IN THE WZ GAUGE (As we have discussed in the general case)

$$\theta^{\check{\underline{\alpha}}} E_{\check{\underline{\alpha}}}{}^{A} = \theta^{\underline{\alpha}} \delta_{\underline{\alpha}}^{A} \quad , \quad \theta^{\underline{\alpha}} w_{\underline{\alpha}}{}^{ab} = 0$$

$$\Rightarrow \quad \theta^{\underline{\alpha}} \mathcal{D}_{\underline{\alpha}} \equiv i_{\theta} \mathcal{D} \equiv \theta\mathcal{D} = \theta\partial \equiv \theta^{\check{\underline{\alpha}}} \partial_{\check{\underline{\alpha}}}$$

all the components in θ-decomposition of $E_M{}^A$ and $\omega_M{}^{ab}$ superfields are expressed through the leading values of TORSION AND CURVATURE SUPER-TENSORS $T_{AB}{}^{C}(z)|_0$, $R_{AB}{}^{cd}(z)|_0$ and of its covariant derivatives. $\mathcal{D}_{[\underline{\beta}_1} \cdots \mathcal{D}_{\underline{\beta}_k]} T_{AB}{}^{C}(z)|_0$, etc.

$$(1+\underline{\theta}\underline{\partial})E^A = \mathcal{D}\theta^{\underline{\beta}} \delta_{\underline{\beta}}{}^{A} + i_{\underline{\theta}} T^A + dx^\mu E_\mu{}^A$$

$$(1+\underline{\theta}\underline{\partial})w^{ab} = i_{\underline{\theta}} R^{ab} + dx^\mu w_\mu{}^{ab}$$

$$\Downarrow$$

$$E_M{}^A(z)|_0 \equiv E_M{}^A(x,0) = \begin{pmatrix} e_\mu{}^a & \psi_\mu{}^{\underline{\alpha}} \\ 0 & \delta_{\underline{\beta}}{}^{\underline{\alpha}} \end{pmatrix}$$

$$E_A{}^N(z)|_0 \equiv E_A{}^N(x,0) = \begin{pmatrix} e_a{}^{\nu}(x) & -\psi_a{}^{\check{\underline{\beta}}}(x) \\ 0 & \delta_{\underline{\alpha}}{}^{\check{\underline{\beta}}} \end{pmatrix}$$

Ex: To obtain $\sim\theta, \bar{\theta}$ contributions to E^a, $E^{\underline{\alpha}}$, ω^{ab}

Ex: To obtain $\sim\theta, \bar\theta$ terms in $E_\mu{}^A$ and $w_\mu{}^{ab}$ dec.s in the WZ gauge in min D=4 N=1 SG

$$E_{(1,0)}^{a} = e^a + 2i\theta\sigma^a\bar\psi - 2i\psi\sigma^a\bar\theta + \sim\underline{\theta\theta}$$

$$E_{(1,0)}^{\alpha} = \psi^\alpha + \frac{i}{8}e^b(\theta\sigma_b\tilde\sigma_c)^\alpha G^c + \frac{i}{8}e^b(\bar\theta\tilde\sigma_b)^\alpha \bar R + \sim\underline{\theta\theta}$$

$$\bar E_{(1,0)}^{\dot\alpha} = \bar\psi^{\dot\alpha} - \frac{i}{8}e^b(\tilde\sigma_c\sigma_b\bar\theta)^{\dot\alpha}G^c - \frac{i}{8}e^b(\tilde\sigma_b\theta)^{\dot\alpha} R + \sim\underline{\theta\theta}$$

$$E_{(0,1)}^{a} = i(\theta\sigma^a)_{\dot\alpha}d\bar\theta^{\dot\alpha} - i\,d\theta^\alpha(\sigma^a\bar\theta)_\alpha + \sim\underline{\theta\theta}$$

$$E_{(0,1)}^{\alpha} = d\theta^\alpha + \sim\underline{\theta\theta}$$

$$\bar E_{(0,1)}^{\dot\alpha} = d\bar\theta^{\dot\alpha} + \sim\underline{\theta\theta}$$

$$w_{(1,0)}^{\alpha\beta} = dx^\mu w_\mu^{\alpha\beta}(x) + \frac{1}{2}\theta^{(\alpha}\psi^{\beta)}\bar R - \frac{i}{8}e^b\,\theta^{(\alpha}\bar{\mathcal{D}}_{\dot\gamma}\bar R\,\tilde\sigma^{\dot\gamma\beta)} + \frac{i}{8}e^b(\theta\sigma_b\tilde\sigma_c)^{(\alpha}\mathcal{D}^{\beta)}G^c -$$
$$- \frac{i}{8}e^b(\sigma_b\bar\theta)_\gamma \cdot W^{\alpha\beta\gamma} + \sim\underline{\theta\theta}$$

$$w_{(0,1)}^{\alpha\beta} = \frac{1}{4}\theta^{(\alpha}d\theta^{\beta)}\bar R\Big| + \sim\underline{\theta\theta}$$

COROLLARY:

$$\boxed{E_{\check\beta}{}^{A} = \delta_{\check\beta}{}^{\alpha}\left(\delta_\alpha{}^A - i(\sigma^a\bar\theta)_\alpha\,\delta_a{}^A\right) + \delta_{\check\beta}{}^{\dot\alpha}\left(\delta_{\dot\alpha}{}^A - i(\theta\sigma^a)_{\dot\alpha}\,\delta_a{}^A\right) + \sim\underline{\theta\theta}}$$

$$\Downarrow$$

$$\partial_{\check\alpha}E_{\check\beta}{}^A = \sim\underline{\theta} \quad \Rightarrow \quad \boxed{\partial_{\check\alpha}E_{\check\beta}{}^A\Big| = 0}$$

THIS IMPLIES THAT, IN WZ GAUGE

$$\mathcal{D}^\alpha\mathcal{D}_\alpha(\mathcal{E}\mathcal{L})\Big|_0 = \varepsilon^{\check\alpha\check\beta}\partial_{\check\beta}\partial_{\check\alpha}(\mathcal{E}\mathcal{L})\Big|_0 = -2\partial_1\partial_2(\mathcal{E}\mathcal{L})\Big|_0 = -2\left[\mathcal{E}\mathcal{L}\right]_F$$

THIS ALLOWS US TO USE $\mathcal{D}^\alpha\mathcal{D}_\alpha$ as GRASSMANNIAN PART OF CHIRAL INTEGRATION MEASURE:

$$\int d^6\zeta_L\,\mathcal{E}\mathcal{L} = \int d^4x\,\mathcal{D}^\alpha\mathcal{D}_\alpha(\mathcal{E}\mathcal{L})\Big|$$

(STAYS BEYOND THE USE OF (4) AS (8) variables decomposition below)

TORSION AND CURVATURE OF MIN SG IN WZ GAUGE.

$$R_{ab}{}^{ab}\Big| = E_b{}^N E_a{}^M R_{MN}{}^{ab}\Big| = e_b{}^\nu e_a{}^\mu R_{\mu\nu}{}^{ab}\Big| - 2\psi_{[a|}{}^{\underline{\alpha}} R_{\underline{\alpha}|b]}{}^{ab}\Big| + \psi_a{}^{\underline{\alpha}}\psi_b{}^{\underline{\beta}} R_{\underline{\alpha}\underline{\beta}}{}^{ab}\Big| =$$

Ex: To obtain

$$= e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}\Big| + \frac{1}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R}\Big| + \frac{1}{2}\bar{\psi}_a\tilde{\sigma}^{ab}\bar{\psi}_b\cdot R\Big| +$$

$$+ \frac{i}{2}\left(\psi_a{}^\alpha + \frac{1}{2}(\psi^b\sigma_b\tilde{\sigma}_a)^\alpha\right)\mathcal{D}_\alpha G^a\Big|$$

$$+ \frac{i}{2}\bar{\mathcal{D}}_{\dot\alpha}G^a\Big|\cdot\left(\bar{\psi}_a{}^{\dot\alpha} + \frac{1}{2}(\tilde{\sigma}_a\sigma_b\bar{\psi}^b)^{\dot\alpha}\right)$$

$$- 2\left(\bar{\Psi}_{\dot\alpha}{}^a\bar{\psi}_a{}^{\dot\alpha} - \frac{1}{2}\bar{\Psi}^a\tilde{\sigma}_a\sigma_b\bar{\psi}^b\right)$$

$$- 2\left(\psi_a{}^\alpha\Psi_\alpha{}^a - \frac{1}{2}\psi^b\sigma_b\tilde{\sigma}_a\Psi^a\right)\Big|$$

Ex: to obtain

$$e_a{}^\mu e_b{}^\nu \mathring{R}_{\mu\nu}{}^{ab} + \frac{i}{2}\psi_a\sigma_b\bar{\psi}_c\,\epsilon^{abcd}G_d\Big| + \frac{3}{32}G^aG_a\Big|$$

TO SPECIFY IT MORE WE NEED TO CALCULATE $T_{ab}{}^\alpha\big|$ and

$$\bar{\Psi}_{\dot\alpha}{}^a\Big| = \epsilon^{abcd}\,T_{bc}{}^\alpha\sigma_{d\alpha\dot\alpha}\Big|$$

We use: • $R_{\gamma c}{}^{\alpha\beta} = \frac{i}{8}(\sigma_c\tilde{\sigma}_d)_\gamma{}^{(\alpha}\mathcal{D}^{\beta)}G^d - \frac{i}{8}\bar{\mathcal{D}}_{\dot\alpha}\bar{R}\,\tilde{\sigma}_c{}^{\dot\alpha(\alpha}\delta_\gamma^{\beta)}$

$R_{\gamma c}{}^{\dot\alpha\dot\beta} = -\frac{i}{8}\sigma_{c\gamma\dot\gamma}\bar{W}^{\dot\alpha\dot\beta\dot\gamma}$

$\Rightarrow R_{\gamma c}{}^{ab} = \frac{i}{8}\sigma_{c\gamma\dot\gamma}\tilde{\sigma}^{ab}_{\dot\alpha\dot\beta}\bar{W}^{\dot\alpha\dot\beta\dot\gamma} - \frac{i}{16}(\sigma_c\tilde{\sigma}_d\sigma^{ab})_\gamma{}^\beta\mathcal{D}_\beta G^d - \frac{i}{16}(\sigma^{ab}\sigma_c)_{\gamma\dot\gamma}\bar{\mathcal{D}}^{\dot\gamma}\bar{R}$

$R_{\gamma c}{}^{ab} = \frac{1}{2}R_{\gamma c}{}^{\alpha\beta}\sigma^{ab}_{\alpha\beta} - \frac{1}{2}R_{\gamma c}{}^{\dot\alpha\dot\beta}\tilde{\sigma}^{ab}_{\dot\alpha\dot\beta} =$

$\Downarrow$

$R_{\gamma b}{}^{ab} = \frac{i}{16}\sigma^{ab}{}_\gamma{}^\beta\mathcal{D}_\beta G_b - \frac{3i}{16}\left(\mathcal{D}_\gamma G^a + \sigma^a_{\gamma\dot\gamma}\bar{\mathcal{D}}^{\dot\gamma}\bar{R}\right) =$

$= -\frac{i}{4}\mathcal{D}_\gamma G^a - \frac{i}{8}\sigma^a_{\gamma\dot\gamma}\bar{\mathcal{D}}^{\dot\gamma}\bar{R} = \Psi_\gamma{}^a - \frac{1}{2}(\sigma^a\tilde{\sigma}_b\Psi^b)_\gamma$

• $dx^\mu w_\mu{}^{ab} = \underset{\substack{\uparrow\\ \text{standard spin connection}}}{\mathring{w}^{ab}} + e^c\underbrace{\Delta w_c{}^{ab}}_{= -\frac{1}{8}\epsilon^{ab}{}_{cd}G^d}$

• $\mathcal{D}_\alpha G^a = 4i\left(\Psi_\alpha{}^a - \frac{1}{3}(\sigma^a\tilde{\sigma}^b\Psi_b)_\alpha\right)$, $\quad\bar{\mathcal{D}}^{\dot\alpha}\bar{R} = -\frac{4i}{3}\tilde{\sigma}^{b\dot\alpha\alpha}\Psi_{\alpha b}$

• $\left(\psi_a{}^\alpha - \frac{1}{4}(\psi^b\sigma_b\tilde{\sigma}_a)^\alpha\right)\sigma^a_{\alpha\dot\alpha} = 0$

Leading component of RS "super-field-strength"

$$T_{ab}{}^{\alpha}| = e_b{}^{\nu} e_a{}^{\mu} T_{\mu\nu}{}^{\alpha}| - 2\psi_{[a|}{}^{\beta} T_{\beta|b]}{}^{\alpha}| - 2\bar\psi_{[a|}{}^{\dot\beta} T_{\dot\beta|b]}{}^{\alpha}| + \cancel{\psi_a{}^{\beta}\psi_b{}^{\gamma} T_{\beta\gamma}{}^{\alpha}} \to 0$$

$$\to -\frac{i}{4}(\bar\psi_{[a}\tilde\sigma_{b]})^{\alpha} R$$

$$\to -\frac{i}{4}(\psi_{[a}\sigma_{b]}\hat\sigma_c)^{\alpha} G^c$$

Ex: to obtain

$$\to \overset{\circ}{\psi}_{ab}{}^{\alpha} + \frac{i}{8}(\psi_{[a}\sigma_{b]c})^{\alpha} G^c|$$

$$\overset{\circ}{\mathcal{D}}\psi^{\alpha} = \frac{1}{2} e^b \wedge e^a \psi_{ab}{}^{\alpha}$$

$$\Delta w_{abc} = -\frac{1}{8}\epsilon_{abcd} G^d$$

$$\to -\frac{i}{8}(\psi_{[a}\sigma_{b]}\hat\sigma_c)^{\alpha} G^c| - \frac{i}{8}\psi_{[a}{}^{\alpha} G_{b]}$$

$$\boxed{T_{ab}{}^{\alpha}| = \overset{\circ}{\psi}_{ab}{}^{\alpha} - \frac{i}{8}(\psi_{[a}\sigma_{b]}\hat\sigma_c)^{\alpha} G^c| - \frac{i}{8}\psi_{[a}{}^{\alpha} G_{b]}| - \frac{i}{4}(\bar\psi_{[a}\hat\sigma_{b]})^{\alpha} R|}$$

Leading component of superfield fermionic eq.

Ex: to obtain

$$\boxed{\Psi_{\dot\alpha}{}^{a}| = \epsilon^{abcd} T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha}| = \overset{\circ}{\Psi}_{\dot\alpha}{}^{a} + \frac{1}{2}R(\bar\psi_b\hat\sigma^{ab})_{\dot\alpha}| - \frac{1}{2}G^{[a}(\psi_b\sigma^{b]})_{\dot\alpha} + \frac{i}{8}\epsilon^{abcd}G_b(\psi_c\sigma_d)_{\dot\alpha}}$$

$$\to \Psi_{\dot\alpha}{}^{a}\tilde\sigma_a{}^{\dot\alpha\alpha}| = (\overset{\circ}{\Psi}{}^{a}\tilde\sigma_a)^{\alpha} - \frac{3}{2}R(\bar\psi^a\hat\sigma_a)^{\alpha} + \frac{3}{4}G^a\psi_a{}^{\alpha}$$

COMBINATION WHICH ENTERS $R_{ab}{}^{ab}|$:

Ex: TO OBTAIN

$$\left(\Psi_{\dot\alpha}{}^{a}\bar\psi_a{}^{\dot\alpha} - \frac{1}{\cancel{4}\,2}\Psi^a\hat\sigma_a\sigma_b\bar\psi^b\right)| = \overset{\circ}{\Psi}{}^{a}\bar\psi_a - \frac{1}{2}\overset{\circ}{\Psi}{}^{a}\hat\sigma_a\sigma_b\bar\psi^b + \frac{1}{4}R\bar\psi_a\tilde\sigma^a\sigma^b\bar\psi_b + R\bar\psi_a\bar\psi^a +$$

$$+\frac{1}{2}G^a\psi_{[a}\sigma^b\bar\psi_{b]} - \frac{3}{8}G^a\psi_a\sigma^b\bar\psi_b - \frac{i}{8}\epsilon^{abcd}G_a\psi_b\sigma_c\bar\psi_d$$

$$\to + \frac{1}{4}R\bar\psi_a\tilde\sigma^{ab}\bar\psi_b + \frac{3}{4}R\bar\psi_a\bar\psi^a$$

no contribution to Re part

$$-2\left(\Psi_{\dot\alpha}{}^{a}\bar\psi_a{}^{\dot\alpha} - \frac{1}{2}\Psi^a\hat\sigma_a\sigma_b\bar\psi^b\right)| + c.c. = -2\left(\overset{\circ}{\Psi}{}^{a}\bar\psi_a - \frac{1}{2}\overset{\circ}{\Psi}{}^{a}\hat\sigma_a\sigma_b\bar\psi^b\right) + c.c. -$$

$$-\frac{i}{2}\epsilon^{abcd}\psi_a\sigma_b\bar\psi_c G_d - \frac{1}{2}R(3\bar\psi_a\bar\psi^a + \bar\psi_a\tilde\sigma^{ab}\bar\psi_b) - \frac{1}{2}\bar R(3\psi_a\psi^a + \psi_a\sigma^{ab}\psi_b)$$

Now we are ready to obtain FINAL expressions for THE LEADING COMPONENT OF THE SCALAR CURVATURE SUPERFIELD (see p. 5.I.26.2 — 1 —)

$$R_{ab}{}^{ab}\Big| = \underline{e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}\Big|} + \cancel{\tfrac{1}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R}}\Big|_{2} + \cancel{\tfrac{1}{2}\bar\psi_a\tilde\sigma^{ab}\bar\psi_b\cdot R}\Big|_{3} -$$

$$\Downarrow\quad e_a{}^\mu e_b{}^\nu \overset{0}{R}_{\mu\nu}{}^{ab} + \cancel{\tfrac{i}{2}\psi_a\sigma_b\bar\psi_c\,\epsilon^{abcd}G_d}\Big|_{1} + \tfrac{3}{32}G^aG_a$$

$$\underline{-2\left(\Psi_\alpha{}^a\psi_a{}^\alpha - \tfrac{1}{2}\psi^a\sigma_a\tilde\sigma_b\bar\psi^b\right)\Big| - 2\left(\bar\psi_{\dot\alpha}\bar\Psi^{\dot\alpha a} - \tfrac{1}{2}\psi^b\sigma_b\tilde\sigma_a\bar\Psi^a\right)\Big|}$$

$\Downarrow$ see p. 5.I.26.2 — 2 —

$$-2\left(\overset{0}{\Psi}{}^a\bar\psi_a - \tfrac{1}{2}\overset{0}{\Psi}{}^a\tilde\sigma_a\sigma_b\bar\psi^b\right) - 2\left(\psi_a\overset{0}{\bar\Psi}{}^a - \tfrac{1}{2}\psi^b\sigma_b\tilde\sigma_a\overset{0}{\bar\Psi}\right) -$$

$$+\tfrac{3}{2}G^a\psi_{(a}\sigma^b\bar\psi_{b)} - \cancel{\tfrac{i}{2}\epsilon^{abcd}\psi_a\sigma_b\bar\psi_c G_d}_{1} - \underline{\tfrac{1}{2}R\left(3\bar\psi_a\bar\psi^a + \cancel{\bar\psi_a\tilde\sigma^{ab}\bar\psi_b}_{3}\right) - \tfrac{1}{2}\bar{R}\left(3\psi_a\psi^a + \cancel{\psi_a\sigma^{ab}\psi_b}_{2}\right)}$$

$$\Downarrow\quad = -\tfrac{3}{2}R\bar\psi_a\bar\psi^a - \tfrac{3}{2}\bar{R}\psi_a\psi^a -$$

$$+\cancel{\tfrac{1}{2}R\bar\psi_a\tilde\sigma^{ab}\bar\psi^b}_{3} - \cancel{\tfrac{1}{2}\bar{R}\psi_a\sigma^{ab}\psi_b}_{2} - \cancel{\tfrac{3}{4}R\,\psi_a\sigma^b\tilde\sigma^a\psi_b}$$

$$\boxed{\begin{aligned} R_{ab}{}^{ab}\Big| = {} & e_a{}^\mu e_b{}^\nu \overset{0}{R}_{\mu\nu}{}^{ab} + \tfrac{3}{32}G^aG_a - \\ & -2\overset{0}{\Psi}{}^a\bar\psi_a - 2\overset{0}{\bar\Psi}_a\psi^a + \underline{\overset{0}{\Psi}{}^a\tilde\sigma_a\sigma_b\bar\psi^b + \psi^b\sigma_b\tilde\sigma_a\overset{0}{\bar\Psi}{}^a} - \\ & -\tfrac{3}{2}R\bar\psi_a\bar\psi^a - \tfrac{3}{2}\bar{R}\psi_a\psi^a + \tfrac{3}{2}G^a\psi_{(a}\sigma^b\bar\psi_{b)} \end{aligned}}$$

$$\Downarrow\quad +4i\left(\psi_{ab}\sigma^a\bar\psi^b - \psi^b\sigma^a\bar\psi_{ab}\right)$$

Ex: to obtain using

$$\overset{0}{\Psi}{}_\alpha{}^a := \epsilon^{abcd}\psi_{bc}^\alpha\sigma_{d\alpha\dot\alpha}$$

$$\Downarrow$$

$$\overset{0}{\Psi}_a\tilde\sigma^a\sigma^b\bar\psi_b = 2\Psi_a\bar\psi^a + 4i\psi_{ab}\sigma^a\bar\psi^b$$

Now we are ready to present the COMPLETE EXPRESSION FOR CHIRAL SUPERFIELD OF MIN SG $= R =$ CHIRAL SUPERFIELD DESCRIBING THE FIELD STRENGTHS SUPERMULTIPLET OF MIN SG

$$R|_0 = \text{complex auxiliary field of min SG}$$

$$\mathcal{D}_\alpha R|_0 = \frac{4i}{3}\sigma_{a\alpha\dot\alpha}\Psi^{\dot\alpha a}|_0 = \underbrace{\frac{4i}{3}\sigma_{a\alpha\dot\alpha}\overset{\circ}{\Psi}{}^{\dot\alpha a}}_{-\frac{8}{3}\sigma^{ab}{}_\alpha{}^\beta\psi_{ab\beta}} - i\sigma^a\bar\psi_a{}^\alpha - 2iR(\sigma^a\bar\psi_a)_\alpha$$

$$\mathcal{D}\mathcal{D}R|_0 = -\frac{8}{3}R_{ab}{}^{ab}|_1 + 2R\cdot\bar R - 2i\mathcal{D}_a G^a|_0$$

$$-\frac{8}{3}e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab} + \cancel{\frac{16}{3}\Psi_\alpha^a\bar\psi_a{}^{\dot\alpha}} + \frac{16}{3}\psi_a{}^\alpha\bar\Psi_\alpha{}^a - \frac{4}{3}\psi_a\sigma^{ab}\psi_b\cdot\bar R - \frac{4}{3}\bar\psi_a\bar\sigma^{ab}\bar\psi_b\cdot R + \cancel{-\frac{8}{3}\Psi^a\sigma_a\bar\sigma_b\bar\psi^b} - \frac{3}{8}\psi^b\cancel{\sigma_b}\bar\sigma_a\bar\Psi^a$$

(3) (2) (1)

$$-2ie_a{}^\mu\mathcal{D}_\mu G^a|_0 + 2i\psi_a{}^\alpha\mathcal{D}_\alpha G^a| + 2i\bar\psi_a{}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}G^a|$$

$$-2ie_a{}^\mu\overset{\circ}{\mathcal{D}}_\mu G^a|_0$$

$$4i\left(\Psi_\alpha{}^a - \frac{1}{3}(\sigma^a\bar\sigma^b\Psi_b)^\alpha\right)$$

$$-8\psi_a{}^\alpha\bar\Psi_\alpha{}^a + 8\Psi_{\dot\alpha}{}^a\bar\psi_a{}^{\dot\alpha} + \cancel{\frac{8}{3}\psi_a\sigma^a\bar\sigma^b\bar\Psi_b} - \frac{8}{3}\Psi^b\bar\sigma_b\sigma^a\bar\psi_a$$

$$-\frac{8}{3}\psi_a{}^\alpha\bar\Psi_\alpha{}^a$$

(2) 16 (1)

$$-\frac{2\cdot 16}{3}\Psi_b\bar\psi^b - \frac{64i}{3}\psi_{ab}\sigma^a\bar\psi^b$$

(3)

$$+\frac{8}{3}\Psi_{\dot\alpha}{}^a\bar\psi_a{}^{\dot\alpha}$$

$$\mathcal{D}\mathcal{D}R|_0 = \underbrace{-\frac{8}{3}e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}}_{-\frac{8}{3}e_a{}^\mu e_b{}^\nu\overset{\circ}{R}_{\mu\nu}{}^{ab} - \frac{1}{4}G^a G_a \mp \frac{4i}{3}\epsilon^{abcd}\psi_a\sigma_b\bar\psi_c G_d} + \frac{8}{3}\Psi_{\dot\alpha}{}^a\bar\psi_a{}^{\dot\alpha} - \frac{8}{3}\psi_a{}^\alpha\bar\Psi_\alpha{}^a - \frac{64i}{3}\underbrace{\psi_{ab}}_{\overset{\circ}{\psi}_{ab}{}^\alpha + \frac{1}{8}(\psi_{[a}G_{b]c})^\alpha G^c}\sigma^a\bar\psi^b -$$

still with deformed connection!

$$+2R\cdot\bar R - 2ie_a{}^\mu\mathcal{D}_\mu G^a| - \frac{4}{3}\psi_a\sigma^{ab}\psi_b\cdot\bar R| - \frac{4}{3}\bar\psi_a\bar\sigma^{ab}\bar\psi_b\cdot R|$$

We can now pass to the standard connection.

$$\boxed{\begin{aligned}\mathcal{D}\mathcal{D}R|_0 = &-\frac{8}{3}e_a{}^\mu e_b{}^\nu\overset{\circ}{R}_{\mu\nu}{}^{ab} + \frac{8}{3}\overset{\circ}{\Psi}_{\dot\alpha}{}^a\bar\psi_a{}^{\dot\alpha} - \frac{8}{3}\psi_a{}^\alpha\overset{\circ}{\bar\Psi}_\alpha{}^a - \frac{64i}{3}\overset{\circ}{\psi}_{ab}\sigma^a\bar\psi^b \\ &-\frac{1}{4}G_a G^a + 2R\cdot\bar R - 2ie_a{}^\mu\overset{\circ}{\mathcal{D}}_\mu G^a + \sim\underline{\psi}\underline{\psi}\times\text{aux.}\end{aligned}}$$

Furthermore, taking into account that

$$T_{ab}{}^{\alpha}\Big| = 2\underset{e_a{}^\mu e_b{}^\nu}{\mathring{D}}_{[\mu}\psi_{\nu]}{}^{\alpha} - \frac{i}{8}(\psi_{[a}\sigma_{b]}\hat{\sigma}_c)^{\alpha}G^c - \frac{i}{8}\psi_{[a}{}^{\alpha}G_{b]}\Big| - \frac{i}{4}(\bar\psi_{[a}\hat\sigma_{b]})^{\alpha}R\Big|$$

$$\Downarrow$$

$$\bar\Psi_{\dot\alpha}{}^{a}\Big| = \mathring{\bar\Psi}_{\dot\alpha}{}^{a} + \frac{i}{8}\epsilon^{abcd}(\psi_b\sigma_c)_{\dot\alpha}G_d + \frac{1}{4}(\psi_b\sigma^b)_{\dot\alpha}G^b - \frac{1}{4}(\psi_b\sigma^b)_{\dot\alpha}G^a\Big| + \frac{1}{2}(\bar\psi_b\hat\sigma^{ab})_{\dot\alpha}R\Big|$$

$$\Leftrightarrow (\bar\Psi^b\tilde\sigma_b)^{\alpha}\Big| = (\mathring{\bar\Psi}^b\tilde\sigma_b)^{\alpha} + \frac{3}{4}\psi_b{}^{\alpha}G^b - \frac{3}{2}(\bar\psi_b\tilde\sigma^b)^{\alpha}R$$

<u>We find:</u>

$$(\sigma_b\Psi^b)_\alpha\Big| = (\sigma_b\mathring\Psi^b)_\alpha - \frac{3}{4}\psi_{\alpha b}G^b - \frac{3}{2}(\sigma^b\bar\psi_b)_\alpha R$$

$$\bar{\mathcal{D}}_{\dot\alpha}G^a\Big| = 4i\left(\mathring{\bar\Psi}_{\dot\alpha}{}^{a} - \frac{1}{3}(\mathring{\bar\Psi}^b\tilde\sigma_b\sigma^a)_{\dot\alpha}\right) - i(\psi_b\sigma^b)_{\dot\alpha}G^a - \frac{1}{2}\epsilon^{abcd}(\psi_b\sigma_c)_{\dot\alpha}G_d + 2i\bar\psi_{\dot\alpha}{}^{a}R$$

$$\Downarrow$$

$$\mathcal{D}_\alpha G^a\Big| = 4i\left(\mathring{\Psi}_\alpha{}^{a} - \frac{1}{3}(\sigma^a\tilde\sigma_b\mathring\Psi^b)_\alpha\right) - i(\sigma^b\bar\psi_b)_{\alpha}G^a + \frac{1}{2}\epsilon^{abcd}(\sigma_b\bar\psi_c)_{\alpha}G_d + 2i\psi_\alpha{}^{a}\bar R$$

$$\mathcal{D}_\alpha R\Big| = \frac{4i}{3}\epsilon\sigma_b\bar\Psi^b\Big| = \frac{4i}{3}(\sigma_b\mathring{\bar\Psi}^b)_\alpha - i\psi_{\alpha b}G^b - 2i(\sigma_b\bar\psi^b)\cdot R$$

$$\bar{\mathcal{D}}_{\dot\alpha}\bar R\Big| = +\frac{4i}{3}(\bar\Psi^b\sigma_b)_{\dot\alpha}\Big| = \frac{4i}{3}(\mathring{\bar\Psi}^b\sigma_b)_{\dot\alpha} - i\bar\psi_{\dot\alpha b}G^b - 2i(\psi^b\sigma_b)_{\dot\alpha}\cdot\bar R$$

THIS ALLOWS US TO OBTAIN THE LOCAL SUSY TRANSFORMATIONS OF THE AUXILIARY FIELDS

$$\delta_\epsilon R\Big| = \epsilon^\alpha\mathcal{D}_\alpha R\Big| = \frac{4i}{3}\epsilon\sigma_b\mathring{\bar\Psi}^b - i\epsilon\psi_b\cdot G^b - 2i\epsilon\sigma_b\bar\psi^b\cdot R$$

$$\delta_\epsilon\bar R\Big| = \bar\epsilon^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\bar R\Big| = -\frac{4i}{3}\mathring{\bar\Psi}^b\sigma_b\bar\epsilon + i\bar\psi_b\bar\epsilon\,G^b + 2i\psi^b\sigma_b\bar\epsilon\cdot\bar R$$

$$\delta_\epsilon G_a\Big| = \epsilon^\alpha\mathcal{D}_\alpha G^a\Big| + \bar\epsilon^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}G^a\Big| =$$

$$= -4i\left(\mathring{\bar\Psi}_a\bar\epsilon - \frac{1}{3}\mathring{\bar\Psi}^b\tilde\sigma_b\sigma_a\bar\epsilon\right) + 4i\left(\epsilon\mathring{\Psi}_a - \frac{1}{3}\epsilon\sigma_a\hat\sigma_b\mathring{\Psi}^b\right)$$

$$+ i(\psi_b\sigma^b\bar\epsilon - \epsilon\sigma^b\bar\psi_b)G^a\Big| + \frac{i}{2}\epsilon^{abcd}(\psi_b\sigma_c\bar\epsilon + \epsilon\sigma_b\bar\psi_c)G_d\Big| +$$

$$+ 2i\epsilon\psi^a\cdot\bar R\Big| - 2i\bar\psi^a\bar\epsilon\cdot R\Big|$$