) < \mathcal{O}_{D_1}(0) >$$

$$\left(x_M \frac{\partial}{\partial x_M} + \beta(g)\frac{\partial}{\partial g} + 2(D + \gamma_{\mathcal{O}_D}(g))\right) C_{2D}(x) = 0$$

$$\left(x_M \frac{\partial}{\partial x_M} + \beta(g)\frac{\partial}{\partial g} + (2D - D_1 + 2\gamma_{\mathcal{O}_D}(g) - \gamma_{\mathcal{O}_{D_1}}(g))\right) C_{2D-D_1}(x) = 0$$

$$C_8(p) \sim p^4(g^2(p) + O(\frac{\Lambda^2_{QCD}}{p^2+\Lambda^2_{QCD}}))$$

$$\sim p^4(g^2(p) + O(\frac{\Lambda^2_{QCD}}{p^2}) + O(\frac{\Lambda^4_{QCD}}{p^4}) + O(\frac{\Lambda^6_{QCD}}{p^6}) + \cdots)$$

$$\sim p^4 g^2(p) + O(p^2\Lambda^2_{QCD}) + O(\Lambda^4_{QCD}) + O(\frac{\Lambda^6_{QCD}}{p^2}) + \cdots$$

$$C_4(p) \sim g^2(p) + O(\frac{\Lambda^2_{QCD}}{p^2+\Lambda^2_{QCD}})$$

$$G^{(2)}(p) \sim C_8(p) + C_4(p)\Lambda^4_{QCD} + \cdots$$

But what is in fact the UV asymptotics of the scalar glueball correlator, that indeed controls the mass gap in large-N Yang-Mills, and more generally the mass gap in the scalar glueball sector of large-N QCD-like confining asymptotically-free theories ?

Specialize the asymptotic theorem to scalar and pseudoscalar glueball propagators (M.B.)

$$\int \langle \frac{\beta(g)}{gN} tr\big(\sum_{\alpha\beta} F^2_{\alpha\beta}(x)\big) \frac{\beta(g)}{gN} tr\big(\sum_{\alpha\beta} F^2_{\alpha\beta}(0)\big)\rangle_{conn} e^{ip\cdot x} d^4x$$

$$= C_S p^4 \left[\frac{1}{\beta_0 \log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log \frac{p^2}{\Lambda^2_{\overline{MS}}}}{\log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) + O\left(\frac{1}{\log^2 \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) \right]$$

$$\int \langle \frac{g^2}{N} tr\big(\sum_{\alpha\beta} F_{\alpha\beta}\tilde{F}_{\alpha\beta}(x)\big) \frac{g^2}{N} tr\big(\sum_{\alpha\beta} F_{\alpha\beta}\tilde{F}_{\alpha\beta}(0)\big)\rangle_{conn} e^{ip\cdot x} d^4x$$

$$= C_{PS} p^4 \left[\frac{1}{\beta_0 \log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log \frac{p^2}{\Lambda^2_{\overline{MS}}}}{\log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) + O\left(\frac{1}{\log^2 \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) \right]$$

$$\int \langle \frac{g^2}{N} tr\big(\sum_{\alpha\beta} F^{-2}_{\alpha\beta}(x)\big) \frac{g^2}{N} tr\big(\sum_{\alpha\beta} F^{-2}_{\alpha\beta}(0)\big)\rangle_{conn} e^{ip\cdot x} d^4x$$

$$= C_{ASD} p^4 \left[\frac{1}{\beta_0 \log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log \frac{p^2}{\Lambda^2_{\overline{MS}}}}{\log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) + O\left(\frac{1}{\log^2 \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) \right]$$

Perturbative check: the 3-loop computation by Chetyrkin et al.

$$< trF^2(p)trF^2(-p) >_{conn} \quad = -\frac{N^2-1}{4\pi^2}p^4 \log\frac{p^2}{\mu^2}\left[1 + g^2(\mu)\left(f_0 - \beta_0 \log\frac{p^2}{\mu^2}\right)\right.$$
$$\left. + g^4(\mu)\left(f_1 + f_2 \log\frac{p^2}{\mu^2} + f_3 \log^2\frac{p^2}{\mu^2}\right)\right]$$

$$\begin{aligned}
f_0 &= \frac{73}{3(4\pi)^2} \\
f_1 - f_3\pi^2 &= \left(\frac{37631}{54} - \frac{242}{3}\zeta(2) - 110\zeta(3)\right)\frac{1}{(4\pi)^4} \\
-2\beta_0 &= -2\frac{11}{3(4\pi)^2} \\
2f_2 &= -\frac{313}{(4\pi)^4} \Rightarrow f_2 = -\frac{313}{2(4\pi)^4} \\
3f_3 &= \frac{121}{3(4\pi)^4} \Rightarrow f_3 = \frac{121}{9(4\pi)^4} \Rightarrow f_3 = \beta_0^2 \\
\Rightarrow f_1 &= \left(\frac{37631}{54} - 110\zeta(3)\right)\frac{1}{(4\pi)^4}
\end{aligned}$$

Perturbative check: the 3-loop computation by Chetyrkin et al.

$$< trF\tilde{F}(p)trF\tilde{F}(-p) >_{conn} = -\frac{(N^2-1)}{4\pi^2} p^4 \log\frac{p^2}{\mu^2}\Big[1 + g^2(\mu)\Big(\tilde{f}_0 - \beta_0 \log\frac{p^2}{\mu^2}\Big)$$
$$+ g^4(\mu)\Big(\tilde{f}_1 + \tilde{f}_2 \log\frac{p^2}{\mu^2} + \beta_0^2 \log^2\frac{p^2}{\mu^2}\Big)\Big]$$

$$\tilde{f}_0 = \frac{97}{3(4\pi)^2}$$
$$\tilde{f}_1 = \Big(\frac{51959}{54} - 110\zeta(3)\Big)\frac{1}{(4\pi)^4}$$
$$-2\beta_0 = -2\frac{11}{3(4\pi)^2}$$
$$2\tilde{f}_2 = -\frac{1135}{3(4\pi)^4} \Rightarrow \tilde{f}_2 = -\frac{1135}{6(4\pi)^4}$$

A two-loop example:

$$< \beta_0 \frac{g^2}{N} trF^2(p) \beta_0 \frac{g^2}{N} trF^2(-p) >_{conn}$$

$$\sim -\beta_0^2 p^4 \log \frac{p^2}{\mu^2} g_a^4(\mu) \left[1 - \beta_0 g_a^2(\mu) \log \frac{p^2}{\mu^2} + O(g^4 \log p) \right]$$

$$\beta_0 \log \frac{p^2}{\mu^2} = \frac{1}{g_a^2(p)} - \frac{1}{g_a^2(\mu)}$$

$$< \beta_0 \frac{g^2}{N} trF^2(p) \beta_0 \frac{g^2}{N} trF^2(-p) >_{conn}$$

$$\sim -\beta_0 p^4 g_a^2(\mu) g_a^2(p) \left(\frac{1}{g_a^2(p)} - \frac{1}{g_a^2(\mu)} \right) \left(1 + O(g^4 \log p) \right)$$

$$= \beta_0 p^4 \left(g_a^2(p) - g_a^2(\mu) \right) \left(1 + O(g^4 \log p) \right)$$

Check of the RG estimate (M.B. and S. Muscinelli, JHEP 08 (2013) 064 [hep-th/1304.6409])

$$\frac{1}{2}\int\langle \frac{g^2}{N} tr\big(\sum_{\alpha\beta} F^{-2}_{\alpha\beta}(x)\big)\, \frac{g^2}{N} tr\big(\sum_{\alpha\beta} F^{-2}_{\alpha\beta}(0)\big)\rangle_{conn}\, e^{-ip\cdot x} d^4x$$
$$= (1-\tfrac{1}{N^2})\,\frac{p^4}{2\pi^2\beta_0}\big(2g^2(p^2) - 2g^2(\mu^2)$$
$$+\big(a+\tilde{a}-\tfrac{\beta_1}{\beta_0}\big)g^4(p^2) - \big(a+\tilde{a}-\tfrac{\beta_1}{\beta_0}\big)g^4(\mu^2)\big) + O(g^6)$$

$$g^2(p^2) = g^2(\mu^2)\big(1-\beta_0 g^2(\mu^2)\log\tfrac{p^2}{\mu^2}$$
$$-\beta_1 g^4(\mu)\log\tfrac{p^2}{\mu^2} + \beta_0^2 g^4(\mu^2)\log^2\tfrac{p^2}{\mu^2}\big) + \cdots$$

Therefore, just as an aside, the asymptotic theorem provides a quantitative understanding of how accurate (approximate or would-be exact) solutions proposed since Migdal paper until our days are ...

In the past years several proposals have been advanced for the glueball propagators of QCD-like theories based on the the AdS-String /Large-N Gauge Theory Correspondence (Witten, Klebanov-Strassler, Maldacena-Nunez, Polchinski-Strassler, and many followers . . .)

The asymptotic theorem implies that

none of the proposals for the scalar glueball propagators based on the AdS String/Large-N Gauge Theory correspondence agrees with the universal RG estimate in the UV for any asymptotically free gauge theory

perhaps as expected, because the AdS-inspired models in the supergravity approximation are in fact strongly coupled in the UV

Migdal (1977)

Polchinski-Strassler (Hard Wall) (2001)

$$\int < trF^2(x)trF^2(0) >_{conn} e^{-ip\cdot x} d^4x \sim \quad p^4\left[2\frac{K_1(\frac{p}{\mu})}{I_1(\frac{p}{\mu})} - \log p\right] \sim -p^4\left[\log p + O(e^{-2\frac{p}{\mu}})\right]$$

Soft Wall (Karch, Katz, Son, Stephanov)(2006)

$$\int < trF^2(x)trF^2(0) >_{conn} e^{-ip\cdot x} d^4x \sim -p^4\left[\log p + O(\frac{\mu^2}{p^2})\right]$$

Klebanov-Strassler (the most interesting and remarkable !):
n=1 cascading SUSY QCD (2000)

At the end of the cascade:
$$\frac{\partial g}{\partial \log \Lambda} = -\frac{\frac{3}{(4\pi)^2} g^3}{1 - \frac{2}{(4\pi)^2} g^2}$$

$$\int < trF^2(x) trF^2(0) >_{conn} e^{-ip\cdot x} d^4x \sim p^4 \log^3 \frac{p^2}{\mu^2}$$ Krasnitz (2002)

Along the cascade:
$$\sim N_{eff}^2 p^4 \log \frac{p^2}{\mu^2}$$

All the previous results, disagree with asymptotic freedom and RG by powers of logarithms

It means that the would-be glueball propagators differ from the correct answer in pure YM or in any AF theory for an infinite number of poles and/or residues,

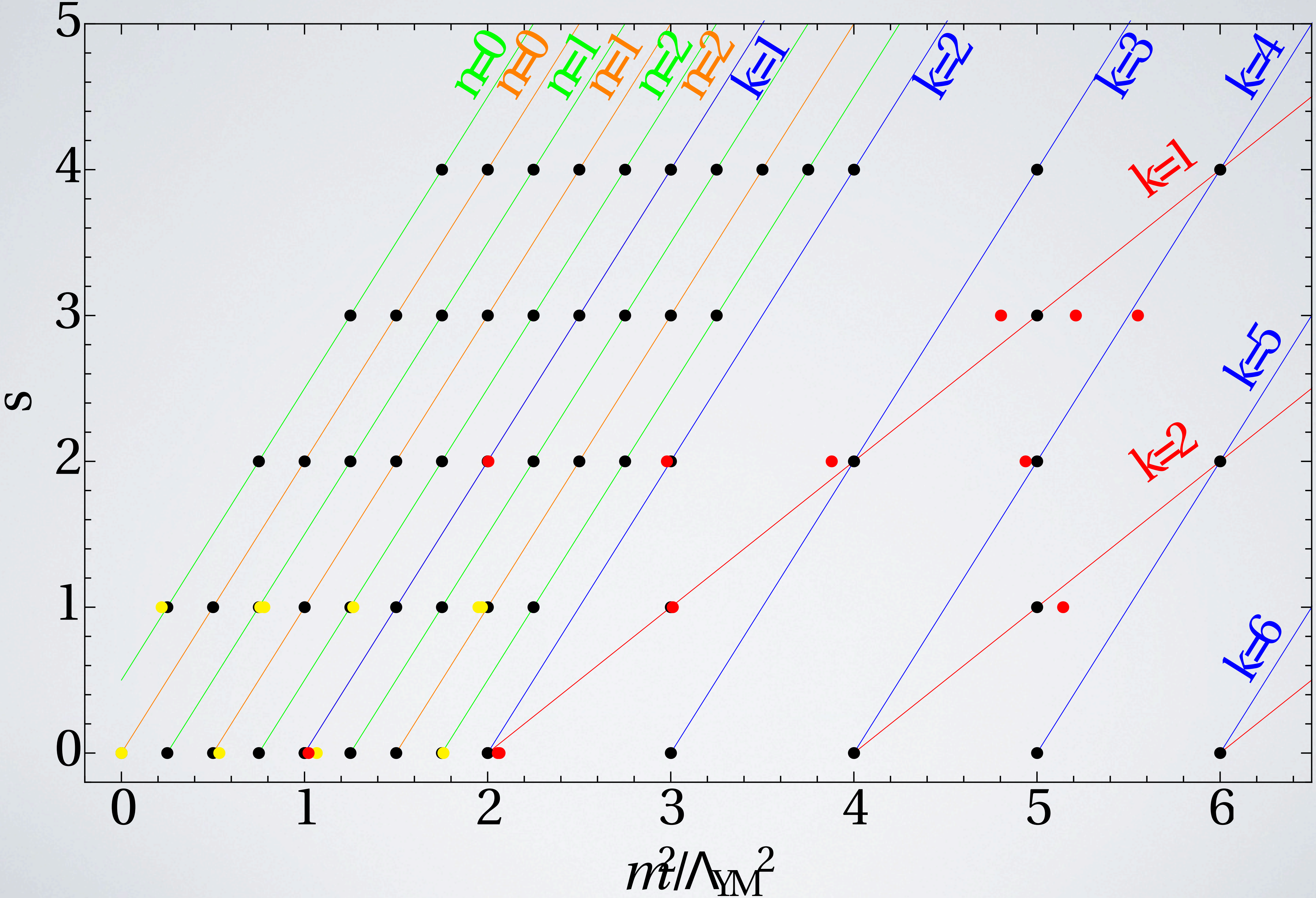
(a fact that raises well motivated doubts on the correctness of the AdS-String inspired spectrum at large-N ... In fact, the AdS-String inspired spectrum disagrees even qualitatively with lattice data at large-N)

All the plots and mass formulae from M.B. [hep-th/1308.2925]
and to appear

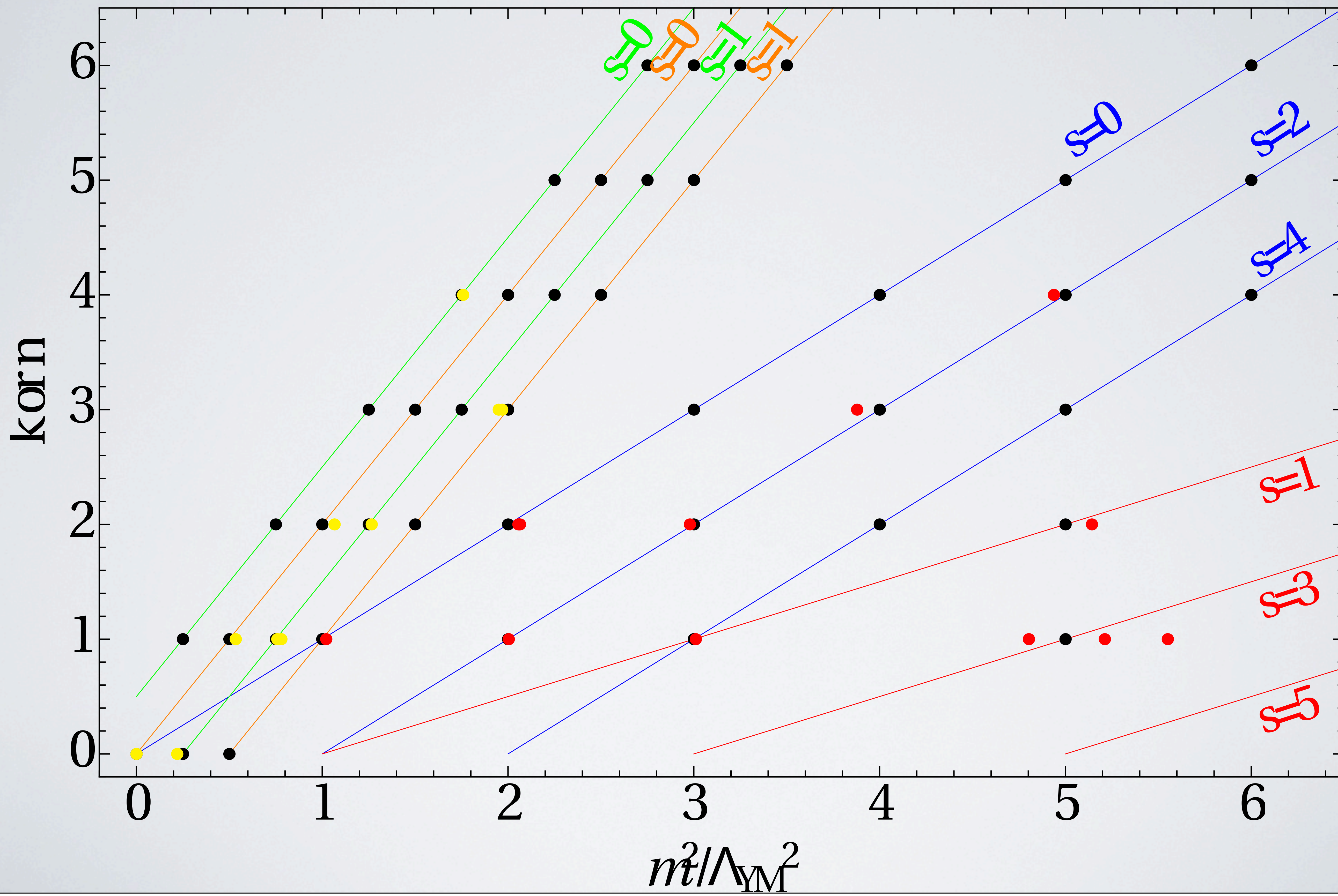
The lattice data are taken from
Meyer-Teper SU(8)
[hep-lat/0409183]
for glueballs (red)

Bali-Bursa-Castagnini-Collins-Del Debbio-Lucini-Panero
SU(17) [hep-lat/1304.4437] for mesons (yellow)

Meson and glueball Regge trajectories in massless large-N QCD from quenched lattice gauge theory



Meson and glueball Regge trajectories in massless large-N QCD from quenched lattice gauge theory



The black points in the plots follow from an actual proposal for the QCD spectrum and collinear S-matrix, based on a twistorial topological string theory (TTST), that we will discuss in the second part of the talk

Why the strong-coupling approximation cannot work ?

The asymptotic theorem for the scalar glueball propagator (that controls the mass gap of large-N Yang-Mills) implies that the more the theory becomes weakly coupled in the UV, the more massive poles occur, with dimensionless residues vanishing in the UV

$$\int \langle TrF^2(x)TrF^2(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x \sim p^4 \sum_{k=1}^{\infty} \frac{g^4(m_k^2)\rho_0^{-1}(m_k^2)}{p^2+m_k^2}$$

Thus dynamical mass generation is not a strong-coupling phenomenon in large-N QCD-like theories !

Besides, the lowest state, i.e. the mass gap, for very large g, is on the same order of the cutoff, i.e. strong coupling is unreliable in QCD-like AF gauge theories

$$\Lambda_{QCD} \sim \Lambda e^{-\frac{1}{2\beta_0 g^2}} \sim \Lambda$$

The TTST is related to a TFT underlying YM (M.B.)

Glueball and meson propagators of any spin in large-N QCD
Nucl. Phys. B 875 (2013) 621 [hep-th/1305.0273]
and
Yang-Mills mass gap, Floer homology, glueball spectrum and conformal window in large-N QCD
[hep-th/1312.1350]

$$O_S = \sum_{\alpha\beta} TrF_{\alpha\beta}F^{\alpha\beta}$$

$$O_P = \sum_{\alpha\beta} Tr(F^{\alpha\beta} {}^{*}F_{\alpha\beta})$$

$$< O_{ASD}(x)O_{ASD}(0) >_{conn} = 4 < O_S(x)O_S(0) >_{conn} +4 < O_P(x)O_P(0) >_{conn}$$

$$\int \langle O_{ASD}(x)O_{ASD}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x$$

$$= \frac{2}{\pi^2}\sum_{k=1}^{\infty}\frac{k^2 g_k^4 \Lambda_{\overline{W}}^6}{p^2 + k\Lambda_{\overline{W}}^2} = \frac{2p^4}{\pi^2}\sum_{k=1}^{\infty}\frac{g_k^4 \Lambda_{\overline{W}}^2}{p^2 + k\Lambda_{\overline{W}}^2} + \text{infinite contact terms}$$

$$\sim C^{(0)}_{ADS}(p^2) + 0 < \frac{1}{N} O_{ASD}(0) > + \text{infinite contact terms}$$

$$C^{(0)}_{ASD}(p^2) = \frac{2p^4}{\pi^2\beta_0}\left[\frac{1}{\beta_0 \log\frac{p^2}{\Lambda^2_{\overline{MS}}}}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log\log\frac{p^2}{\Lambda^2_{\overline{MS}}}}{\log\frac{p^2}{\Lambda^2_{\overline{MS}}}}\right) + O\left(\frac{1}{\log^2\frac{p^2}{\Lambda^2_{\overline{MS}}}}\right)\right]$$

We now look for a vast generalization of the Asymptotic Theorem to r-point correlators

that we call the Asymptotically-Free Bootstrap

The extension to multi-point correlators gives us information about the interaction,

and thus it may suggest appropriate string candidates for AF QCD-like theories, as opposed to gauge/gravity at strong coupling

The basic idea of the asymptotically-free bootstrap is based on the following estimate for 3-point scalar correlators:

$$< O(x_1)O(x_2)O(x_3) >_{conn} = G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1)$$

at leading 1/N order, i.e. planar:

$$< O(x_1)O(x_2)O(x_3) >_{conn} \sim C(x_1 - x_2) < O(x_2)O(x_3) >_{conn}$$
$$\sim C(x_1 - x_2) G^{(2)}(x_2 - x_3)$$
$$\sim N\, C(x_1 - x_2) C^2(x_2 - x_3)$$

with: $G^{(2)} \sim O(N^0)$ and $C \sim O(\frac{1}{N})$

at 1/N^3 order (i.e. non-planar):

$$G^{(3)}(\lambda(x_1 - x_2), \lambda(x_2 - x_3), \lambda(x_3 - x_1))$$
$$\sim C(\lambda(x_1 - x_2)) C(\lambda(x_2 - x_3)) C(\lambda(x_3 - x_1)) + O(g^2(\lambda x_0)) \frac{Z^3(\lambda, g(\mu))}{\lambda^{3D}}$$

$$O(x)O(0) \sim C(x)O(0) + \cdots$$

but the new ingredient is

the Kallen-Lehmann representation for OPE coefficients

This is the new crucial feature, that extends to OPE the aforementioned asymptotic theorem for 2-point correlators

$$O(x)O(0) \sim C(x)O(0) + \cdots$$

$$C(x_1 - x_2) \sim \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_n^{D-4} Z_n \rho^{-1}(m_n^2)}{p^2 + m_n^2} e^{ip\cdot(x_1 - x_2)} d^4p$$

$$\sim \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_n^{D-4} (\frac{g(m_n)}{g(\mu)})^{\frac{\gamma_0}{\beta_0}} \rho^{-1}(m_n^2)}{p^2 + m_n^2} e^{ip\cdot(x_1 - x_2)} d^4p$$

$$\sim \frac{(\frac{g(x_1 - x_2)}{g(\mu)})^{\frac{\gamma_0}{\beta_0}}}{(x_1 - x_2)^D}$$

The asymptotic spectral representation of 2 and 3-point scalar correlators follows:

at N^0 order:

$$< O(q_1)O(q_2) >_{conn} \sim \delta(q_1+q_2) \sum_{n=1}^{\infty} \frac{m_n^{2D-4} Z_n^2 \rho^{-1}(m_n^2)}{q_1^2+m_n^2}$$

at 1/N order:

$$< \mathcal{O}_{D,\gamma_0}(q_1)\mathcal{O}_{D,\gamma_0}(q_2)\mathcal{O}_{D,\gamma_0}(q_3) >_{conn}$$
$$\sim \delta(q_1+q_2+q_3) C(q_2) G^{(2)}(q_3)$$
$$\sim \delta(q_1+q_2+q_3) \sum_{n_1=1}^{\infty} \frac{m_{n_1}^{D-4} Z_{n_1} \rho^{-1}(m_{n_1}^2)}{q_2^2+m_{n_1}^2} \sum_{n_2=1}^{\infty} \frac{m_{n_2}^{2D-4} Z_{n_2}^2 \rho^{-1}(m_{n_2}^2)}{q_3^2+m_{n_2}^2}$$

at 1/N^3 order:

$$< O_{D,\gamma_0}(q_1)O_{D,\gamma_0}(q_2)O_{D,\gamma_0}(q_3) >_{conn}$$
$$\sim \delta(q_1+q_2+q_3) \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \int \frac{m_{n_1}^{D-4} Z_{n_1} \rho^{-1}(m_{n_1}^2)}{p^2+m_{n_1}^2} \frac{m_{n_2}^{D-4} Z_{n_2} \rho^{-1}(m_{n_2}^2)}{(p+q_2)^2+m_{n_2}^2} \frac{m_{n_3}^{D-4} Z_{n_3} \rho^{-1}(m_{n_3}^2)}{(p+q_2+q_3)^2+m_{n_3}^2} d^4p$$

Let us see how it works in detail ...

The Asymptotically-Free Bootstrap (for any spin)

1. Conformal invariance of correlators at lowest order of perturbation theory. For 2 an 3 point correlators structure is fixed uniquely by conformal invariance

2. RG improvement by Callan-Symanzik + asymptyotic freedom ; 1+2 imply that 3 point correlators factorize asymptotically on products of certain coefficients of OPE

3. Kallen-Lehmann representation of coefficients of OPE; This is the new crucial feature, that extends to OPE the aforementioned asymptotic theorem for 2 point correlators

4. 1+2+3 fix uniquely the glueball and meson 3-point correlators asymptotically in the UV

5. primitive r-point correlators follow by iterating the OPE

The asymptotically-free bootstrap (scalar case, positive charge conjugation)

$$< O(x_1)O(x_2) >_{conn} = G^{(2)}(x_1 - x_2)$$

$$< O(x_1)O(x_2)O(x_3) >_{conn} = G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1)$$

$$\left(\sum_{i=1}^{i=2} x_i \cdot \frac{\partial}{\partial x_i} + \beta(g)\frac{\partial}{\partial g} + 2(D + \gamma(g))\right) G^{(2)}(x_1 - x_2) = 0$$

$$\left(\sum_{i=1}^{i=3} x_i \cdot \frac{\partial}{\partial x_i} + \beta(g)\frac{\partial}{\partial g} + 3(D + \gamma(g))\right) G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1) = 0$$

1. Conformal invariance of correlators at lowest order of perturbation theory. For 2 an 3 point correlators structure is fixed uniquely by conformal invariance

$$\left(\sum_{i=1}^{i=2} x_i \cdot \frac{\partial}{\partial x_i} + 2D\right) G^{(2)}(x_1 - x_2) = 0$$

$$\left(\sum_{i=1}^{i=3} x_i \cdot \frac{\partial}{\partial x_i} + 3D\right) G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1) = 0$$

$$G^{(2)}(x_1 - x_2) = C_2 \frac{1}{(x_1 - x_2)^{2D}}$$

$$G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1) = C_3 \frac{1}{(x_1 - x_2)^D} \frac{1}{(x_2 - x_3)^D} \frac{1}{(x_3 - x_1)^D}$$

2. RG improvement by Callan-Symanzik +asymptyotic freedom

$$\left(\sum_{i=1}^{i=2} x_i \cdot \frac{\partial}{\partial x_i} + 2(D - \gamma_0 g^2)\right) G^{(2)}(x_1 - x_2) = 0$$

$$\left(\sum_{i=1}^{i=3} x_i \cdot \frac{\partial}{\partial x_i} + 3(D - \gamma_0 g^2)\right) G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1) = 0$$

Thus at next-to-leading perturbative order:

$$G^{(2)}(x_1 - x_2) = C_2(1 + O(g^2(\mu))) \frac{1}{(x_1 - x_2)^{2D - \gamma_0 g^2(\mu)}}$$

$$\sim C_2(1 + O(g^2(\mu))) \frac{1}{(x_1 - x_2)^{2D}} (1 + \gamma_0 g^2(\mu) \log(|x_1 - x_2|\mu))$$

$$G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1) \sim C_3(1 + O(g^2(\mu))) \frac{(1 + \gamma_0 g^2(\mu) \log(|x_1 - x_2|\mu))}{(x_1 - x_2)^D}$$

$$\frac{(1 + \gamma_0 g^2(\mu) \log(|x_2 - x_3|\mu))}{(x_2 - x_3)^D} \frac{(1 + \gamma_0 g^2(\mu) \log(|x_3 - x_1|\mu))}{(x_3 - x_1)^D}$$

Renormalization-group improvement:

$$1+\gamma_0 g^2(\mu)\log(|x|\mu) \sim (1+\beta_0 g^2(\mu)\log(|x|\mu))^{\frac{\gamma_0}{\beta_0}} \sim (\frac{g(x)}{g(\mu)})^{\frac{\gamma_0}{\beta_0}}$$

$$g(x)^2 \sim \frac{1}{\beta_0 \log(\frac{1}{x^2\Lambda^2_{QCD}})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{1}{x^2\Lambda^2_{QCD}})}{\log(\frac{1}{x^2\Lambda^2_{QCD}})}+O(\frac{1}{\log(\frac{1}{x^2\Lambda^2_{QCD}})})\right)$$

$$G^{(2)}(x_1-x_2) \sim C_2(1+O(g^2))\frac{(\frac{g(x_1-x_2)}{g(\mu)})^{\frac{2\gamma_0}{\beta_0}}}{(x_1-x_2)^{2D}}$$

$$G^{(3)}(x_1-x_2, x_2-x_3, x_3-x_1) \sim C_3(1+O(g^2))\frac{(\frac{g(x_1-x_2)}{g(\mu)})^{\frac{\gamma_0}{\beta_0}}}{(x_1-x_2)^D}\frac{(\frac{g(x_2-x_3)}{g(\mu)})^{\frac{\gamma_0}{\beta_0}}}{(x_2-x_3)^D}\frac{(\frac{g(x_3-x_1)}{g(\mu)})^{\frac{\gamma_0}{\beta_0}}}{(x_3-x_1)^D}$$

1+2 imply that 3-point correlators factorize asymptotically on products of certain coefficients of OPE

$$O(x)O(0) \sim C(x)O(0) + \cdots$$

$$\left(x \cdot \frac{\partial}{\partial x} + \beta(g)\frac{\partial}{\partial g} + (D + \gamma(g))\right) C(x) = 0$$

$$C(x) \sim \frac{\left(\frac{g(x)}{g(\mu)}\right)^{\frac{\gamma_0}{\beta_0}}}{x^D}$$

$$\begin{aligned} < O(x_1)O(x_2)O(x_3) >_{conn} &\sim C(x_1 - x_2) < O(x_2)O(x_3) >_{conn} \\ &\sim C(x_1 - x_2)G^{(2)}(x_2 - x_3) \\ &\sim C(x_1 - x_2)C^2(x_2 - x_3) \end{aligned}$$

$$G^{(3)}(x_1 - x_2, x_2 - x_3, x_3 - x_1) \sim C(x_1 - x_2)C(x_2 - x_3)C(x_3 - x_1)$$

The precise statement being

$$G^{(3)}(\lambda(x_1 - x_2), \lambda(x_2 - x_3), \lambda(x_3 - x_1))$$
$$\sim C(\lambda(x_1 - x_2))C(\lambda(x_2 - x_3))C(\lambda(x_3 - x_1)) + O(g^2(\lambda x_0))\frac{Z^3(\lambda, g(\mu))}{\lambda^{3D}}$$

or its slightly stronger version

$$< \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3) >_{conn}$$
$$\sim C(x_1 - x_2) < \mathcal{O}(x_2)\mathcal{O}(x_3) >_{conn}$$
$$\sim C(x_1 - x_2)G^{(2)}(x_2 - x_3)$$
$$\sim C(x_1 - x_2)C^2(x_2 - x_3)$$
$$\sim \frac{\left(\frac{g(x_1 - x_2)}{g(\mu)}\right)^{\frac{\gamma_0}{\beta_0}}}{(x_1 - x_2)^D}\frac{\left(\frac{g(x_2 - x_3)}{g(\mu)}\right)^{\frac{2\gamma_0}{\beta_0}}}{(x_2 - x_3)^{2D}}$$

$$\mathcal{O}_0(x)\mathcal{O}_0(0) \sim C_0(x)1 + C(x)\mathcal{O}_0(0) + \sum_{i\neq 0} C_i(x)\mathcal{O}_i(0)$$

$$+ \sum_{D'\neq D} C_{D'}(x)\mathcal{O}_{D'}(0) + \sum_{s\neq 0} C_s(x)\mathcal{O}^{(s)}(0) + \cdots$$

$$C_{2D-D_1}(x) = \frac{1}{x^{2D-D_1}}\,\mathcal{G}_{2D-D_1}(g(x))\,Z^2_{\mathcal{O}_D}(x\mu, g(\mu))Z^{-1}_{\mathcal{O}_{D_1}}(x\mu, g(\mu))$$

3. Kallen-Lehmann (KL) representation of coefficients of OPE; This is the new crucial feature, that extends to OPE the aforementioned asymptotic theorem for 2 point-correlators

$$
\begin{aligned}
C(x_1 - x_2) &\sim \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_n^{D-4} Z_n \rho^{-1}(m_n^2)}{p^2 + m_n^2} \, e^{ip\cdot(x_1 - x_2)} d^4p \\
&\sim \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_n^{D-4} \left(\frac{g(m_n)}{g(\mu)}\right)^{\frac{\gamma_0}{\beta_0}} \rho^{-1}(m_n^2)}{p^2 + m_n^2} \, e^{ip\cdot(x_1 - x_2)} d^4p \\
&\sim \frac{\left(\frac{g(x_1 - x_2)}{g(\mu)}\right)^{\frac{\gamma_0}{\beta_0}}}{(x_1 - x_2)^D}
\end{aligned}
$$

4. 1+2+3 fix uniquely the glueball and meson 3-point scalar vertices asymptotically in the UV

at 1/N^3 order (non planar):

$$< O_{D,\gamma_0}(x_1) O_{D,\gamma_0}(x_2) O_{D,\gamma_0}(x_3) >_{conn} \sim$$

$$\sum_{n_1=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_{n_1}^{D-4} Z_{n_1} \rho^{-1}(m_{n_1}^2)}{p_1^2 + m_{n_1}^2} e^{ip_1 \cdot (x_1 - x_2)} d^4 p_1$$

$$\sum_{n_2=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_{n_2}^{D-4} Z_{n_2} \rho^{-1}(m_{n_2}^2)}{p_2^2 + m_{n_2}^2} e^{ip_2 \cdot (x_2 - x_3)} d^4 p_2$$

$$\sum_{n_3=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{m_{n_3}^{D-4} Z_{n_3} \rho^{-1}(m_{n_3}^2)}{p_3^2 + m_{n_3}^2} e^{ip_3 \cdot (x_3 - x_1)} d^4 p_3$$

5. r-point asymptotic correlators follow by iterating the OPE

$$< O(q_1)O(q_2) >_{conn} \sim \delta(q_1+q_2)\sum_{n=1}^{\infty}\frac{m_n^{2D-4}Z_n^2\rho^{-1}(m_n^2)}{q_1^2+m_n^2}$$

at 1/N order:

$$< \mathcal{O}_{D,\gamma_0}(q_1)\mathcal{O}_{D,\gamma_0}(q_2)\mathcal{O}_{D,\gamma_0}(q_3) >_{conn}$$

$$\sim \delta(q_1+q_2+q_3)C(q_2)G^{(2)}(q_3)$$

$$\sim \delta(q_1+q_2+q_3)\sum_{n_1=1}^{\infty}\frac{m_{n_1}^{D-4}Z_{n_1}\rho^{-1}(m_{n_1}^2)}{q_2^2+m_{n_1}^2}\sum_{n_2=1}^{\infty}\frac{m_{n_2}^{2D-4}Z_{n_2}^2\rho^{-1}(m_{n_2}^2)}{q_3^2+m_{n_2}^2}$$

at 1/N^3 order:

$$< O_{D,\gamma_0}(q_1)O_{D,\gamma_0}(q_2)O_{D,\gamma_0}(q_3) >_{conn}$$

$$\sim \delta(q_1+q_2+q_3)\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\int\frac{m_{n_1}^{D-4}Z_{n_1}\rho^{-1}(m_{n_1}^2)}{p^2+m_{n_1}^2}\frac{m_{n_2}^{D-4}Z_{n_2}\rho^{-1}(m_{n_2}^2)}{(p+q_2)^2+m_{n_2}^2}\frac{m_{n_3}^{D-4}Z_{n_3}\rho^{-1}(m_{n_3}^2)}{(p+q_2+q_3)^2+m_{n_3}^2}d^4p$$

Some higher-order contributions are obtained iterating the OPE

$$< O(x_1)O(x_2)O(x_3)O(x_4) >_{conn} \sim C(x_1 - x_2)C(x_2 - x_3)C(x_3 - x_4)C(x_4 - x_2)$$
$$\sim C(x_1 - x_2)C(x_2 - x_3)C(x_3 - x_4)C(x_4 - x_1)$$

$$< \mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_r) >_{conn} \sim C(x_1 - x_2)\cdots C(x_r - x_1)$$

$$C(x_i - x_j) \sim Z((x_i - x_j)\mu, g(\mu))\frac{1}{(x_i - x_j)^D}$$

But for multi-point correlators these are leading contributions for a given anomalous dimension, but not necessarily the only ones, as opposed to 3-point correlators

But this is not the whole story !

We want to find the asymptotic effective action and asymptotic S-matrix

i.e. we want to go from

propagators and correlators

to

kinetic terms and vertices

this requires some more not-completely-trivial work

as a result we find some surprises for the S-matrix

The structure of the 1PI effective action for scalar correlation functions in massless large-N QCD and n=1 SUSY YM, asymptotically in the UV

$$
\begin{aligned}
S_{eff} =& \frac{1}{2!} \sum_n \int dq_1 dq_2 \delta(q_1 + q_2) m_n^{4-2D} Z_n^{-2} \rho(m_n^2) \Phi_n(q_1)(q_1^2 + m_n^2) \Phi_n(q_2) \\
&+ \frac{c_3^{(1)}(N)}{3!} \int dq_1 dq_2 dq_3 \delta(q_1 + q_2 + q_3) \sum_{n_1=1}^{\infty} m_{n_1}^{2-D} Z_{n_1}^{-1} \Phi_{n_1}(q_1) \\
&m_{n_1}^{4-D} Z_{n_1}^{-1} \Phi_{n_1}(q_2) \rho(m_{n_1}^2) \sum_{n_3=1}^{\infty} m_{n_3}^{-D} Z_{n_3}^{-1} \Phi_{n_3}(q_3) \\
&+ \frac{c_3^{(2)}(N)}{3!} \int dq_1 dq_2 dq_3 \delta(q_1 + q_2 + q_3) \int \sum_{n_1=1}^{\infty} m_{n_1}^2 \frac{m_{n_1}^{-D} Z_{n_1}^{-1} \Phi_{n_1}(q_2)}{p^2 + m_{n_1}^2} \\
&\sum_{n_2=1}^{\infty} m_{n_2}^2 \frac{m_{n_2}^{-D} Z_{n_2}^{-1} \Phi_{n_2}(q_3)}{(p+q_2)^2 + m_{n_2}^2} \sum_{n_3=1}^{\infty} m_{n_3}^2 \frac{m_{n_3}^{-D} Z_{n_3}^{-1} \Phi_{n_3}(q_1)}{(p+q_2+q_3)^2 + m_{n_3}^2} dp \\
&+ \cdots
\end{aligned}
$$

$c_3^{(1)}(N) \sim \frac{1}{N}$ for glueballs and gluinoballs

$c_3^{(1)}(N) \sim \frac{1}{\sqrt{N}}$ for mesons

$c_3^{(2)}(N) \sim \frac{1}{N^3}$ for glueballs and gluinoballs

$c_3^{(2)}(N) \sim \frac{1}{\sqrt{N^3}}$ for mesons

higher order r-point asymptotic vertices in the 1PI effective action from the OPE

$$\int dq_1 dq_2 \cdots dq_r \delta(q_1 + q_2 + \cdots + q_r) \int \sum_{n_1=1}^{\infty} m_{n_1}^2 \frac{m_{n_1}^{-D} Z_{n_1}^{-1} \Phi_{n_1}(q_2)}{p^2 + m_{n_1}^2} \sum_{n_2=1}^{\infty} m_{n_2}^2 \frac{m_{n_2}^{-D} Z_{n_2}^{-1} \Phi_{n_2}(q_3)}{(p+q_2)^2 + m_{n_2}^2}$$
$$\cdots \sum_{n_r=1}^{\infty} m_{n_r}^2 \frac{m_{n_r}^{-D} Z_{n_r}^{-1} \Phi_{n_r}(q_1)}{(p+q_2+\cdots+q_r)^2 + m_{n_r}^2} dp$$

OPE

$$< \mathcal{O}_{D,\gamma_0}(q_1)\mathcal{O}_{D,\gamma_0}(q_2)\mathcal{O}_{D,\gamma_0}(q_3) >_{conn}$$

$$\sim \delta(q_1+q_2+q_3)C(q_2)G^{(2)}(q_3)$$

$$\sim \delta(q_1+q_2+q_3)\sum_{n_1=1}^{\infty}\frac{m_{n_1}^{D-4}Z_{n_1}\rho^{-1}(m_{n_1}^2)}{q_2^2+m_{n_1}^2}\sum_{n_2=1}^{\infty}\frac{m_{n_2}^{2D-4}Z_{n_2}^2\rho^{-1}(m_{n_2}^2)}{q_3^2+m_{n_2}^2}$$

The asymptotic planar 3-point correlator implied by the vertex in the 1PI effective action agrees with the OPE asymptotically

$$< \mathcal{O}_{D,\gamma_0}(q_1)\mathcal{O}_{D,\gamma_0}(q_2)\mathcal{O}_{D,\gamma_0}(q_3) >_{conn}$$

$$\sim \delta(q_1+q_2+q_3)\sum_{n_1=1}^{\infty}\sum_{n_3=1}^{\infty}\frac{m_{n_1}^{D-2}Z_{n_1}}{q_1^2+m_{n_1}^2}\frac{m_{n_1}^{D-2}Z_{n_1}\rho^{-1}(m_{n_1}^2)}{q_2^2+m_{n_1}^2}\frac{m_{n_1}^2}{m_{n_3}^2}\frac{m_{n_3}^{D-2}Z_{n_3}\rho^{-1}(m_{n_3}^2)}{q_3^2+m_{n_3}^2}$$

$$\sim \delta(q_1+q_2+q_3)\sum_{n_1=1}^{\infty}\sum_{n_3=1}^{\infty}\frac{m_{n_1}^{2D-4}Z_{n_1}^2\rho^{-1}(m_{n_1}^2)}{q_1^2+m_{n_1}^2}\frac{m_{n_1}^2}{q_2^2+m_{n_1}^2}\frac{m_{n_3}^{D-4}Z_{n_3}\rho^{-1}(m_{n_3}^2)}{q_3^2+m_{n_3}^2}$$

Improved 3-point non-planar correlator

$$< \mathcal{O}_{D,\gamma_0}(q_1)\mathcal{O}_{D,\gamma_0}(q_2)\mathcal{O}_{D,\gamma_0}(q_3) >_{conn}$$

$$\sim \delta(q_1+q_2+q_3)\int \sum_{n_1=1}^{\infty} \frac{m_{n_1}^{D-4} Z_{n_1}\rho^{-1}(m_{n_1}^2)}{p^2+m_{n_1}^2}\frac{m_{n_1}^2}{q_2^2+m_{n_1}^2}$$

$$\sum_{n_2=1}^{\infty} \frac{m_{n_2}^{D-4} Z_{n_2}\rho^{-1}(m_{n_2}^2)}{(p+q_2)^2+m_{n_2}^2}\frac{m_{n_2}^2}{q_3^2+m_{n_2}^2}\sum_{n_3=1}^{\infty} \frac{m_{n_3}^{D-4} Z_{n_3}\rho^{-1}(m_{n_3}^2)}{(p+q_2+q_3)^2+m_{n_3}^2}\frac{m_{n_3}^2}{q_1^2+m_{n_3}^2}dp$$

$$\lim_{n\to\infty}\frac{m_n^2}{q^2+m_n^2}=1$$

$$< O_{D,\gamma_0}(q_1)O_{D,\gamma_0}(q_2)O_{D,\gamma_0}(q_3) >_{conn}$$

$$\sim \delta(q_1+q_2+q_3)\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\int \frac{m_{n_1}^{D-4} Z_{n_1}\rho^{-1}(m_{n_1}^2)}{p^2+m_{n_1}^2}\frac{m_{n_2}^{D-4} Z_{n_2}\rho^{-1}(m_{n_2}^2)}{(p+q_2)^2+m_{n_2}^2}\frac{m_{n_3}^{D-4} Z_{n_3}\rho^{-1}(m_{n_3}^2)}{(p+q_2+q_3)^2+m_{n_3}^2}d^4p$$

The generating functional of scalar S-matrix amplitudes (1PI) in massless large-N QCD and n=1 SUSY YM, asymptotically in the UV

$$S_{can} = \frac{1}{2!}\sum_n \int dq_1 dq_2 \delta(q_1+q_2)\Phi_n(q_1)(q_1^2+m_n^2)\Phi_n(q_2)$$
$$+\frac{c_3^{(1)}(N)}{3!}\int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3)\sum_{n_1,n_3=1}^{\infty}\Phi_{n_1}(q_1)\Phi_{n_1}(q_2)\frac{m_{n_1}^2}{m_{n_3}^2}\Phi_{n_3}(q_3)\rho^{-\frac{1}{2}}(m_{n_3}^2)$$
$$+\frac{c_3^{(2)}(N)}{3!}\int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3)\int\sum_{n_1=1}^{\infty}\frac{\rho^{-\frac{1}{2}}(m_{n_1}^2)\Phi_{n_1}(q_2)}{p^2+m_{n_1}^2}$$
$$\sum_{n_2=1}^{\infty}\frac{\rho^{-\frac{1}{2}}(m_{n_2}^2)\Phi_{n_2}(q_3)}{(p+q_2)^2+m_{n_2}^2}\sum_{n_3=1}^{\infty}\frac{\rho^{-\frac{1}{2}}(m_{n_3}^2)\Phi_{n_3}(q_1)}{(p+q_2+q_3)^2+m_{n_3}^2}dp+\cdots$$

The S-matrix depends only on the spectrum but not on the anomalous dimensions ! No conventional string theory has this S-matrix, since vertices are non-local but very much field theoretical (as in super-renormalizable field theories).

Thus the (unknown) spectrum determines the asymptotic S-matrix

This is the asymptotically-free version

of the old-fashioned bootstrap

i.e.

the asymptotically-free bootstrap !

But a different choice of contact terms for C(x) would lead to a non-renormalizable asymptotic generating functional of the scalar S matrix !

$$C(x_1 - x_2) \sim \sum_{n=1}^{\infty} \frac{1}{(2\pi)^4} \int \frac{p^{D-4} Z_n \rho^{-1}(m_n^2)}{p^2 + m_n^2} \, e^{ip\cdot(x_1 - x_2)} dp$$

Given the Kallen-Lehmann representation, extension of the Asymptotic Theorem to other coefficients of OPE is straightforward, taking into account different naive dimensions and anomalous dimensions of each coefficient, but in the spectral representation for the coefficient functions in the OPE the residues need not to be positive, i.e. the spectral sum need not to converge absolutely, but everything works asymptotically to the extent the Euler-McLaurin formula works asymptotically. This is a mathematical assumption ... that requires that residues do not oscillate wildly. In the latter case our statements hold for the inclusive meson and glueball S-matrix for fixed spin, rather than for individual amplitudes; thus at worse by the asymptotically-free bootstrap we get a version of gluon-quark/ glueball-meson duality

For vector and axial flavor currents (or gluinoball (flavor-singlet) chiral currents) in spinor notation

$$< j^a_{R\alpha_1\dot\beta_1}(x) j^b_{R\alpha_2\dot\beta_2}(0) > \sim \delta^{ab} \frac{x_{\alpha_1\dot\beta_2} x_{\alpha_2\dot\beta_1}}{x^8}$$

$$< j_V^{a\alpha_1\dot\beta_1}(x_1) j_V^{b\alpha_2\dot\beta_2}(x_2) j_V^{c\alpha_3\dot\beta_3}(x_3) >$$

$$\sim < j_V^{a\alpha_1\dot\beta_1}(x_1) j_A^{b\alpha_2\dot\beta_2}(x_2) j_A^{c\alpha_3\dot\beta_3}(x_3) >$$

$$\sim f^{abc} \left(\frac{x_{12}^{\alpha_1\dot\beta_2}}{x_{12}^4} \frac{x_{23}^{\alpha_2\dot\beta_3}}{x_{23}^4} \frac{x_{31}^{\alpha_3\dot\beta_1}}{x_{31}^4} - \frac{x_{13}^{\alpha_1\dot\beta_3}}{x_{13}^4} \frac{x_{32}^{\alpha_3\dot\beta_2}}{x_{32}^4} \frac{x_{21}^{\alpha_2\dot\beta_1}}{x_{21}^4} \right)$$

$$< j_A^{a\alpha_1\dot\beta_1}(x_1) j_A^{b\alpha_2\dot\beta_2}(x_2) j_A^{c\alpha_3\dot\beta_3}(x_3) >$$

$$\sim < j_A^{a\alpha_1\dot\beta_1}(x_1) j_V^{b\alpha_2\dot\beta_2}(x_2) j_V^{c\alpha_3\dot\beta_3}(x_3) >$$

$$\sim d^{abc} \left(\frac{x_{12}^{\alpha_1\dot\beta_2}}{x_{12}^4} \frac{x_{23}^{\alpha_2\dot\beta_3}}{x_{23}^4} \frac{x_{31}^{\alpha_3\dot\beta_1}}{x_{31}^4} + \frac{x_{13}^{\alpha_1\dot\beta_3}}{x_{13}^4} \frac{x_{32}^{\alpha_3\dot\beta_2}}{x_{32}^4} \frac{x_{21}^{\alpha_2\dot\beta_1}}{x_{21}^4} \right)$$

$$< j_-^{a\alpha_1\dot\beta_1}(x_1) j_-^{b\alpha_2\dot\beta_2}(x_2) j_-^{c\alpha_3\dot\beta_3}(x_3) >$$

$$\sim -Tr(T^c T^b T^a) \frac{x_{12}^{\alpha_1\dot\beta_2}}{x_{12}^4} \frac{x_{23}^{\alpha_2\dot\beta_3}}{x_{23}^4} \frac{x_{31}^{\alpha_3\dot\beta_1}}{x_{31}^4} - Tr(T^b T^c T^a) \frac{x_{13}^{\alpha_1\dot\beta_3}}{x_{13}^4} \frac{x_{32}^{\alpha_3\dot\beta_2}}{x_{32}^4} \frac{x_{21}^{\alpha_2\dot\beta_1}}{x_{31}^4}$$

2- point planar, and 3- point non-planar correlators

$$< j^a_{R\alpha_1\dot{\beta}_1}(x) j^b_{R\alpha_2\dot{\beta}_2}(0) >$$

$$\sim \delta^{ab} \sum_n \frac{1}{(2\pi)^4} \int \big(\epsilon_{\alpha_1\alpha_2}\epsilon_{\dot{\beta}_1\dot{\beta}_2} + \frac{p_{\alpha_1\dot{\beta}_1}p_{\alpha_2\dot{\beta}_2}}{m^2_{Rn}}\big) \frac{m^2_{Rn} Z^2_{Rn} \rho^{-1}_{1R}(m^2_{Rn})}{p^2 + m^2_{Rn}} e^{ip\cdot x} dp$$

$$= \delta^{ab} \frac{1}{(2\pi)^4} \int p^2 \big(\epsilon_{\alpha_1\alpha_2}\epsilon_{\dot{\beta}_1\dot{\beta}_2} - \frac{p_{\alpha_1\dot{\beta}_1}p_{\alpha_2\dot{\beta}_2}}{p^2}\big) \sum_n \frac{Z^2_{Rn} \rho^{-1}_{1R}(m^2_{Rn})}{p^2 + m^2_{Rn}} e^{ip\cdot x} dp + \cdots$$

$$< j^{a\alpha_1\dot{\beta}_1}_R(q_1) j^{b\alpha_2\dot{\beta}_2}_R(q_2) j^{c\alpha_3\dot{\beta}_3}_R(q_3) >$$

$$\frac{1}{N^g} \int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3) Tr^{(R)}(a,b,c) \int \sum_{n_1=1}^{\infty} \frac{p_{\alpha_1\dot{\beta}_2} m^{-2}_{Rn_1} z^{-1}_{Rn_1} \rho^{-1}_{1R}(m^2_{Rn_1})}{p^2 + m^2_{Rn_1}} \frac{m^2_{n_1}}{q_2^2 + m^2_{n_1}}$$

$$\sum_{n_2=1}^{\infty} \frac{(p+q_2)_{\alpha_2\dot{\beta}_3} m^{-2}_{Rn_2} z^{-1}_{Rn_2} \rho^{-1}_{1R}(m^2_{Rn_2})}{(p+q_2)^2 + m^2_{Rn_2}} \frac{m^2_{n_2}}{q_3^2 + m^2_{n_2}} \sum_{n_3=1}^{\infty} \frac{(p+q_2+q_3)_{\alpha_3\dot{\beta}_1} m^{-2}_{Rn_3} z^{-1}_{Rn_3} \rho^{-1}_{1R}(m^2_{Rn_3})}{(p+q_2+q_3)^2 + m^2_{Rn_3}} \frac{m^2_{n_3}}{q_1^2 + m^2_{n_3}} dp$$

$$+ opposite\ orientation$$

1PI Effective action (no planar 3-point vertex depicted)

$$\Gamma_{1R} = \frac{1}{2}\sum_n \int dq_1 dq_2 \delta(q_1+q_2) m_{Rn}^{-2} Z_{Rn}^{-2} \rho_{1R}(m_{Rn}^2)\delta_{ab}$$

$$\Phi_{Rn}^{a\alpha_1\dot\beta_1}(q_1)\left(\epsilon_{\alpha_1\alpha_2}\epsilon_{\dot\beta_1\dot\beta_2}(q_1^2+m_{Rn}^2) - q_{1\alpha_1\dot\beta_1} q_{1\alpha_2\dot\beta_2}\right)\Phi_{Rn}^{b\alpha_2\dot\beta_2}(q_2)$$

$$+\frac{1}{3N^g}\int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3) Tr^{(R)}(a,b,c)\int \sum_{n_1=1}^{\infty} \frac{p_{\alpha_1\dot\beta_2} m_{Rn_1}^{-2} z_{Rn_1}^{-1} \Phi_{Rn_1}^{a\alpha_1\dot\beta_1}(q_2)}{p^2+m_{Rn_1}^2}$$

$$\sum_{n_2=1}^{\infty}\frac{(p+q_2)_{\alpha_2\dot\beta_3} m_{Rn_2}^{-2} z_{Rn_2}^{-1}\Phi_{Rn_2}^{b\alpha_2\dot\beta_2}(q_3)}{(p+q_2)^2+m_{Rn_2}^2}\sum_{n_3=1}^{\infty}\frac{(p+q_2+q_3)_{\alpha_3\dot\beta_1} m_{Rn_3}^{-2} z_{Rn_3}^{-1}\Phi_{Rn_3}^{c\alpha_3\dot\beta_3}(q_1)}{(p+q_2+q_3)^2+m_{Rn_3}^2} dp + \cdots$$

$$\lim_{n\to\infty} Z_{Rn} = 1$$

$$\sum_n z_{Rn}^{-1} m_{Rn}^{-2} \rho_1^{-1}(m_{Rn}^2) = c_R$$

$$Tr^{(+)}(a,b,c) = Tr(T^a T^b T^c)$$

$$Tr^{(-)}(a,b,c) = -Tr(T^c T^b T^a)$$

$$Tr^{(V)}(a,b,c) = f^{abc}$$

$$Tr^{(A)}(a,b,c) = d^{abc}$$

1PI S-matrix generating functional (no planar 3-point vertex depicted)

$$S_{1R} = \frac{1}{2}\sum_n \int dq_1 dq_2 \delta(q_1+q_2)\delta_{ab}\Phi^{a\alpha_1\dot\beta_1}_{Rn}(q_1)\big(\epsilon_{\alpha_1\alpha_2}\epsilon_{\dot\beta_1\dot\beta_2}(q_1^2+m^2_{Rn}) - q_{1\alpha_1\dot\beta_1}q_{1\alpha_2\dot\beta_2}\big)\Phi^{b\alpha_2\dot\beta_2}_{Rn}(q_2)$$

$$+ \frac{C}{3N^g}\int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3) Tr^{(R)}(a,b,c)$$

$$\int \sum_{n_1=1}^{\infty} \frac{p_{\alpha_1\dot\beta_2} Z_{Rn_1} z^{-1}_{Rn_1} m^{-1}_{Rn_1} \rho^{-\frac{1}{2}}_{1R}(m^2_{Rn_1})\Phi^{a\alpha_1\dot\beta_1}_{Rn_1}(q_2)}{p^2+m^2_{Rn_1}}$$

$$\sum_{n_2=1}^{\infty} \frac{(p+q_2)_{\alpha_2\dot\beta_3} Z_{Rn_2} z^{-1}_{Rn_2} m^{-1}_{Rn_2} \rho^{-\frac{1}{2}}_{1R}(m^2_{Rn_2})\Phi^{b\alpha_2\dot\beta_2}_{Rn_2}(q_3)}{(p+q_2)^2+m^2_{Rn_2}}$$

$$\sum_{n_3=1}^{\infty} \frac{(p+q_2+q_3)_{\alpha_3\dot\beta_1} Z_{Rn_3} z^{-1}_{Rn_3} m^{-1}_{Rn_3} \rho^{-\frac{1}{2}}_{1R}(m^2_{Rn_3})\Phi^{c\alpha_3\dot\beta_3}_{Rn_3}(q_1)}{(p+q_2+q_3)^2+m^2_{Rn_3}} dp + \cdots$$

In the s=1 sector the theory looks renormalizable by power counting but not super-renormalizable

Possible applications

3-point current correlators: pion form factor

and

vector dominance

4-point vector current correlator: light by light scattering

The main limitation of the asymptotically-free bootstrap is that it does not contain spectral information,

but in fact it provides a guide to find out an exact large-N solution,
perhaps only for the spectrum and S-matrix

by other methods,

say a String Solution for the spectrum and the S-matrix

In the pure-glueball sector of positive charge conjugation of large-N QCD the higher-order contributions to the 1PI effective action can be re-summed into a functional determinant, that is compatible with the planar local 3-point vertex, in the sense that the functional determinant arises by the (one-loop) quantization of the purely local Phi^3 theory !

$$S = \frac{1}{2} tr \int \Phi(-\Delta + M^2)\Phi \, d^4x + \frac{c'}{3!N} \int tr(\Phi^2 M^2) tr(\frac{\Phi}{M^2}\rho_0^{-\frac{1}{2}}) \, d^4x$$
$$+ \frac{\kappa}{2} \log Det_3(-\Delta + M^2 + \frac{c'}{N}\rho_0^{-\frac{1}{2}}\Phi\otimes)$$

$$Tr_3 \log(1 - \frac{c'}{N}((-\Delta + M^2)^{-1}\rho_0^{-\frac{1}{2}}\Phi)) = \hat{T}r_3 \log(1 - \frac{c'}{N} tr((-\Delta + M^2)^{-1}\rho_0^{-\frac{1}{2}}\Phi))$$

Can we lift the asymptotic structure of the S-matrix to an actual String solution ?

Holomorphic Chern-Simons on twistor space= String of n=4 SUSY YM Witten (2004): massless particles = Dolbeault cohomology thanks to Penrose construction

Nair (1987) Boels, Mason, Skinner (2006)

$$S(A) = \frac{1}{2\pi} \int \Omega \wedge tr(A \wedge \bar{\partial} A + \frac{2}{3} A \wedge A \wedge A) - \kappa \int d\mu \ \log\det \left((\bar{\partial} + A)_{L(x_-, \tilde{\theta})} \right)$$

Neitzke-Vafa (2004), M.B. (2008)

Conjecturally: Complex Chern-Simons on Lagrangian submanifolds of non-commutative twistor space= String of large-N YM: massive glueball Regge trajectories = infinite non-Abelian Non-commutative Hodge structure

$$S(B) = \frac{1}{2\pi} \int tr(B \wedge dB + \frac{2}{3} B \wedge B \wedge B) - \kappa \int d\mu \ \log\det\left(d + B_\lambda\right)$$

Why does the Topological String Theory have chances to work ?

Because S-matrix amplitudes for topological strings arise by summing on D-branes, as in Witten Topological Twistor String of n=4 SUSY YM

or by summing on world-sheet instantons as in the Twistorial A-model that is dual to the TFT (M.B.)

and not by summing on Riemann surfaces, as for conventional strings, that in general implies very soft behavior in the UV, more soft than in super-renormalizable field theories

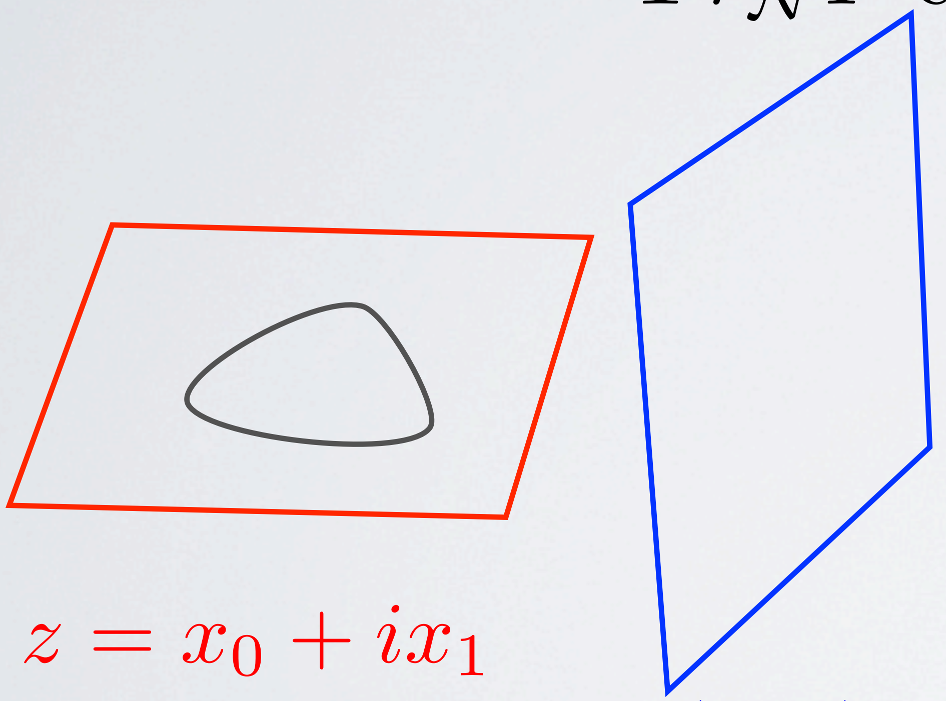
In Witten topological string the field theoretical MHV amplitudes of n=4 YM are exactly reproduced, i.e. they are hard in the UV

The TTST is related to a TFT underlying pure large-N YM

Twistor Wilson loops are first defined in U(N) YM on non-commutative space-time, for large non-commutativity:

$$Tr_{\mathcal{N}} \Psi(\hat{B}_{\lambda}; L_{ww}) =$$

$$Tr_{\mathcal{N}} P \exp i \int_{L_{ww}} (\hat{A}_z + \lambda \hat{D}_u) dz + (\hat{A}_{\bar{z}} + \lambda^{-1} \hat{D}_{\bar{u}}) d\bar{z}$$



$$z = x_0 + i x_1$$

$$\bar{z} = x_0 - i x_1$$

$$\hat{u} = \hat{x}_2 + i \hat{x}_3$$

$$\hat{\bar{u}} = \hat{x}_2 - i \hat{x}_3$$

$$\hat{D}_u = \hat{\partial}_u + i \hat{A}_u$$

$$[\hat{\partial}_u, \hat{\partial}_{\bar{u}}] = \theta^{-1} 1$$

$$[\hat{u}, \hat{\bar{u}}] = \theta 1$$

NC YM in the limit of large non-commutativity is equivalent to SU(N)YM in large N limit on commutative space-time

V.e.v. of twistor Wilson loops is trivial

$$
\begin{aligned}
< \frac{1}{\mathcal{N}} Tr_{\mathcal{N}} \Psi(\hat{B}_\lambda; L_{ww}) > &= < \frac{1}{\mathcal{N}} Tr_{\mathcal{N}} \Psi(\hat{B}_1; L_{ww}) > \\
\lim_{\theta \to \infty} < \frac{1}{\mathcal{N}} Tr_{\mathcal{N}} \Psi(\hat{B}_\lambda; L_{ww}) > &= 1
\end{aligned}
$$

Hint: at lowest order in perturbation theory

$$
\begin{aligned}
&< Tr_{\mathcal{N}} \Big(\int_{L_{ww}} (\hat{A}_z + \lambda \hat{D}_u) dz + (\hat{A}_{\bar{z}} + \lambda^{-1} \hat{D}_{\bar{u}}) d\bar{z} \int_{L_{ww}} (\hat{A}_z + \lambda \hat{D}_u) dz + (\hat{A}_{\bar{z}} + \lambda^{-1} \hat{D}_{\bar{u}}) d\bar{z} \Big) > \\
&= 2 \int_{L_{ww}} dz \int_{L_{ww}} d\bar{z} (< Tr_{\mathcal{N}} (\hat{A}_z \hat{A}_{\bar{z}}) > + i^2 < Tr_{\mathcal{N}} (\hat{A}_u \hat{A}_{\bar{u}}) >) \\
&= 0
\end{aligned}
$$

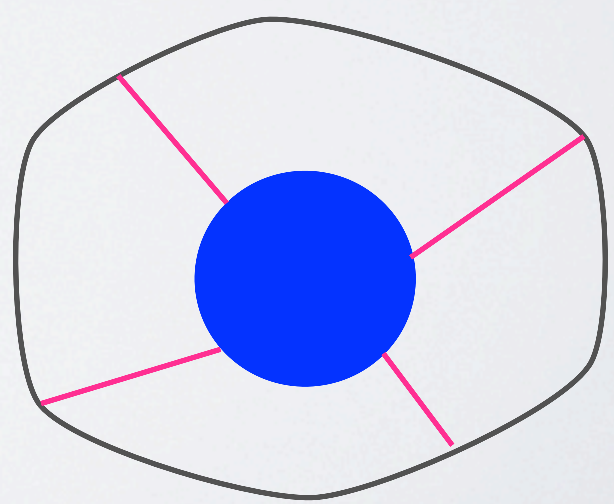
NC twistor loops are gauge equivalent for large theta to the following loops of the large-N commutative gauge theory

$$< Tr_{\mathcal{N}} \exp i \int_{L_{ww}} (A_z(z,\bar z, i\lambda z, i\lambda^{-1}\bar z) + i\lambda A_u(z,\bar z, i\lambda z, i\lambda^{-1}\bar z))dz$$
$$+(A_z(z,\bar z, i\lambda z, i\lambda^{-1}\bar z) + i\lambda^{-1} A_u(z,\bar z, i\lambda z, i\lambda^{-1}\bar z))d\bar z >$$

Thus they are supported on Lagrangian submanifolds of twistor space of complexification of Euclidean space-time

Triviality proof is based on vanishing of coefficients of propagators:

$$\dot z \dot{\bar z} + i^2 \lambda \dot z \lambda^{-1} \dot{\bar z} = 0$$
$$\dot z \bar z + i^2 \lambda \dot z \lambda^{-1} \dot{\bar z} = 0$$
$$z \dot{\bar z} + i^2 \lambda z \lambda^{-1} \dot{\bar z} = 0$$



Wilson loop in NC YM theory, translations can be reabsorbed by gauge transformations = modern Eguchi-Kawai reduction

$$
\begin{aligned}
\frac{1}{\mathcal{N}} tr_N Tr_{\hat{N}} \Psi(\hat{A}; L_{ww}) &= \frac{1}{\mathcal{N}} tr_N Tr_{\hat{N}} P \exp \int_{L_{ww}} (\hat{\partial}_\alpha + i\hat{A}_\alpha) dx_\alpha \\
\hat{U}(x) &= e^{x_\alpha \hat{\partial}_\alpha} \\
\hat{A}^{\hat{U}}_\alpha &= \hat{U}(x)\hat{A}_\alpha \hat{U}(x)^{-1} + i\partial_\alpha \hat{U}(x)\hat{U}(x)^{-1} \\
\hat{\partial}^{\hat{U}}_\alpha &= \hat{U}(x)\hat{\partial}_\alpha \hat{U}(x)^{-1} \\
\Psi(\hat{A}; L_{yz}) &= P \exp i \int_{L_{yz}} (-i\hat{\partial}_\alpha + \hat{A}_\alpha) dx_\alpha \\
& \hat{U}(y)\Psi(\hat{A}; L_{yz})\hat{U}(z)^{-1} \\
&= P \exp i \int_{L_{yz}} (-i\hat{\partial}^U_\alpha + \hat{A}^U_\alpha) dx_\alpha \\
&= P \exp i \int_{L_{yz}} (\hat{U}(x)\hat{A}_\alpha \hat{U}(x)^{-1} - iU(x)\hat{\partial}_\alpha U(x)^{-1} + i\partial_\alpha \hat{U}(x)\hat{U}(x)^{-1}) dx_\alpha \\
&= P \exp i \int_{L_{yz}} \hat{U}(x)\hat{A}_\alpha \hat{U}(x)^{-1} dx_\alpha \\
&= P_\star \exp i \int_{L_{yz}} A_\alpha(x) dx_\alpha
\end{aligned}
$$

NC EGUCHI-KAWAI REDUCTION

Eguchi-Kawai reduction (1982), Gonzalez Arroyo - Korthals Altes (1983), Minwalla-Ramsdonk-Seiberg (1999), Makeenko (2000), Szabo (2001), Douglas-Nekrasov (2001), Dhar-Kitazawa (2001), Alvarez-Gaume'-Barbon (2002)... NCYM is equivalent to a matrix model with rescaled action, because translations can be absorbed into gauge transformations

Operator/function correspondence:

$$
\begin{aligned}
[\hat{x}^\alpha, \hat{x}^\beta] &= i\theta^{\alpha\beta} 1 \\
\hat{\Delta}(x) &= \int \frac{d^d k}{(2\pi)^d} e^{ik\hat{x}} e^{-ikx} \\
\hat{f} &= \int d^d x f(x) \hat{\Delta}(x) \\
\hat{f}\hat{g} &= \widehat{f \star g} \\
(f \star g)(x) &= f(x) \exp(\frac{i}{2} \partial_x^\alpha \theta^{\alpha\beta} \partial_y^\beta) g(y)|_{y=x} \\
f_1(x_1) \star ... \star f_n(x_n) &= \prod_{i<k} \exp(\frac{i}{2} \partial_{x^i}^\alpha \theta^{\alpha\beta} \partial_{x^k}^\beta) f_1(x_1)...f_n(x_n)
\end{aligned}
$$

$$
\begin{aligned}
e^{a\hat{\partial}} \hat{\Delta}(x) e^{-a\hat{\partial}} &= \hat{\Delta}(x+a) \\
\hat{\partial}^i(\hat{x}^j) &= \delta^{ij} 1 \\
(2\pi)^{\frac{d}{2}} Pf(\theta) \hat{T}r \hat{f} &= \int d^d x f(x) \\
(2\pi)^{\frac{d}{2}} Pf(\theta) \hat{T}r(\hat{\Delta}(x)\hat{\Delta}(y)) &= \delta^d(x-y) \\
\int d^d x (f \star g)(x) &= \int d^d x f(x) g(x)
\end{aligned}
$$

$$[\hat{\partial}_\alpha, \hat{\partial}_\beta] = i\theta^{-1}_{\alpha\beta} 1$$

$$
\begin{aligned}
e^{a\hat{\partial}}\hat{\Delta}(x)e^{-a\hat{\partial}} &= \hat{\Delta}(x+a) \\
\hat{\partial}^i(\hat{x}^j) &= \delta^{ij}1 \\
(2\pi)^{\frac{d}{2}}Pf(\theta)\hat{T}r\hat{f} &= \int d^dxf(x) \\
(2\pi)^{\frac{d}{2}}Pf(\theta)\hat{T}r(\hat{\Delta}(x)\hat{\Delta}(y)) &= \delta^d(x-y) \\
\int d^dx(f\star g)(x) &= \int d^dxf(x)g(x)
\end{aligned}
$$

$$
\frac{N}{2g^2}\int d^dxtr_N(F_{\alpha\beta}\star F_{\alpha\beta})(x)
$$

$$
=\frac{N}{2g^2}(2\pi)^{\frac{d}{2}}Pf(\theta)tr_N\hat{T}r(-i[\hat{\partial}_\alpha+i\hat{A}_\alpha,\hat{\partial}_\beta+i\hat{A}_\beta]+\theta^{-1}_{\alpha\beta}1)^2
$$

$$
=\frac{N}{2g^2}\hat{N}(\frac{2\pi}{\Lambda})^dtr_NTr_{\hat{N}}(-i[\hat{\partial}_\alpha+i\hat{A}_\alpha,\hat{\partial}_\beta+i\hat{A}_\beta]+\theta^{-1}_{\alpha\beta}1)^2
$$

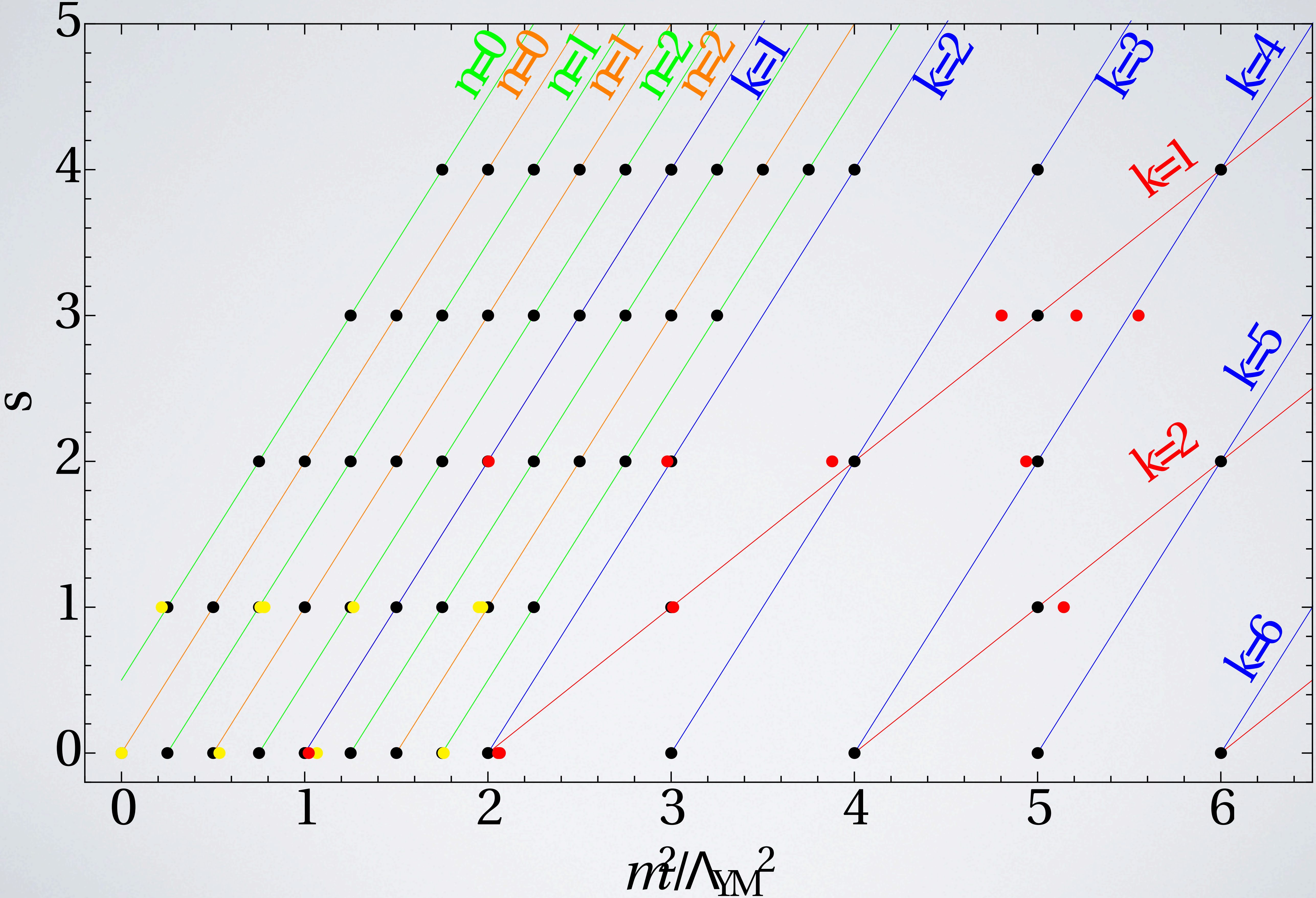
$$
\hat{N}(\frac{2\pi}{\Lambda})^d=(2\pi)^{\frac{d}{2}}Pf(\theta)
$$

The TTST is a blackboard talk

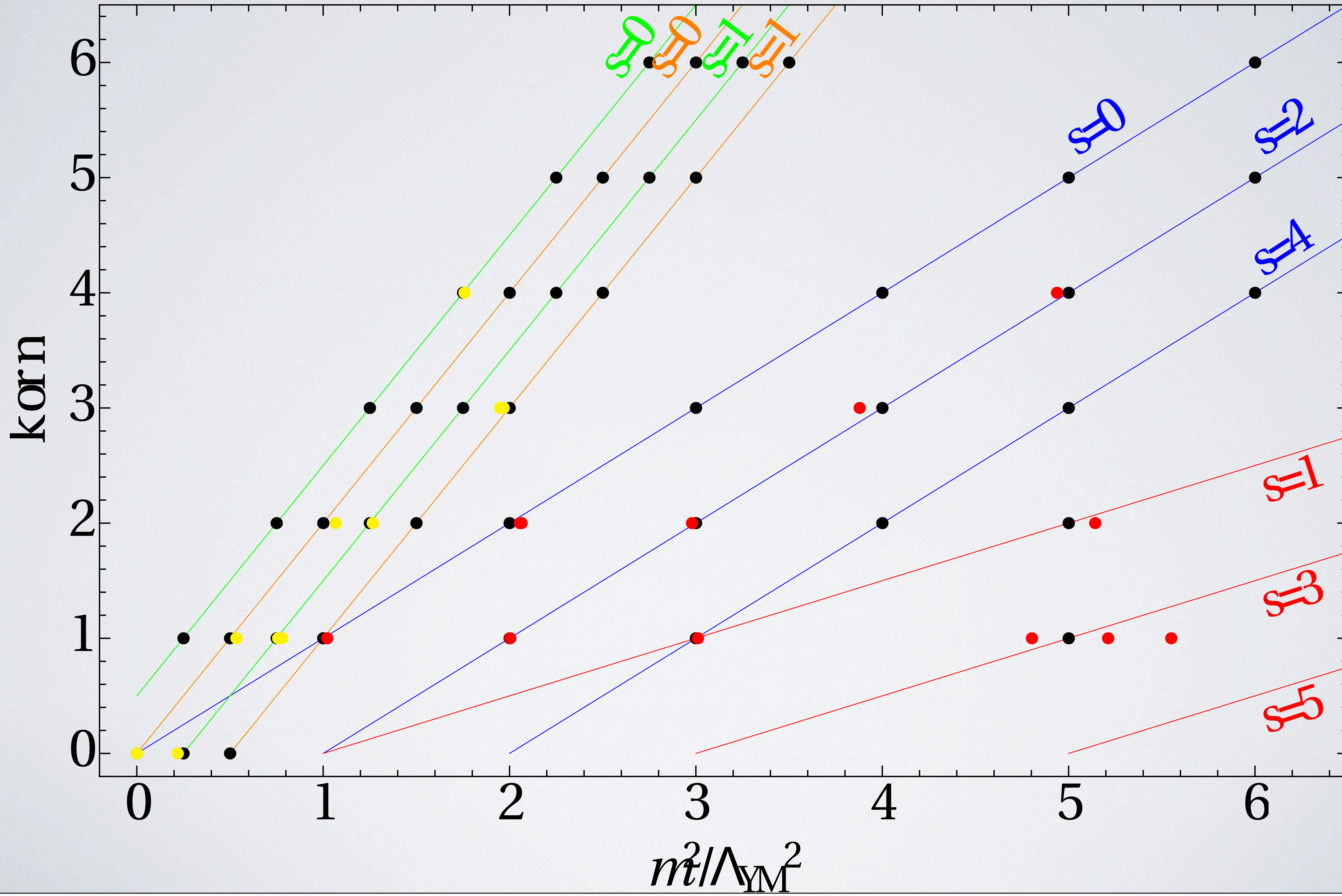
In the TTST the Hodge structure implies (half-)integer valued spectrum in units of 1/2 Lambda_QCD^2

$$
\begin{aligned}
m_k^{(s)2} &= (k+\frac{s}{2})\Lambda_{QCD}^2 \ ; s \ even; \ \ k = 1,2,\cdots \\
m_k^{(s)2} &= 2(k+\frac{s}{2})\Lambda_{QCD}^2 \ ; s \ odd; \ \ k = 1,2,\cdots \\
m_n^{(s)2} &= \frac{1}{2}(n+s)\Lambda_{QCD}^2 \ ; s = 0,1,\cdots \\
m_n^{(s)2} &= \frac{1}{2}(n+s-\frac{1}{2})\Lambda_{QCD}^2 \ ; s = 1,\cdots
\end{aligned}
$$

Meson and glueball Regge trajectories in massless large-N QCD from quenched lattice gauge theory



Meson and glueball Regge trajectories in massless large-N QCD from quenched lattice gauge theory

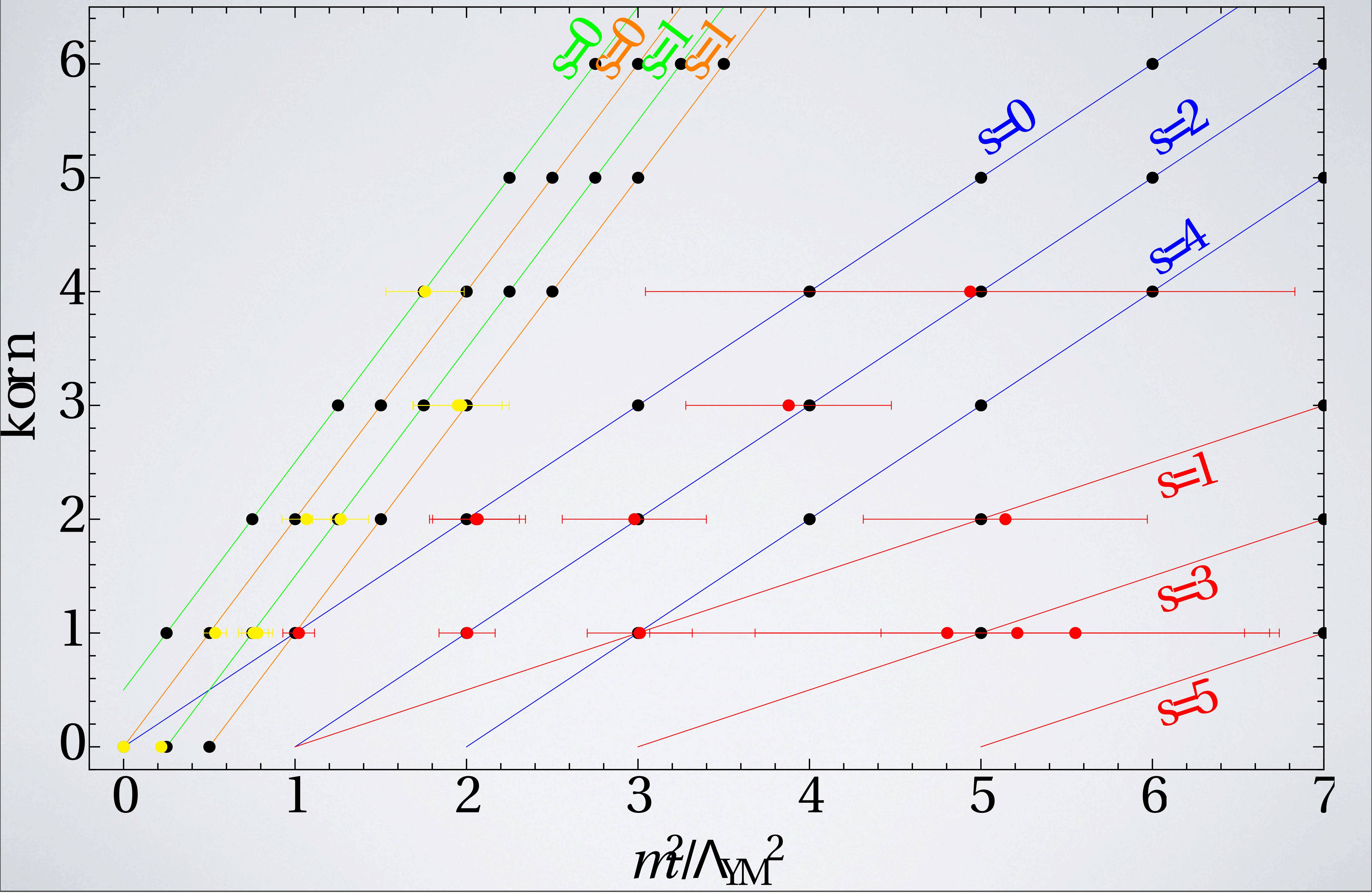


All the plots and mass formulae from M.B. [hep-th/1308.2925]
and to appear

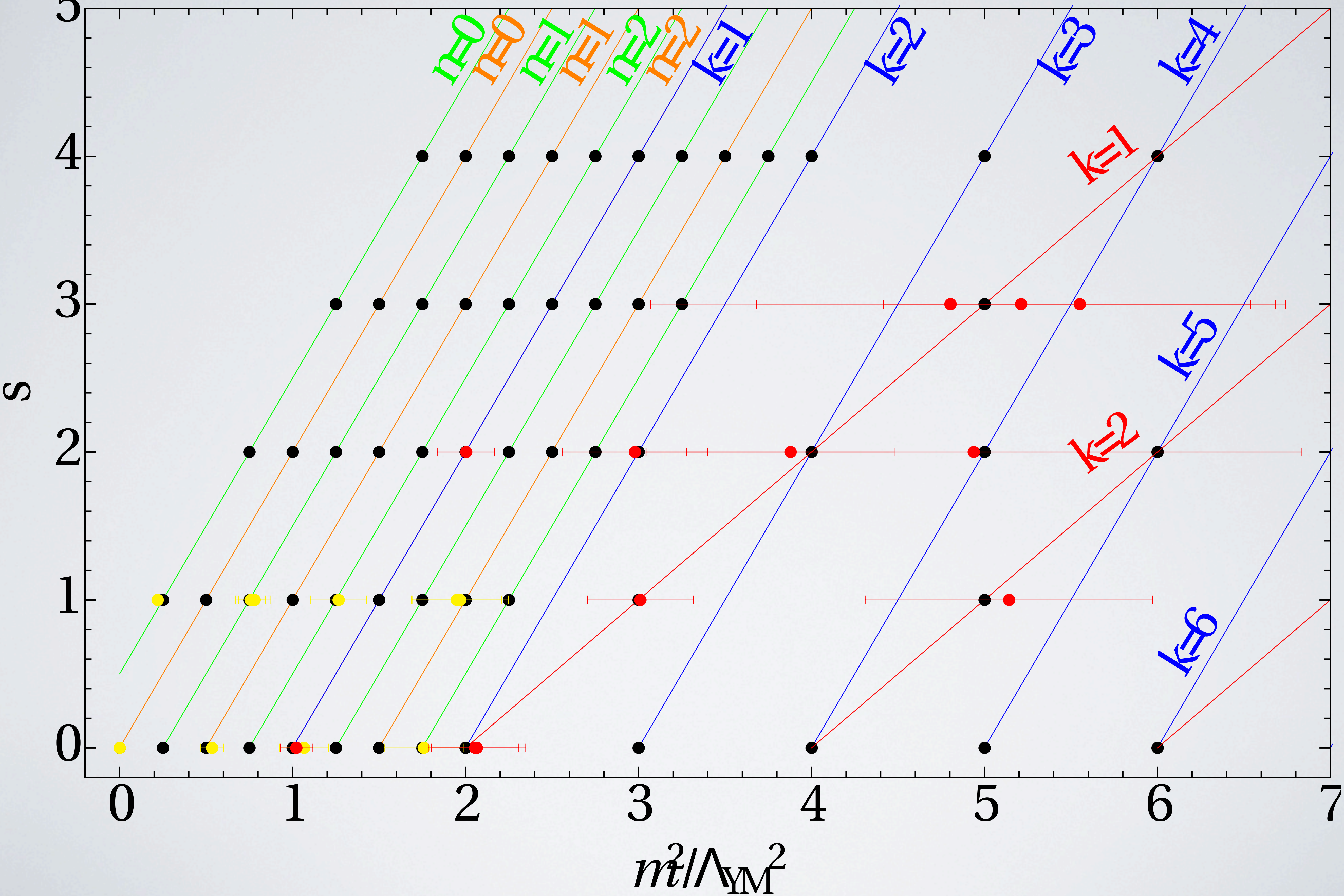
The lattice data are taken from
Meyer-Teper SU(8)
[hep-lat/0409183]
for glueballs (red)

Bali-Bursa-Castagnini-Collins-Del Debbio-Lucini-Panero
SU(17) [hep-lat/1304.4437] for mesons (yellow)

Meson and glueball Regge trajectories in massless large-N QCD from quenched lattice gauge theory



Meson and glueball Regge trajectories in massless large-N QCD from quenched lattice gauge theory



From the TTST spectrum and the lattice data it follows that there are two different slopes in the glueball sector

This is incompatible with the universally accepted picture that glueballs are vibrations of closed string only

but it might be explained by the TTST, since it contains both an open and a closed string sector

Meson slope is twice larger (open strings in the TTST interpretation) than the largest glueball slope: usual interpretation, fundamental versus adjoint strings

Infrared test: SU(8) lattice YM by Meyer-Teper (state of the art = presently smallest gauge coupling)

$$r_s = \frac{m_{0^{++*}}}{m_{0^{++}}}$$

$$r_{ps} = \frac{m_{0^{-+}}}{m_{0^{++}}}$$

$$r_s = r_{ps} = 1.42$$

$$r_2 = \frac{m_{2^{++}}}{m_{0^{++}}} = 1.40$$

TFT:

$$r_s = r_{ps} = \sqrt{2} = 1.4142\cdots$$

Witten model:

The lowest Kaluza-Klein (KK) mass=cutoff is lower than the would-be mass gap. Otherwise, ignoring the KK, the scalar mass gap is degenerate with spin 2 = qualitative disagreement with lattice gauge theory and actual PDG spectrum

$$r_s = 1.5860 \qquad r_s = 1.7388$$

$$r_{ps} = 1.2031 \qquad r_{ps} = 2.092$$

$$r_2 = 1 \qquad r_2 = 1.7388$$

Hard Wall (Polchinski-Strassler):

D $$r_s = 1.64; r_2 = 1.48$$

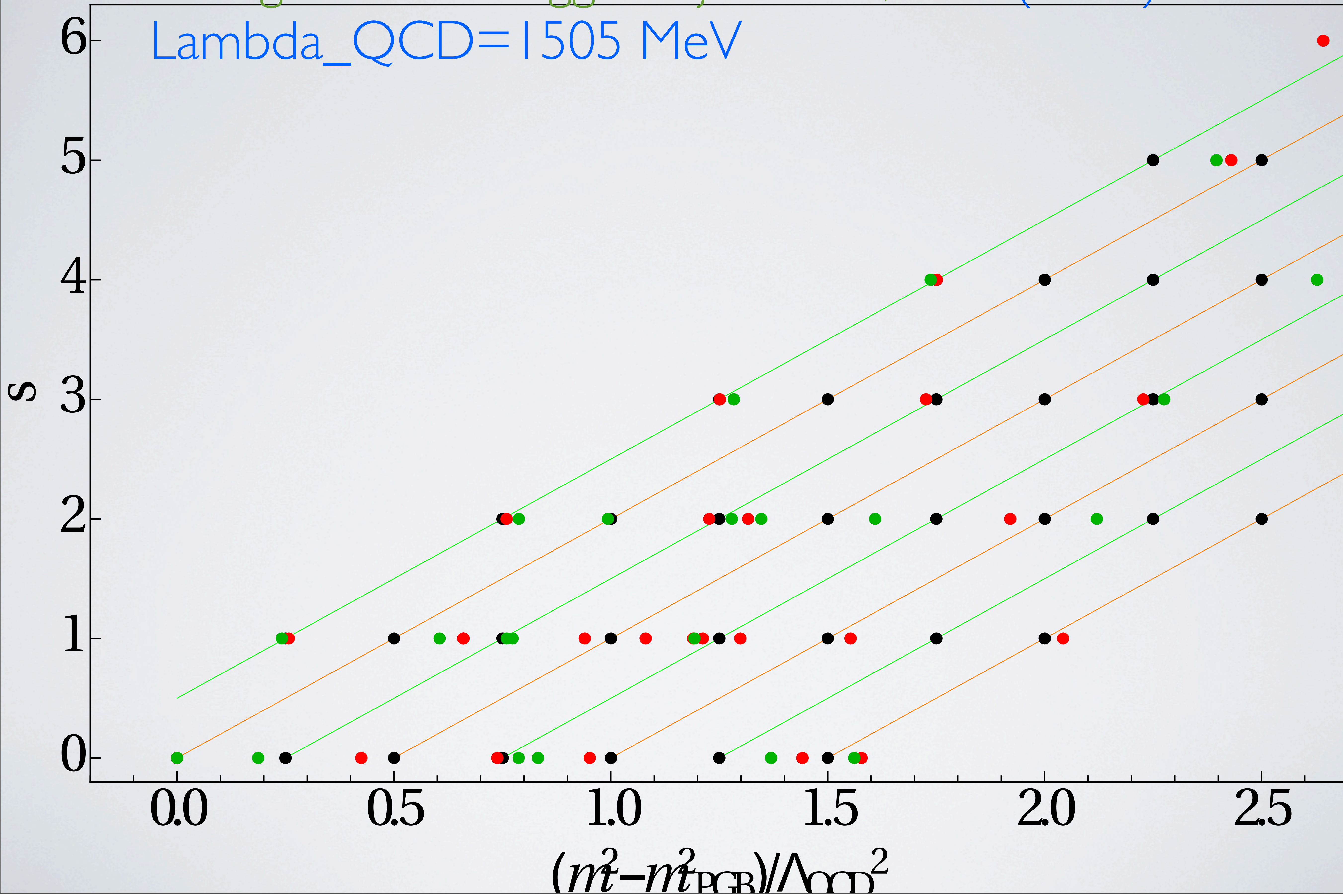
N $$r_s = 1.83; r_2 = 1.56$$

S $$r_s = 2.19; r_{ps} = 1.25; r_2 = 1.2$$

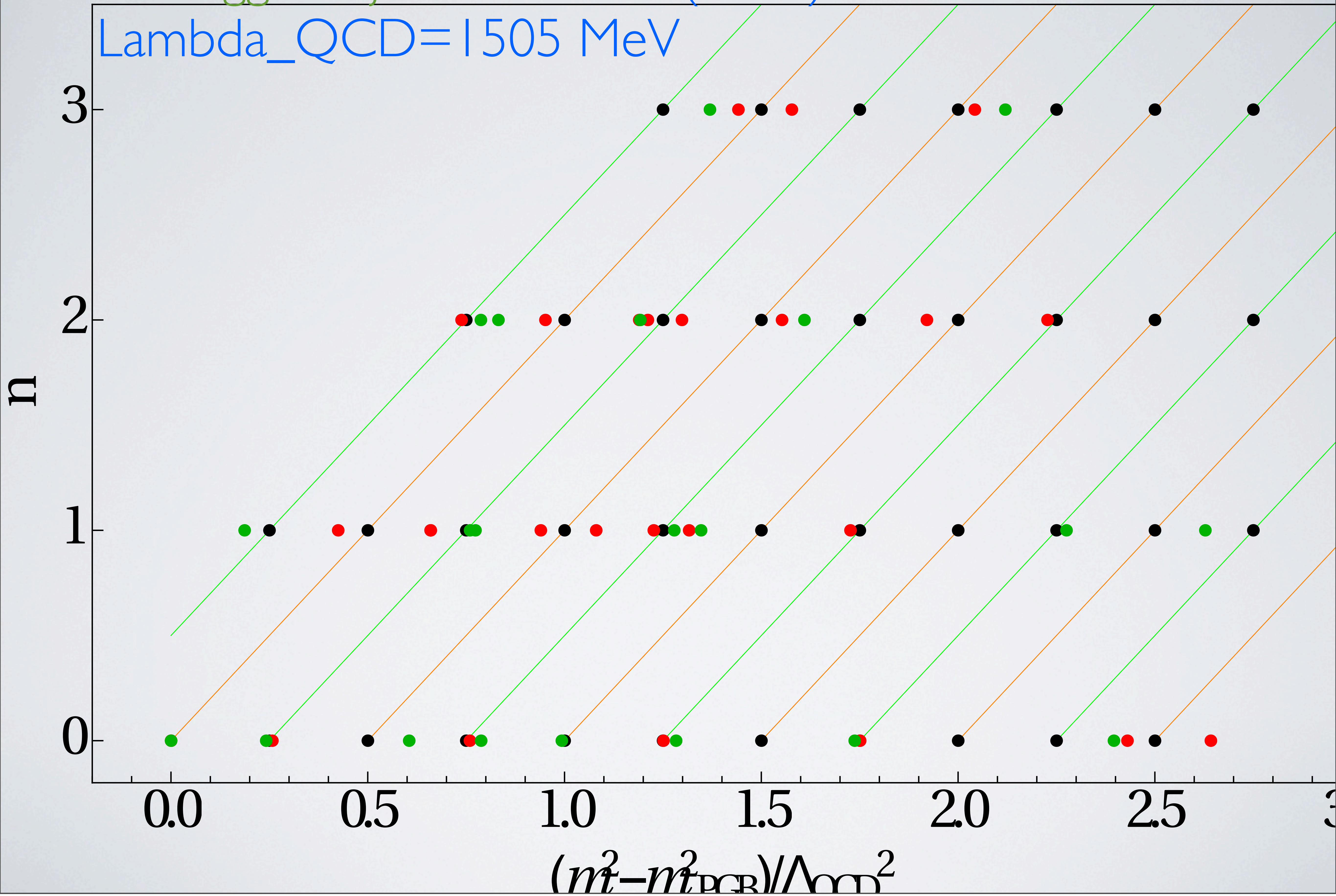
Soft Wall:

$$r_s = \sqrt{\frac{3}{2}} = 1.2247\cdots$$

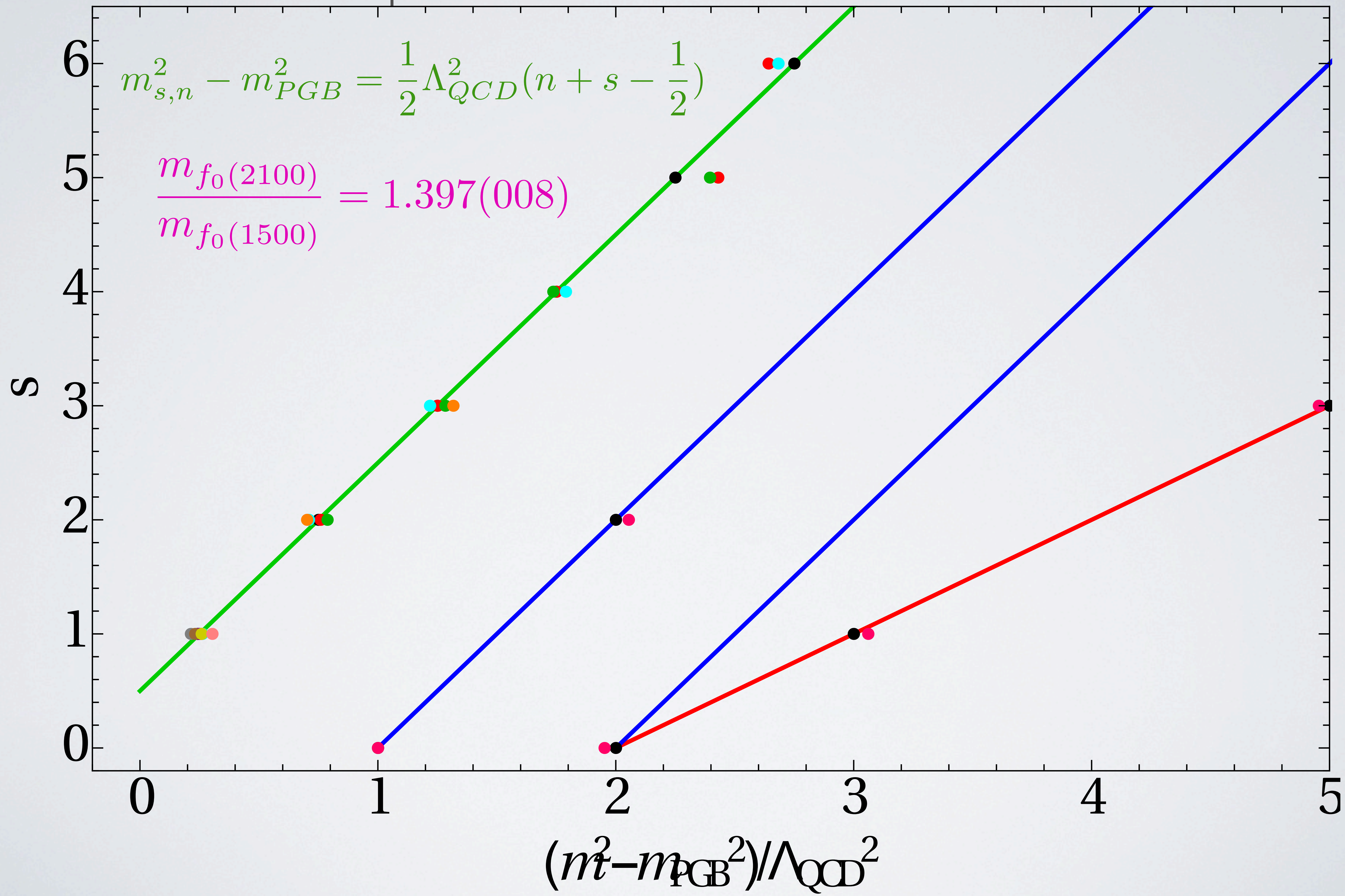
SU(3) unflavored I=1 meson Regge trajectories,
and strange-meson Regge trajectories, PDG(2014)



SU(3) unflavored I=1 meson Regge trajectories, and strange-meson Regge trajectories, PDG(2014)



The actual glueball (purple and blue) and meson leading Regge trajectories for any flavor (other colors) implied by Particle Data Group and BES collaboration versus the TTST



Prediction for glueball masses

f_0(2100) th 2128 exp 2103 err 1%

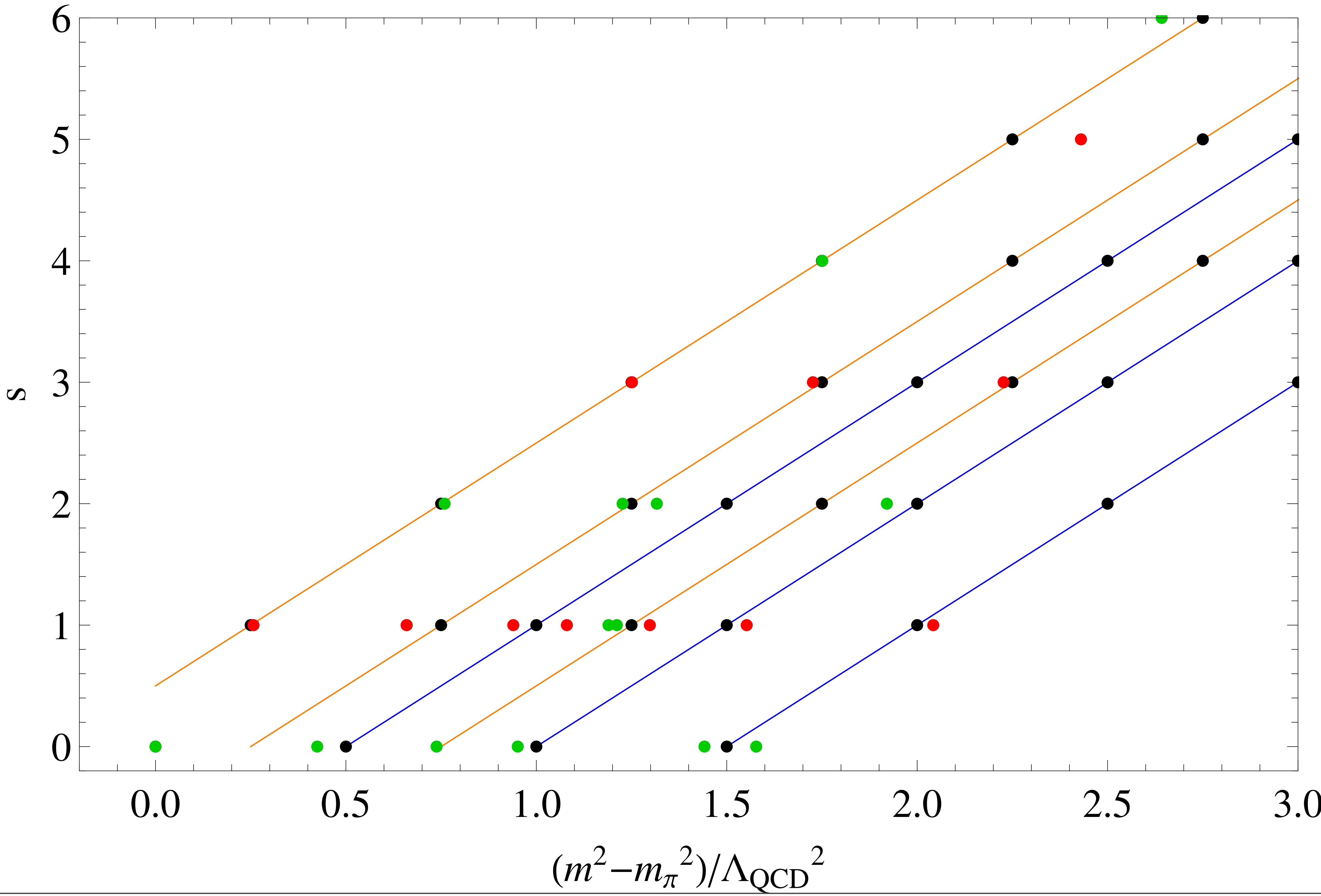
f_2(2150) th 2128 exp 2157 err 1%

X_1(?)(2632) th 2607 exp 2633 err 1%

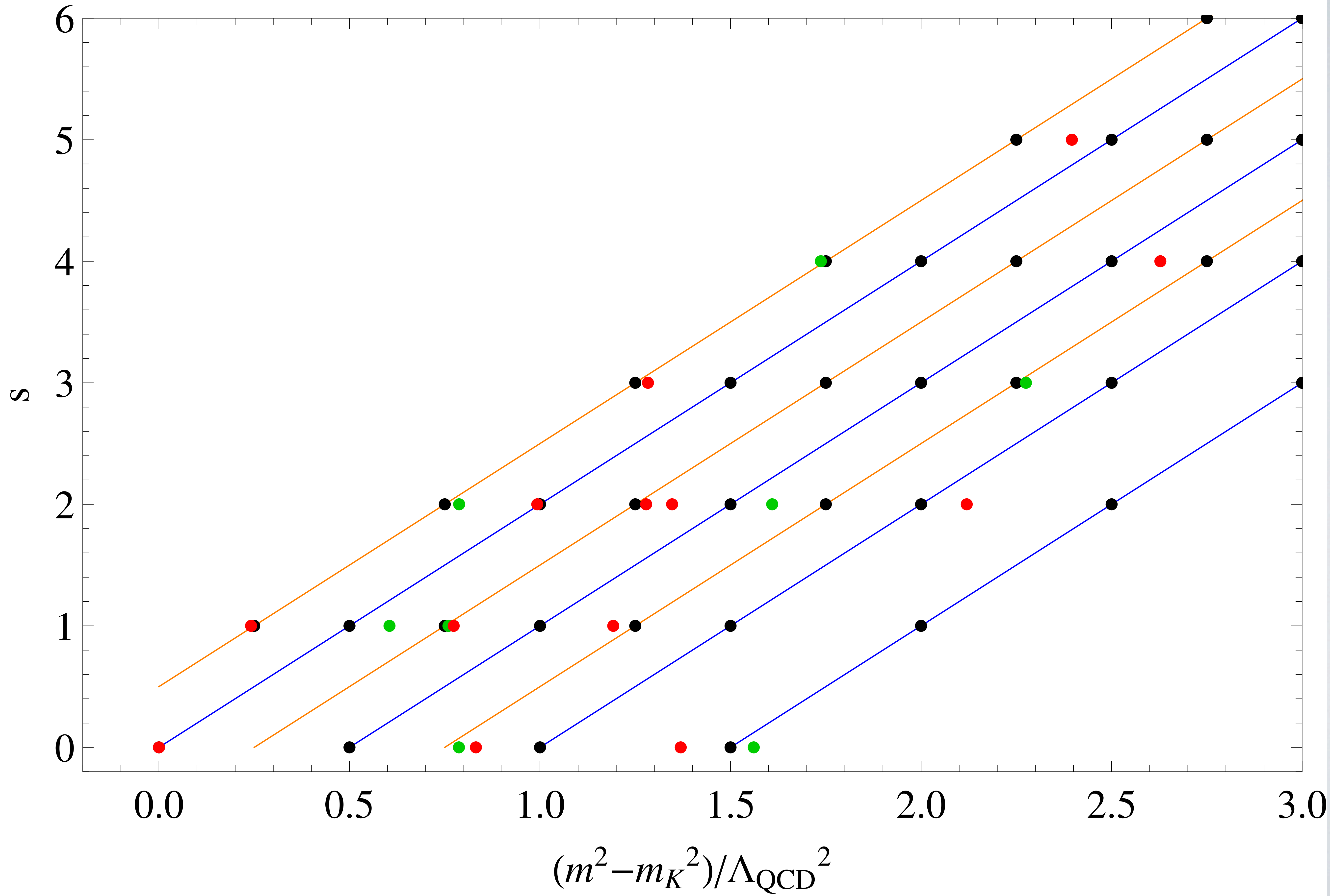
X_3(?)(3350) th 3365 exp 3350 err 1 %

Name	J^{PC}	$m_{\rm exp}$ (MeV)	$m_{\rm th}$ (MeV)	Error (%)
π^0	0^{-+}	134.98	0	–
$\rho(770)^0$	1^{--}	775.26	753	3.0
$a_0(980)$	0^{++}	990	1064	7.0
$b_1(1235)$	1^{+-}	1229.5	1303	5.7
$a_1(1260)$	1^{++}	1230	1303	5.6
$\pi(1300)$	0^{-+}	1300	1505	13.6
$a_2(1320)$	2^{++}	1318.3	1303	1.1
$\rho(1450)$	1^{--}	1465	1683	12.9
$a_0(1450)$	0^{++}	1474	1505	2.1
$\rho(1570)$	1^{--}	1570	1683	6.7
$\eta_2(1645)$	2^{++}	1617	1683	3.9
$a_1(1640)$	1^{++}	1647	1683	2.1
$\pi_2(1670)$	2^{-+}	1672.2	1683	0.6
$\rho_3(1690)$	3^{--}	1688.8	1683	0.4
$\rho(1700)$	1^{--}	1720	1683	2.2
$a_2(1700)$	2^{++}	1732	1683	2.9
$\pi(1800)$	0^{-+}	1812	1843	1.7
$\eta_2(1870)$	2^{++}	1842	1991	7.5
$\rho(1900)$	1^{--}	1880	1991	5.6
$\pi(1880)$	0^{-+}	1895	1843	2.8
$\rho_3(1990)$	3^{--}	1982	1991	0.4
$a_4(2040)$	4^{++}	1996	1991	0.3
$\pi_2(2100)$	2^{-+}	2090	1991	5.0
$\rho(2150)$	1^{--}	2155	2258	4.5
$\rho_3(2250)$	3^{--}	2250	2258	0.3
$\rho_5(2350)$	5^{--}	2350	2258	4.1
$a_6(2450)$	6^{++}	2450	2496	1.8

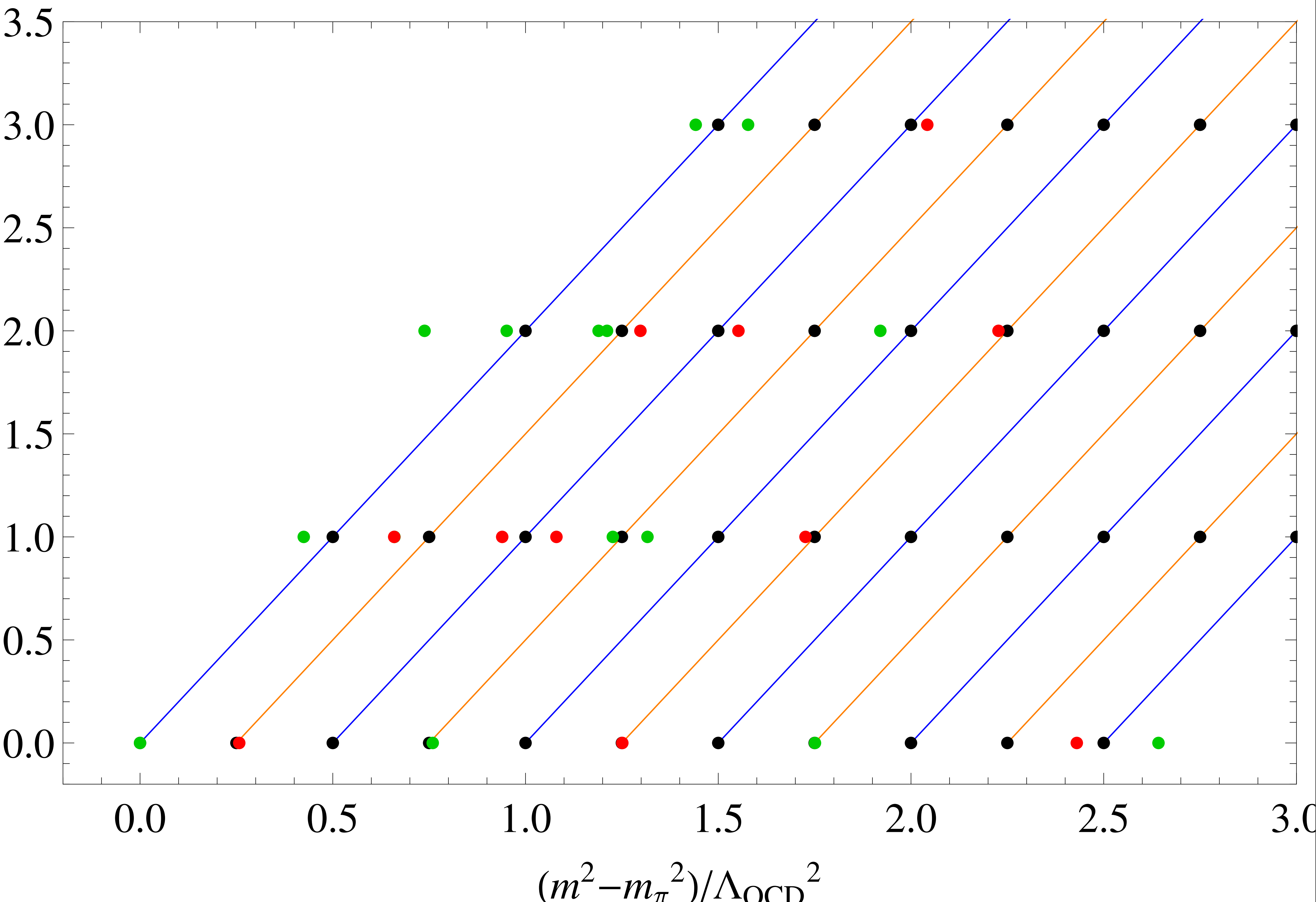
SU(3) unflavored I=1 meson Regge trajectories, Lambda YM=1505 MeV



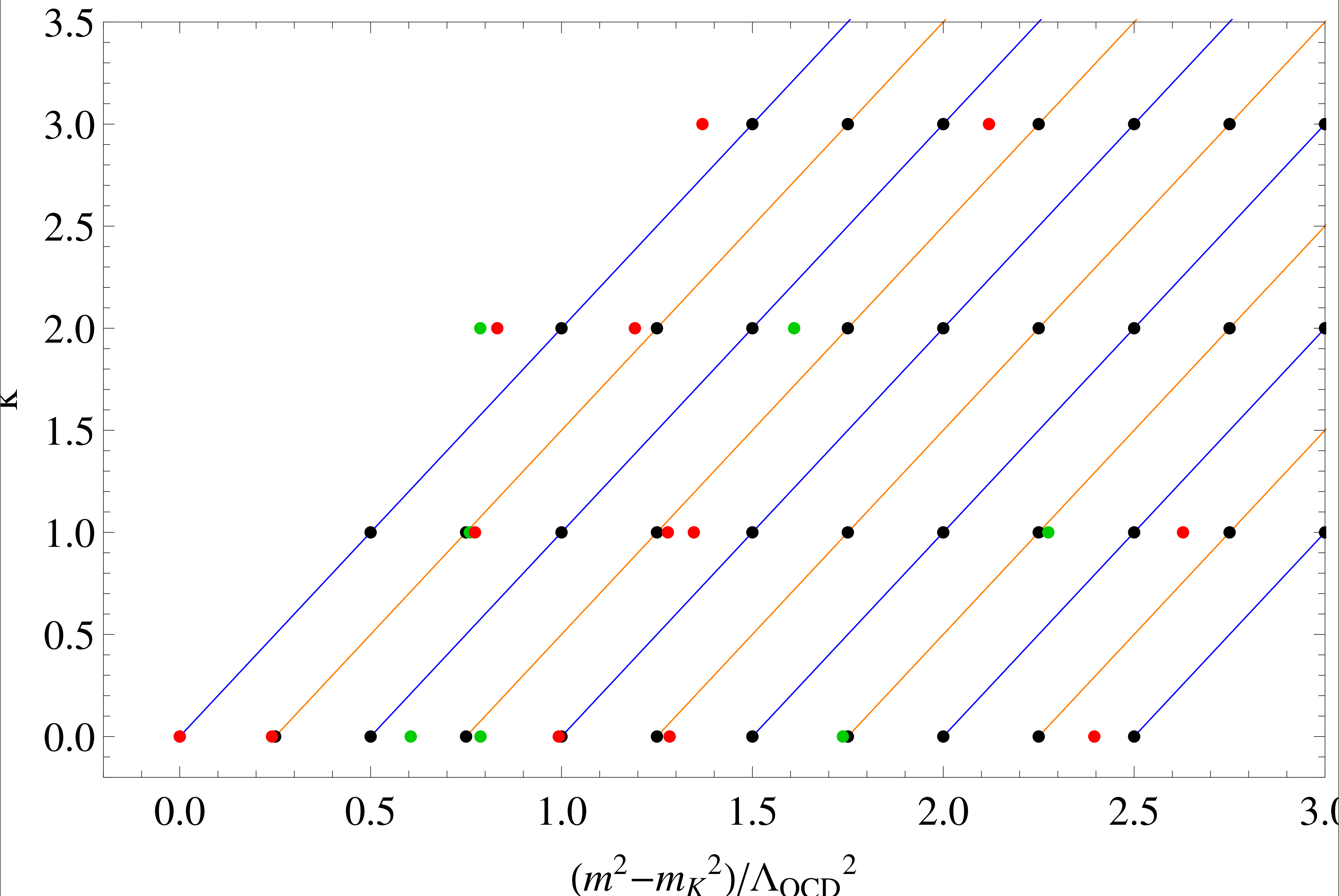
SU(3) strange-meson Regge trajectories, Lambda YM=1505 MeV



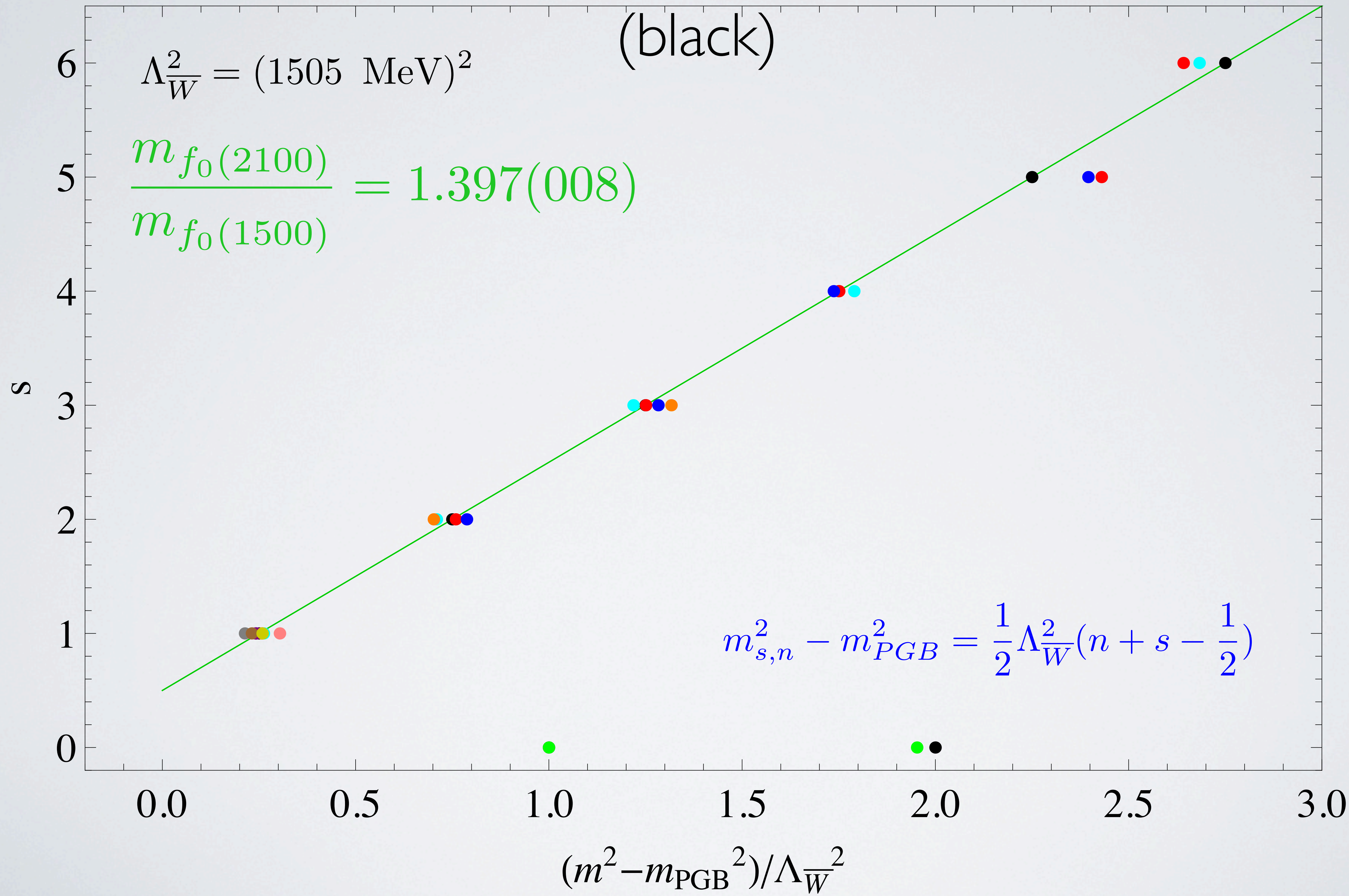
SU(3) QCD unflavored I=1 meson spectrum



SU(3) QCD strange-meson spectrum



The actual glueball (green) and meson leading Regge trajectories for any flavor (other colors) implied by Particle Data Group and BES collaboration versus the TFT theory (black)



Does the Twistorial A-model on non-commutative twistor space solve really QCD in ‘t Hooft limit,
only for the spectrum and the collinear S-matrix ?

We will see ... (to appear shortly)