Lecture 4-3, Oct. 2014

D=10 type II SUPERGRAVITY

Dimensional reduction of 11D SUGRA to D=10. Type IIA supergravity.

Separate one of 11 coordinates $X^{\underline{\mu}} = (x^\mu, x^\#)$.
Dim. reduction (or compactification on S^1 with $R_{S^1} \mapsto 0$):

$$\boxed{\partial_\#(all\ fields) = 0}$$

11D fields carrying $SO(1,10)$ indices $\mapsto$ irreps. of $SO(1,9)$

$$e_{\underline{\mu}}{}^{\underline{a}}(x) = \begin{pmatrix} e^{\Phi(x)/12} e_\mu{}^a(x) & A_\mu(x) \\ 0 & e^{-2\Phi(x)/3} \end{pmatrix}$$

$\Phi(x)$- dilaton. Powers of dilaton are chosen to have the $d = 10$ Einstein-Hilbert action without $e^{n\Phi(x)}$ multipliers (so–called Einstein frame). To resume this and other decompositions:

$$\begin{aligned} e_{\underline{\mu}}{}^{\underline{a}}(X) &\mapsto e_\mu{}^a(x), A_\mu(x), \Phi(x) \\ A_{\underline{\mu\nu\rho}}(X) &\mapsto A_{\mu\nu\rho}(x), B_{\mu\nu}(x) (= A_{\mu\nu\#}(x)) \\ \psi_{\underline{\mu}}{}^{\underline{\alpha}}(X) &\mapsto \psi_\mu{}^{\underline{\alpha}}(x), \chi^{\underline{\alpha}}(x) \ (=\propto \psi_\#{}^{\underline{\alpha}}(x)) \ . \end{aligned}$$

10D Majorana spinor is also 32 component. However it can be decomposed on two irreducible chiral 16 dim. Majorana-Weyl representations (of opposite chirality):

$$\psi_\mu{}^{\underline{\alpha}}(x) = (\psi_\mu{}^\alpha(x)\, ,\, \psi_{\mu\alpha}(x)) \ .$$

Hence the **field content of type IIA supergravity** is:

$$NS-NS\ sector\ :\qquad e_\mu^a\ ,\Phi(x)\ ,B_{\mu\nu}(x)$$

Fermions:

$$gravitini\qquad \psi_\mu^{\alpha 1}, \psi_{\mu\alpha}{}^2$$

and

$$dilatini\qquad \chi_\alpha{}^1, \chi^{\alpha 2}\ ,$$

as well as

$$RR\ sector\ :\qquad R_2 = dA_1\ ,\qquad R_4 = dA_3 - A_1 \wedge H_3$$

where

$$H_3 = dB_2\ .$$

One can also introduce dual field strengths

$$RR^*\ sector\ :\qquad R_6 = dA_5 - A_3 \wedge H_3\ ,\qquad R_8 = dA_7 - A_5 \wedge H_3$$

such that $R_6 = *R_2$ and $R_8 = *R_4$.

Roman's massive type IIA SUGRA also include

$$R_{10} = dA_9 - A_7 \wedge dB_2\ .$$

This is dual to a 0-form which plays the role of a mass parameter. Curiously, the type IIA SG allows to add some kind of mass term to the action. This massive type IIA has no apparent D=11 origin.

Type IIB SUGRA (D=10, N=2 chiral SUGRA)

has no apparent D=11 origin. The field content is

$$NS-NS\ sector\ :\qquad e_\mu^a\ ,\Phi(x)\ ,B_{\mu\nu}(x)$$

THE SAME NS-NS sector as type IIA.

Chiral fermionic sector: $\boxed{\text{gravitini }\psi_\mu^{\alpha 1},\psi_\mu{}^{\alpha 2}}$ and $\boxed{\text{dilatini }\chi_\alpha{}^1,\chi_\alpha^2}$
The RR sector contains even-form potentials; the field strengths are

$$RR\ sector\ :\qquad R_1=dA_0\ ,\qquad R_3=dA_2-A_0\,H_3\ ,$$

$$R_5=dA_4-A_2\wedge H_3\ ,\qquad where\qquad H_3=dB_2$$

where 5-form field strength is self–dual $R_5=dA_4-A_2\wedge dB_2=*R_5$

[*This properties during 14 years hampered the way to construct type IIB SUGRA action. The problem was resolved in 1998 by Padova collaboration: Dall'Agata, Lechner, Sorokin CQG 1998 and Dall'Agata, Lechner, Tonin , JHEP 1998].

One can also introduce dual field strengths

$$RR^*\ sector\ :\qquad R_7=dA_6-A_4\wedge dB_2\ ,\qquad R_9=dA_8-A_6\wedge dB_2$$

such that $R_7=*R_3$ and $R_9=*R_1$. It will be also useful to introduce A_{10}, although its field strength in 11D *spacetime* is equal to zero (*notice that $R_{11}=dA_{10}-A_8\wedge dB_2$ in superspace can be different from zero due to the fermionic vielbein contributions).

Type IIA and type IIB supergravity are related by the so–called T-duality transformations. Reason: these SUGRAS are limits of type IIA and type IIB string model and that T-duality is the relation between these two (in a space with an isometry direction).

IIA— IIB SUPERGRAVITY T-duality in a space with one isometry

T-duality rules for the bosonic fields from NS-NS sector were found by Buscher in 1985:

$$g^{(s)}_{yy} = \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \, , \qquad g^{(s)}_{\tilde{m}y} = \frac{1}{\hat{g}_{\hat{z}\hat{z}}} \hat{B}_{\hat{z}\tilde{m}} \, ,$$
$$g^{(s)}_{\tilde{m}\tilde{n}} = \hat{g}^{(s)}_{\tilde{m}\tilde{n}} + \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \left(\hat{B}_{\tilde{m}\hat{z}} \hat{B}_{\tilde{n}\hat{z}} - \hat{g}^{(s)}_{\tilde{m}\hat{z}} \hat{g}^{(s)}_{\tilde{n}\hat{z}} \right) \, ,$$
$$e^{2\Phi} = -\frac{e^{2\hat{\Phi}}}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \, ,$$
$$B_{\tilde{m}\tilde{n}} = \hat{B}_{\tilde{m}\tilde{n}} + \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \left(\hat{g}^{(s)}_{\tilde{m}\hat{z}} \hat{B}_{\tilde{n}\hat{z}} - \hat{g}^{(s)}_{\tilde{n}\hat{z}} \hat{B}_{\tilde{m}\hat{z}} \right) , \qquad B_{y\tilde{m}} = \frac{1}{\hat{g}^{(s)}_{\hat{z}\hat{z}}} \hat{g}^{(s)}_{\tilde{m}\hat{z}} \, .$$

T-duality for the RR sector – by Bergshjoeff, Hull & Ortin in 1995 and, for higher forms, by Meessen & Ortin in 1999: $C^{(0)} = \hat{C}^{(1)}_{\hat{z}}$,

$$\begin{aligned} C^{(2n)}_{y\tilde{m}_1 \ldots \tilde{m}_{2n-1}} &= \hat{C}^{(2n-1)}_{\tilde{m}_1 \ldots \tilde{m}_{2n-1}} + \frac{(2n-1)}{\hat{g}_{\hat{z}\hat{z}}} \hat{C}^{(2n-1)}_{\hat{z}[\tilde{m}_1 \ldots \tilde{m}_{2n-2}} \hat{g}_{\tilde{m}_{2n-1}]\hat{z}} \, , \\ C^{(2n)}_{\tilde{m}_1 \ldots \tilde{m}_{2n}} &= \hat{C}^{(2n+1)}_{\hat{z}\tilde{m}_1 \ldots \tilde{m}_{2n}} + 2n \hat{C}^{(2n-1)}_{[\tilde{m}_1 \ldots \tilde{m}_{2n-1}} \hat{B}_{\tilde{m}_{2n}]\hat{z}} + \\ &+ \frac{2n(2n-1)}{\hat{g}_{\hat{z}\hat{z}}} \hat{C}^{(2n-1)}_{\hat{z}[\tilde{m}_1 \ldots \tilde{m}_{2n-2}} \hat{B}_{\hat{z}\tilde{m}_{2n-1}} \hat{g}_{\tilde{m}_{2n}]\hat{z}} \, . \end{aligned}$$

T-duality for fermionic fields – by Hassan in 2000.

Superfield T-duality rules: [Kulik and Roiban 2002], [I.B. and B. Julia 2003]. In particular include the superfield (superform) generalization of the T–duality rules for the RR superform potentials, which looks quite straightforward

$$i_y C_{2n} = -\hat{C}^{(-)}_{2n-1} + \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}} \wedge i_{\hat{z}} \hat{C}_{2n-1} \, , \quad n = 1, 2, 3, 4, 5 \, ,$$

$$C^{(-)}_{2n} = i_{\hat{z}} \hat{C}_{2n+1} + i_{\hat{z}} \hat{B}_2 \wedge \left(\hat{C}^{(-)}_{2n-1} - \frac{\hat{E}^{\#(-)}}{\hat{E}^{\#}_{\hat{z}}} \wedge i_{\hat{z}} \hat{C}_{2n-1} \right) \, , \quad n = 0, 1, ..., 4.$$

D=10 type IIB SUPERGRAVITY in superspace [Howe& West 84]

Type IIB superspace $\Sigma^{(10|16+16)}$ with local coordinates $Z^M = (x^\mu, \theta^{\underline{\check{\alpha}}})$ is supplied by the bosonic and fermionic supervielbein 1-forms,

$$E^A = (E^a, E^{\alpha 1}, E^{\alpha 2}) = dZ^M E_M^A(Z) \, , \qquad a = 0, 1, ..., 9, \qquad \alpha = 1, ..., 16 \, ,$$

and SO(1,10) connection ($\omega^{ab} = -\omega^{ba}$, $\omega_\beta{}^\alpha = \frac{1}{4}\omega^{ab}\sigma_{ab\beta}{}^\alpha$)

$$\omega_B{}^A = diag(\omega_b{}^a, \omega_\beta{}^\alpha, \omega_\beta{}^\alpha) = dZ^M \omega_{MB}{}^A(Z) \, ,$$

These are subject to the SSP constraints. The most important is

$$T^{\underline{a}} = -iE^{\alpha 1} \wedge E^{\beta 1}\sigma^{\underline{a}}_{\alpha\beta} - iE^{\alpha 2} \wedge E^{\beta 2}\sigma^{\underline{a}}_{\alpha\beta}$$

which implies the standard transformation rule for 10D graviton:

$$\delta_\epsilon e^a(x) = i_\epsilon T^a|_{\theta=0} = -2i(\psi^1 \sigma_{\underline{a}} \epsilon^1 + \psi^2 \sigma_{\underline{a}} \epsilon^2)$$

Notice that this constraints essentially determines the dynamics of SUGRA. Other constraints are conventional and the equations of motion follow from the constraints.

Actually all the fields of type II SG are collected in the dilaton superfield $\Phi(Z)$ which enters the fermionic torsion 2-forms

$$T^{\alpha 1} = -E^{\alpha 1} \wedge E^{\beta 1}\nabla_{\beta 1} e^{-\Phi} + \tfrac{1}{2} E^1 \sigma^{\underline{a}} \wedge E^1 \, \tilde{\sigma}^{\alpha\beta}_{\underline{a}} \nabla_{\beta 1} e^{-\Phi} + \propto E^{\underline{a}} \, ,$$
$$T^{\alpha 2} = -E^{\alpha 2} \wedge E^{\beta 2}\nabla_{\beta 2} e^{-\Phi} + \tfrac{1}{2} E^2 \sigma^{\underline{a}} \wedge E^2 \, \tilde{\sigma}^{\alpha\beta}_{\underline{a}} \nabla_{\beta 2} e^{-\Phi} + \propto E^{\underline{a}} \, .$$

One can also introduce the NS-NS 2-form potential and RR potentials. The NS-NS 3-form field strength satisfies the constraints

$$H_3 = -iE^{\underline{a}} \wedge (E^1 \wedge \sigma_{\underline{a}} E^1 - E^2 \wedge \sigma_{\underline{a}} E^2) + \frac{1}{3!} E^{\underline{c}_3} \wedge E^{\underline{c}_2} \wedge E^{\underline{c}_1} H_{\underline{c}_1 \underline{c}_2 \underline{c}_3} \, .$$

from which one can extract the SUSY transformations

$$\delta_\epsilon B_2(x) = i_\epsilon H_3|_{\theta=0} = -2ie^{\underline{a}} \wedge (\psi^1 \sigma_{\underline{a}} \epsilon^1 - \psi^2 \sigma_{\underline{a}} \epsilon^2) \, .$$

RR superform potentials of type IIB SG and their field strength superforms can be collected in the formal sum of even forms

$$C = C_0 \oplus C_2 \oplus C_4 \oplus C_6 \oplus C_8 \oplus C_{10}$$
$$C_{2n} = \frac{1}{2n!} dZ^{M_{2n}} \wedge \ldots \wedge dZ^{M_1} C^{(2n)}_{M_1 \ldots M_{2n}}(Z) \; .$$

Their field strength superforms, which can be collected in the formal sum

$$R = dC - C \wedge H_3 = R_1 \oplus R_3 \oplus R_5 \oplus R_7 \oplus R_9 \; ,$$
$$R_{2n+1} = \frac{1}{(2n+1)!} dZ^{M_{2n+1}} \wedge \ldots \wedge dZ^{M_1} R^{(2n+1)}_{M_1 \ldots M_{2n+1}}(Z) \; ,$$

are subject to the constraints

$$\begin{aligned} R_{2n+1} &= 2ie^{-\Phi} E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}^{(2n-1)}_{\alpha\beta} - \\ &\quad -e^{-\Phi} \left(E^2 \wedge \bar{\sigma}^{(2n)} \nabla_1 \Phi - (-)^n E^1 \wedge \bar{\sigma}^{(2n)} \nabla_2 \Phi \right) + \\ &\quad + \frac{1}{(2n+1)!} E^{a_{2n+1}} \wedge \ldots \wedge E^{a_1} R_{a_1 \ldots\, a_{2n+1}} \; . \end{aligned}$$

Notice the universal dependence on dilaton superfield $\Phi(Z)$.

The above constraints are *on-shell* as the set of their consequences include all the equations of motion. For the 4-form potential equations have the from of self-duality condition

$$R_{a_1 \ldots a_5} = (*R)_{a_1 \ldots a_5} := \tfrac{1}{5!} \varepsilon_{\; a_1 \ldots a_5 b_1 \ldots b_5} R^{b_1 \ldots b_5}$$

When the higher superforms are introduced, the other RR equations acquire the form of duality conditions

$$R_{a_1 \ldots a_{9-2n}} = \tfrac{(-)^n}{(2n+1)!} \, \varepsilon_{\; a_1 \ldots a_{9-2n} b_1 \ldots b_{2n+1}} R^{b_1 \ldots b_{2n+1}} \qquad ,$$
$$R_{a_1 \ldots a_9} = \varepsilon_{\; a_1 \ldots a_9 b} R^b \; , \qquad R_{a_1 \ldots a_7} = -\tfrac{1}{3!} \varepsilon_{\; a_1 \ldots a_7 b_1 \ldots b_3} R^{b_1 \ldots b_3} \; .$$

Ex.: *To extract the SUSY transformations for the RR fields from the above superspace constraints.*

Studying BIs one finds ($\underline{\beta} := (\beta I) = 1, ..., 32$, $\beta = 1, ..., 16$, etc.)

$$D_{\underline{\beta}} D_{\underline{\gamma}} e^{-\Phi} = i\sigma^a_{\underline{\beta\gamma}} D_a e^{-\Phi} - i(\not{R}^{(1)} i\tau_2)_{\underline{\beta\gamma}} + \frac{i}{2}(\not{R}^{(3)}\tau_1)_{\underline{\beta\gamma}} + \frac{i}{2}(\not{H}^{(3)}\tau_3)_{\underline{\beta\gamma}} ,$$

where τ_1, τ_2, τ_3 are Pauli matrices, $\not{R}^{(1)} = R_a\tilde{\sigma}^{a\beta\gamma} = D_a C_0 \sigma^a_{\beta\gamma}$, $\not{R}^{(3)} = \frac{1}{3!} R_{abc}\sigma^{abc}_{\beta\gamma}$, $\quad (\tilde{\not{R}}^{(3)}\tau_1)_{\underline{\beta\gamma}} = \begin{pmatrix} \frac{1}{3!}0 & R_{abc}\tilde{\sigma}^{abc}_{\beta\gamma} \\ \frac{1}{3!}R_{abc}\tilde{\sigma}^{abc}_{\beta\gamma} & 0 \end{pmatrix}$ etc.

Notice $-\frac{1}{2}T_{\underline{\beta\gamma}}{}^{\underline{\alpha}} D_{\underline{\alpha}} e^{-\Phi} = 0$, due to thestructuire of dim 1/2 torsion

$$T_{\underline{\hat{\beta}\hat{\gamma}}}{}^{\underline{\alpha}} D_{\underline{\alpha}} = \begin{pmatrix} -2\left(\delta_{(\beta}{}^{\alpha} D_{\gamma)1} e^{-\Phi} - \frac{1}{2}\sigma^a_{\beta\gamma}\tilde{\sigma}^{\alpha\delta}_a D_{\delta 1} e^{-\Phi}\right) D_{\alpha 1} & 0 \\ 0 & -2\left(\delta_{(\beta}{}^{\alpha} D_{\gamma)2} e^{-\Phi} - \frac{1}{2}\sigma^a_{\beta\gamma}\tilde{\sigma}^{\alpha\delta}_a D_{\delta 2} e^{-\Phi}\right) D_{\alpha 2} \end{pmatrix} .$$

We are interesting in the dim 1 fermionic torsion component,

$$E^{\underline{\alpha}} T_{\underline{\alpha} b}{}^{\beta 1} = -\frac{1}{8} E^{\alpha 1} H_{b\underline{cd}} (\sigma^{\underline{cd}})_{\alpha}{}^{\beta} + \frac{1}{16} E^{\alpha 2} \sum_{n=0}^{4} (-)^n (\sigma_b \tilde{\not{R}}^{(2n+1)})_{\alpha}{}^{\beta} ,$$

$$E^{\underline{\alpha}} T_{\underline{\alpha} b}{}^{\beta 2} = \frac{1}{16} E^{\alpha 1} \sum_{n=0}^{4} (\sigma_b \tilde{\not{R}}^{(2n+1)})_{\alpha}{}^{\beta} + \frac{1}{8} E^{\alpha 2} H_{b\underline{cd}} (\sigma^{\underline{cd}})_{\alpha}{}^{\beta} .$$

[up to the terms of higher order in fermions]. These fix the form of the supersymmetry transformations of gravitino $\underline{\psi}^{\underline{\alpha}}_b = (\psi^{\alpha 1}_b, \psi^{\alpha 2}_b)$ and dilatino $\underline{\chi}_{\underline{\alpha}} = (\chi^1_\alpha, \chi^2_\alpha)$

$$\begin{aligned} \delta_\varepsilon \underline{\psi}_b &:= \mathcal{D}_b \underline{\varepsilon} := D_b \underline{\varepsilon} - \frac{1}{8} H_{b\underline{c_1 c_2}} \underline{\varepsilon} (\sigma^{\underline{c_1 c_2}} \otimes \tau_3) + \\ &\quad + \frac{1}{8} e^{\Phi}\, \underline{\varepsilon} \left(-\sigma_b \hat{\tilde{\not{R}}}^{(1)} \otimes i\tau_2 + \sigma_b \hat{\tilde{\not{R}}}^{(3)} \otimes \tau_1 - \frac{1}{2}\sigma_b \hat{\tilde{\not{R}}}^{(5)} \otimes i\tau_2 \right) , \\ \delta_\varepsilon \underline{\chi} &= \underline{\varepsilon} \left[\frac{1}{2} \not{\nabla}\Phi \otimes I + \frac{1}{4}\not{H} \otimes \tau_3 - \frac{1}{2} e^{\Phi} \hat{\not{R}}^{(1)} \otimes i\tau_2 + \frac{1}{4} e^{\Phi} \hat{\not{R}}^{(3)} \otimes \tau_1 \right] . \end{aligned}$$

Superfield T-duality rules

[Kulik and Roiban 2002], [I.B. and B. Julia 2003] $(Z^{\underline{M}} = (\tilde{Z}^M, y), ...)$

$$type\ IIA\ :\ \hat{E}^{\hat{a}} = (\hat{E}^{\tilde{a}}, \hat{E}^{\#})\ , \qquad \hat{E}^{\tilde{a}} = \hat{E}^{\tilde{a}(-)} = d\tilde{Z}^M \hat{E}_M^{\tilde{a}}(\tilde{Z}) \qquad ...$$

$$type\ IIB\ :\ E^{a} = (E^{\tilde{a}}, E^{*})\ , \qquad E^{\tilde{a}} = E^{\tilde{a}(-)} = d\tilde{Z}^M E_M^{\tilde{a}}(\tilde{Z})\ , \qquad ...$$

The T–duality rules for NS-NS superfields are (Einstein frame)

$$e^{\frac{\Phi(\tilde{Z})}{4}} E^{\tilde{a}(-)} = e^{\frac{\hat{\Phi}(\tilde{Z})}{4}} \hat{E}^{\tilde{a}(-)}\ , \qquad e^{\frac{\Phi}{4}} E_y^{*} = \frac{1}{e^{\frac{\hat{\Phi}}{4}} \hat{E}_{\hat{z}}^{\#}}\ , \qquad e^{\frac{\Phi}{4}} E^{*(-)} = \frac{i_{\hat{z}} \hat{B}_2}{e^{\frac{\hat{\Phi}}{4}} \hat{E}_{\hat{z}}^{\#}}\ ,$$

$$e^{\Phi(\tilde{Z})} = \frac{e^{\hat{\Phi}(\tilde{Z})}}{e^{\frac{\hat{\Phi}}{4}} \hat{E}_{\hat{z}}^{\#}}\ ,$$

$$i_y B_2 = \frac{\hat{E}^{\#(-)}}{\hat{E}_{\hat{z}}^{\#}}\ , \qquad B_2^{(-)} = \hat{B}_2^{(-)} - i_{\hat{z}} \hat{B}_2 \wedge \frac{\hat{E}^{\#(-)}}{\hat{E}_{\hat{z}}^{\#}}\ .$$

The T–duality rules for fermionic supervielbeins are

$$e^{-\frac{1}{8}\Phi} \frac{E_y^{\beta 1}}{E_y^{*}} = -\, e^{-\frac{1}{8}\hat{\Phi}} \left(\frac{\hat{E}_{\hat{z}}^{\beta 1}}{\hat{E}_{\hat{z}}^{\#} + \frac{i}{4} \tilde{\sigma}^{\# \beta\gamma} \hat{\nabla}_{\gamma 1} \hat{\Phi} - \frac{i}{8} \tilde{\sigma}^{\# \beta\gamma} \hat{\nabla}_{\gamma 1} ln \left(e^{\frac{\hat{\Phi}}{4}} E_{\hat{z}}^{\#} \right)} \right)\ ,$$

$$e^{-\frac{1}{8}\Phi} \frac{E_y^{\beta 2}}{E_y^{*}} = e^{-\frac{1}{8}\hat{\Phi}}\ \tilde{\sigma}^{\# \beta\gamma} \left(\frac{\hat{E}_{\hat{z}\gamma}^{\ 2}}{\hat{E}_{\hat{z}}^{\#}} + \frac{i}{4} \sigma_{\beta\gamma}^{\#} \hat{\nabla}_2^{\gamma} \hat{\Phi} - \frac{i}{8} \sigma_{\beta\gamma}^{\#} \hat{\nabla}_2^{\gamma} ln \left(e^{\frac{\hat{\Phi}}{4}} E_{\hat{z}}^{\#} \right) \right)\ ,$$

$$e^{\frac{1}{8}\Phi} E^{\beta 1[-]} = e^{\frac{1}{8}\hat{\Phi}} \left(\hat{E}^{\beta 1[-]} - \frac{i}{8} \hat{E}^{\tilde{a}(-)} \tilde{\sigma}_{\tilde{a}}^{\ \beta\gamma} \hat{\nabla}_{\gamma 1} ln \left(e^{\frac{\hat{\Phi}}{4}} E_{\hat{z}}^{\#} \right) \right)\ ,$$

$$e^{\frac{1}{8}\Phi} E^{\beta 2[-]} = e^{\frac{1}{8}\hat{\Phi}}\ \tilde{\sigma}^{\# \beta\gamma} \left(\hat{E}_{\gamma}^{2[-]} - \frac{i}{8} \hat{E}^{\tilde{a}(-)} \sigma_{\tilde{a}\beta\gamma} \hat{\nabla}_2^{\gamma} ln \left(e^{\frac{\hat{\Phi}}{4}} E_{\hat{z}}^{\#} \right) \right)\ .$$

Finally, the T–duality rules for the RR superform potentials are

$$C^{(0)} = i_{\hat{z}} \hat{C}_1 \equiv \hat{C}_{\hat{z}}^{(1)}\ ,$$

$$i_y C_{2n} = -\hat{C}_{2n-1}^{(-)} + \frac{\hat{E}^{\#(-)}}{\hat{E}_{\hat{z}}^{\#}} \wedge i_{\hat{z}} \hat{C}_{2n-1}\ , \quad n = 1, 2, 3, 4, 5\ ,$$

$$C_{2n}^{(-)} = i_{\hat{z}} \hat{C}_{2n+1} + i_{\hat{z}} \hat{B}_2 \wedge \left(\hat{C}_{2n-1}^{(-)} - \frac{\hat{E}^{\#(-)}}{\hat{E}_{\hat{z}}^{\#}} \wedge i_{\hat{z}} \hat{C}_{2n-1} \right)\ , \quad n = 1, ..., 4.$$