Confining Strings On $R^3 \times S^1$

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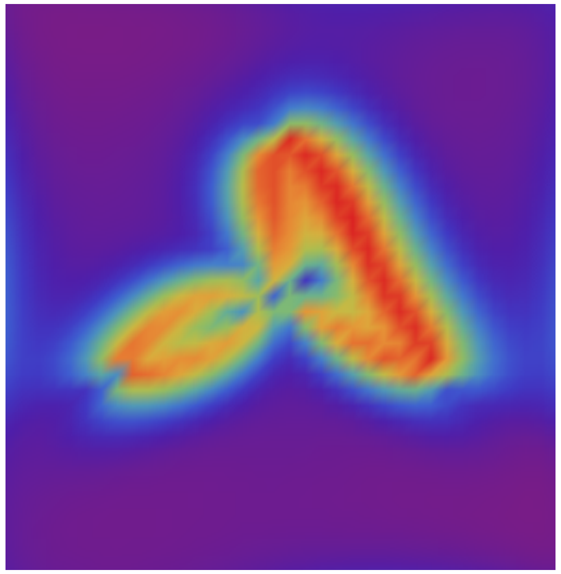
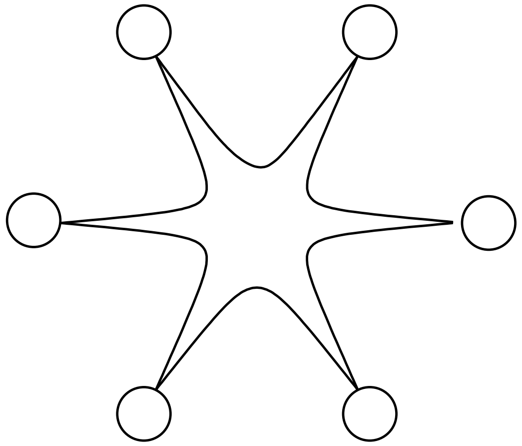
and in progress



3-quark SU(3) baryon in QCD(adj)



6-monopole SU(6) "dual baryon", from Shifman-Yung 0703267
(ref. also motivated by other analogies to their work...)

Motivation/Summary/Outline I:

Confining strings may seem ubiquitous and 'old'... but are analytically understood - within continuum QFT, starting from the microscopic QFT degrees of freedom, and in a controlled manner - only in a few cases.

- Seiberg-Witten theory: N=2 super YM with N=1 soft mass, abelian confinement Douglas Shenker, Hanany Strassler Zaffaroni mid/late 1990s

- monopole confinement in abelian Higgs model and in related (dual) models with nonabelian strings Gorsky, Shifman, Yung 2004-2014-

$\longrightarrow$ (here) confinement on $R^3 \times S^1$, abelian Unsal, Shifman, Yaffe,... 2007-

Lattice - numerical experiment - confining flux tubes exist, for sure, spectrum etc.

String theory - strings are there in dual theory, to begin with

one only has to work to make them give linear potential (so they don't fall to horizon)

- under control in regimes quite far from asymptotically-free QFT

It is interesting to study the few understood QFT cases, their relations to each other, to string, and to lattice...

Motivation/Summary/Outline II:

In this talk, I will study the last case above:

- confinement on $R^3 \times S^1$, abelian Unsal, Shifman, Yaffe,...

Many properties of theories with semiclassical confinement in this setup have been understood

SYM: Seiberg, Witten/Aharony, Intriligator, Hanany, Seiberg, Strassler late 1990s

SYM, with new insight, & non-SYM: Unsal w/ Yaffe, Shifman... since 2007

but confining strings have not been studied in any detail.

We shall see that confining strings in these theories have properties distinct from other theories with abelian confinement (e.g. SW) and show surprising similarities to various dual theories with (non-) abelian confinement of monopoles discussed previously.

Motivation/Summary/Outline III:

1. a lightning review of confinement on R^3 x S^1:
   deformed Yang-Mills theory and QCD(adjoint)/SYM

   Unsal w/ Yaffe, Shifman...

   experts: hopefully not too bored

   non-experts: can't explain all, will assert a few facts
   - but if these are accepted, study of strings will be clear

2. confining strings in deformed YM and QCD(adj):
   domain walls, mesons, and baryons

3. comparison to other understood cases and the transition to
   the nonabelian regime

4. for the future:
   lattice
   uses to study global structure

1. confinement on R^3 x S^1, size of circle- L:

We study SU(N) in the regime $NL\Lambda \ll 1$

QCD(adj): YM with n_f adjoint Weyl fermions; $n_f = 1$ is SYM

dYM: pure YM with particular double-trace "deformation"

Assertions...

i.) in each case, the theory abelianizes at a scale 1/(NL)

$SU(N) \to U(1)^{N-1}$ W-bosons' mass $\frac{1}{NL} \gg \Lambda$

no light states charged under the N-1 massless "photons"

since only adjoint fields, massless states after breaking neutral under Cartan

in the regime we study, perturbative IR dynamics boring:
free U(1)s + light neutral Cartan subalgebra "gauginos" in QCD(adj)

1. confinement on R^3 x S^1, size of circle- L:

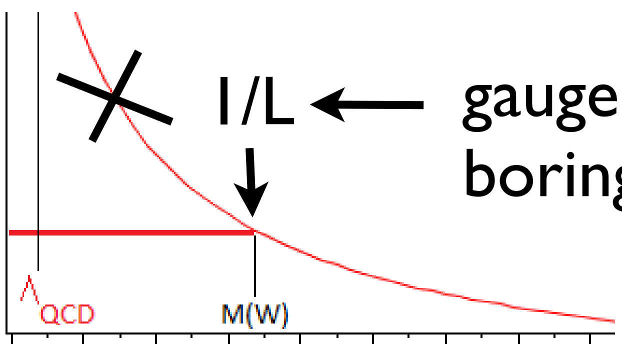
Assertions, contd.:

i.) in each case, the theory abelianizes at a scale 1/(NL)

in the regime we study, perturbative IR dynamics boring:

free U(1)s + Cartan components of gauginos in QCD(adj) $\frac{1}{NL} \gg \Lambda$

$$W = P e^{i \oint_{S_1} A_4 \, dx^4} \qquad \text{SU(2)}: \langle A_4 \rangle^{\text{Cartan}} \sim \frac{\pi}{L}$$



gauge coupling "freezes" (Higgsed) to a small value a scale 1/L

boring perturbative IR dynamics

energy scale

SYM: Seiberg, Witten/Aharony, Intriligator, Hanany, Seiberg, Strassler

SYM & non-SYM: Unsal w/ Yaffe, Shifman...

ii.) nonperturbatively, however, the dynamics is quite rich

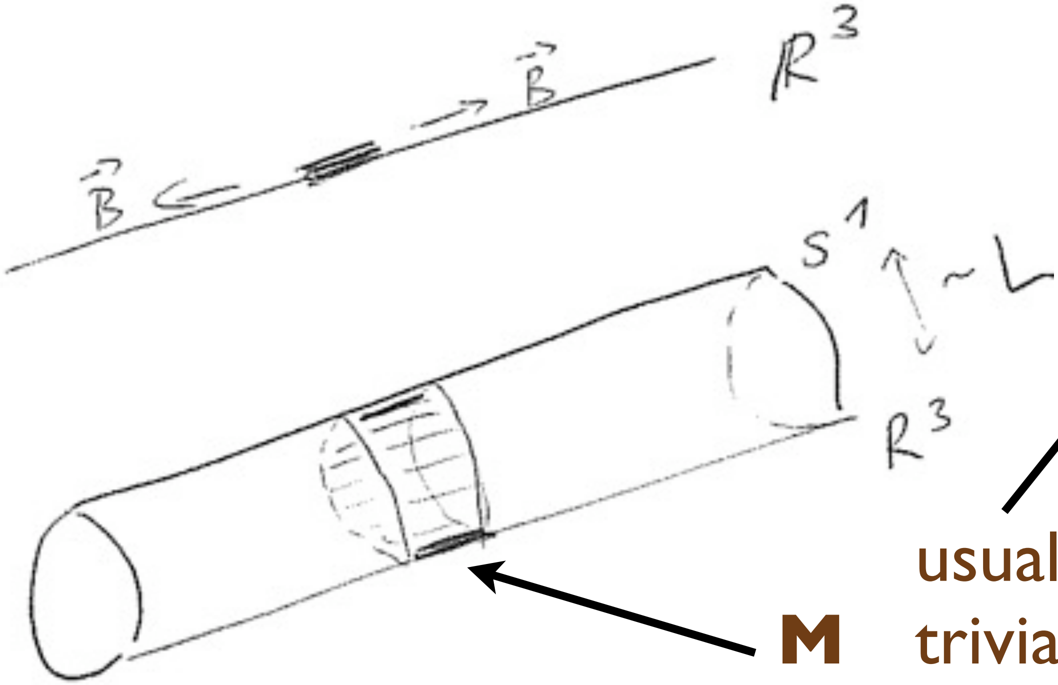
the $SU(N) \to U(1)^{N-1}$ theory has instanton solutions

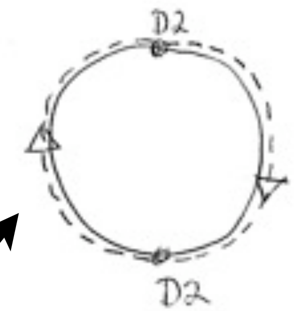
these change the IR behavior of the theory and generate a mass gap (Polyakov mechanism in a locally 4d setting)

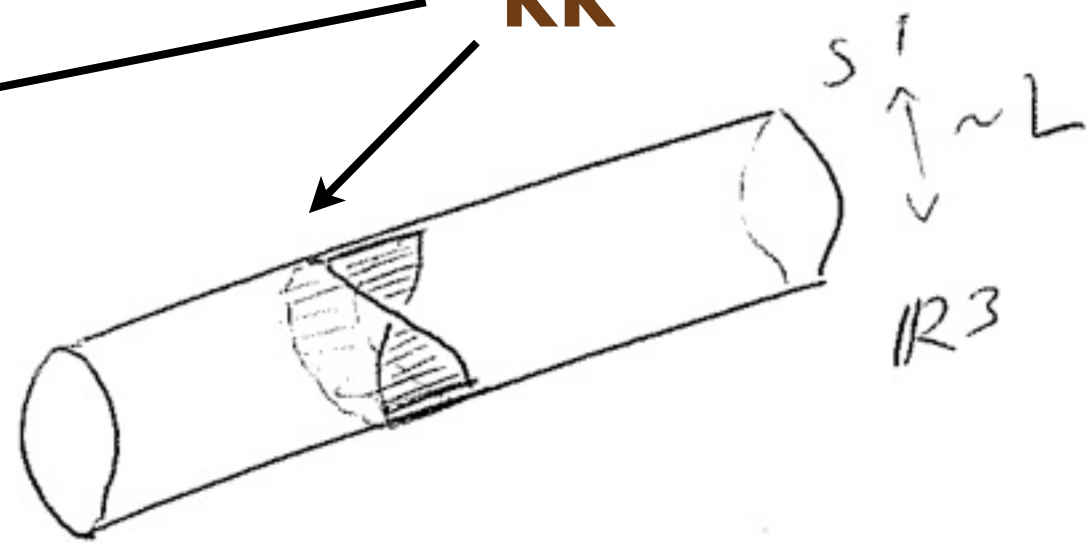
1. confinement on R^3 x S^1, size of circle- L:

Wilson line breaks SU(2) to U(1) so there are monopole-instantons

KK







M

usual monopole
trivially
embedded in 4d

For SU(N), 4d BPST instanton dissociates into N constituents:

$$SU(N):\quad e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L)\,N}}$$

(large-N survive!)

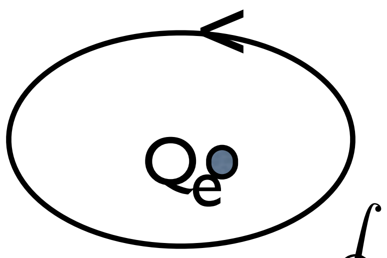
As opposed to 4d BPST instantons, have long-range "magnetic field".
Dilute monopole-instanton gas - as in SM to obtain 't Hooft vertex (qqql)^3 = 3d dilute - but Coulomb! - gas
[this is all non-experts need to accept to understand study of strings]

1. confinement on R^3 x S^1, size of circle- L:

to write Z - the partition function of dYM/QCD(adj), need

sigma = dual photon field $e^2 d\sigma = *F$

$$e^2 = \frac{g^2}{L}, \quad g^2 \sim g_4^2(1/L)$$



$$\oint d\sigma = Q_e$$

electric coupling ~ 4d coupling at 1/L

$$\partial_0 \sigma \sim \frac{L}{g^2} F_{12} \qquad \partial_i \sigma \sim \frac{L}{g^2} \epsilon_{ij} E_j, \quad j = 1, 2$$

time derivative = 3d magnetic field

spatial gradient = 3d electric field

monodromy of sigma around a spatial loop = electric charge inside

Main result

[Polyakov, 1970's]:

$$Z \sim \int \mathcal{D}\sigma \; e^{-\int dx L_{eff}(x)}$$

$$Z[j] = \langle e^{i \int dx j(x) \rho_m(x)} \rangle, \quad L_{eff}(x) = e^2 (\partial_i \sigma)^2 - \zeta \cos(\sigma(x) + j(x))$$

for SU(2), only one dual photon (Cartan)

I. confinement on R^3 x S^I, size of circle- L:

$$L_{eff}^{dYM} = \frac{g^2}{L}(\partial_i \vec{\sigma})^2 - \sum_i^N \zeta \cos \vec{\alpha}_i \cdot \vec{\sigma} \qquad \zeta \sim L^{-3} e^{-\frac{4\pi^2}{g^2}\frac{2}{\mathsf{N}}}$$

monopole-instanton fugacity

two important scales!

$M \sim \frac{1}{L}$ W-boson mass

$m \sim M e^{-\frac{\mathcal{O}(1)\pi^2}{g^2}}$ dual photon mass

$$L_{eff}^{dYM} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos \vec{\alpha}_i \cdot \vec{\sigma} \right]$$

same as before, except N-I dual photons and N monopole-instantons

$\vec{\alpha}_1 = (1, -1, 0, 0, ...0)$ $\vec{\alpha}_2 = (0, 1, -1, 0, ...0)$ ... $\vec{\alpha}_{N-1} = (0, 0, 0, ...0, 1, -1)$ $\vec{\alpha}_N = (-1, 0, 0, ...0, 1)$

monopole-instanton charges (under U(I)$^{\mathsf{N}}$, convenient basis)=all simple+lowest root

$$L_{eff}^{QCD(adj)} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod} N)}) \cdot \vec{\sigma} \right]$$

1. confinement on R^3 x S^1, size of circle- L:

formulae reveal different confinement mechanisms in dYM and QCD(adj):

monopole instantons vs "magnetic bions"

monopole instantons

$$L_{eff}^{dYM} = M\left[(\partial_i\vec{\sigma})^2 - \sum_i^N m^2 \cos\vec{\alpha}_i\cdot\vec{\sigma}\right]$$

same as before, except N-1 dual photons and N monopole-instantons

$\vec{\alpha}_1 = (1,-1,0,0,...0)$ $\vec{\alpha}_2 = (0,1,-1,0,...0)$ ... $\vec{\alpha}_{N-1} = (0,0,0,...0,1,-1)$ $\vec{\alpha}_N = (-1,0,0,...0,1)$

monopole-instanton charges (under $U(1)^N$, convenient basis)=all simple+lowest root

$$L_{eff}^{QCD(adj)} = M\left[(\partial_i\vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot\vec{\sigma}\right]$$

magnetic bions

"bound states" of monopole instantons and anti-monopole instantons

1. confinement on R^3 x S^1, size of circle- L:

formulae reveal different confinement mechanisms in dYM and QCD(adj):

monopole instantons vs "magnetic bions"

monopole instantons

$$L_{eff}^{dYM} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos \vec{\alpha}_i \cdot \vec{\sigma} \right]$$

a crucial - for strings - property, most easily seen QCD(adj) L_eff:

$\vec{\alpha}_j \cdot \vec{\sigma} \rightarrow \vec{\alpha}_{j+1(\mathrm{mod} N)} \cdot \vec{\sigma}$ Z_N Weyl symmetry (due to center stability)

$$\vec{\sigma} \rightarrow P\vec{\sigma}, \quad P = s_{\alpha_{N-1}} s_{\alpha_{N-2}} \ldots s_{\alpha_2} s_{\alpha_1}, \quad s_\alpha \vec{v} = \vec{v} - 2\vec{\alpha} \, \frac{\vec{v} \cdot \vec{\alpha}}{\vec{\alpha} \cdot \vec{\alpha}}$$

$$L_{eff}^{QCD(adj)} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod} N)}) \cdot \vec{\sigma} \right]$$

magnetic bions

"bound states" of monopole instantons and anti-monopole instantons

II. confining strings in QCD(adj) and dYM:

$$L_{eff}^{dYM} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos \vec{\alpha}_i \cdot \vec{\sigma} \right]$$

$$L_{eff}^{QCD(adj)} = M \left[(\partial_i \vec{\sigma})^2 - \sum_i^N m^2 \cos(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod} N)}) \cdot \vec{\sigma} \right]$$

probe for confinement - area law for quarks in representation $\mathcal{R}$

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}\, P e^{i \oint\limits_C A_k dx^k} \sim e^{-\mathrm{Area(C)}\Sigma_{str.}}$$

in the abelian regime of small L, simplify:

$$W_{\mathcal{R}}(C) = \sum_{H \in \mathcal{R}} \mathrm{tr}_{\mathcal{R}}\, e^{i\vec{H} \cdot \oint\limits_C \vec{A}_k dx^k} = \sum_{\vec{\nu} \in \mathcal{R}} e^{i\vec{\nu} \cdot \oint\limits_C \vec{A}_k dx^k} =$$

$$= \sum_{\vec{\nu} \in \mathcal{R}} e^{i\vec{\nu} \cdot \int\limits_{S:\partial S = C} \vec{B}_{\mathrm{normal}} d^2 x} = \sum_{\vec{\nu} \in \mathcal{R}} e^{i\vec{\nu} \cdot \vec{\Phi}(S(C))}$$

: all we need is magnetic flux through C

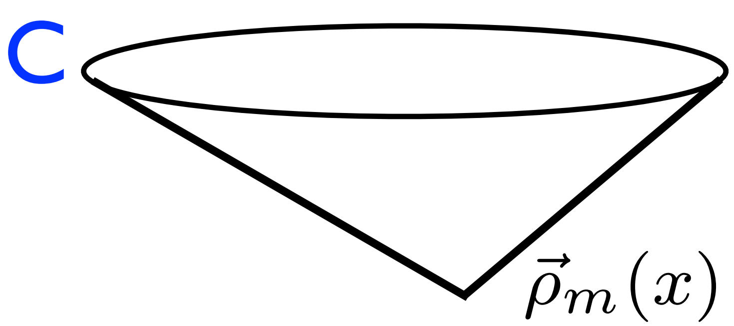
II. confining strings in QCD(adj) and dYM:

$$\langle W_{\mathcal{R}}(C)\rangle = \sum_{\vec{\nu}\in\mathcal{R}} \langle e^{i\vec{\nu}\cdot\vec{\Phi}(S(C))}\rangle$$: all we need is magnetic flux through C

but in the monopole gas, magnetic flux is due to monopole-instantons

$$\langle e^{i\vec{\nu}\cdot\vec{\Phi}(S(C))}\rangle = \langle e^{i\vec{\nu}\cdot\int d^3x\,\vec{\rho}_m(x)\eta_C(x)}\rangle$$

$$\vec{\Phi}(C,x) = \vec{\rho}_m(x)\eta_C(x)$$



monopole
-instanton
density

solid angle that C spans from x

4pi jumps don't matter - Dirac/GNO:

$$\vec{\rho}\cdot\vec{\nu} \in \frac{1}{2}Z$$

now recall that correlation functions of the density are generated by

$$Z[j] = \langle e^{i\int dx j(x)\rho_m(x)}\rangle, \quad L_{eff}(x) = e^2(\partial_i\sigma)^2 - \zeta\cos(\sigma(x) + j(x))$$

$$Z \sim \int \mathcal{D}\sigma\; e^{-\int dx L_{eff}(x)}$$

all goes through for a multimonopole gas:

$\vec{\sigma}(x) \to \vec{\sigma}(x) + \vec{\nu}\eta_C(x)$ in potential term

II. confining strings in QCD(adj) and dYM:

$$\langle W_{\mathcal{R}}(C)\rangle = \sum_{\vec\nu\in\mathcal{R}} \langle e^{i\vec\nu\cdot\vec\Phi(S(C))}\rangle = \sum_{\vec\nu\in\mathcal{R}} \langle W(\vec\nu)\rangle$$

dYM

$$\langle W(\vec\nu)\rangle = \int \mathcal{D}\sigma\ \exp\left[-M\int_{R^3}(\partial\vec\sigma)^2 - Mm^2\int_{R^3}\sum_{i=1}^{N}\left\{\begin{array}{c}\cos\vec\alpha_i\cdot(\vec\sigma+\vec\nu\eta_C)\\ \cos(\vec\alpha_i-\vec\alpha_{i-1(\mathrm{mod}N)})\cdot(\vec\sigma+\vec\nu\eta_C)\end{array}\right.\right]$$

Wilson loop-quarks with charges $\vec\nu$

QCD(adj)

Semiclassically, $\langle W(\vec\nu)\rangle \sim e^{-S[\vec\sigma_{class.}]}$, where $\vec\sigma_{class.}$ solves:

dYM $\quad \nabla^2\vec\sigma - m^2\sum_{i=1}^{N}\vec\alpha_i\sin\vec\alpha_i\cdot(\vec\sigma+\vec\nu\eta_C) = 0$

QCD(adj) $\quad \nabla^2\vec\sigma - m^2\sum_{i=1}^{N}(\vec\alpha_i-\vec\alpha_{i-1(\mathrm{mod}N)})\sin(\vec\alpha_i-\vec\alpha_{i-1(\mathrm{mod}N)})\cdot(\vec\sigma+\vec\nu\eta_C) = 0$

These equations are great for numerics, for any contour C, via Gauss-Seidel relaxation - diffusion process in (discrete, fictitious) "time" t relaxes to minimum of action $\frac{\partial\sigma}{\partial t} = -\frac{\delta S}{\delta\sigma} = \nabla^2\sigma - 2m^2\sin(\sigma+\frac{1}{2}\eta_C)$

II. confining strings in QCD(adj) and dYM:

$$\langle W_{\mathcal{R}}(C)\rangle = \sum_{\vec{\nu}\in\mathcal{R}} \langle e^{i\vec{\nu}\cdot\vec{\Phi}(S(C))}\rangle = \sum_{\vec{\nu}\in\mathcal{R}} \langle W(\vec{\nu})\rangle$$

dYM

$$\langle W(\vec{\nu})\rangle = \int \mathcal{D}\sigma \; \exp\left[-M\int_{R^3}(\partial\vec{\sigma})^2 - Mm^2\int_{R^3}\sum_{i=1}^{N}\left\{ \begin{array}{c} \cos\vec{\alpha}_i\cdot(\vec{\sigma}+\vec{\nu}\eta_C) \\ \cos(\vec{\alpha}_i-\vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot(\vec{\sigma}+\vec{\nu}\eta_C) \end{array}\right.\right]$$

Wilson loop-quarks with charges $\vec{\nu}$

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QCD(adj) $\quad \nabla^2\vec{\sigma} - m^2\sum_{i=1}^{N}(\vec{\alpha}_i-\vec{\alpha}_{i-1(\mathrm{mod}N)})\sin(\vec{\alpha}_i-\vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot(\vec{\sigma}+\vec{\nu}\eta_C) = 0$

Simply put, we are looking for solutions of the equations of motion with dual photon monodromy $\vec{\nu}$ around C (recall monodromy=electric charge!)

- let's get some intuition from simple cases...

II. confining strings in QCD(adj) and dYM:

$$\langle W(\vec{\nu})\rangle \sim e^{-S[\vec{\sigma}_{class.}]}\ ,$$ where $\vec{\sigma}_{class.}$ solves:

dYM $$\nabla^2\vec{\sigma} - m^2\sum_{i=1}^{N}\vec{\alpha}_i \sin\vec{\alpha}_i\cdot(\vec{\sigma}+\vec{\nu}\eta_C) = 0$$

QCD(adj) $$\nabla^2\vec{\sigma} - m^2\sum_{i=1}^{N}(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod}N)})\sin(\vec{\alpha}_i - \vec{\alpha}_{i-1(\mathrm{mod}N)})\cdot(\vec{\sigma}+\vec{\nu}\eta_C) = 0$$

some intuition from simple cases: SU(2) $\vec{\sigma}$ is one-dimensional vector

$\alpha_1 = -\alpha_2 = 1$ magnetic charge of monopoles (electric charge of W bosons)

$\nu_1 = -\nu_2 = \frac{1}{2}$ electric charge of fundamental quarks (consider + sources only)

dYM $$\nabla^2\sigma - 2m^2\sin(\sigma + \frac{1}{2}\eta_C) = 0$$

QCD(adj) $$\nabla^2\sigma - 4m^2\sin 2(\sigma + \frac{1}{2}\eta_C) = 0$$

II. confining strings in QCD(adj) and dYM:

some intuition from simple cases: SU(2) $\vec{\sigma}$ is one-dimensional vector

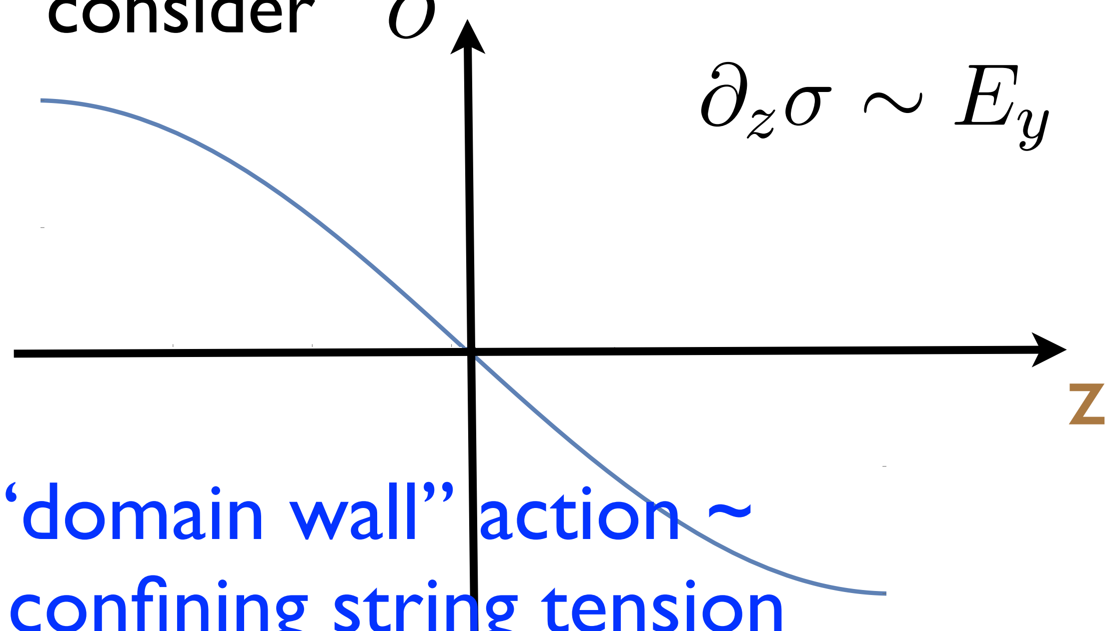
dYM	QCD(adj)
$\nabla^2\sigma - 2m^2\sin(\sigma + \frac{1}{2}\eta_C) = 0$	$\nabla^2\sigma - 4m^2\sin 2(\sigma + \frac{1}{2}\eta_C) = 0$

dYM first:

let C be an infinitely large contour in the x-y plane and take the solid angle be $+2\pi$ above the plane and -2π below the plane

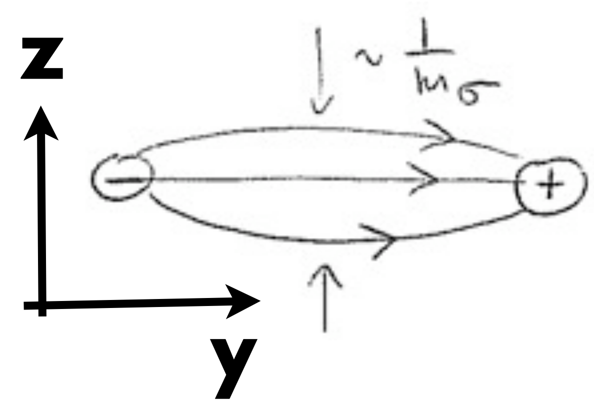
thus σ should have 2π "monodromy" across z=0

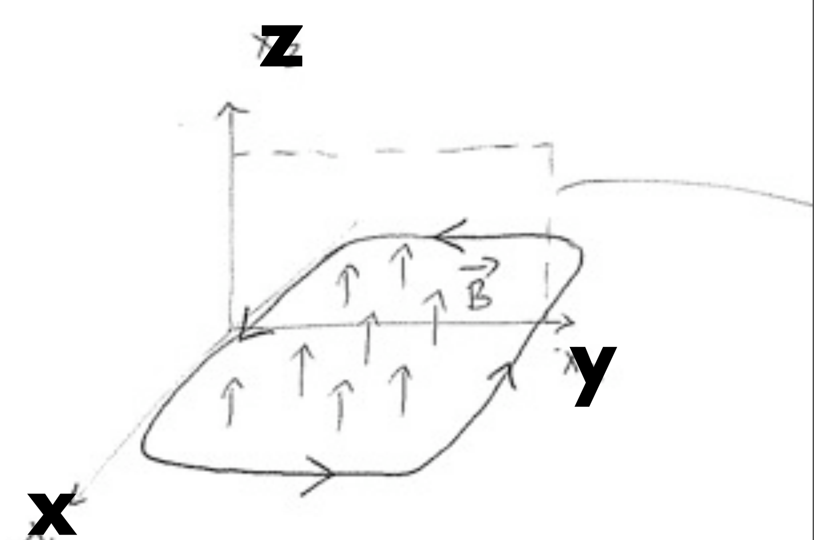
consider



$\partial_z\sigma \sim E_y$

"domain wall" action ~
confining string tension





$\partial_z\sigma \sim E_y$

σ "monodromy" around C=electric flux of confining string

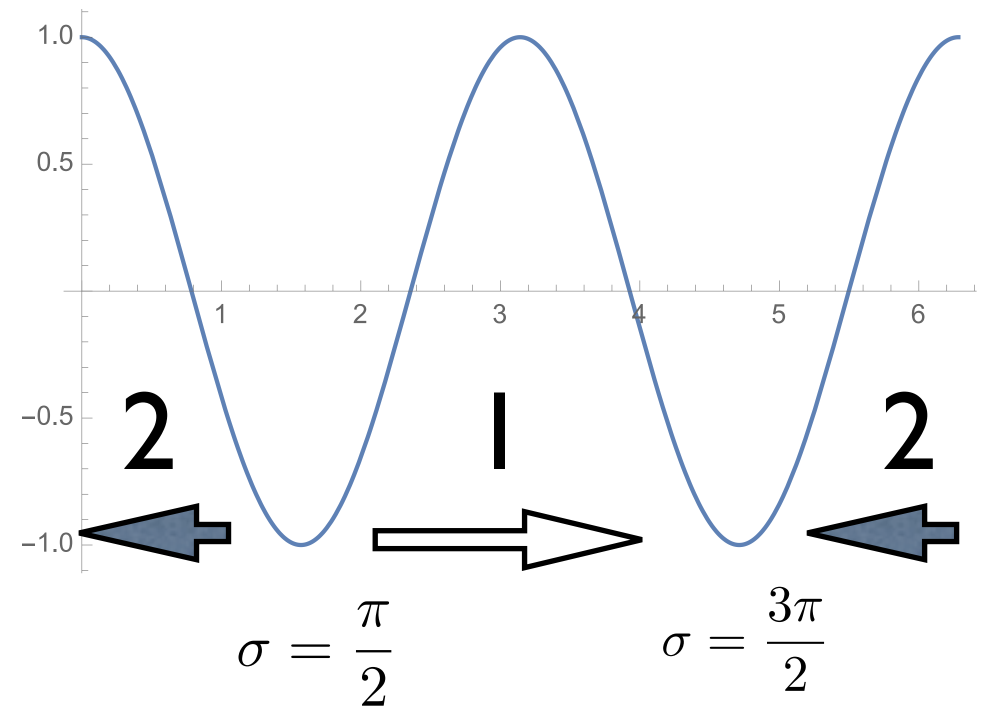
for quark,
$+2\pi$

II. confining strings in QCD(adj) and dYM:

QCD(adj)

$$\nabla^2\sigma - 4m^2 \sin 2(\sigma + \frac{1}{2}\eta_C) = 0$$

$$V(\sigma) \sim \cos 2\sigma$$



two vacua (broken chiral Z_2)

DW 1: el. flux π

DW 2: el. flux $-\pi$

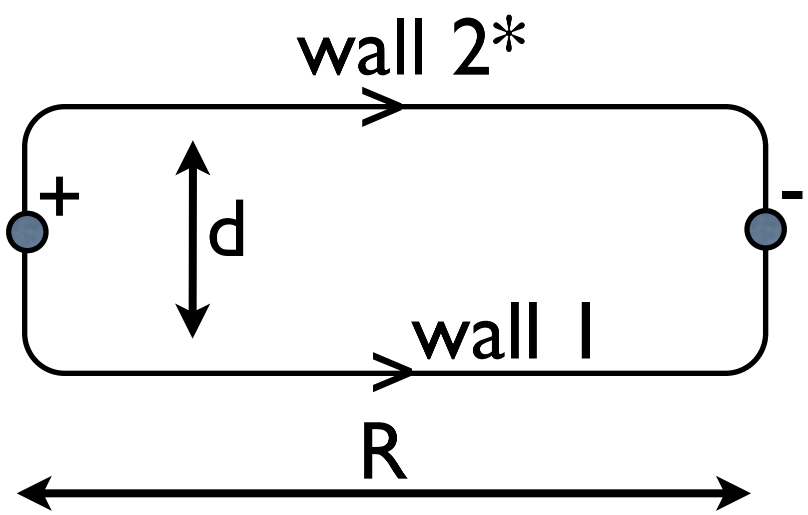
in SYM, both 1 and 2 DWs are BPS

e.g., both DWs have "1/2-quark" fluxes, π not 2π

no such charges allowed by Dirac;
(in fact these are genuine DWs separating Z_2 vacua)

So, whatever configuration has 2π monodromy - to confine quarks - must be composed of two walls... wall 1 followed by anti-wall 2* has correct flux

II. confining strings in QCD(adj) and dYM:



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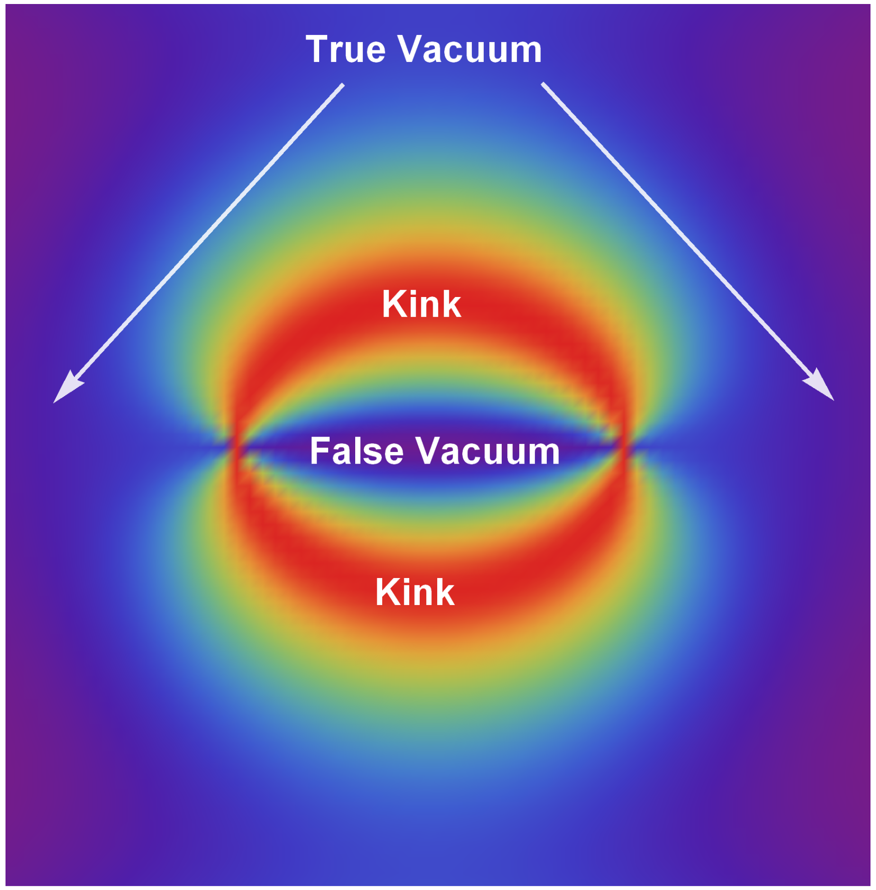
wall action (model) 1-2* repulsion

$$S \sim MmT(R+d) + MmTRe^{-md}$$

$$md_* \sim \log mR$$ (semiclassically, w/out massless fermion exchange)

or via numerical minimization via Gauss-Seidel (logR growth of d holds)



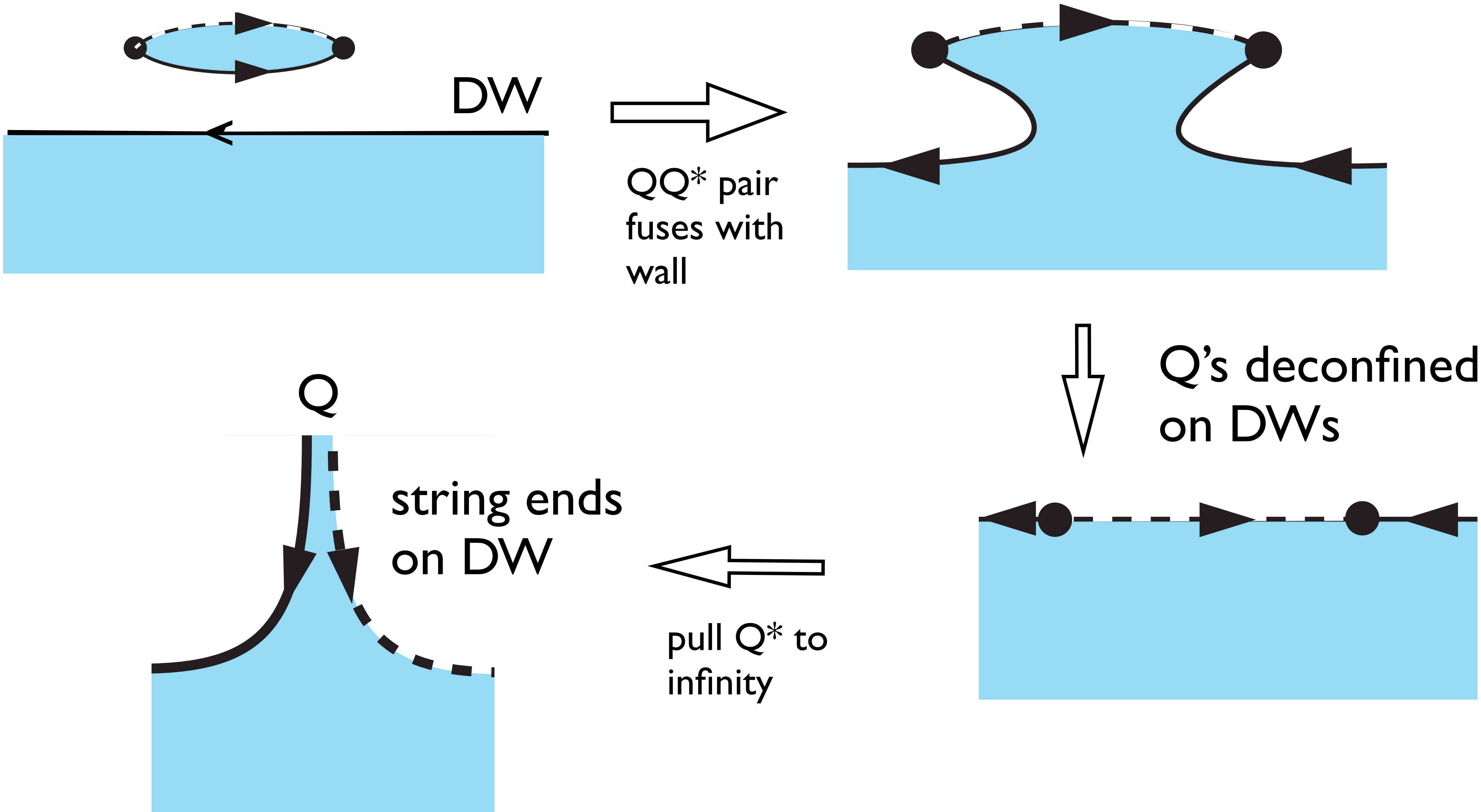
physically, the reason for the compositeness of the string is the composite nature of magnetic bions

(also, for all SU(N), as we'll see)

implications for DWs and strings... next:

II. confining strings in QCD(adj) and dYM:

the picture or strings made out of DWs also implies that, as suggested a long time ago by S.-J. Rey /Witten 1997/ [via ideas of 4d oblique confinement or large-N arguments], confining strings can end on DWs



an electric example of strings and branes "from flesh and blood" (Shifman-Yung)

II. confining strings in QCD(adj) and dYM:

The story is even more fun in SU(N). Here, we don't know the solutions for single DWs (for SU(2), DWs 1 and 2 are explicitly known, SYM or QCD(adj)).

Recall, the crucial - for strings - property

$\vec{\alpha}_j \cdot \vec{\sigma} \to \vec{\alpha}_{j+1(\mathrm{mod} N)} \cdot \vec{\sigma}$ Z_N Weyl symmetry (due to center stability)

More abstractly [in SU(3), this is a 120° rotation in weight space]

$$\vec{\sigma} \to P\vec{\sigma}\,, \quad P = s_{\alpha_{N-1}} s_{\alpha_{N-2}} \ldots s_{\alpha_2} s_{\alpha_1}, \quad s_\alpha \vec{v} = \vec{v} - 2\vec{\alpha}\, \frac{\vec{v}\cdot\vec{\alpha}}{\vec{\alpha}\cdot\vec{\alpha}}$$

This implies that $\langle W(\vec{\nu})\rangle = \langle W(P\vec{\nu})\rangle$

i.e. confining string tensions for quarks with weights in the same Z_N Weyl orbit are the same, both for QCD(adj) and dYM.
Since P permutes the N weights of the fundamental, all strings confining fundamental quarks have the same tension.

II. confining strings in QCD(adj) and dYM:

Without details [can explain], in QCD(adj)/SYM, elementary DWs have monodromy $\frac{2\pi}{N_c}\vec{\rho}$ (the Weyl vector/N)

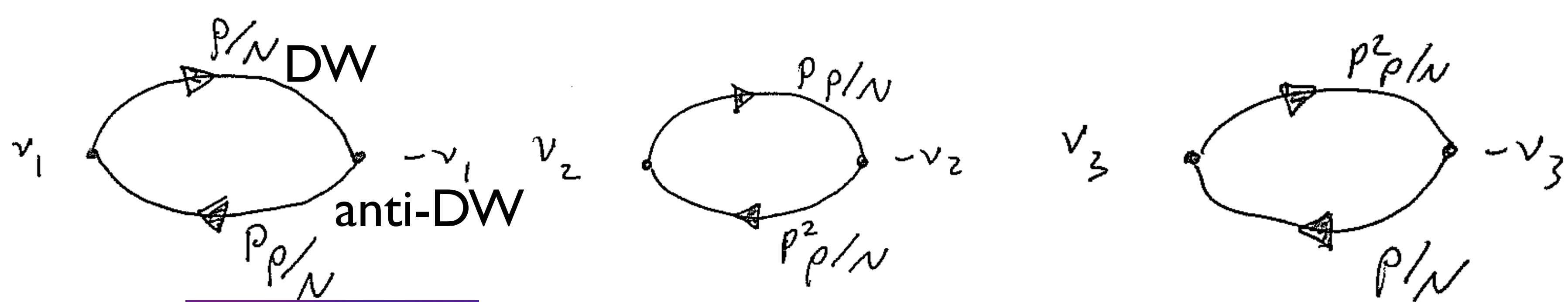
at the same time, the highest weight of the fundamental is

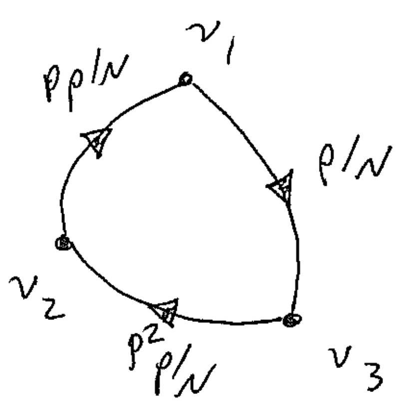
$$2\pi\vec{w}_1 = \frac{2\pi}{N_c}\vec{\rho} - \frac{2\pi}{N_c}P\vec{\rho}.$$

thus a string confining quarks (in the 0 vacuum) with charges $2\pi\vec{w}_1$ can be made of a wall and an P-antiwall
(this generalizes the SU(2) construction)

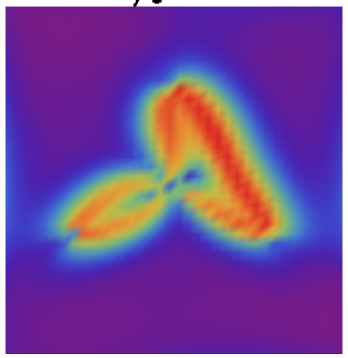
Strings confining the other two weights of the SU(3) fundamental are similarly constructed:

mesons -

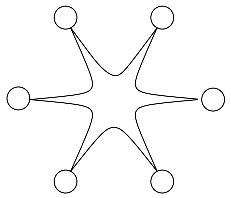




- baryon



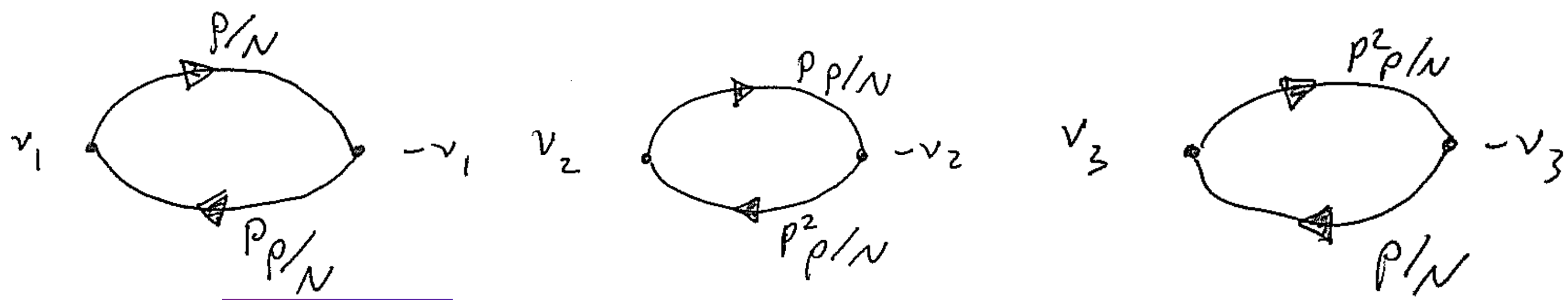
more like

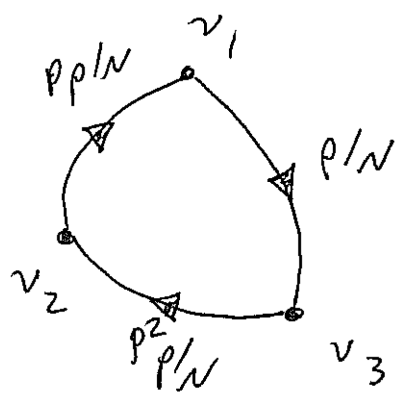


(N=3 here, but story really continues)

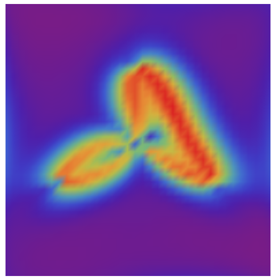
III. comparisons with other abelian and nonabelian confining strings...

mesons -



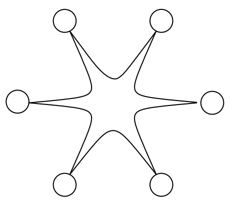


- baryon

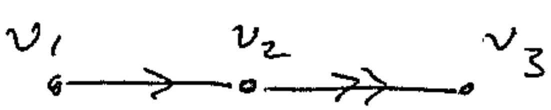


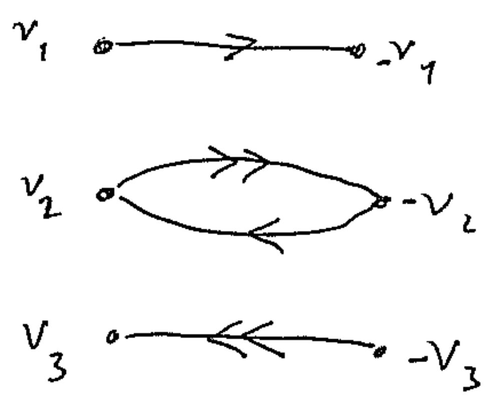
(our one crude attempt at constructing baryons)

cf the dual ones from 1st page



for fun, let's compare with Seiberg-Witten:





- nondegenerate mesons

(k-th component bound by k-string and an anti k-1-string)

only linear baryons

(more dramatic for N>3)

qualitative difference is because:

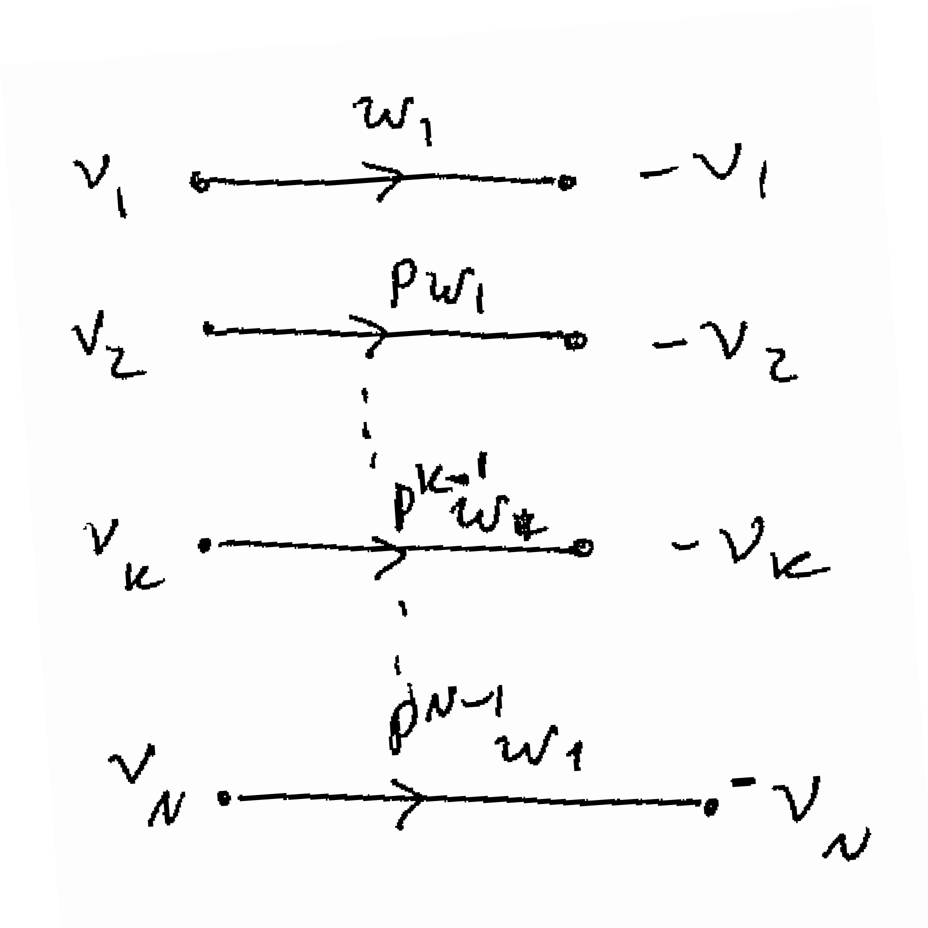
1. in SW there are N-1 condensing objects, in QCD(adj)/dYM there are N "condensing" monopole instantons
2. in SW Weyl group totally broken, in QCD(adj)/dYM a Z_N subgroup exact, due to center stability

III. comparisons with other abelian and nonabelian confining strings...

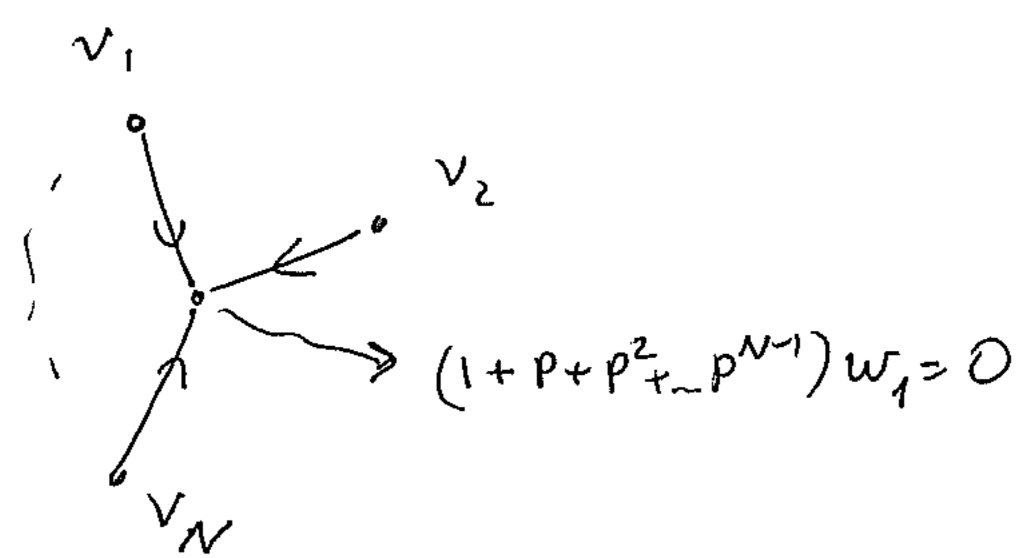
In dYM, we have "DWs" with flux $w_1, w_2, \ldots w_{N-1}$ [the fundamental weights].

The vacuum is unique and these "DWs" are, in fact, confining strings.

For fundamental quarks, we also have Z_N degeneracy of strings:



also, "Y"-baryons exist, since
the sum of the N fluxes vanishes:



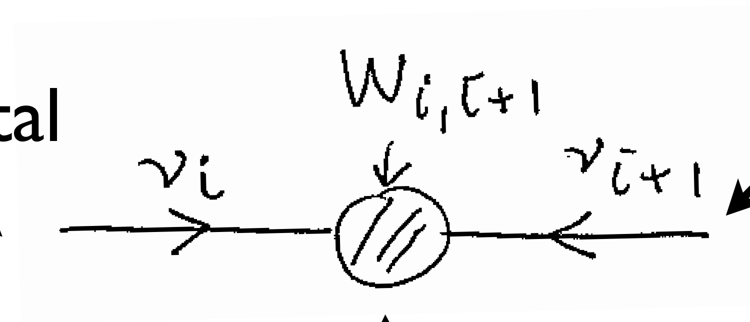
To be sure, just like in SW and QCD(adj), these are still abelian strings - distinct (if degenerate) meson Regge trajectories.

One can speculate about "integrating in" W-bosons, as entire heavy spectrum known- cf SW

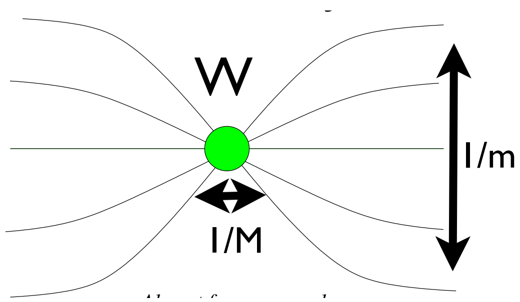
III. comparisons with other abelian and nonabelian confining strings...

One can speculate about "integrating in" W-bosons...[qualitatively similar in QCD(adj)/dYM]

a string confining i-th component of fundamental

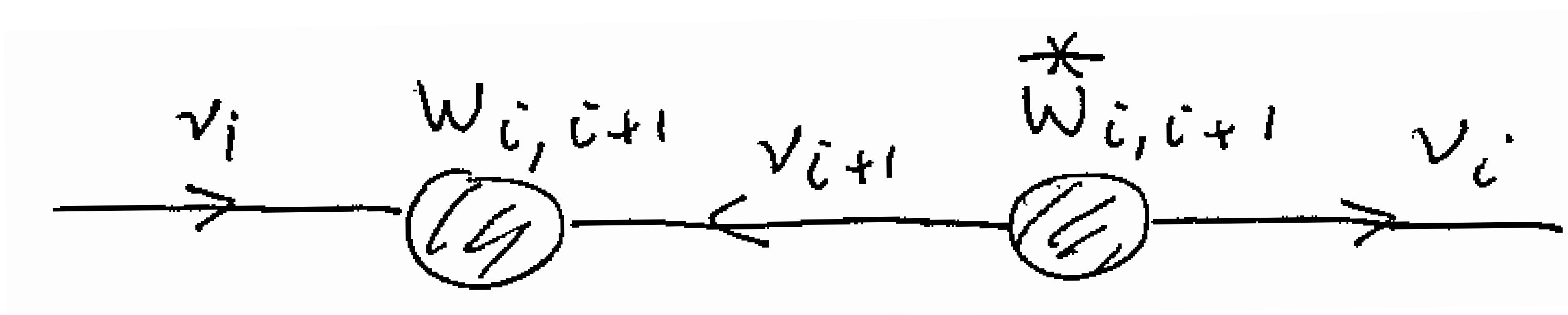


a degenerate anti-string confining i+1-th component of fundamental



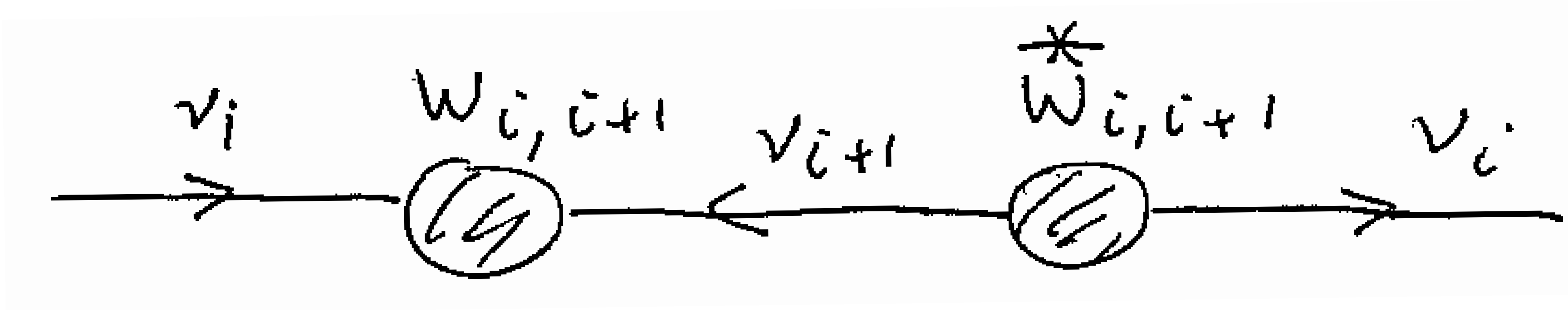
flux is exactly absorbed by W boson (no tension imbalance) - off-diagonal massive gauge boson - "nearest-neighbor" W's are the lightest, stable, and there are N degenerate species

Thus - like quarks on DWs in QCD(adj) - W-bosons in QCD(adj) and dYM are not confined on strings (at scales larger than the Debye screening length, 1/m):



III. comparisons with other abelian and nonabelian confining strings...

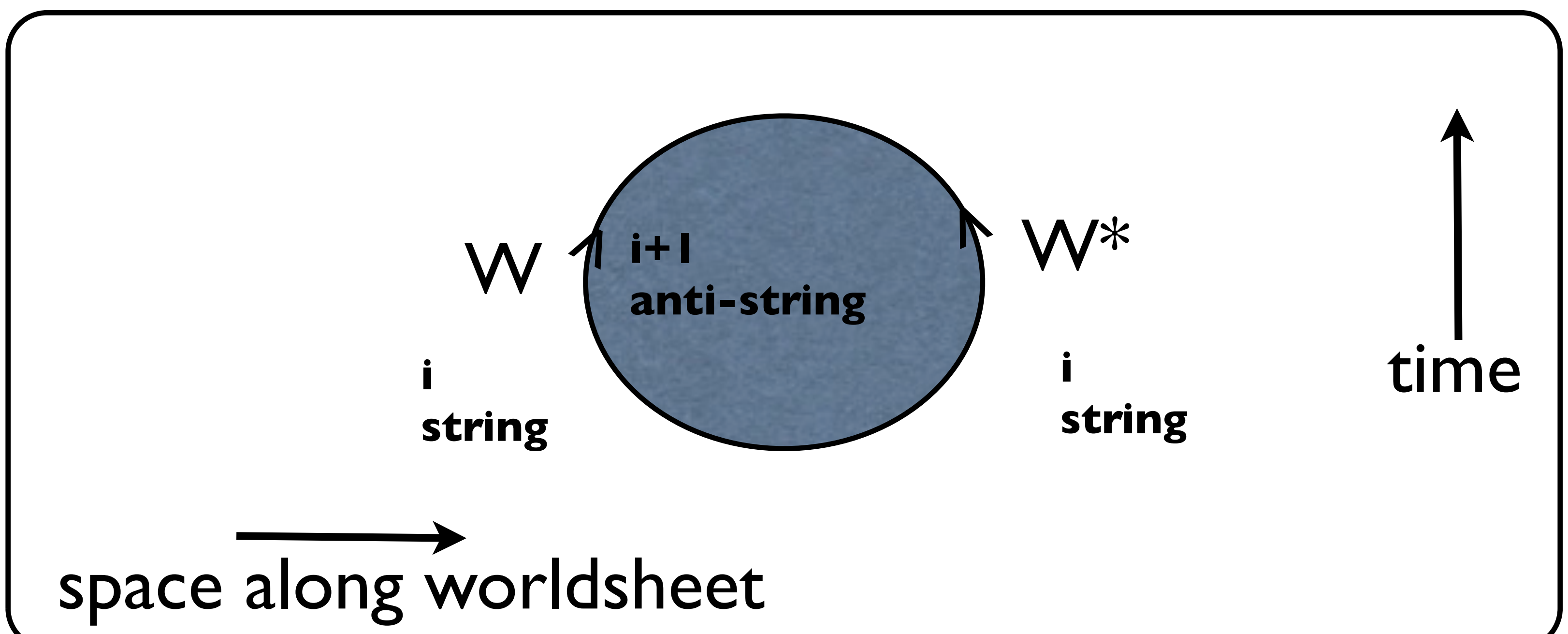
One can speculate about "integrating in" W-bosons...[qualitatively similar in QCD(adj)/dYM]



W-W* pairs on the string are massive (order M) excitations on the worldsheet

A W is a "bead" on the string converting an i-string to an i+1 anti-string

On the Euclidean worldsheet, virtual W worldlines on the string look like boundaries separating regions with an i-string flux to an i+1 anti-string flux



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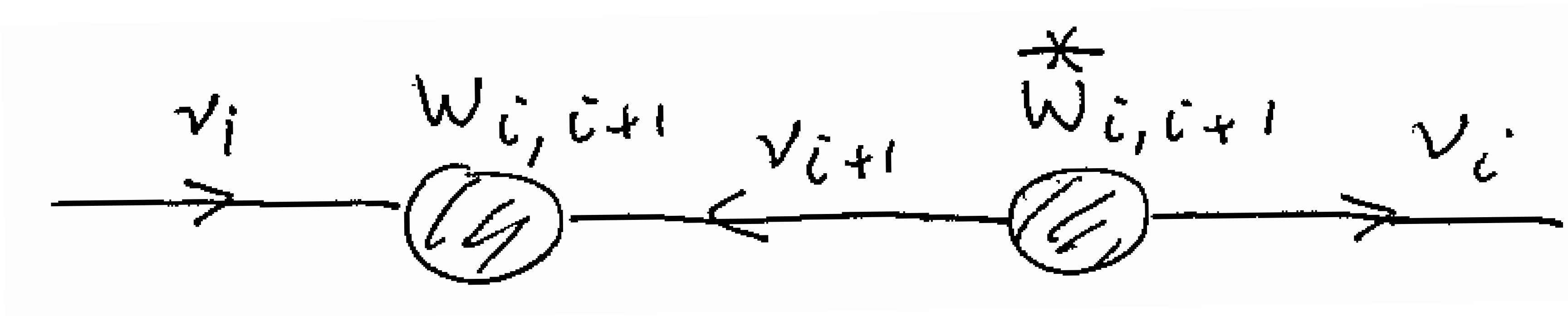

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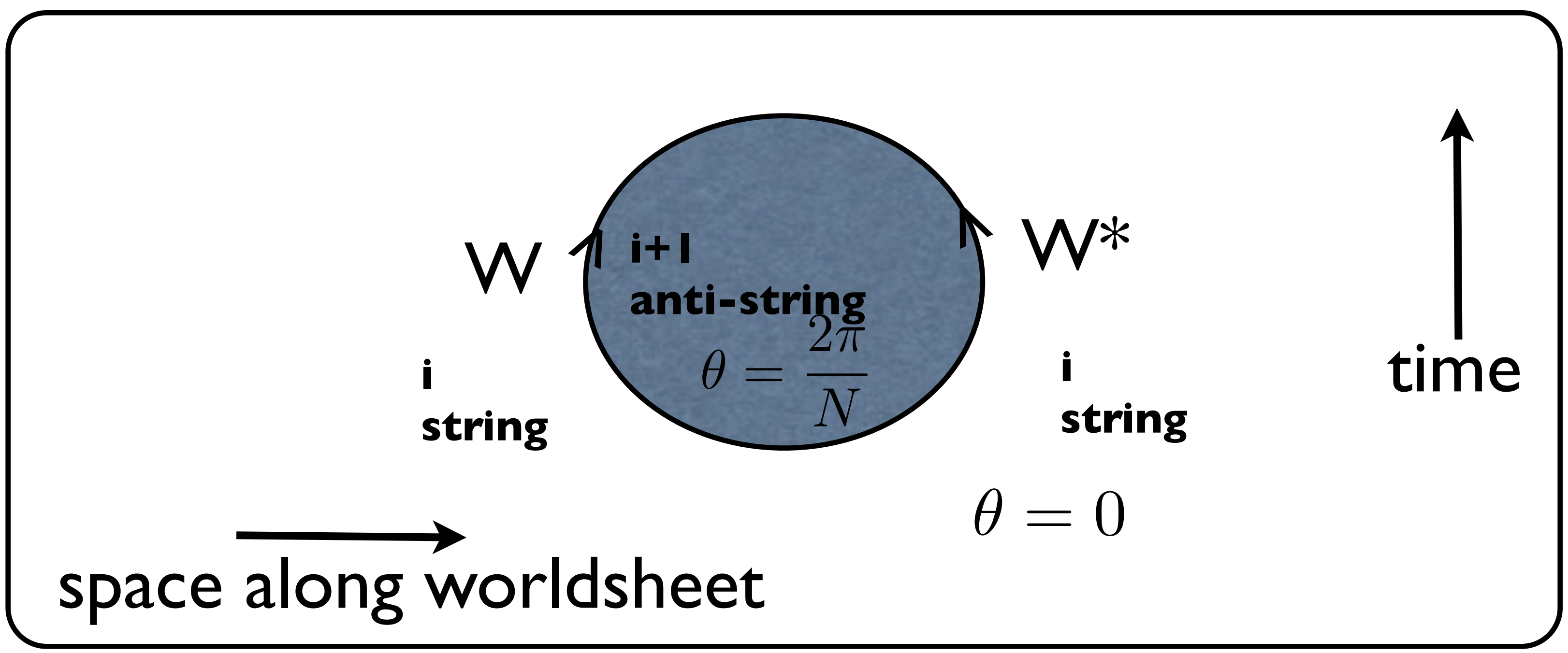


label "vacua" by discrete

$$\theta_{\vec{x}} = \frac{2\pi k}{N}$$

with $a^2\kappa = \frac{M}{m}$ N

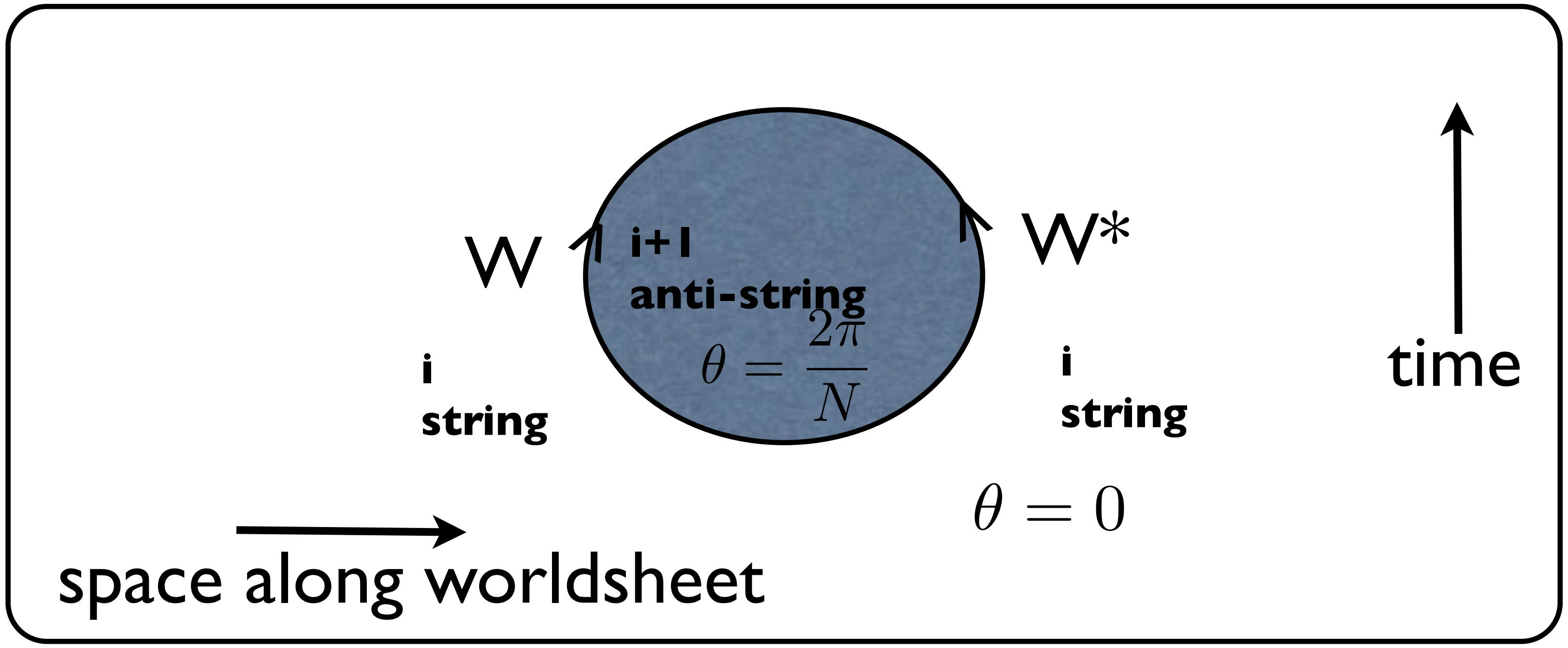
$$S_{clock} = a^2\kappa \sum_{\vec{x},\mu=1,2} |\theta_{\vec{x}} - \theta_{\vec{x}+\vec{\mu}}(\mathrm{mod}2\pi)|$$

W-W* wordline action is correct:

S ~ length x M x width (=1/m)

III. comparisons with other abelian and nonabelian confining strings...

One can speculate about "integrating in" W-bosons...[qualitatively similar in QCD(adj)/dYM]



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III. comparisons with other abelian and nonabelian confining strings...

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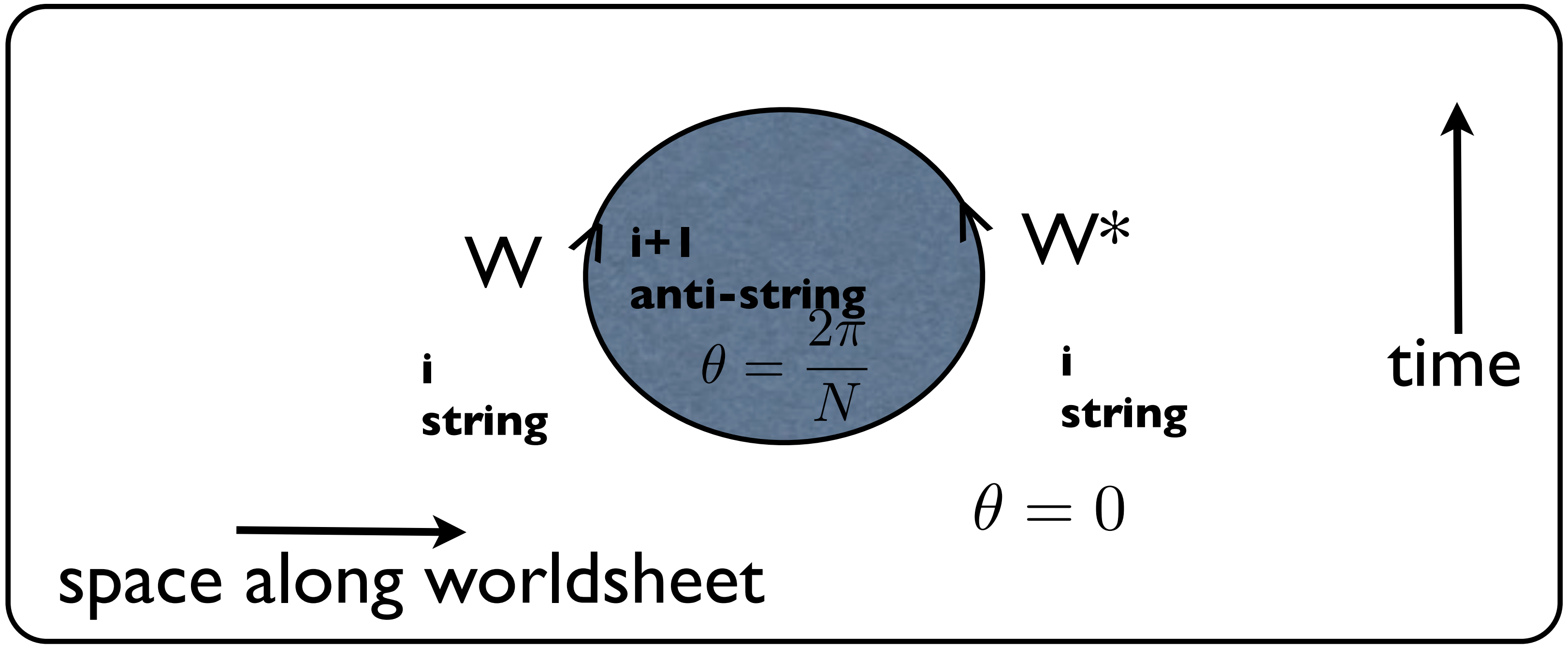

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The 2d clock (aka "cyclic Potts") model has a phase transition to a Z_N restored phase with unique vacuum at $a^2\kappa \sim \log\sqrt{N}$,
but this occurs beyond the validity of our worldsheet theory, requires M/m>>1.
(N M/m)$_{cr.}$~logN is not helpful as one can more carefully check in the abelian large-N limit
[furthermore, ignored Goldstones, A_4, and shape deformations of the (thick) string of mass ~ m]

Nonetheless, it is tempting to speculate that the transition to nonabelian confinement is accompanied by a worldsheet phase transition...

cf Gorsky, Shifman, Yung 2004 in a dual theory confining monopoles, a Z_N transition; monopoles=DWs...
- we, however, lack their nice theoretical control (but our Z_N symmetry "automatic" from center)

IV. future...

We've seen that even abelian confinement can be quite rich and diverse.

Interesting doable questions:

Taxonomy and properties of k-strings in this setup?

The picture of strings and DWs in dYM and QCD(adj) can be used to elucidate the recently discovered distinct global structure - discrete theta angles "p" Aharony Seiberg Tachikawa, Kapustin Seiberg 2013-2014 -of $[SU(N)/Z_k]_p$ theories in a physical manner.

As an application, the low-T/high-T Kramers-Wannier-like e/m-duality, emerging near T_c on $R^2 x S^1 x S^1$ in dYM [Simic,Unsal/Anber,Unsal,EP,2011]: M >> T >> m

$\hat{S}$: $\frac{8\pi LT}{g(L)^2} \longleftrightarrow \frac{g(L)^2}{2\pi LT}$ $\quad \zeta_{mon.} \sim e^{-S_0} \longleftrightarrow \zeta_W \sim e^{-\frac{m_W}{T}}$ (T_c is self-dual T)

can be shown to be consistent with global structure and $(\hat{S}\hat{T})^3 = 1$, eliminating some existing puzzles (as in Ising! Kapustin Seiberg), e.g. for rank one:

$$SU(2) \overset{\longleftrightarrow}{\hat{S}} SO(3)_+ \overset{\longleftrightarrow}{\hat{T}} SO(3)_-$$

$\hat{T}$ (loop on $SU(2)$), $\hat{S}$ (loop on $SO(3)_-$)

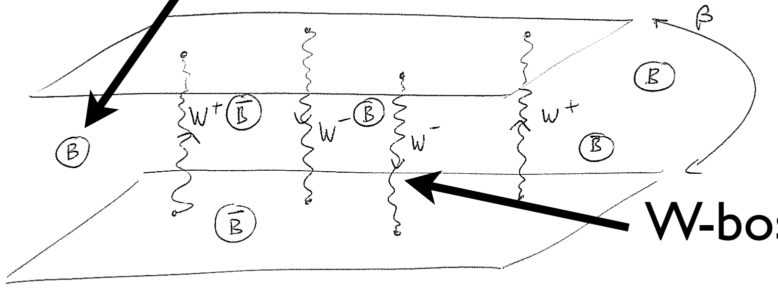
2Pi shift of theta: $\hat{T}$

etc., as in S-duality of N=4 SYM, detail--->

IV. future...

We've seen that even abelian confinement can be quite rich and diverse.

monopole-instanton gas



W-boson gas

$R^2 \times S^1_L \times S^1_\beta \xrightarrow{\text{near Tc}} R^2$ self-dual electric-magnetic Coulomb gas

T<Tc

$\frac{8\pi LT}{g(L)^2} \overset{\hat{S}}{\longleftrightarrow} \frac{g(L)^2}{2\pi LT}$

T>Tc

electric charge N-ality

magnetic charge (dual) N-ality

$\hat{T}$ SU(2) $\overset{\hat{S}}{\longleftrightarrow}$ $SO(3)_+$

W=Wilson S^1_β (1,0) $\overset{\hat{S}}{\longleftrightarrow}$ H='t Hooft S^1_L(0,1)

(unique vacuum) (unique vacuum)

$SO(3)_+$ $\overset{\hat{S}}{\longleftrightarrow}$ SU(2) $\hat{T}$

<H> = +/-1 <W> = +/-1

(broken dual center) (broken center)

$\hat{T}$ $\hat{T}$

$SO(3)_-$ $\overset{\hat{S}}{\longleftrightarrow}$ $SO(3)_-$

WH (1,1) WH (1,1)

(unique vacuum) (unique vacuum)

IV. future...

We've seen that even abelian confinement can be quite rich and diverse.

Other interesting questions:

Can the "double strings" in SYM be seen on the lattice?
perhaps less of a fantasy goal then massless QCD(adj) - e.g. Bergner,Piemonte 2014

How do the "double strings" in SYM morph into the ones in SW theory?

Is there a phase transition on the worldsheet upon transition from abelian to non-abelian regime? How would lattice look for one?
- Gorsky/Shifman/Yung nonabelian-abelian string transition
or roughening transition 'similarities'