$D = 4$, $\mathcal{N} = 8$ SUPERGRAVITY

$\mathcal{N} = 8$ SUGRA by dim. reduction of 11D SUGRA to D=4 [Cremmer & Julia NPB 79]

We assume that all the fields depend only on 4 of 11 coordinates

Triangle gauge

$\mu = 0, 1, 2, 3$

$a = 0, 1, 2, 3$

$\hat{I}, \hat{M} = 1, \dots, 7$

$\underline{\mu} = 0, 1, \dots, 9, \#$

$\underline{a} = 0, 1, \dots, 9, \#$

$$e_{\underline{\mu}}{}^{\underline{a}}(x) = \begin{pmatrix} e_\mu{}^a(x) & A_\mu{}^{\hat{I}}(x) \\ 0 & h^{\hat{I}\hat{J}}(x) \end{pmatrix}$$

$\underline{\alpha} = 1, \dots, 32$

$\alpha = 1, 2$

$\dot{\alpha} = 1, 2$

$i, j = 1, \dots, 8$

$$\psi_{\underline{\mu}}{}^{\underline{\alpha}}(x) \mapsto (\psi_{\underline{\mu} i}^{\alpha}(x), \bar{\psi}_{\underline{\mu}}{}^{\dot{\alpha} i}(x)\,,\ \chi_{\hat{I} i}^{\alpha}(x), \bar{\chi}_{\hat{I}}{}^{\dot{\alpha} i}(x)\,)$$

$$A_{\underline{\mu\nu\rho}}(x) \mapsto A_{\mu\nu\rho}(x), A_{\mu\nu}{}^{\hat{I}}(x), A_\mu{}^{\hat{I}\hat{J}}(x), A^{\hat{I}\hat{J}\hat{K}}(x)$$

Thus the fields of the dimensionally reduced theory are

- graviton $e_\mu{}^a(x)$
- 8 Weyl gravitini $\psi_{\underline{\mu} i}^{\alpha}(x), \bar{\psi}_{\underline{\mu}}{}^{\dot{\alpha} i}(x)$; $i = 1, \dots, 8$ is the SU(8) index.
- 28=7+21 vector fields $A_\mu{}^{\hat{I}}(x)$ and $A_\mu{}^{\hat{I}\hat{J}}(x) = A_\mu{}^{[\hat{I}\hat{J}]}(x)$, $I = 1, \dots, 7$. It appears that they can be combined in $A_\mu{}^{ij}(x) = A_\mu{}^{[ij]}(x)$ with $i, j = 1, \dots, 8$
- 56=7x8 spinor fields $\chi_{\hat{I} i}^{\alpha}(x), \bar{\chi}_{\hat{I}}{}^{\dot{\alpha} i}(x)$
- $A_{\mu\nu\rho}(x)$ remains auxiliary.
- The most complicated story is about scalars: 70 = 28+35+7 scalars in the theory

- 70 = 28+35+7 scalars:
  - $h^{\hat{I}\hat{J}} = h^{(\hat{I}\hat{J})}$,
  - $A^{\hat{I}\hat{J}\hat{K}}(x) = A^{[\hat{I}\hat{J}\hat{K}]}(x)$ $\hat{I},\hat{J},\hat{K}=1,..,7$
  - plus coming from 7 'notophs' $A_{\mu\nu}{}^{\hat{I}}(x)$ (in D=4 antisymmetric tensor field of rank 2 is equivalent to scalar [Ogievetskij & Polubarinov 1967]) after dualization.

- After a nontrivial field redefinition Cremmer & Julia [NPB 1979] succeed in showing that these scalars parametrize the coset $E_{7(+7)}/SU(8)$ of an exceptional Lie group $E_{7(+7)}$.

70 Scalar fields of D=4, $\mathcal{N}=8$ SUGRA can be described by the $E_{7(+7)}$ valued matrix [4] 56×56 $[IJ] \leftarrow 28$ values $(=\frac{8\cdot 7}{2})$ $[ij]$

$$\mathcal{V} := \mathcal{V}^{\underline{\Sigma}}{}_{(\underline{\Pi})} = (\mathcal{V}^{\underline{\Sigma}}{}_{kl}, \bar{\mathcal{V}}^{\underline{\Sigma}\,kl}) = \begin{pmatrix} u^{IJ}{}_{kl} & \bar{v}^{IJ\,kl} \\ v_{IJ\,kl} & \bar{u}_{IJ}{}^{kl} \end{pmatrix} \quad \in \quad E_{7(+7)}\,.$$

$I,J=1,..,8$
$i,j=1,..,8$

defined modulo $SU(8)$ transf-s of the diagonal $SU(8)\subset E_{7(+7)}$. $\boxed{70 = 133_{dim(E_{7(+7)})} - 63_{dim(SU(8))}}$.

$E_{7(+7)}$ is a subgroup of a symplectic group, $E_{7(+7)}\subset Sp(56,\mathbb{R})$ which implies

$$\mathcal{V}\in Sp(56,\mathbb{R}) \quad \Rightarrow \quad \begin{aligned} \mathcal{V}^{\underline{\Sigma}}{}_{ij}\bar{\mathcal{V}}^{\underline{\Pi}ij} - \mathcal{V}^{\underline{\Pi}}{}_{ij}\bar{\mathcal{V}}^{\underline{\Sigma}ij} &= i\Xi^{\underline{\Sigma\Pi}}\,, \\ \mathcal{V}^{\underline{\Sigma}}{}_{ij}\mathcal{V}^{\underline{\Pi}}{}_{kl}\Xi_{\underline{\Sigma\Pi}} &= 0\,, \\ \mathcal{V}^{\underline{\Sigma}}{}_{ij}\bar{\mathcal{V}}^{\underline{\Pi}kl}\Xi_{\underline{\Sigma\Pi}} &= -i\delta_{[i}{}^{k}\delta_{j]}{}^{l}\,, \end{aligned}$$

where $\Xi^{\underline{\Sigma\Pi}} = -\Xi^{\underline{\Pi\Sigma}}$ is the symplectic metric, usually [4, 5, 6, 7, 8]

$$\Xi^{\underline{\Pi\Sigma}} := \begin{pmatrix} 0 & \mathbb{I}_{28\times 28} \\ -\mathbb{I}_{28\times 28} & 0 \end{pmatrix} := \begin{pmatrix} 0 & \delta_{[I}{}^{K}\delta_{J]}{}^{L} \\ -\delta^{[I}{}_{K}\delta^{J]}{}_{L} & 0 \end{pmatrix} = -\Xi_{\underline{\Sigma\Pi}}\,.$$

The fact that $\mathcal{V} \in E_{7(+7)}$ (and not just $Sp(56, \mathbb{R})$) can be expressed by stating that the variation of the 56×28 blocks of $\mathcal{V}$ can be decomposed on $SU(8)$ transformations and

$$\delta \mathcal{V}^{\Sigma}{}_{ij} = \bar{\mathcal{V}}^{\Sigma\, kl}\; i_\delta P_{ijkl} \;, \qquad \delta \bar{\mathcal{V}}^{\Sigma\, ij} = \mathcal{V}^{\Sigma}{}_{kl}\; i_\delta \bar{P}^{ijkl} \;.$$

The later 70 variations are determined by antisymmetric forth rank $SU(8)$ tensor parameter $i_\delta P_{ijkl}$ which obeys

$$i_\delta P_{ijkl} = i_\delta P_{[ijkl]} = \frac{1}{4!} \epsilon_{ijkli'j'k'l'} i_\delta \bar{P}^{i'j'k'l'} \;.$$

Similarly, the derivatives of the scalar fields $(\mathcal{V}^{\Sigma}{}_{kl}, \bar{\mathcal{V}}^{\Sigma\, kl})$ are not arbitrary but expressed through $SU(8)$ Cartan forms $\Omega_i{}^j = -(\Omega_j{}^i)^*$, and the complex self-dual antisymmetric 4th rank tensor 1-forms parameterizing the space co-tangent to $E_{7(+7)}/SU(8)$ coset,

$$P_{ijkl} = P_{[ijkl]} = \frac{1}{4!} \epsilon_{ijkli'j'k'l'} \bar{P}^{i'j'k'l'} \;.$$

Actually it is convenient to work with $SU(8)$ covariant derivatives

$$\begin{aligned} D\mathcal{V}^{\Sigma}{}_{ij} &= d\mathcal{V}^{\Sigma}{}_{ij} + 2\Omega_{[i|}{}^{k} \mathcal{V}^{\Sigma}{}_{k|j]} = P_{ijkl} \bar{\mathcal{V}}^{\Sigma\, kl} \;, \\ D\bar{\mathcal{V}}^{\Sigma\, ij} &= d\bar{\mathcal{V}}^{\Sigma\, ij} - 2\bar{\mathcal{V}}^{\Sigma\, [i|k} \Omega_k{}^{|j]} = \bar{P}^{ijkl} \mathcal{V}^{\Sigma}{}_{kl} \;. \end{aligned}$$

The kin terms for the scalars in the $\mathcal{N}$=8 SUGRA Lagrangian

$$L^{CJ} = e\mathcal{R} + \propto \epsilon^{\mu\nu\rho\sigma} \psi_\mu \sigma_\nu \mathcal{D}_\rho \bar{\psi}_\sigma + L_{vect} + L_{spin\ 1/2} + L_{scalar} + \ldots$$

is written in terms of P^{μ}_{ijkl},

$$L_{scalar} = \propto\ e\ P^{\mu}_{ijkl} \bar{P}^{ijkl}_{\mu}$$

which also enters in a specific scalar-spin 3/2-spin 1/2 interaction.

The composed SU(8) connection connection $\Omega_{\mu i}{}^{j} = -(\Omega_{\mu j}{}^{i})^*$ enters the covariant derivatives of the fermionic fields,

$$\mathcal{D}\psi^{\alpha i} = d\psi^{\alpha i} - \psi^{\beta i} \omega_\beta{}^\alpha - \psi^{\alpha j} \Omega_j{}^i \;.$$

$D = 4$, $\mathcal{N} = 8$ SUGRA IN $8\mathcal{N}$ SUPERSPACE

Superspace of $\mathcal{N} = 8$ supergravity $\Sigma^{(4|32)}$

$\Sigma^{(4|32)}$ with local coordinates $Z^M = (x^\mu, \theta^{\check{\alpha}})$ is supplied by the bosonic and fermionic supervielbein 1-forms,

$$E^A = (E^a, E^\alpha) = dZ^M E_M^A(Z) \,, \qquad a = 0, 1, ..., 9, \#, \qquad \alpha = 1, ..., 32 \,,$$

SO(1,10) connection

$$\omega_B{}^A = diag(\omega_b{}^a, \omega_\beta{}^\alpha, \omega_{\dot\beta}{}^{\dot\alpha}) = dZ^M \omega_{MB}{}^A(Z) \,,$$

$$\omega^{ab} = -\omega^{ba} \,, \qquad \omega_\beta{}^\alpha = \frac{1}{4}\omega^{ab}\sigma_{ab\beta}{}^\alpha \,, \qquad \omega_{\dot\beta}{}^{\dot\alpha} = -\frac{1}{4}\omega^{ab}\tilde\sigma_{ab}{}^{\dot\alpha}{}_{\dot\beta} \,,$$

and with an induced $SU(8)$ connection

$$\Omega_i{}^j = -(\Omega_j{}^i)^* = dZ^M \Omega_{Mi}{}^j(Z) = E^A \Omega_{Ai}{}^j(Z)$$

constructed from the scalar superfields

$$\mathcal{V}^{\Sigma}{}_{(\underline{\Pi})}(Z) = (\mathcal{V}^{\Sigma}{}_{kl}(Z), \bar{\mathcal{V}}^{\Sigma\, kl}(Z)) = \begin{pmatrix} u^{IJ}{}_{kl} & \bar{v}^{IJ\, kl} \\ v_{IJ\, kl} & \bar{u}_{IJ}{}^{kl} \end{pmatrix} \quad \in \quad E_{7(+7)} \,.$$

according to

$$d\mathcal{V}^{\Sigma}{}_{ij}\, \bar{\mathcal{V}}^{\underline{\Pi}\, kl}\Xi_{\underline{\Sigma\Pi}} = 2i\Omega_{[i}{}^k \delta_{j]}{}^l \,.$$

This composed nature of the $SU(8)$ connection can be extraced from the expression for the $SO(1,10)\otimes SU(8)$ covariant derivatives

$$\begin{aligned} D\mathcal{V}^{\Sigma}{}_{ij} &= d\mathcal{V}^{\Sigma}{}_{ij} + 2\Omega_{[i|}{}^k \mathcal{V}^{\Sigma}{}_{k|j]} = P_{ijkl}\bar{\mathcal{V}}^{\Sigma\, kl} \,, \\ D\bar{\mathcal{V}}^{\Sigma\, ij} &= d\bar{\mathcal{V}}^{\Sigma\, ij} - 2\bar{\mathcal{V}}^{\Sigma\, [i|k}\Omega_k{}^{|j]} = \bar{P}^{ijkl}\mathcal{V}^{\Sigma}{}_{kl} \,. \end{aligned}$$

which also show that the scalar superfields parametrize $E_{7(+7)}$ and define the covariant $E_{7(+7)}/SU(8)$ Cartan super-1-form

$$P_{ijkl} = P_{[ijkl]} = \frac{1}{4!}\epsilon_{ijkli'j'k'l'}\bar{P}^{i'j'k'l'} = dZ^M P_{M\, ijkl}(Z) = E^A P_{A\, ijkl}(Z)$$

now as a form in superspace.

The $\mathcal{N}=8$ Superspace torsion constraints [Brink&Howe 1979] are collected in the following expressions for bosonic and fermionic torsion 2-forms T^a and $T^{\underline{\alpha}} = (T^\alpha_i, T^{\dot\alpha i})$:

$$
\begin{aligned}
T^a &= DE^a = dE^a - E^b \wedge \omega_b{}^a = -iE^\alpha_i \wedge \bar E^{\dot\beta j}\sigma^a_{\alpha\dot\beta}\,,\\
T^\alpha_i &= DE^\alpha_i = dE^\alpha_i - E^\beta_i \wedge \omega_\beta{}^\alpha - E^\alpha_j \wedge \Omega_i{}^j = \\
&= \frac{1}{2}\bar E^{\dot\beta j}\wedge \bar E^{\dot\gamma k}\epsilon_{\dot\beta\dot\gamma}\bar\chi^\alpha_{ijk} + E^c \wedge E^{\underline{\beta}}T_{\underline{\beta}\,c\,i}{}^\alpha + \frac{1}{2}E^c\wedge E^b T_{bc\,i}{}^\alpha\,,\\
T^{\dot\alpha i} &= -\frac{1}{2}E^\beta_j \wedge E^\gamma_k \epsilon_{\beta\gamma}\bar\chi^{\dot\alpha ijk} + E^c\wedge E^{\underline{\beta}}T_{\underline{\beta}\,c}{}^{\dot\alpha i} + \frac{1}{2}E^c\wedge E^b T_{bc}{}^{\dot\alpha i}.
\end{aligned}
$$

Their consequences obtained by studying the Bianchi identities include the expressions for the curvature 2-form $\boxed{R_a{}^b = (d\omega - \omega\wedge\omega)_a{}^b}$

$$
\sigma^a_{\alpha\dot\alpha}\tilde\sigma_b^{\dot\beta\beta}R_a{}^b = 2\delta_\alpha{}^\beta R_{\dot\alpha}{}^{\dot\beta} + 2\delta_{\dot\alpha}{}^{\dot\beta}R_\alpha{}^\beta
$$

$$
\begin{aligned}
R^{\alpha\beta} &= \frac{1}{4}R^{ab}\sigma^{\alpha\beta}_{ab} = \frac{1}{2}E^\gamma_i\wedge E^\delta_j\epsilon_{\gamma\delta}N^{\alpha\beta ij} + \frac{1}{2}\bar E^{\dot\beta i}\wedge\bar E^{\dot\gamma j}\epsilon_{\dot\gamma\dot\delta}M^{\alpha\beta}_{ij} + \\
&\quad + E^\gamma_i\wedge\bar E^{\dot\gamma j}R^i_{\gamma\,\dot\gamma j}{}^{\alpha\beta} + E^c\wedge E^{\underline{\beta}}R_{\underline{\beta}\,c}{}^{\alpha\beta} + \frac{1}{2}E^c\wedge E^b R_{bc}{}^{\alpha\beta},\\
R^{\dot\alpha\dot\beta} &= -\frac{1}{4}R^{ab}\tilde\sigma^{\dot\alpha\dot\beta}_{ab} = -\frac{1}{2}E^\gamma_i\wedge E^\delta_j\epsilon_{\gamma\delta}\bar M^{\dot\alpha\dot\beta\, ij} - \frac{1}{2}\bar E^{\dot\beta i}\wedge\bar E^{\dot\gamma j}\epsilon_{\gamma\delta}\bar N^{\dot\gamma\dot\delta}_{ij} + \\
&\quad + E^\gamma_i\wedge\bar E^{\dot\gamma j}R^i_{\gamma\,\dot\gamma j}{}^{\dot\alpha\dot\beta} + E^c\wedge E^{\underline{\beta}}R_{\underline{\beta}\,c}{}^{\dot\alpha\dot\beta} + \frac{1}{2}E^c\wedge E^b R_{bc}{}^{\dot\alpha\dot\beta}
\end{aligned}
$$

as well as the conditions for **main fermionic superfields** $\chi_{\alpha jkl}$,

$$
\begin{aligned}
D^{\alpha i}\chi_{\alpha jkl} &= -\frac{e^{i\beta}}{6}\epsilon_{jkl[2][3]}\bar\chi^{\dot\alpha i[2]}\bar\chi^{[3]}_{\dot\alpha}\,, &\qquad \bar D^{\dot\alpha}_i\bar\chi^{jkl}_{\dot\alpha} &= \frac{e^{-i\beta}}{6}\epsilon^{jkl[2][3]}\chi^\alpha_{[2]}\chi_{\alpha[3]}\,,\\
D^i_{(\alpha}\chi_{\beta)jkl} &= -\frac{3}{4}\delta^i_{[j}M_{kl]\alpha\beta}\,, &\qquad \bar D_{i(\dot\alpha}\bar\chi^{jkl}_{\dot\beta)} &= -\frac{3}{4}\delta^{[j}_i\bar M^{kl]}_{\dot\alpha\dot\beta}\,.
\end{aligned}
$$

These latter expressions define $M_{ij\alpha\beta} = (\bar M^{ij}_{\dot\alpha\dot\beta})^*$ while

$$
N^{ij}_{\alpha\beta} = \frac{e^{-i\beta}}{6\cdot 4!}\epsilon^{ij[3][3']}\chi_{\alpha[3]}\chi_{\beta[3']}\,,\qquad \bar N_{\dot\alpha\dot\beta ij} = -\frac{e^{-i\beta}}{6\cdot 4!}\epsilon_{ij[3][3']}\bar\chi^{[3]}_{\dot\alpha}\bar\chi^{[3']}_{\dot\beta}\,.
$$

The curvature of the induced $SU(8)$ connection,

$$R_i{}^j := d\Omega_i{}^j - \Omega_i{}^k \wedge \Omega_k{}^j$$

is expressed by

$$R_i{}^j = -(R_j{}^i)^* = \frac{1}{3}\mathbb{P}_{iklp} \wedge \bar{\mathbb{P}}^{jklp},$$

through the wedge product of two covariantly closed 1–forms,

$$D\mathbb{P}_{ijkl} := d\mathbb{P}_{ijkl} - 4\Omega_{[i|}{}^p \wedge \mathbb{P}_{p|jkl]} = 0 \,, \qquad D\bar{\mathbb{P}}^{ijkl} = 0 \,,$$

which obey the constraints

$$\mathbb{P}_{ijkl} = 2E^{\alpha}_{[i}\chi_{jkl]\alpha} - \frac{e^{i\beta}}{4!}\bar{E}^{\dot{\alpha}p}\epsilon_{ijklp[3]}\bar{\chi}^{[3]}_{\dot{\alpha}} + E^a\mathbb{P}_{a\,ijkl} \,,$$

$$\bar{\mathbb{P}}^{ijkl} = \frac{e^{-i\beta}}{4!}E^{\alpha}_p\epsilon^{ijklp[3]}\chi_{\alpha[3]} - 2\bar{E}^{\dot{\alpha}[i}\bar{\chi}^{jkl]}_{\dot{\alpha}} + E^a\bar{\mathbb{P}}^{ijkl}_a \,,$$

where β is a constant phase parameter and complex fermionic superfields $\boxed{\chi_{jkl\alpha} = \chi_{[jkl]\alpha} := \chi_{[3]\alpha}}$ are **the main superfields of the $\mathcal{N} = 8$ supergravity**, $\bar{\chi}^{jkl]}_{\dot{\alpha}} = (\chi_{jkl\alpha})^*$.

The covariant constancy conditions $D\mathbb{P}_{ijkl} = 0$, $D\bar{\mathbb{P}}^{ijkl} = 0$ imply

$$\bar{D}_{\dot{\alpha}i}\chi_{\alpha\,jkl} = 2i\sigma^a_{\alpha\dot{\alpha}}\mathbb{P}_{aijkl} \,, \qquad D^i_{\alpha}\bar{\chi}^{jkl}_{\dot{\alpha}} = -2i\sigma^a_{\alpha\dot{\alpha}}\bar{\mathbb{P}}^{ijkl}_a \,,$$

see above

and also also helps to find the expression for $D^{\alpha i}\chi_{\alpha jkl}$ and the duality relation between the vector $\mathbb{P}_{aijkl}$ and its conjugate $\bar{\mathbb{P}}^{ijkl}_a$,

$$\mathbb{P}_{aijkl} = \frac{e^{i\beta}}{4!}\epsilon_{ijklpqrs}\bar{\mathbb{P}}^{pqrs}_a \,.$$

Notice the presence of constant phase $e^{i\beta}$ in this duality relation. This also makes the superspace one forms to be related by

$$\mathbb{P}_{ijkl} = \frac{e^{i\beta}}{4!}\epsilon_{ijklpqrs}\bar{\mathbb{P}}^{pqrs} \,.$$

The dim 1 fermionic torsions are expressed through the bilinears and derivatives of fermionic main superfields $\chi^{\alpha}_{ijk} = (\bar\chi^{\dot\alpha ijk})^*$

$$
\begin{aligned}
T^{j}_{\beta b\, i}{}^{\alpha} &= \frac{1}{4}\chi_{ikl\beta}(\bar\chi^{jkl}\tilde\sigma_b)^{\alpha} , \qquad T_{\dot\beta j\, b}{}^{\dot\alpha i} = \frac{1}{4}\bar\chi^{ikl}_{\dot\beta}(\tilde\sigma_b\chi_{jkl})_{\alpha} , \\
T_{\dot\beta j\, b}{}_{i}{}^{\alpha} &= -\frac{i}{2}\sigma_{b\beta\dot\beta}M^{\alpha\beta}_{ij} + \frac{ie^{-i\beta}}{288}\,\epsilon_{ij[3][3']}\bar\chi^{[3]}_{\dot\alpha}\bar\chi^{[3']}_{\dot\beta}\tilde\sigma_b^{\dot\alpha\alpha} = (T^{j}_{\beta b}{}^{\dot\alpha i})^* ,
\end{aligned}
$$

where $M_{kl\alpha\beta} = -\frac{2}{3}D^{j}{}_{(\alpha}\chi_{\beta)jkl} = (\bar M^{kl}_{\dot\alpha\dot\beta})^*$.

In the $\mathcal{N} = 8$ superspace one can also define the Abelian 1-form potentials in 28 and $\bar{28}$ of $SU(N)$:

$$
A^{ij} = dZ^M A^{ij}_M(Z) = (\bar A_{ij})^*
$$

with two form field strengths obeying the BIs

$$
DF_{ij} = \mathbb{P}_{ijkl}\wedge \bar F^{kl} , \qquad D\bar F^{ij} = \bar{\mathbb{P}}^{ijkl}\wedge F_{kl} ,
$$

and restricted by the constraints

$$
F_{ij} = -iE^{\alpha}_i \wedge E^{\beta}_j \epsilon_{\alpha\beta} - \frac{1}{2}E^a\wedge \bar E^{\dot\gamma k}\sigma_{a\gamma\dot\gamma}\chi^{\gamma}_{ijk} + \frac{1}{2}E^a \wedge E^b F_{bc\, ij} = (\bar F^{ij})^* .
$$

With this constraints the BIs imply, in particular, that

$$
\begin{aligned}
\sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta}F_{ab\, ij} &= 2\epsilon_{\alpha\beta}F_{\dot\alpha\dot\beta\, ij} - 2\epsilon_{\dot\alpha\dot\beta}F_{\alpha\beta\, ij} , \\
F_{\alpha\beta\, ij} &= \frac{i}{32}M_{\alpha\beta\, ij} = -\frac{i}{48}D^{k}{}_{(\alpha}\chi_{\beta)ijk} , \\
F_{\dot\alpha\dot\beta\, ij} &= \frac{i}{2}\bar N_{\dot\alpha\dot\beta\, ij} = -i\frac{e^{i\beta}}{6\cdot 4!}\epsilon_{ij[3][3']}\bar\chi^{[3]}_{\dot\alpha}\bar\chi^{[3']}_{\dot\beta} .
\end{aligned}
$$

To conclude: in the superfield formalism all the fields of the theory are contained inside of the fermionic main superfield $\boxed{\chi^{\gamma}_{ijk}(Z) = (\bar\chi^{\dot\gamma ijk})^*}$ (a counterpart of $W_{\alpha\beta\gamma}$ in the $\mathcal{N} = 1$ case).

Equations of motion also follow from these on–shell constraints.

4.4. –(7')–

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