Holographic Renyi Entropy in Lovelock Gravity

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Outline

- Holography and AdS/CFT duality
- A geometric construction to calculate EE using holography
- Renyi entropy from holography
- Lovelcok gravity
- Holographic Renyi entropy for Lovelock gravity

Holographic principle

The whole information in a region of the spacetime encoded in the area which has bounded that region. d+1-dim bulk $\longleftrightarrow$ d-dim boundary

AdS/CFT duality

A conformal field theory in flat spacetime (no gravity) in d-dimensions is equivalent to a gravitational theory in anti-de Sitter spacetime in d+1 dimensions!

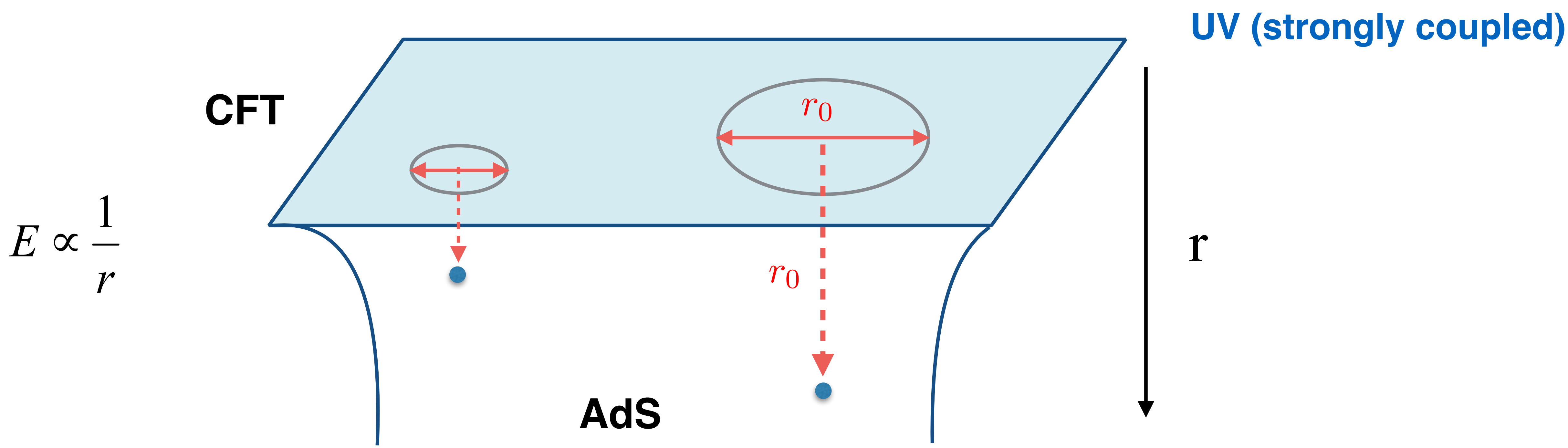
CFT: a QFT which enjoys Poincare symmetry + special conf. trans. + dilatation

AdS: a maximally symmetric spacetime with a constant negative curvature

Why one more dimension?
To match the DOF on both sides: e.g., 4-dim CFT has 15 DOF, then the minimum dimensions required for a maximally symmetric spacetime to have the same number of DOF is 5, remember d(d+1)/2.

Why AdS geometry?
CFT enjoys the scaling symmetry: physics is the same in all scales.
AdS geometry is invariant under the scaling symmetry.

Taking a point in AdS corresponds to a position in the filed theory. However in the AdS side we have one more dimension, usually called the radial direction: how to interpret this extra DOF in the field theory side?



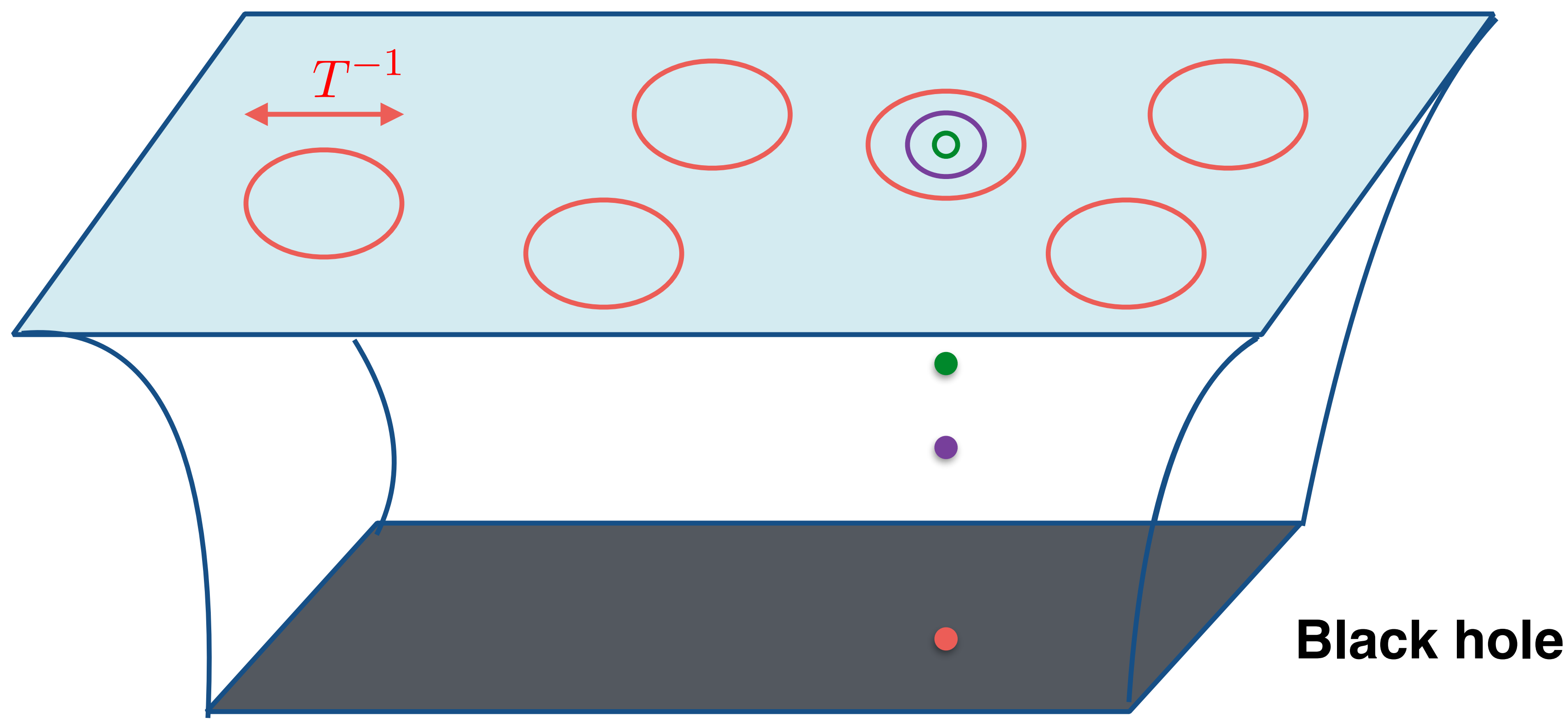
✓ The extra dimension corresponds to the range of interaction for the particle in CFT.

✓ Any radial slice is conformal to Minkowski space in d-dimensions.

✓ The boundary corresponds then to the field theory where no degrees of freedom have been integrated out, i.e., the UV limit in the gauge theory but the IR limit for the AdS bulk.

✓ This is the limit that is normally discussed, one often says that the FT lives on the boundary. However, the FT really lives everywhere. A slice of *constant r* corresponds to a particular effective theory at that cutoff.

CFT at finite temperature

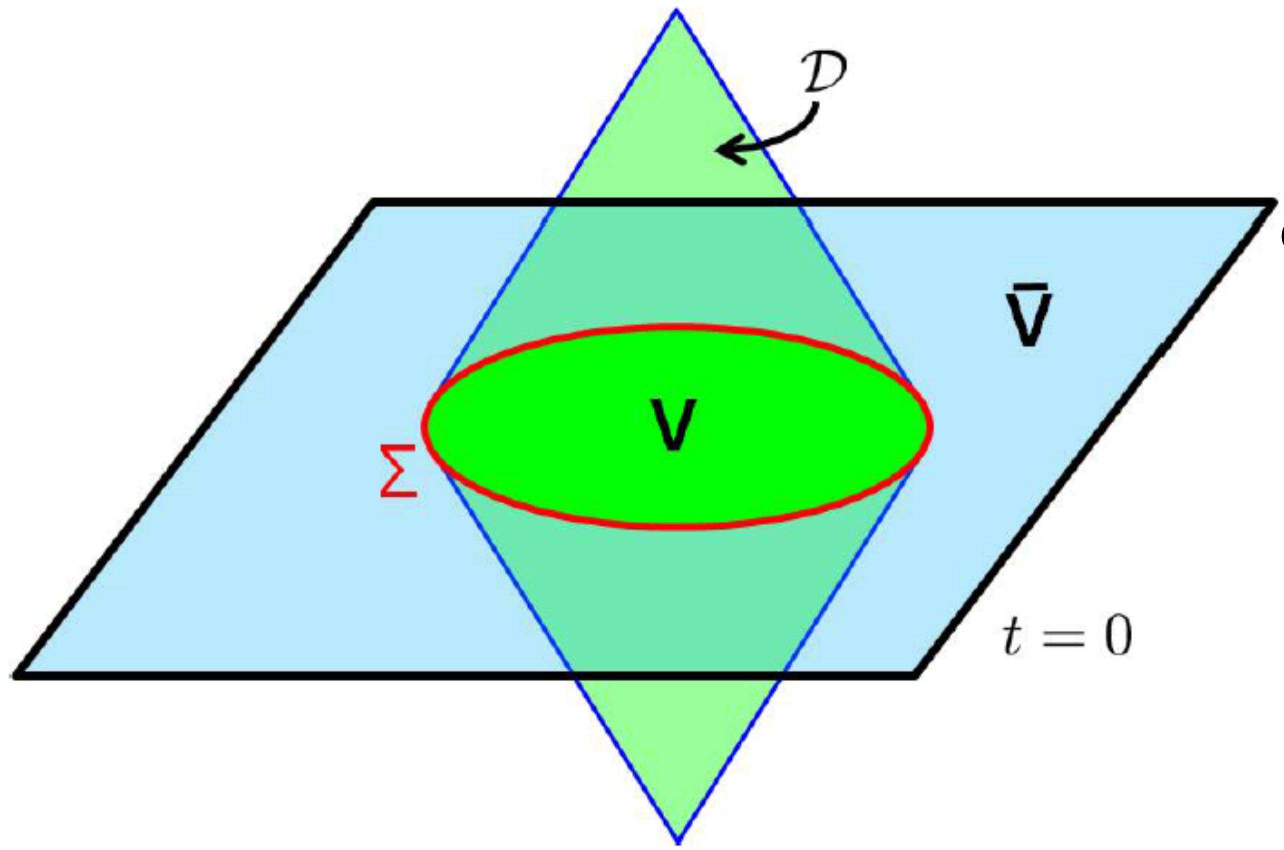


Thermal state in the Boundary CFT ⟷ **Black hole in the AdS bulk**

Thermal entropy of the CFT can be calculated as the horizon entropy of the BH which is easily calculated using Wald's formula.

Mapping the vacuum state to a thermal state in CFT

Casini, Huerta, Myers '11



Σ : (d-1)-dimensional spherical entangling surface

$\mathcal{D}$: causal development of the region inside the entangling surface

- They show beginning with the Minkowski vacuum for an arbitrary CFT, the density matrix on $\mathcal{D}$ becomes a thermal density matrix on a hyperbolic space.

start with the flat space in polar coordinate

$$ds^2 = -dt^2 + dr^2 + r^2\, d\Omega_{d-2}^2$$

do the coordinate transformation

$$t = R\,\frac{\sinh(\tau/R)}{\cosh u + \cosh(\tau/R)}\,, \qquad r = R\,\frac{\sinh u}{\cosh u + \cosh(\tau/R)}$$

$$\Rightarrow \quad ds^2 = \Omega^2\, \underbrace{\left[-d\tau^2 + R^2\,\left(du^2 + \sinh^2 u\, d\Omega_{d-2}^2\right)\right]}_{R \times H^{d-1}}$$

1. with a conformal transformation the pre-factor is eliminated and the resulting line element is the metric on a hyperbolic geometry $\mathcal{H}$.

2. for convention the curvature scale of the hyperbolic space is fixed to match the radius of the spherical entangling surface.

3. Note
$$\tau \to \pm\infty : \qquad (t,r) \to (\pm R, 0)$$
$$u \to \infty : \qquad (t,r) \to (0, R)$$
$\Rightarrow$ The new coordinates precisely cover $\mathcal{D}$, the causal development inside the entangling surface Σ .

Density matrices of CFTs on two geometries

$\mathcal{D}$

causal development of an arbitrary CFT on the Minkowski vacuum

$\downarrow$

$\rho_{\mathcal{D}}$

reduced density matrix describing the vacuum state

$\longleftrightarrow$

$\mathcal{H}$

a CFT on a hyperbolic geometry

$R \times H^{d-1}$

$\downarrow$

$\rho_{\mathcal{H}}$

$\downarrow$

Thermal density matrix with

$$T = \frac{1}{2\pi R}$$

✓ **Hence in order to calculate EE for a spherical entangling surface one could calculate the thermal entropy of a state in a hyperbolic geometry, where the temperature is of the same order as the curvature scale of the hyperbolic geometry.**

replacing a difficult problem with another equally difficult problem?!

However, from the AdS/CFT duality we know that thermal entropy of the CFT can be calculated as the horizon entropy of BH.

What kind of BH in the bulk?

We need to approach AdS asymptotically with hyperbolic foliation. This requires having a topological BH in the bulk, i.e., a BH with hyperbolic event horizon.

Renyi Entropy

Interesting measure of entanglement for a quantum system: $$S_n = \frac{1}{1-n} \ln tr(\rho^n)$$

$n \to 1$ **Von Neumann entropy(EE):** $S(\rho) = -tr(\rho \ln \rho)$

reduced density matrix

Recently it has drawn some attention in the CM community.
H. Li, F. D. M. Haldane '08 / S. T. Flammia, A. Hamma, T. L. Hughes, X.-G. Wen '09 / M. A. Metlitski, C. A. Fuertes, S. Sachdev '09

Essentially, the field theoretical approach to calculate Renyi entropy is *replica trick* which requires evaluating the partition function on a n-fold cover of the original background geometry. This approach combined with the limit n→1 is regarded as the standard calculation of entanglement entropy.

However, it is not always easy to use replica trick, even sometimes it may fail.
J. Stéphan, G. Misguich, V. Pasquier '11

Is there a holographic approach?
Holography geometrizes calculations in the boundary theory and so a natural holographic implementation of the replica trick involves a singular bulk geometry, reflecting the singular nature of the n-fold cover in the boundary.

Working with this singular bulk space in a straightforward way produces incorrect results.
D. V. Fursaev '06 / M. Headrick '10

Hence it seems without a *full understanding of string theory or quantum gravity* in the bulk, we will not be able to work with such singular bulk space in a controlled way.

Holographic Renyi Entropy

New solution has been found for the bulk geometry which remains smooth despite the singularity in the boundary metric.

G. Michalogiorgakis ’08 / M. Headrick ’10 / J. Hung, R. C. Myers, M. Smolkin, A. Yale ’11 & ’14

We showed that for a spherical entangling surface the density matrix is essentially thermal:

$$\rho = \frac{1}{Z(T)} e^{-H/T} \qquad \text{with} \qquad Z(T) = tr(e^{-E/T})$$

Let’s calculate the RE for a boundary CFT at a desired temperature T_0

$$\boxed{S_n = \frac{1}{1-n} \ln tr(\rho^n)} \qquad tr(\rho^n) = \frac{Z(T_0/n)}{Z(T_0)^n}$$

Now if we use the usual definition for the free energy: $F(T) = -T \log Z(T)$

and the standard identity: $S = -\partial F/\partial T$

$$\boxed{S_n(T_0) = \frac{n}{n-1}\frac{1}{T_0}\int_{T_0/n}^{T_0} S_{thermal}(T)dT}$$

horizon entropy of a BH with hyperbolic horizon in the AAdS bulk

Lovelock gravity

Recap: **basic properties of standard general relativity**

1. **Field equations are generally covariant,**
2. **They contain at most the second order derivatives of the metric.**

In a spacetime with d > 4, Einstein gravity is not the most general gravity theory sharing these properties but the so-called Lovelock gravity with the Lagrangian:

$$\mathcal{L} = \sum^{[d/2]} \alpha_p \mathcal{L}_p$$

Here we consider up to third order with the action:

$$I = \frac{1}{2\ell_p^{d-1}} \int d^{d+1}x\sqrt{-g}\left(\frac{d(d-1)}{L^2} + R + \alpha_2\mathcal{L}_2 + \alpha_3\mathcal{L}_3\right)$$

cosmological constant

Ricci scalar

Lovelock coefficients << 1

$$\alpha_2 = \frac{L^2\lambda}{(d-2)(d-3)}, \qquad \alpha_3 = \frac{L^4\mu}{(d-2)(d-3)(d-4)(d-5)}$$

Note that in 3,4 dim we recover Einstein gravity: Lovelock is a natural generalization of GR in higher dimensions. The minimum dimensions required for third order Lovelock is 7.

Topological Lovelock black hole

One possible solution to the EOM of 3rd order Lovelock is:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{-1,5}$$

metric of 5d hyperboloid

$$\mu = \lambda^2 \quad \Rightarrow \quad f(r) = -1 + \frac{r^2}{L^2\lambda}\left(1 - \sqrt[3]{1 - 3\lambda + \frac{6L^6\lambda m}{5r^6}}\right)$$

constant of integration

To calculate the Renyi entropy holographically, what we need is a hyperbolic foliation in a pure asymptotic AdS bulk, therefore we demand:

$$\frac{1}{L^2} = \frac{\lambda}{1 - \sqrt[3]{1 - 3\lambda}} \frac{1}{\tilde{L}^2}$$

radius of pure AdS

The black hole entropy could be obtained from Wald's formula:

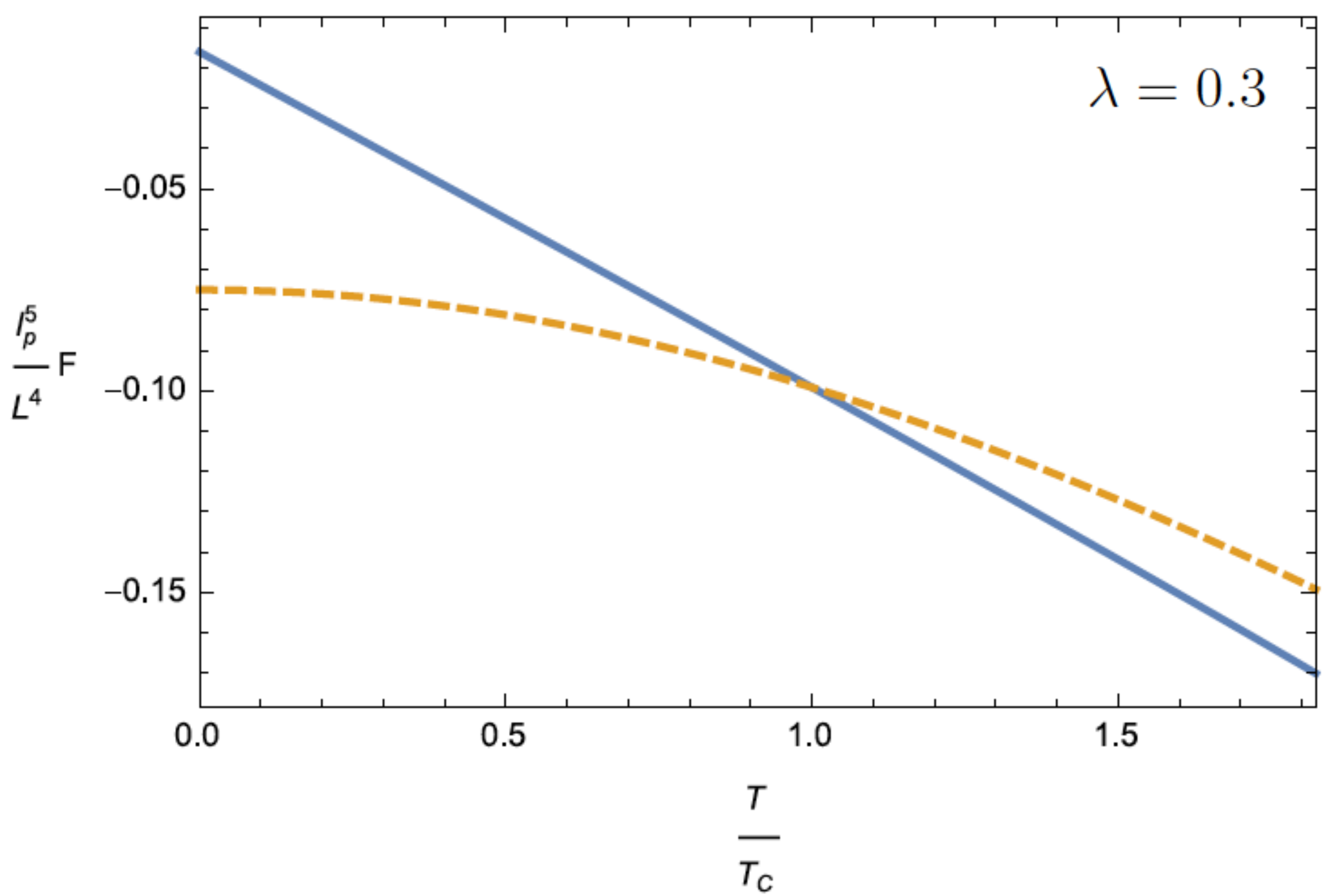
$$\boxed{S = \frac{2\pi L^5}{3\ell_p^5}\left(3r_+^5 - 10\lambda r_+^3 + 15\lambda^2 r_+\right)}$$

This is the entropy we now use in the holographic formula to calculate Renyi entropy.

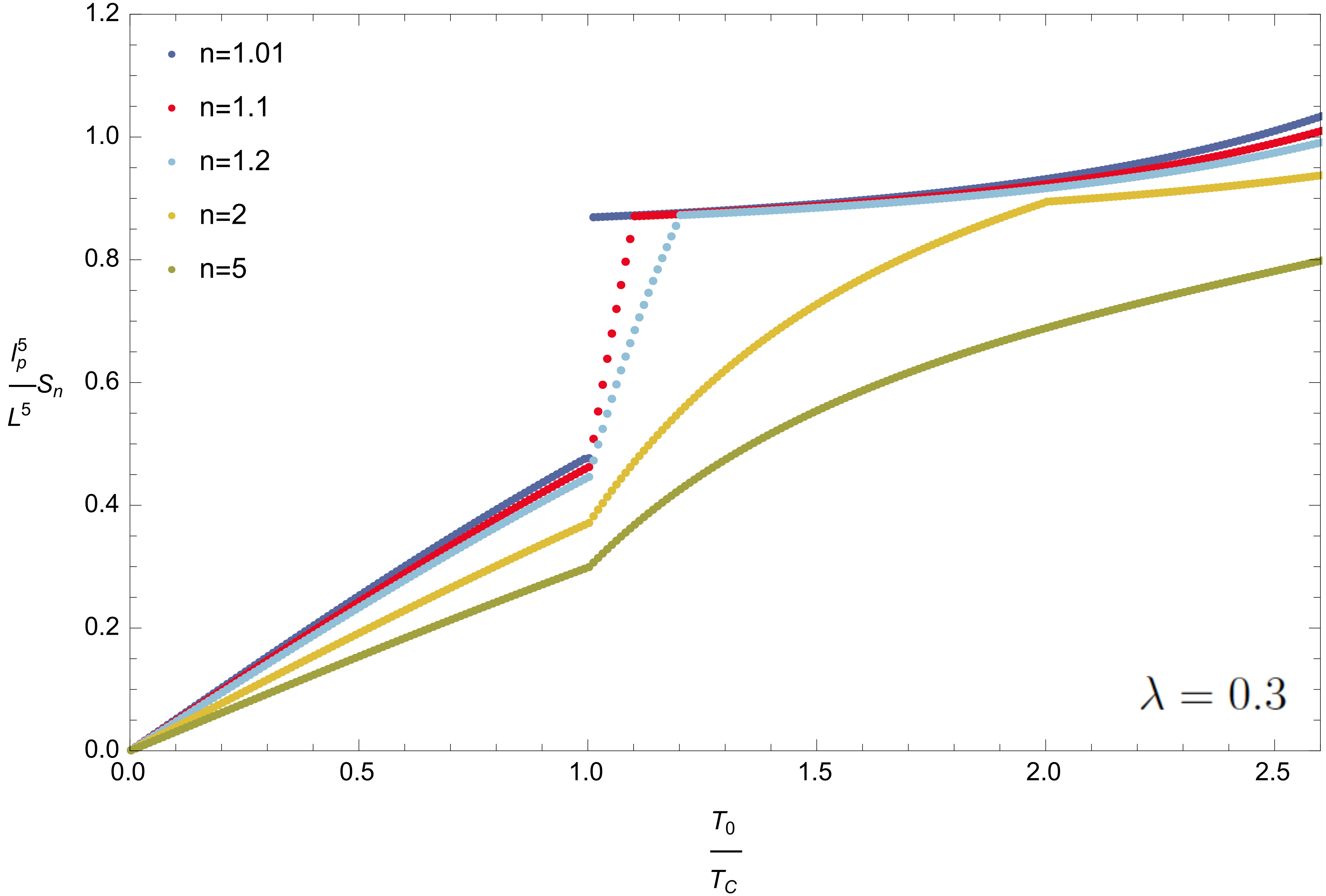
Why are 3rd Lovelock BHs interesting?

$\frac{1}{4} < \lambda < \frac{1}{3}$ **: there exist two BHs with different masses which have the same temperature.**

Comparison of the free energies

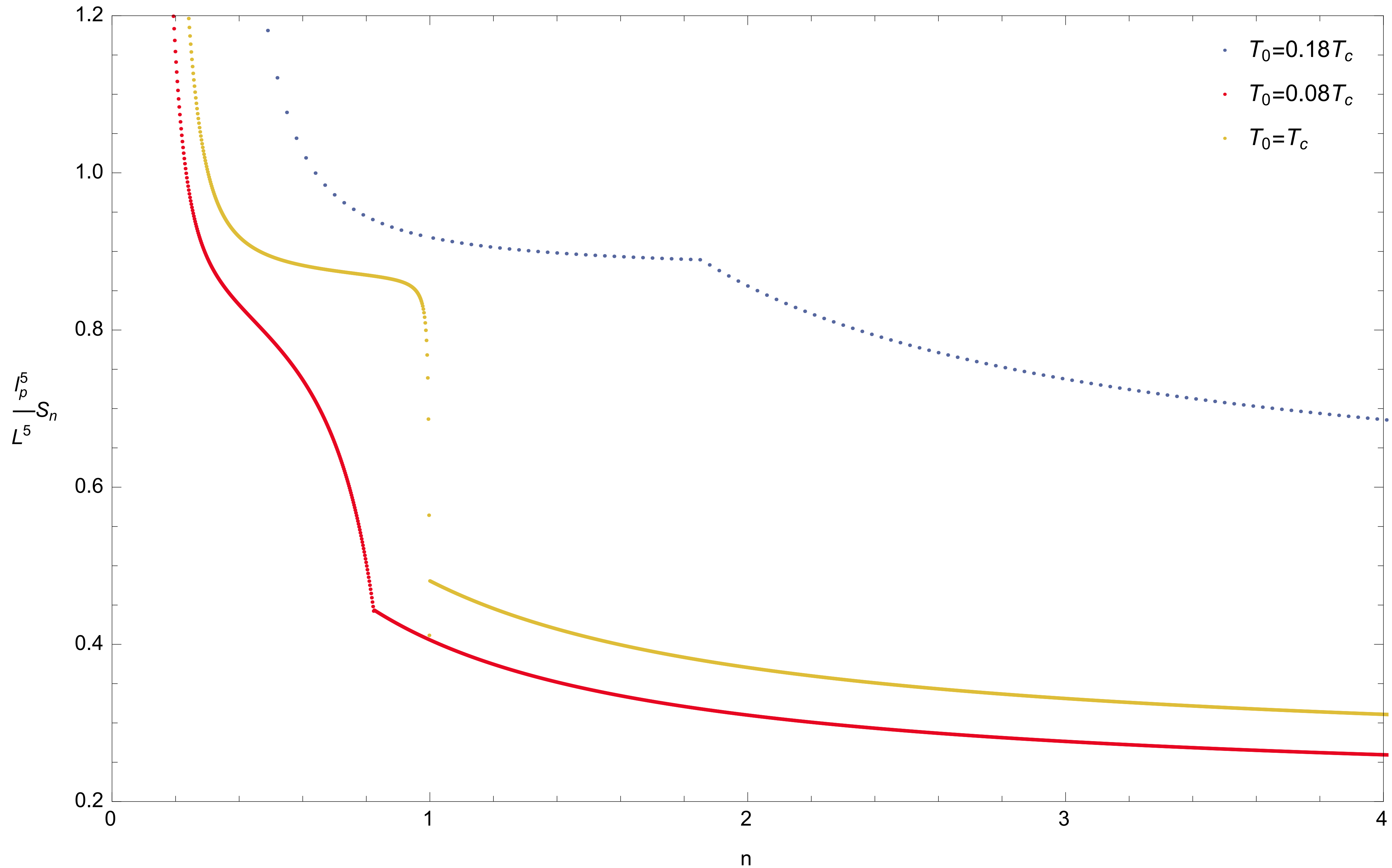


Holographic Renyi entropy for Lovelock gravity



$$T_0 = 1/2\pi R$$

radius of the spherical entangling surface on the boundary CFT



1.2
1.0
0.8
0.6
0.4
0.2
$\frac{l_p^5}{L^5} S_n$
0
1
2
3
4
n
T_0=0.18T_c
T_0=0.08T_c
T_0=T_c

Summary

- A geometric construction presented which relates the reduced density matrix of a CFT in the Minkowski vacuum to the thermal density of the CFT on a hyperbolic space.

- Following this construction one could easily calculate the Renyi entropy holographically provided the dual bulk gravity is known, simply by replacing thermal entropy of the boundary CFT with the horizon entropy of topological black hole in the AdS bulk.

- Investigating thermodynamics of topological Lovelock BHs in AAdS bulk, we observed a phase transition in the bulk theory for some values of Lovelock coefficients.

- The behaviour of the Renyi entropy was examined for the boundary CFT at temperature T0 in such a bulk gravity. We observed a kink in the Renyi entropy of any order n and a discontinuity for n=1.

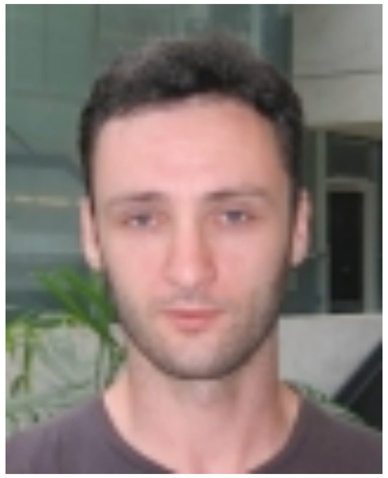
- For the boundary CFT with T=Tc, the derivative of Renyi entropy wrt n is discontinuous which is expected to imply a discontinuity in some of the CFT parameters/central charges.

More about this is ongoing…

hopefully this preview encourages you to read the paper when is published!

Thanks to:

Rob Myers



Misha Smolkin

Larus Thorlacius

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Thank You!