Phase of Det D in 3D QED

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Phase of Fermion Determinant and Parity Anomaly

- Two component fermion: $D = \sigma_\mu(\partial_\mu + iA_\mu) + m$
- Effective gauge action induced by the fermion:

$$\int \mathcal{D}\overline{\psi}\mathcal{D}\psi e^{-\overline{\psi}D(m,A)\psi} = \left|\det D(m,A)\right| e^{i\Gamma(A)},$$

on fixed gauge field background A_μ in 3d Euclidean space.

- $\Gamma_{\rm odd} \to -\Gamma_{\rm odd}$ under parity:

$$x_\mu \to -x_\mu \qquad A_\mu \to -A_\mu \qquad \psi \to \psi \qquad \overline{\psi} \to -\overline{\psi}.$$

- $\Gamma_{\rm odd} \neq 0$ when $m \to \infty$ and $m \to 0$ (Niemi & Semenoff '83, Redlich '84).
- Reason in perturbation theory: Induced local Chern-Simons action

$$\Gamma_{\rm CS} = \frac{\kappa}{4\pi} \int F^*_\mu A_\mu d^3x.$$

What are the non-perturbative aspects of Γ? Is there a parity-even phase? Is the phase still a local $\Gamma_{\rm CS}$ at finite m with $\kappa = \kappa(m)$?

Gauge-Invariant Lattice Formulation

- Choice: preserve either parity (Narayanan & Nishimura '97) or gauge symmetry (Kikukawa & Neuberger '98).
- This work: Wilson-Dirac fermions which are gauge-invariant (Coste & Lüscher '89).
- Wilson-Dirac operator chosen such that

$$D_w = \begin{cases} D_n - B + M & \text{if} \quad M > 0 \\ D_n + B - M & \text{if} \quad M < 0. \end{cases}$$

- Parity covariant: $\Gamma(-M) = -\Gamma(M)$.

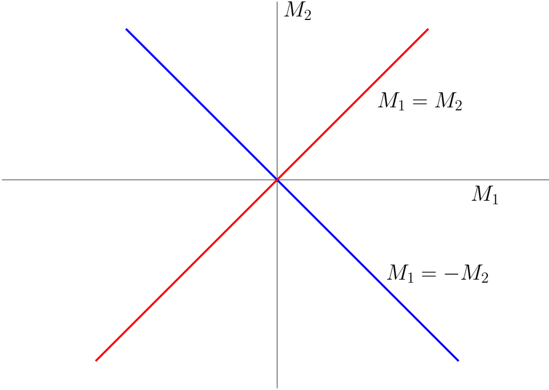
Many Flavours of Two Component Fermions

- Two-flavours with masses M_1 and M_2: (Sign-problem for Monte-Carlo)

$$\det\left\{D_w(M_1)D_w(M_2)\right\} = \det\left\{D_w(M_1)D_w^\dagger(-M_2)\right\}.$$

- $\{$2-component fermion $M_2 = -M_1\} \equiv \{$ 1 flavour Dirac fermion$\}$
- $\det\left\{D_w(M_1)D_w^\dagger(M_1)\right\} > 0$ (Tractable by MC)

- **Condensates:** $\langle\overline{\psi_1}\psi_1 - \overline{\psi_2}\psi_2\rangle$, $\langle\overline{\psi_1}\psi_2\rangle$ and $\langle\overline{\psi_2}\psi_1\rangle \neq 0$?
- Mild **Sign problem** when $M_1 \approx -M_2$?
- **Vacuum structure**: Lines of constant free-energy? (Vafa and Witten '84)



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Gauge-field background on $l \times l \times \beta$ torus

- Gauge-fields periodic up to a gauge transformation:

$$A_2 = \frac{2\pi q_3}{l^2} x + \frac{2\pi h_2}{l} + A_2^p .$$

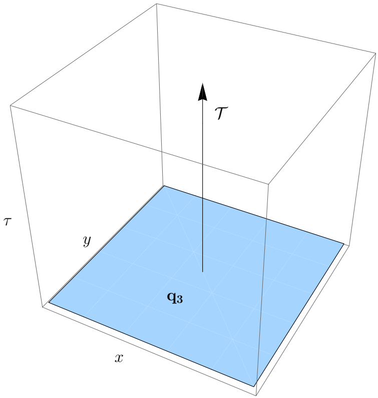
Constant flux → $\frac{2\pi q_3}{l^2}$; Toron → $\frac{2\pi h_2}{l}$; Perturbations → A_2^p

- "Periodicity" for fermions

$$e.g., \qquad \psi(l, y, \tau) = e^{-i\frac{2\pi q_3 y}{l}} \psi(0, y, \tau).$$

A Conventional Example: Static Magnetic Field

- Only magnetic flux q_3.
- Transfer matrix: $\mathcal{T}(t) = e^{-H_{2d}(t)\Delta t}$
- Eigenvalues of $\mathcal{T}$: $e^{\pm\lambda^{\pm}}$



$$\det D = \det\left(1 - \prod_{t=0}^{\beta} \mathcal{T}(t)\right)$$

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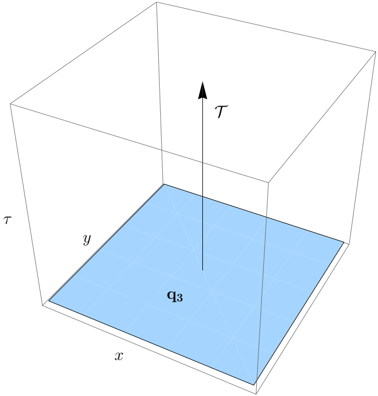

$$\det D = \prod_{i=1}^{V+q_3} \left(1 - e^{\beta\lambda_i^+}\right) \prod_{j=1}^{V-q_3} \left(1 - e^{-\beta\lambda_j^-}\right); \quad M > 0$$

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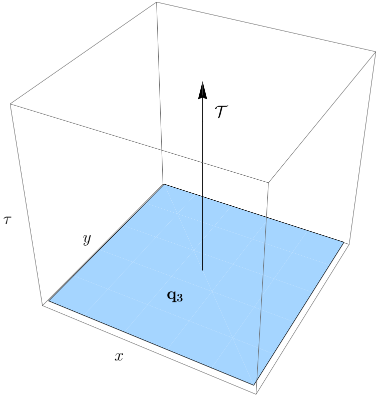
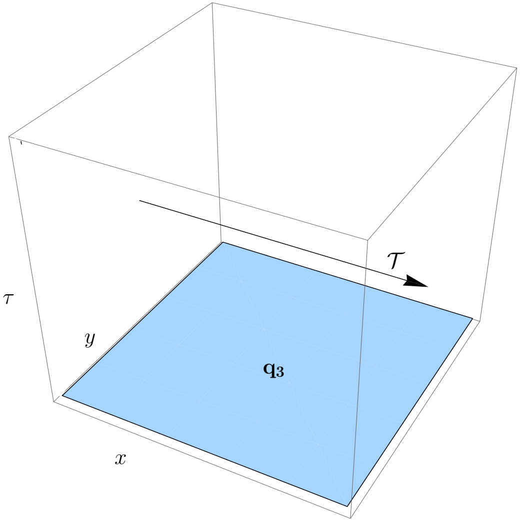


$$\frac{\mathbf{det}D}{|\mathbf{det}D|} = (-1)^{q_3}$$

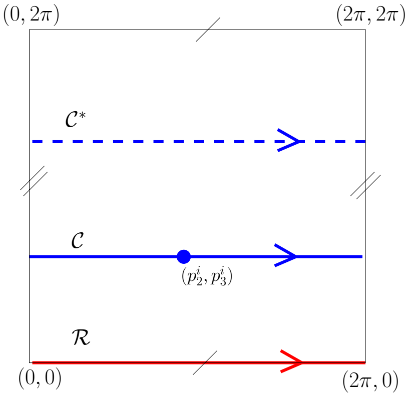
A Lateral Perspective



A Lateral Perspective

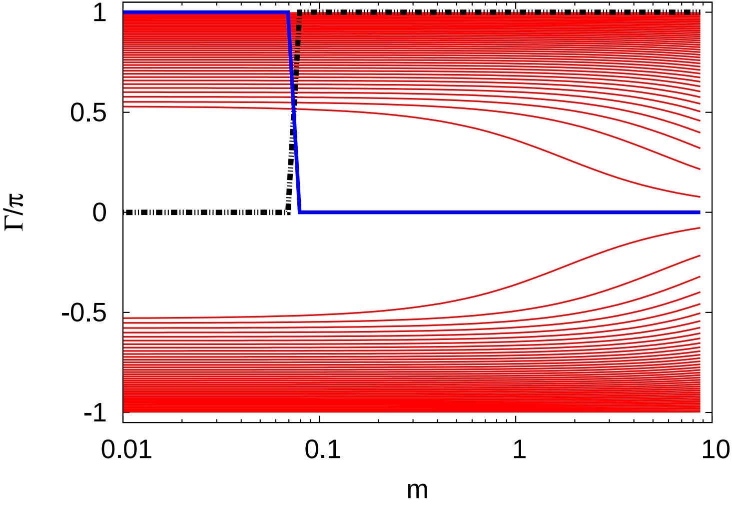
- The transfer matrix block-diagonalizes into cycles $\mathcal{C}$ in p_y-p_t space.
- $[\det D]_{\mathcal{C}} = \det \left\{ 1 - \prod_{p \in \mathcal{C}} \widetilde{\mathcal{T}}(p) \right\}$

  where $\mathcal{C}$ is a closed path in momentum space: $\left(p_2^{\mathrm{i}} - \frac{2\pi q_3 x}{l}, p_3^{\mathrm{i}} \right)$.



Eigenvalues of cycles (p_1^i, p_2^i) and $(2\pi - p_1^i, 2\pi - p_2^i)$ are complex conjugates.
$\Rightarrow$ Cycle $\mathcal{R}$ passing through $(0,0)$ is real.

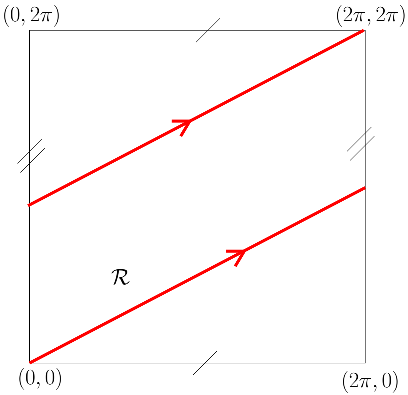
Sign flip in real cycle⇒ $(-1)^{q_3}$



Static Electric and Magnetic Fields (q_2 and q_3)

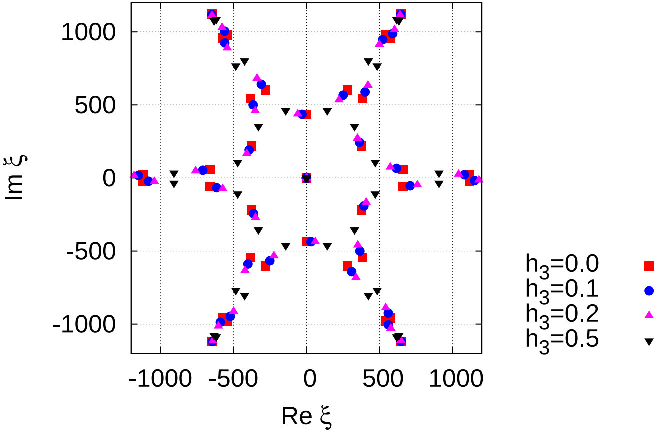
- Cycles still pair into complex conjugates
- Cycles now wind around the torus. The slope is q_2/q_3.
- When q_2 and q_3 have a common factor r, then there are r cycles with the same slope.

$$\Rightarrow \frac{\det D}{|\det D|} = (-1)^{q_2+q_3+\mathbf{q_2 q_3}}$$



Eigenvalue Flow due to Toron in Magnetic Field

$\xi \rightarrow$ Eigenvalues of $\prod_{x=1}^{L} \mathcal{T}(x)$.



$m \rightarrow \infty \Rightarrow \Gamma = -2\pi h_3 q_3$.

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Form-factor G

- Time dependent perturbative fields $A_1^p(t)$ and $A_2^p(t)$ on top of uniform magnetic field q_3.
- At zero temperature,

$$\Gamma = -2\pi h_3 q_3 - \int d\tau d\tau' G(\tau - \tau') A_1^p(\tau) A_2^p(\tau').$$

- $G(\tau) = G_{\mathrm{reg}}(\tau) + \frac{1}{L}\left(\frac{c}{\tau^3}\right) + \mathcal{O}\left(\frac{1}{L^2}\right)$
- $\Rightarrow \Gamma_{\mathrm{odd}} = \Gamma_{\mathrm{reg}}(m, q_3, h_3) + \Gamma_{\mathrm{sing}}(m)$
- Γ_{reg} is non-local and vanishes when $m \to 0$ and $m \to \infty$
- Γ_{sing} is a pure Chern-Simons term with $\kappa = \kappa(m)$.

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Conclusions

- Possible to introduce parity covariant 2-component lattice fermions in 3d. Wilson fermions are sufficient for this purpose.
- Using gauge-field backgrounds with both non-vanishing electric and magnetic fluxes, we studied the phase of $\det D$.
- In the presence of electric and magnetic fluxes, transfer matrix block-diagonalizes into closed cycles in momentum space.
- Crossing of eigenvalue of $2d$ Dirac Hamiltonian $\Rightarrow$ Flip in sign for the cycle passing through zero momentum.
- A parity-even phase $q_1 q_2$.
- A dominant non-local contribution to phase exists at generic m. At $m = 0$ and ∞, the phase is only a regulator dependent local Chern-Simons term.