• FIRST LOOK ON THE SUPERFIELD SUGRA ACTION

$$S^{SG} = \sim \frac{1}{k^2}\int d^4x \left(e R_{ab}{}^{ab} + e L^{RS} \right) =$$

$[\ ] = L^{-2}$ $\quad [\ ] = L^4$ $\quad [\ ] = L^{-2}$ ← Length (dim of length)

$$= \sim \frac{1}{k^2}\int d^8z\, E = \sim \frac{1}{k^2}\int d^4x\, \hat{d}^2\theta\, \hat{d}^2\bar{\theta}\quad E$$

$\frac{\partial}{\partial\theta^1}\frac{\partial}{\partial\theta^2}$ $\quad [\ ] = L^{-1}$ $\quad [\ ] = L^{-1}$ $\quad [\ ] = L^0$ DIMENSIONLESS

Wess & Zumino 77 : $\quad E = \mathrm{Sdet}\, E_M{}^A(Z) \equiv \mathrm{Ber}.\, E_M{}^A(Z)$

Berezenian for Felix Berezin

$$\mathrm{Sdet}\begin{pmatrix} M_{bb'} & 0 \\ 0 & M_{ff'} \end{pmatrix} = \frac{\det M_{bb'}}{\det M_{ff'}}$$

or $\quad \mathrm{Sdet}\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} = \frac{\det A}{\det B}$

$$\mathrm{Sdet}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det A / \det(D - CA^{-1}B) = \frac{\det(A - BD^{-1}C)}{\det D}$$

$$\begin{pmatrix} A & 0 \\ C & 1 \end{pmatrix}\begin{pmatrix} 1 & A^{-1}B \\ 0 & D - CA^{-1}B \end{pmatrix}$$

$$\begin{pmatrix} 1 & B \\ 0 & D \end{pmatrix}\begin{pmatrix} A - BD^{-1}C & 0 \\ D^{-1}C & 1 \end{pmatrix}$$

Sdet $E_M{}^A$

$$\boxed{S^{SG} = \sim \int d^8z\, E}$$ ← INV. UNDER SSP DIFFS.

UNDER THE SSP DIFF-S : $\quad \boxed{d^8z' = d^8z\ \mathrm{Sdet}\frac{\partial z'^M}{\partial z^N}}$

Because $\int d\theta^1 \ldots = \frac{\partial}{\partial\theta^1}\ldots$

$$E'^A(z') = dz'^M E'_M{}^A(z') = dz^N E_N{}^A(z) = E^A(z)$$

$$\Rightarrow \quad \boxed{E'(z') = E(z) / \mathrm{Sdet}\frac{\partial z'}{\partial z}}$$

$$S^{SG} = \sim \frac{1}{\kappa^2}\int d^8z\, \overset{\overset{\text{Sdet}\,E_M{}^A}{\ulcorner}}{E}$$

$$\delta S^{SG} = \sim \frac{1}{\kappa^2}\int d^8z\, \delta E = \sim \frac{1}{\kappa^2}\int d^8z\, E\, \underbrace{E_A{}^M (-)^A \delta E_M{}^A}_{\Leftarrow\ \text{Str}\, E^{-1}\delta E}$$

$$\boxed{\ln \underset{\sqcup}{\text{Sdet}}\, M = \underset{\sqcup}{\text{Str}}\, \ln M}$$

BUT $E_M{}^A$ ARE CONSTRAINED VARIABLES:

$T^A = DE^A$ OBEY SOME CONSTRAINTS.

ONE CAN EITHER

- FIND THE VARIATIONS $\delta E_M{}^A$, $\delta\omega_M{}^{ab}$ WHICH PRESERVE CONSTRAINTS AND USE THEM TO VARY THE ACTION [WESS-ZUMINO APPROACH – SEE BELOW] ((WE WILL TURN TO IT LATER))

OR

- TO SOLVE CONSTRAINTS IN TERMS OF UNCONSTRAINED SUPERFIELDS – PREPOTENTIALS.

FOR MINIMAL SG THE MINIMAL SET OF THESE INCLUDES THE Ogievetsky–Sokatchev axial superfield $\mathcal{H}^\Gamma(z)$ and Siegel's compensator $\Phi(z)$ (obeying $\bar{D}_{\dot\alpha}\Phi = 0$)

SOLUTION OF THE MIN SG CONSTRAINTS

$$T_{\alpha\beta}{}^{q} = 0\ , \qquad T_{\alpha\beta}{}^{\dot\gamma} = 0 \quad \Rightarrow$$

$$\Rightarrow \quad \{\mathcal{D}_\alpha, \mathcal{D}_\beta\}\Phi = -T_{\alpha\beta}{}^{\gamma}\nabla_\gamma \Phi$$

$$\mathcal{D}_\alpha \varphi = \nabla_\alpha \varphi = E_\alpha{}^{M}\partial_M \varphi$$

$$\Downarrow$$

$$\boxed{\{\nabla_\alpha, \nabla_\beta\}\Phi = -S_{\alpha\beta}{}^{\gamma}\nabla_\gamma \Phi}$$

Let us denote $\nabla_\alpha = V^{-1}{}_\alpha{}^{\beta}\underbrace{(\partial_\beta + k_\beta{}^{\dot\gamma}\bar\partial_{\dot\gamma} + \chi_\beta{}^{m}\partial_m)}_{\hat\Delta_\beta}$

$$\{\nabla_\alpha, \nabla_\beta\} = V^{-1}{}_\alpha{}^{\alpha'} V^{-1}{}_\beta{}^{\beta'}\{\hat\Delta_{\alpha'}, \hat\Delta_{\beta'}\} + 2\nabla_{(\alpha}V^{-1}{}_{\beta)}{}^{\gamma}\Delta_\gamma = -S_{\alpha\beta}{}^{\gamma}\nabla_\gamma$$

let us apply this to θ^γ: $\quad \{\nabla_\alpha, \nabla_\beta\}\theta^\gamma = 2\nabla_{(\alpha}V^{-1}{}_{\beta)}{}^{\gamma} = -S_{\alpha\beta}{}^{\delta}V^{-1}{}_\delta{}^{\gamma}$

$$\Rightarrow \quad \boxed{S_{\alpha\beta}{}^{\delta} = -2\nabla_{(\alpha}V^{-1}{}_{\beta)}{}^{\gamma}V_\gamma{}^{\delta}}$$

$$\Downarrow \quad V^{-1}{}_\alpha{}^{\alpha'}V^{-1}{}_\beta{}^{\beta'}\{\Delta_{\alpha'}, \Delta_{\beta'}\} + \underbrace{\cancel{2\nabla_{(\alpha}V^{-1}{}_{\beta)}{}^{\gamma}V_\gamma{}^{\delta}\nabla_\delta}}_{= -S_{\alpha\beta}{}^{\delta}\nabla_\delta} = \cancel{-S_{\alpha\beta}{}^{\gamma}\nabla_\gamma}$$

$$\boxed{\{\hat\Delta_\alpha, \hat\Delta_\beta\} = 0}$$

$$\Downarrow \quad \hat\Delta_{(\alpha}k_{\beta)}{}^{\dot\gamma} = 0\ , \qquad \hat\Delta_{(\alpha}\chi_{\beta)}{}^{m} = 0$$

SOLUTION ["Ectoplasm has no topology"]

$$k_\beta{}^{\dot\gamma} = \hat\Delta_\beta h^{\dot\gamma}\ , \qquad \chi_\beta{}^{m} = \hat\Delta_\beta(h^m + i\mathcal{H}^m)$$

$$\boxed{\hat\Delta_\alpha = \partial_\alpha + \hat\Delta_\alpha(h^\mu + i\mathcal{H}^\mu)\partial_\mu + \hat\Delta_\alpha \bar h^{\dot\beta}\bar\partial_{\dot\beta}}$$

$$\hat\Delta_\alpha(h + i\mathcal{H})^\mu =$$

Under superspace gen. coord. transf-s

$$\partial_M = \partial_M Z'^N \partial'_N$$

$$\nabla_\alpha = L^{-1}{}_\alpha{}^{\beta}\nabla_\beta$$

$$\hat\Delta_\alpha = \hat\Delta_\alpha Z'^M \partial'_M = \overbrace{\hat\Delta_\alpha\theta'^\beta}^{=\Omega^{-1}{}_\alpha{}^{\beta}}\hat\Delta'_\beta + (\hat\Delta_\alpha\bar\theta'^{\dot\beta} - \partial_\alpha\theta'^\beta\hat\Delta'_\beta\bar h'^{\dot\beta})\bar\partial'_{\dot\beta} + (\hat\Delta_\alpha x'^\mu - \hat\Delta_\alpha(h'^\mu + i\mathcal{H}'^\mu))\partial'_\mu$$

$$= \Omega^{-1}{}_\alpha{}^{\beta}\hat\Delta'_\beta$$

$\uparrow$

as constraints are solved in an arbitrary frame

WRITING THE SSP G.C. TR. AS

$$x'^{\mu} = x^{\mu} + \Lambda^{\mu}(x,\theta,\bar\theta)$$

$$\theta'^{\alpha} = \theta^{\alpha} + \Lambda^{\alpha}(x,\theta,\bar\theta) \quad , \quad \bar\theta'^{\dot\alpha} = \bar\theta^{\dot\alpha} + \bar\Lambda^{\dot\alpha}(x,\theta,\bar\theta)$$

we find from $\hat\Delta_\alpha = \Omega^{-1}{}_\alpha{}^\beta \hat\Delta'_\beta = \Delta_\alpha \theta'^\beta \hat\Delta_\beta$

h^μ, h^α are pure gauge superfields $\Leftarrow$

$$\begin{cases} h'^{\mu}(z') = h^{\mu}(z) + \Lambda^{\mu} - \frac{1}{2}(\eta^{\mu} + \bar\eta^{\mu}) \\ \bar h'^{\dot\beta}(z') = \bar h^{\dot\beta}(z) + \bar\Lambda^{\dot\beta} - \bar\eta^{\dot\beta} \\ h'^{\beta}(z') = h^{\beta}(z) + \Lambda^{\beta} - \eta^{\beta} \end{cases}$$

c.c.

$$\boxed{\Delta_\alpha \bar\eta^\mu = 0 = \bar\Delta_{\dot\alpha} \eta^\mu \\ \Delta_\alpha \bar\eta^{\dot\beta} = 0 = \bar\Delta_{\dot\alpha} \eta^\beta}$$

$$\boxed{\mathcal{H}'^{\mu}(z') = \mathcal{H}^{\mu}(z) + \frac{1}{2i}(\eta^{\mu} - \bar\eta^{\mu})}$$

$$\boxed{\bar\Delta_{\dot\alpha} \eta^\mu = 0} \; , \; \boxed{\Delta_\alpha \bar\eta^\mu = 0}$$

ONLY $\mathcal{H}^\mu(z)$ is NOT PURE GAUGE (NOT Stückelberg) superfield

$$\boxed{\hat\Delta'_\alpha = \Omega_\alpha{}^\beta \hat\Delta_\beta} \qquad \Omega^{-1}{}_\alpha{}^\beta = \delta_\alpha{}^\beta + \hat\Delta_\alpha \Lambda^\beta$$

ONE CAN FIX THE WZ[(1)] gauge where

$$\boxed{\mathcal{H}^{\mu}(z) = \theta\sigma^a\bar\theta\, \tilde{e}_a{}^{\mu}(x) + \theta\theta\bar\theta_{\dot\alpha} \tilde{\bar\psi}^{\dot\alpha\mu} + \bar\theta\bar\theta\, \theta^\alpha \tilde\psi_\alpha{}^\mu + \theta\theta\bar\theta\bar\theta \tilde{A}^\mu}$$

TO MAKE IT COINCIDENT WITH THE WZ gauge of the "potential approach" we need to gauge

$$\boxed{h^{\mu} = -\frac{i}{3}\theta\theta\bar\theta\tilde{\bar\psi}^{\mu} + \frac{i}{3}\bar\theta\bar\theta\,\theta\tilde\psi^{\mu} + \frac{1}{4}\theta\theta\bar\theta\bar\theta\; \hat{e}_a{}^{\nu}\partial_\nu \tilde{e}^{a\mu}}$$

(but not $h^\mu = 0$)

and vanishing $\boxed{h^\alpha = 0}$, $\boxed{\bar h^{\dot\alpha} = 0}$

If we fix the gauge $\bar{h}^{\dot\alpha}=0$, $h^\mu=0$

$$\nabla_\alpha = V^{-1}{}_\alpha{}^\beta \Delta_\beta \,, \qquad \Delta_\beta = \partial_\beta + i\Delta_\alpha \mathcal{H}^\mu \partial_\mu$$

Ogievetskij – Sokatchev. $\Delta_\alpha \mathcal{H}^\mu = \partial_\alpha \mathcal{H}^\nu (\delta - i\partial \mathcal{H})^{-1}_\nu{}^\mu$

But to use the "potential" form of the WZ gauge

$\theta^{\dot\alpha} E_{\dot\alpha}{}^A = \theta^{\dot\alpha} \delta_{\dot\alpha}{}^A$.

we need to keep a nonzero h^m, but can set $\bar{h}^{\dot\alpha}=0$

$$\nabla_\alpha = V^{-1}{}_\alpha{}^\beta \hat\Delta_\beta \,,$$

$$\boxed{\hat\Delta_\beta = \partial_\beta + \hat\Delta_\beta (h^\mu + i\mathcal{H}^\mu)\partial_\mu = \partial_\beta + \partial_\beta (h+i\mathcal{H})^\nu (\delta - \partial(h+i\mathcal{H}))^{-1}_\nu{}^\mu}$$

$$\hat{\bar\Delta}_{\dot\beta} = \bar\partial_{\dot\beta} + \hat{\bar\Delta}_{\dot\beta} (h^\mu - i\mathcal{H}^\mu)\partial_\mu = \bar\partial_{\dot\beta} + \bar\partial_{\dot\beta} (h-i\mathcal{H})^\nu (\delta - \partial(h-i\mathcal{H}))^{-1}_\nu{}^\mu$$

AS FAR AS $\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = 2i\sigma^a_{\alpha\dot\alpha}\mathcal{D}_a$ WHICH IMPLIES

$$\{\nabla_\alpha, \bar\nabla_{\dot\beta}\} + \omega_{\alpha\dot\beta}{}^{\dot\gamma}\bar\nabla_{\dot\gamma} + \omega_{\dot\beta\alpha}{}^\gamma \nabla_\gamma = 2i\sigma^a_{\alpha\dot\beta}\nabla_a$$

$$\{\nabla_\alpha, \bar\nabla_{\dot\beta}\} = V^{-1}{}_\alpha{}^\gamma \bar V^{-1}{}_{\dot\beta}{}^{\dot\delta} \underbrace{\{\hat\Delta_\gamma, \hat{\bar\Delta}_{\dot\delta}\}}_{2i\sigma^a_{\gamma\dot\delta}\hat\Delta_a} + \nabla_\alpha \bar V^{-1}{}_{\dot\beta}{}^{\dot\gamma}\hat{\bar\Delta}_{\dot\gamma} + \bar\nabla_{\dot\beta} V^{-1}{}_\alpha{}^\gamma \hat\Delta_\gamma$$

We find $\qquad \omega_{\alpha\dot\beta}{}^{\dot\gamma} = ?$

$$\sigma^a_{\alpha\dot\alpha}\nabla_a = V^{-1}{}_\alpha{}^\beta \bar V^{-1}{}_{\dot\alpha}{}^{\dot\beta}\sigma^a_{\beta\dot\beta}\hat\Delta_a + \sim\nabla_\gamma + \sim\bar\nabla_{\dot\gamma}$$

$$\sigma^a_{\alpha\dot\alpha}\hat\Delta_a = -\frac{i}{2}\{\hat\Delta_\alpha, \hat{\bar\Delta}_{\dot\alpha}\}$$

$$\boxed{\hat\Delta_a = \frac{i}{4}\bar\sigma_a^{\dot\alpha\alpha}[\hat\Delta_\alpha, \hat{\bar\Delta}_{\dot\alpha}]\mathcal{H}^\mu\partial_\mu + \hat\Delta_a h^\mu\partial_\mu \equiv \hat C_a{}^\mu\partial_\mu + \hat\Delta_a h^\mu\partial_\mu}$$

$$\boxed{\hat C_a{}^\mu = \frac{\bar\sigma_a^{\dot\alpha\alpha}}{4}[\hat\Delta_\alpha, \hat{\bar\Delta}_{\dot\alpha}]\mathcal{H}^\mu}$$

We will be back to $E_a{}^M$ calculations below flat: $x^\mu_L = x^\mu + i\theta\sigma^\mu\bar\theta$

curved, WZ gauge $\mathcal{H}^\mu = \theta\sigma^a\bar\theta\,\tilde{e}_a{}^\mu + \theta\theta\bar\theta_{\dot\alpha}\hat{\bar\varphi}^{\dot\alpha\mu} + \text{c.c.} + \theta\theta\bar\theta\bar\theta\,\hat{A}^\mu$

$h^\mu =$

$$\begin{cases} x^\mu_L = x^\mu + (h^\mu + i\mathcal{H}^\mu) \\ \theta^\alpha_L = \theta^\alpha \\ \bar\theta^{\dot\alpha}_L = \bar\theta^{\dot\alpha} \end{cases}$$

$x^{\mu\prime}_L = x^\mu_L + \lambda^\mu$

$\theta^{\alpha\prime}_L = \lambda^\alpha$

$\bar\theta^{\dot\alpha}_L = \bar\lambda^{\dot\alpha}$

$$\boxed{\hat{\bar\Delta}_{\dot\beta}\lambda^\alpha = 0 = \hat{\bar\Delta}_{\dot\beta}\lambda^\mu}$$

$h^\alpha = 0$, (but not obligatory $\bar h^{\dot\alpha}$!)

$$dx^\mu_L\partial^L_\mu + d\theta^\alpha_L\partial^L_\alpha + d\bar\theta^{\dot\alpha}\bar\partial^L_{\dot\alpha} = dx^\mu\partial_\mu + d\theta^\alpha\partial_\alpha + d\bar\theta^{\dot\alpha}\bar\partial_{\dot\alpha}$$

$$\partial^L_\mu = (\delta - \partial(h - i\mathcal{H}))^{-1}{}_\mu{}^\nu\partial_\nu$$

$$\boxed{\bar\partial^L_{\dot\alpha} = \hat{\bar\Delta}_{\dot\alpha}} = \bar\partial_{\dot\alpha} + \bar\partial_{\dot\alpha}(h - i\mathcal{H})^\nu(\delta - \partial(h - i\mathcal{H}))^{-1}{}_\nu{}^\mu\partial_\mu$$

$\hat{\bar\Delta}_{\dot\beta}\lambda^\alpha = 0 = \hat{\bar\Delta}_{\dot\beta}\lambda^\mu \Rightarrow$ $\left.\begin{matrix}\bar\partial^L_{\dot\beta}\lambda^\alpha = 0 \\ \bar\partial^L_{\dot\beta}\lambda^\mu = 0\end{matrix}\right\} \Rightarrow$ $\lambda^\alpha = \lambda^\alpha(x^\mu_L, \theta^\alpha)$

$\lambda^\mu = \lambda^\mu(x^\mu_L, \theta^\alpha)$

Inv. chiral SSP

$\zeta^M = (x^\mu_L, \theta^\alpha)$

$\delta x^\mu_L := x^{\mu\prime}_L - x^\mu_L = \lambda^\mu(\zeta_L)$

$\delta\theta^\alpha := \theta^{\alpha\prime} - \theta^\alpha = \lambda^\alpha(\zeta_L)$

$x^\mu_L + x^\mu_R = x^\mu\,2h^\mu$

$x^\mu_L - i x^\mu_R = 2i\mathcal{H}^\mu$

axial gravitational superfield by Ogievetski & Sokatchev.

Embedding of $\mathbb{M}^{(4|4)}$ into $\mathbb{C}^{(4,2)}$

$$\boxed{\hat\Delta_\alpha = \partial_\alpha^R = \underbrace{\partial_\alpha}_{} + \underbrace{\hat\Delta_\alpha (h+i\mathcal{H})^\mu}_{} \underbrace{\partial_\mu}_{} =}$$

$$\partial_\alpha^L - \partial_\alpha (h - i\mathcal{H})^\mu \partial_\mu^L \qquad \left(\delta - \partial(h-i\mathcal{H})\right)_\mu{}^\nu \partial_\nu^L$$

$$\partial_\alpha (h+i\mathcal{H})^\nu \left(\delta - \partial(h+i\mathcal{H})\right)^{-1}_\nu{}^\mu$$

$$\partial_\alpha (h+i\mathcal{H})^\mu \partial_\mu^L + 2i\hat\Delta_\alpha (h+i\mathcal{H})^\nu \partial\mathcal{H}^\mu \partial_\mu^L \qquad \partial_\nu^L + 2i\left(\delta - \partial(h+i\mathcal{H})\right)_\nu^{-1\,\rho} \partial_\rho \mathcal{H}^\mu \partial_\mu^L$$

$$2i\partial_\alpha \mathcal{H}^\mu \partial_\mu^L \qquad 2i\Delta_\alpha \mathcal{H}^\mu \partial_\mu^L$$

$$\boxed{\hat\Delta_\alpha = \partial_\alpha^L + 2i\hat\Delta_\alpha \mathcal{H}^\mu \partial_\mu^L} = \partial_\alpha^R \qquad \boxed{\hat{\bar\Delta}_{\dot\alpha} = \bar\partial_{\dot\alpha}^R - 2i\hat{\bar\Delta}_{\dot\alpha} \mathcal{H}^\mu \partial_\mu^R} = \bar\partial_{\dot\alpha}^L$$

Ex: To obtain

Flat

$$D_\alpha = \partial_\alpha + i(\sigma^a\bar\theta)_\alpha \partial_a = \partial_\alpha^L + 2i(\sigma^a\bar\theta)_\alpha \partial_a$$

Notice disappearence of h^μ!

$$\hat\Delta_a = -\frac{i}{4}\tilde\sigma_a^{\dot\alpha\alpha}\{\hat\Delta_\alpha, \hat{\bar\Delta}_{\dot\alpha}\} = \frac{1}{2}\tilde\sigma_a^{\dot\alpha\alpha}\hat{\bar\Delta}_{\dot\alpha}\hat\Delta_\alpha \mathcal{H}^\mu \partial_\mu^L =: \hat e_a{}^\mu \partial_\mu^L$$

$$\bar\partial_{\dot\alpha}^L$$

$$\partial_\alpha^L + 2i\hat\Delta_\alpha \mathcal{H}^\mu \partial_\mu^L$$

$$= \frac{1}{2}\tilde\sigma_a^{\dot\alpha\alpha}\bar\partial_{\dot\alpha}^L \hat\Delta_\alpha \mathcal{H}^\mu \cdot \partial_\mu^L$$

$$\boxed{\hat\Delta_a = \hat e_a{}^\mu \partial_\mu^L}, \qquad \hat e_a{}^\mu = \frac{1}{2}\tilde\sigma_a^{\dot\alpha\alpha}\hat e^\mu_{\alpha\dot\alpha}, \qquad \boxed{\hat e_{\alpha\dot\alpha}{}^\mu = \hat{\bar\Delta}_{\dot\alpha}\hat\Delta_\alpha \mathcal{H}^\mu}$$

As $\quad \nabla_\alpha = V^{-L}_\alpha{}^\beta \hat\Delta_\beta, \qquad \bar\nabla_{\dot\alpha} = \bar V^{-L}_{\dot\alpha}{}^{\dot\beta}\hat{\bar\Delta}_{\dot\beta}$

$$\boxed{\hat\Delta_a = \hat\tau_a{}^\mu \partial_\mu^R} \qquad \hat\tau_a{}^\mu = \frac{1}{2}\tilde\sigma_a^{\dot\alpha\alpha}\hat\tau^\mu_{\alpha\dot\alpha} \qquad \boxed{\hat\tau_{\alpha\dot\alpha}{}^\mu = -\hat\Delta_\alpha \hat{\bar\Delta}_{\dot\alpha}\mathcal{H}^\mu}$$

THESE REPRESENTATIONS ARE CONVENIENT TO CALCULATE THE ALGEBRA

$$[\hat\Delta_a, \hat\Delta_\alpha] = -(\hat\Delta_\alpha \hat\tau_a{}^\mu \cdot \hat\tau_\mu{}^b)\hat\Delta_b$$

$$\hat\tau_a{}^\mu \partial_\mu^R \qquad \partial_\alpha^R \qquad \partial_\alpha^R \tau_a{}^\mu \partial_\mu^R$$

$$[\hat\Delta_a, \hat{\bar\Delta}_{\dot\alpha}] = -(\hat{\bar\Delta}_{\dot\alpha}\hat e_a{}^\mu \cdot e_\mu{}^b)\hat\Delta_b$$

Fix the gauge (by SSP Lorentz transf-s)

$$\nabla_\alpha = F\hat{\Delta}_\alpha , \qquad \bar{\nabla}_{\dot\alpha} = \bar{F}\hat{\bar{\Delta}}_{\dot\alpha} .$$

$$\{\nabla_\alpha, \nabla_\beta\} = -S_{\alpha\beta}{}^{\gamma}\nabla_\gamma = 2\nabla_{(\alpha}F \cdot F^{-1}\nabla_{\beta)}$$

$$E_\alpha{}^M = F\delta_\alpha{}^M + F\hat{\Delta}_\alpha (h+iH)^\mu \delta_\mu{}^M$$

$$\underbrace{\mathcal{D}_\alpha \nabla_\beta + \mathcal{D}_\beta \nabla_\alpha}_{\{\nabla_\alpha,\nabla_\beta\} + 2\omega_{(\alpha\beta)}{}^{\gamma}\nabla_\gamma} = -\underbrace{T_{\alpha\beta}{}^{A}}_{=0}\nabla_A = 0 \quad \text{for min SUGRA}$$

$$\{\nabla_\alpha,\nabla_\beta\} = 2\hat{\Delta}_{(\alpha}F \cdot \nabla_{\beta)}$$

$$\Rightarrow \quad \omega_{(\alpha\beta)}{}^{\gamma} = -\hat{\Delta}_{(\alpha}F\delta_{\beta)}{}^{\gamma}$$

$$\Downarrow$$

$$\boxed{\omega_{\alpha\beta}{}^{\gamma} = +\epsilon_{\alpha\beta}\hat{\Delta}^{\gamma}F - \hat{\Delta}_\beta F\,\delta_\alpha{}^{\gamma}} \iff \boxed{\omega_{\alpha\beta\gamma} = +2\epsilon_{\alpha(\beta}\hat{\Delta}_{\gamma)}F}$$

$$T_{\gamma\delta}{}^{A} = 0 \qquad \text{min SG}$$

$$R_{\gamma\delta\beta}{}^{\alpha} = 2\underbrace{\nabla_{(\gamma}\omega_{\delta)\beta}{}^{\alpha}}_{F\hat{\Delta}_{(\gamma}} + 2\omega_{(\gamma|\beta}{}^{\varepsilon}\omega_{|\delta)\varepsilon}{}^{\alpha} + \underbrace{S_{\gamma\delta}{}^{\varepsilon}\omega_{\varepsilon\beta}{}^{\alpha}}_{-2\nabla_{(\gamma}F \cdot F^{-1}\omega_{\delta)\beta}{}^{\alpha}} = -\epsilon_{\beta(\gamma}\delta_{\delta)}{}^{\alpha}\bar{R}$$

$$\Downarrow$$

$$\boxed{\bar{R} = -2F\hat{\Delta}^{\alpha}\hat{\Delta}_{\alpha}F - 2\hat{\Delta}^{\alpha}F \cdot \hat{\Delta}_\alpha F}$$

$$= -F^2\hat{\Delta}\hat{\Delta}\ln F^2 - F^2\hat{\Delta}^{\varepsilon}\ln F^2\,\hat{\Delta}_\varepsilon \ln F^2$$

Ex: To show that $\forall\, \upsilon$

$$\boxed{(\mathcal{D}^\alpha\mathcal{D}_\alpha - \bar{R})\upsilon = \hat{\Delta}^\alpha\hat{\Delta}_\alpha(F^2\upsilon)}$$

$$\mathcal{D}_\alpha\bar{\nabla}_{\dot\alpha} + \bar{\mathcal{D}}_{\dot\alpha}\nabla_\alpha = \underbrace{\{\nabla_\alpha,\bar{\nabla}_{\dot\alpha}\}}_{\{F\hat{\Delta}_\alpha,\bar{F}\hat{\bar{\Delta}}_{\dot\alpha}\}} + \omega_{\alpha\dot\alpha}{}^{\dot\beta}\underbrace{\bar{\nabla}_{\dot\beta}}_{\bar{F}\bar{\Delta}_{\dot\beta}} + \omega_{\dot\alpha\alpha}{}^{\beta}\underbrace{\nabla_\beta}_{F\Delta_\beta} = 2i\delta_{\alpha\dot\alpha}^{a}\nabla_a - \cancel{T_{\alpha\dot\alpha}{}^{\beta}\nabla_\beta}$$

$$\boxed{\nabla_{\alpha\dot\alpha} = F\bar{F}\hat{\Delta}_{\alpha\dot\alpha} - \frac{i}{2}\left(\bar{F}\omega_{\alpha\dot\alpha}{}^{\dot\beta} + F\hat{\Delta}_\alpha\bar{F}\delta_{\dot\alpha}{}^{\dot\beta}\right)\hat{\bar{\Delta}}_{\dot\beta} - \frac{i}{2}\left(F\omega_{\dot\alpha\alpha}{}^{\beta} + \bar{F}\hat{\bar{\Delta}}_{\dot\alpha}F\delta_\alpha{}^{\beta}\right)\hat{\Delta}_\beta}$$

$$= F\bar{F}\hat{\Delta}_{\alpha\dot\alpha} + k_{\alpha\dot\alpha}{}^{\dot\beta}\hat{\bar{\Delta}}_{\dot\beta} + k_{\alpha\dot\alpha}{}^{\beta}\hat{\Delta}_\beta$$

$$0 = R_{\dot\gamma\dot\delta\beta}{}^{\alpha} = 2\underbrace{\bar{\nabla}_{(\dot\gamma}\omega_{\dot\delta)\beta}{}^{\alpha}}_{2\bar{F}\bar{\Delta}_{(\dot\gamma}\omega_{\dot\delta)\beta}{}^{\alpha}} + 2\omega_{(\dot\gamma|\beta}{}^{\varepsilon}\omega_{|\dot\delta)\varepsilon}{}^{\alpha} + \underbrace{S_{\dot\gamma\dot\delta}{}^{\dot\varepsilon}\omega_{\dot\varepsilon\beta}{}^{\alpha}}_{-2\bar{\Delta}_{(\dot\gamma}\bar{F}\cdot\omega_{\dot\delta)\beta}{}^{\alpha}} \Rightarrow \boxed{\omega_{\dot\gamma\beta}{}^{\alpha} = u_\beta^{-1\varepsilon}\bar{\nabla}_{\dot\gamma}u_\varepsilon{}^{\alpha}}$$

$$0 = R_{\gamma\dot\delta\beta}{}^{\alpha} \Rightarrow \boxed{W_{\gamma\dot\gamma}{}^{\alpha\beta} = -\frac{i}{2}\left(\nabla_\gamma W_{\dot\gamma}{}^{\alpha\beta} + \bar{\nabla}_{\dot\gamma}W_\gamma{}^{\alpha\beta} + W_\gamma{}^{\alpha\varepsilon}W_{\dot\gamma\varepsilon}{}^{\beta} + W_{\dot\gamma}{}^{\alpha\varepsilon}W_{\gamma\varepsilon}{}^{\beta}\right)}$$

THIRD CLASS CONSTRAINT OF MINIMAL SG, $T_{\alpha b}{}^{b}=0$ ($\Rightarrow T_{\dot\alpha b}{}^{a}=0$)

DETERMINES THE FORM OF $F, \bar F$

Let us calculate $T_{\alpha b}{}^{a}=-[\nabla_\alpha,\nabla_b]^a - W_{\alpha b}{}^{a}$, or better its trace

$$T_{\alpha a}{}^{a}=-[\nabla_\alpha,\nabla_a]^a=[\nabla_a,\nabla_\alpha]^a$$

$$[\nabla_a,\nabla_\alpha]=[F\bar F\hat\Delta_a+k_a{}^{\alpha}\hat\Delta_\alpha+\underbrace{k_a{}^{\dot\alpha}}_{-\frac14\bar F\tilde\sigma_a{}^{\dot\beta\beta}\omega_{\beta\dot\beta}{}^{\dot\alpha}-\frac14\tilde\sigma_a{}^{\dot\alpha\alpha}\nabla_\alpha\bar F}\hat{\bar\Delta}_{\dot\alpha},\ F\hat\Delta_\alpha]=$$

$$=-(\nabla_\alpha F\cdot\bar F+\nabla_\alpha\bar F\cdot F)\hat\Delta_a+F^2\bar F\underbrace{[\hat\Delta_a,\hat\Delta_\alpha]}_{\text{see p. 5: }-\hat\Delta_\alpha\hat\varepsilon_a{}^{\mu}\cdot\hat\varepsilon_\mu{}^{b}\cdot\hat\Delta_b}+Fk_a{}^{\dot\alpha}\underbrace{\{\hat{\bar\Delta}_{\dot\alpha},\hat\Delta_\alpha\}}_{2i\sigma^{b}_{\alpha\dot\alpha}\hat\Delta_b}+\sim\hat\Delta_\beta,\hat{\bar\Delta}_{\dot\beta}$$

$$[\nabla_a,\nabla_\alpha]^b=-(\nabla_\alpha\ln F+\nabla_\alpha\ln\bar F)\delta_a^b-\nabla_\alpha\hat\varepsilon_a{}^{\mu}\hat\varepsilon_\mu{}^{b}+\frac{2i}{F\bar F}F\underbrace{k_a{}^{\dot\alpha}}_{-\frac14\tilde\sigma_a{}^{\dot\alpha\gamma}\nabla_\gamma\bar F-\frac14\tilde\sigma_a{}^{\dot\beta\gamma}\omega_{\gamma\dot\beta}{}^{\dot\alpha}\bar F}\sigma^{b}_{\alpha\dot\alpha}$$

$$T_{\alpha a}{}^{a}=[\nabla_a,\nabla_\alpha]^a=-4(\nabla_\alpha\ln F+\nabla_\alpha\ln\bar F)-\nabla_\alpha\hat\varepsilon_a{}^{\mu}\hat\varepsilon_\mu{}^{a}+\underbrace{\frac12(\sigma^a\tilde\sigma_a)_\alpha{}^{\gamma}\nabla_\gamma\bar F/\bar F}_{2\nabla_\alpha\ln\bar F}+\emptyset=$$

$$=-\nabla_\alpha\ln F^4\bar F^2\hat\varepsilon$$

$$\boxed{T_{\alpha b}{}^{b}=-\nabla_\alpha\ln F^4\bar F^2\hat\varepsilon}\qquad\boxed{\hat\varepsilon=\det\hat\varepsilon_a{}^{\mu}}\qquad\boxed{\hat\varepsilon_a{}^{\mu}=-\frac12\tilde\sigma_a{}^{\dot\alpha\alpha}\{\hat\Delta_\alpha,\hat{\bar\Delta}_{\dot\alpha}\}}$$

SOLUTION:

$$\boxed{\hat\varepsilon F^4\bar F^2=\bar\Phi^{-3}}\qquad \nabla_\alpha\bar\Phi=0$$

$$\boxed{\hat{\bar\varepsilon} F^2\bar F^4=\Phi^{-3}}\qquad \bar\nabla_{\dot\alpha}\Phi=0$$

SIEGEL'S CHIRAL COMPENSATORS: $\Phi(\zeta_L)$ and $\bar\Phi(\bar\zeta_R)=(\Phi(\zeta_R))^*$

FORMALLY, CAN BE REMOVED BY GENERAL COORD TRANSF-S OF $\mathbb{C}^{(4|2)}$ SUPERSPACE

$$\boxed{\begin{aligned}F^2&=\hat{\bar\varepsilon}^{1/3}\hat\varepsilon^{-2/3}\Phi\bar\Phi^{-2},\\ \bar F^2&=\hat{\bar\varepsilon}^{-2/3}\hat\varepsilon^{1/3}\Phi^{-2}\bar\Phi.\end{aligned}}$$

$$E_\alpha{}^M \partial_M = \nabla_\alpha = F \hat{\Delta}_\alpha = F\,\partial_\alpha{}^R = F\left(\partial_\alpha{}^L + 2i\hat{\Delta}_\alpha \mathcal{H}^\mu \partial_\mu{}^L\right) = \mathcal{L}_\alpha{}^M \partial_M{}^L = \mathcal{R}_\alpha{}^M \partial_M{}^R$$

$$\bar{E}_{\dot\alpha}{}^M \partial_M = \bar\nabla_{\dot\alpha} = \bar F \bar{\hat\Delta}_{\dot\alpha} = \bar F\left(\bar\partial_{\dot\alpha}{}^R - 2i\bar{\hat\Delta}_{\dot\alpha}\mathcal{H}^\mu\partial_\mu{}^R\right) = \bar F\,\bar\partial_{\dot\alpha}{}^L = \mathcal{L}_{\dot\alpha}{}^M \partial_M{}^L = \mathcal{R}_{\dot\alpha}{}^M \partial_M{}^R$$

$$E_a{}^M \partial_M = \nabla_a = \underbrace{F\bar F \hat\Delta_a}_{\hat e_a{}^\mu \partial_\mu{}^L = \hat e_a{}^\mu \partial_\mu{}^R} + \underbrace{F\chi_a{}^\beta}_{-\frac{i}{4}\bar\sigma_a{}^{\dot\alpha\alpha}(\bar F \omega_{\dot\alpha\alpha}{}^\beta + \bar F \bar{\hat\Delta}_{\dot\alpha} F \delta_\alpha{}^\beta)} \hat\Delta_\beta + \underbrace{\bar F \bar\chi_a{}^{\dot\beta}}_{-\frac{i}{4}\bar\sigma_a{}^{\dot\alpha\alpha}(\bar F \omega_{\alpha\dot\alpha}{}^{\dot\beta} + F\hat\Delta_\alpha \bar F \delta_{\dot\alpha}{}^{\dot\beta})} \bar{\hat\Delta}_{\dot\beta} = \mathcal{L}_a{}^M \partial_M{}^L = \mathcal{R}_a{}^M \partial_M{}^R$$

$$E^{-1} = \operatorname{Ber} E_A{}^M = \operatorname{Ber}\mathcal{L}_A{}^M \Big/ \operatorname{Ber}\frac{\partial z_L}{\partial z} = \mathcal{L}^{-1} \Big/ \det(\delta - \chi(h - i\mathcal{H}))$$

$$\mathcal{L}^{-1} = \operatorname{Ber}\mathcal{L}_A{}^M = \operatorname{Ber}\begin{pmatrix} F\bar F\,\hat e_a{}^\mu + 2iF\chi_a{}^\beta\hat\Delta_\beta \mathcal{H}^\mu & F\chi_a{}^\beta & \bar F\bar\chi_a{}^{\dot\beta} \\ 2iF\hat\Delta_\alpha \mathcal{H}^\mu & F\delta_\alpha{}^\beta & 0 \\ 0 & 0 & \bar F \bar\chi_{\dot\alpha}{}^{\dot\beta} \end{pmatrix} \begin{matrix} \\ -\chi_a{}^\beta \\ -\bar\chi_a{}^{\dot\beta} \end{matrix}$$

$$= \operatorname{Ber}\begin{pmatrix} F\bar F\,\hat e_a{}^\mu & 0 & 0 \\ 0 & F\delta_\alpha{}^\beta & 0 \\ 0 & 0 & \bar F\delta_{\dot\alpha}{}^{\dot\beta} \end{pmatrix} = \hat e\, F^2 \bar F^2$$

$$E = \hat e^{-1} F^{-2} \bar F^{-2} \cdot \det\frac{\partial x_L}{\partial x} = \hat e^{-1} F^{-2}\bar F^{-2} \operatorname{Ber}\frac{\partial z_L}{\partial z}$$

Thus the Wess–Zumino action for SUGRA is

$$\boxed{\int d^8 z\, E = \int d^8 z_L\, \hat e^{-1} F^{-2} \bar F^{-2} = \int d^8 z_R\, \hat e^{-1} F^{-2} \bar F^{-2}}$$

WHERE F^2 AND $\bar F^2$ ARE DETERMINED BY THE 3RD CLASS CONSTRAINTS

IN THE CASE OF MIN SG THESE ARE

$$T_\alpha = -\nabla_\alpha \ln \hat e F^4 \bar F^2 = 0$$

$$\Rightarrow \quad F^2 = \hat e^{1/3}\,\hat{\bar e}^{-2/3}\,\varphi\bar\varphi^{-2}, \qquad \bar F^2 = \hat e^{-2/3}\,\hat{\bar e}^{1/3}\,\varphi^{-2}\bar\varphi$$

$$\Rightarrow \quad F^2 \bar F^2 = \hat e^{-1/3}\,\hat{\bar e}^{-1/3}\,\varphi^{-1}\bar\varphi^{-1}$$

$$\boxed{\hat e^{-1} F^{-2} \bar F^{-2} = \hat e^{-2/3}\,\hat{\bar e}^{1/3}\,\varphi\bar\varphi}$$

MIN. SG.

$$\boxed{\int d^8 z\, E = \int d^8 z_L\, \hat e^{-2/3}\,\hat{\bar e}^{1/3}\,\varphi\bar\varphi = \int d^8 z_R\, \hat e^{1/3}\,\hat{\bar e}^{-2/3}\,\varphi\bar\varphi}$$

"FIRST ORDER FORM" OF THE MIN. SG ACTION

[V. Zima, JETP Lett. 40(1984) 928-930]
(2 citations for 2014...)
BUT WILL BE VERY USEFUL (not here)

Let us consider F AS INDEPENDENT SUPERFIELD AND WRITE THE ACTION

$$\boxed{S'(n=-1/3) = \frac{1}{3\kappa^2}\left(\underbrace{\int d^8 z_L\, \hat{\ell}^{-1} F^{-2} \bar{F}^{-2}} + \int d^8 z_L\, \bar{F}^2 \Phi^3 + \int d^8 z_R\, F^2 \bar{\Phi}^3\right)}$$

$$\int d^8 z_L\, \hat{\ell}^{-1} F^{-2} \bar{F}^{-2} = \int d^8 z_R\, \hat{r}^{-1} F^{-2} \bar{F}^{-2} = \int d^8 z\, \tilde{E}$$

$\tilde{E}$ ← superviel fach superdeterminant of SUGRA without 3d class constraints

$$\delta F^2 \Rightarrow \boxed{\hat{r}\bar{F}^2 F^4 = \bar{\Phi}^{-3}} \quad \text{or} \quad \hat{r} F^2 \bar{F}^2 = F^{-2}\bar{\Phi}^{-3}$$

$$\text{or} \quad \hat{r}^{-1} F^{-2} \bar{F}^{-2} = F^2 \bar{\Phi}^3$$

$$\delta \bar{F}^2 \Rightarrow \boxed{\hat{\ell} F^2 \bar{F}^4 = \Phi^{-3}} \quad \text{or} \quad \hat{\ell} F^2 \bar{F}^2 = \bar{F}^{-2}\Phi^{-3}$$

$$\boxed{S(n=-1/3)\Big|_{\delta S/\delta F^2=0} = \frac{1}{\kappa^2}\int d^8 z_L\, \hat{\ell}^{-2/3}\hat{r}^{1/3}\Phi\bar{\Phi} = \int d^8 z_R\, \hat{\ell}^{1/3}\hat{r}^{-4/3}\Phi\bar{\Phi} = \frac{1}{\kappa^2}\int d^8 z\, \tilde{E}\Big|_{F^2 = \hat{\ell}^{+1/3}\hat{r}^{-2/3}\Phi\bar{\Phi}^{-2}}}$$

RESUME OF THE CONSTRAINT SOLUTION

SOLUTION OF THE SG CONSTRAINTS. MIN. SG.

$$\nabla_\alpha = F\,\Delta_\alpha\,, \qquad \bar\nabla_{\dot\alpha} = \bar F\,\bar\Delta_{\dot\alpha} \qquad \{\hat\Delta_\alpha, \hat\Delta_\beta\} = 0 = \{\hat{\bar\Delta}_{\dot\alpha}, \hat{\bar\Delta}_{\dot\beta}\}$$

gauge fixed

$$\{\nabla_\alpha, \nabla_\beta\} = -S_{\alpha\beta}{}^\gamma \nabla_\gamma\,, \qquad S_{\alpha\beta}{}^\gamma = -2\,\nabla_{(\alpha} \ln F\; \delta_{\beta)}{}^\gamma$$

WZ gauge

$$\mathcal{H}^r(z) = \theta\sigma^a\bar\theta\, e_a{}^r + \theta\theta\bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha r} + \bar\theta\bar\theta\theta^\alpha\psi_\alpha{}^r + \theta\theta\bar\theta\bar\theta A^r$$

$$h^r = -\tfrac{i}{3}\theta\theta\bar\theta\bar\psi^r + \tfrac{i}{3}\bar\theta\bar\theta\theta\psi^r + \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\, e_a{}^{\nu}\partial_\nu e^r$$

$$\boxed{\hat\Delta_\alpha = \partial_\alpha + \hat\Delta_\alpha(h^r + i\mathcal{H}^r)\partial_r = \partial_\alpha^L + 2i\hat\Delta_\alpha\mathcal{H}^r\partial_r^L = \partial_\alpha^R}$$

$$\boxed{\hat{\bar\Delta}_{\dot\alpha} = \bar\partial_{\dot\alpha} + \hat{\bar\Delta}_{\dot\alpha}(h^r - i\mathcal{H}^r)\partial_r = \bar\partial_{\dot\alpha}^L = \bar\partial_{\dot\alpha}^R - 2i\hat{\bar\Delta}_{\dot\alpha}\mathcal{H}^r\partial_r^R}$$

$$\mathcal{H}^r(z') = \mathcal{H}^r(z) + \tfrac{1}{2i}(\Lambda^r - \bar\Lambda^r)$$
$$\delta x^r = \tfrac{1}{2}(\Lambda^r + \bar\Lambda^r)$$
$$\hat\Delta_\alpha\bar\Lambda^r = 0 = \hat{\bar\Delta}_{\dot\alpha}\Lambda^r$$

$$x^r_L = x^r - (h^r - i\mathcal{H}^r) \qquad x'^r_L = x^r_L + \Lambda^r(x_L,\theta)$$
$$\theta^\alpha_L = \theta^\alpha \qquad \theta'^\alpha_L = \theta^\alpha_L + \Lambda^\alpha(x_L,\theta)$$
$$\bar\theta^{\dot\alpha}_L = \bar\theta^{\dot\alpha} \qquad \bar\theta'^{\dot\alpha}_L = \bar\Lambda^{\dot\alpha}$$

$$\{\hat\Delta_\alpha, \hat{\bar\Delta}_{\dot\alpha}\} = 2i\sigma^a_{\alpha\dot\alpha}\hat\Delta_a$$

$$\boxed{\hat\Delta_a = \hat e_a{}^r\partial_r^L = \hat\varepsilon_a{}^r\partial_r^R = \hat C_a{}^r\partial_r + \hat\Delta_a h^r\partial_r}$$

$$\hat C_a{}^r = \tfrac{1}{4}\hat\sigma_a^{\dot\alpha\alpha}[\hat\Delta_\alpha, \hat{\bar\Delta}_{\dot\alpha}]\mathcal{H}^r\,, \qquad \boxed{\hat\varepsilon_a{}^r = \tfrac{1}{2}\hat\sigma_a^{\dot\alpha\alpha}\hat\varepsilon_{\alpha\dot\alpha}{}^r = -\tfrac{1}{2}\hat\sigma_a^{\dot\alpha\alpha}\hat\Delta_\alpha\hat{\bar\Delta}_{\dot\alpha}\mathcal{H}^r}\,, \qquad \boxed{\hat e_a{}^r = \tfrac{1}{2}\hat\sigma_a^{\dot\alpha\alpha}\hat e_{\alpha\dot\alpha}{}^r = +\tfrac{1}{2}\hat\sigma_a^{\dot\alpha\alpha}\hat{\bar\Delta}_{\dot\alpha}\hat\Delta_\alpha\mathcal{H}^r}$$

$$\boxed{\begin{aligned}[][\hat\Delta_a, \hat\Delta_\alpha] &= -\hat\Delta_\alpha\hat\varepsilon_a{}^r\cdot\hat\varepsilon_r{}^b\cdot\hat\Delta_b\\ [\hat\Delta_a, \hat{\bar\Delta}_{\dot\alpha}] &= -\hat{\bar\Delta}_{\dot\alpha}\hat e_a{}^r\,\hat e_r{}^b\cdot\hat\Delta_b\end{aligned}}$$

p.S.I.16-5 - to correct!

$$\nabla_{\alpha\dot\alpha} = F\bar F\,\hat\Delta_{\alpha\dot\alpha} - \tfrac{i}{2}(\bar F\omega_{\alpha\dot\alpha}{}^{\dot\beta} + F\hat\Delta_\alpha\bar F\delta_{\dot\alpha}{}^{\dot\beta})\hat{\bar\Delta}_{\dot\beta} - \tfrac{i}{2}(F\omega_{\dot\alpha\alpha}{}^\beta + \bar F\hat{\bar\Delta}_{\dot\alpha}F\cdot\delta_\alpha{}^\beta)\hat\Delta_\beta$$

$$\sigma^a_{\alpha\dot\alpha}(F\chi_a{}^\beta\hat\Delta_\beta + \bar F\bar\chi_a{}^\beta\hat{\bar\Delta}_\beta$$

$$\omega_{\alpha\beta\gamma} = 2\epsilon_{\alpha(\beta}\hat\Delta_{\gamma)}F\,, \qquad \omega_{\gamma\dot\gamma}{}^{\alpha\beta} = -\tfrac{i}{2}(\nabla_\gamma w_{\dot\gamma}{}^{\alpha\beta} + \bar\nabla_{\dot\gamma}\omega_\gamma{}^{\alpha\beta} + w_\gamma{}^{\alpha\varepsilon}w_{\dot\gamma\varepsilon}{}^\beta + w_{\dot\gamma}{}^{\alpha\varepsilon}w_{\gamma\varepsilon}{}^\beta)$$

$$\omega_{\dot\gamma\beta}{}^\alpha = u^{-1}_\beta{}^\varepsilon\bar\nabla_{\dot\gamma}u_\varepsilon{}^\alpha\,, \qquad \det u = 1$$

$$\bar R = -2F\hat\Delta\hat\Delta F - 2\hat\Delta^\alpha F\cdot\hat\Delta_\alpha F = -F^2\hat\Delta\hat\Delta\ln F^2 - F^2\hat\Delta^\alpha\ln F^2\cdot\hat\Delta_\alpha\ln F^2$$

$$T_\alpha := T_{\alpha b}{}^b = -\nabla_\alpha\ln\hat\varepsilon F^4\bar F^2\,. \qquad T_\alpha = 0 \Rightarrow \hat\varepsilon F^4\bar F^2 = \bar\Phi^{-3} \qquad \nabla_\alpha\bar\Phi = 0$$