FROM SUPERSPACE / SUPERFIELD TO COMPONENTS

"Ectoplasm method"

- SUSY action in spacetime as $S = \int_{M^4} L_4 = \frac{1}{4!}\int d^4x \; \epsilon^{\mu\nu\rho\sigma} E_\mu^A E_\nu^B E_\rho^C E_\sigma^D \, \mathcal{L}_{DCBA}$

- $E_\mu^A = \left(E_\mu^a(x,\theta),\ E_\mu^\alpha(x,\theta),\ E_\mu^{\dot\alpha}(x,\theta) \right)$

  $E_\mu^a(x,\theta) = e_\mu^a(x) + \sim\theta$, $E_\mu^\alpha(x,\theta) = \psi_\mu^\alpha(x) + \sim\theta$, $E_\mu^{\dot\alpha}(x,\theta) = \psi_\mu^{\dot\alpha}(x) + \sim\theta$

  $\mathcal{L}_{DCBA} = \mathcal{L}_{DCBA}(x,\theta)$

- Generically $S = S(\underline{\theta})$, but, if $d\mathcal{L}_4 = 0$, then $\frac{\partial}{\partial\theta}\int_{M^4} \mathcal{L}_4 = 0$

$$d\int_{M^4} \mathcal{L}_4 = d\theta^\alpha \frac{\partial}{\partial\theta^\alpha}\int_{M^4} \mathcal{L}_4 = \int_{M^4} d\mathcal{L}_4$$

$$\int_{M^4} dx\, \partial_\mu(\ldots) = 0$$

Thus we can use $\theta = 0$ value, i.e.

$$\boxed{S = \int \mathcal{L}_4(x,\theta)\Big|_{\theta=0,\ E^a = dx^\mu e_\mu^a(x),\ E^\alpha = dx^\mu \psi_\mu^\alpha(x)}}$$

CLOSED 4-FORM IN SSP OF MINIMAL SUGRA

SUGGESTIONS

CLOSED 4-FORM IN FLAT SSP

$$\boxed{dE^a = -2i\, E^\alpha \wedge \bar{E}^{\dot\alpha} \sigma^a_{\alpha\dot\alpha}}$$

$$h^0_{4L} = +\frac{1}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\,\alpha\beta}\ , \qquad dh^0_{4L} = 0$$

$$\left(h_4 = -i h_{4L} + i \bar{h}_{4R} = dC_3 \leftarrow \text{WZ term of supermembrane} \right)$$

Ex: To prove that $dh_{4L} = 0$

Suggestion: $\sigma^a_{(\alpha|\dot\alpha}(\sigma_{ab})_{|\beta\gamma)} = 0$ (TO PROVE)

CLOSED 4-FORM CAN BE CONSTRUCTED FROM THE SCALAR SUPERFIELD

$$\mathcal{L}_0\ , \qquad \bar{D}_{\dot\alpha} \mathcal{L}_0 = 0$$

$$\bar{\mathcal{L}}_0\ , \qquad D_\alpha \bar{\mathcal{L}}_0 = 0$$

$$H^0_{4L} = \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\,\alpha\beta} \bar{\mathcal{L}}_0 + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge E^\alpha \epsilon_{abcd} \sigma^d_{\alpha\dot\beta} \bar{D}^{\dot\beta} \bar{\mathcal{L}}_0$$
$$+ \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a \frac{i}{16} \epsilon_{abcd} \bar{D}\bar{D} \bar{\mathcal{L}}_0$$

Ex: To prove that →

$$dH^0_{4L} = 0 \quad \Longrightarrow \quad \boxed{D_\alpha \bar{\mathcal{L}}_0 = 0}$$

CLOSED 4-FORM IN PURE MINIMAL SG SSP (NO MATTER)

$$h_{4L} = \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\,\alpha\beta} - \frac{i}{128} E^d \wedge E^c \wedge E^b \wedge E^a \epsilon_{abcd} R$$

THESE EXAMPLES SUGGEST TO SEARCH FOR $\mathcal{L}_4$ USING THE ANSATZ (CONSTRAINTS)

$$\mathcal{L}_4 = \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta (\sigma_{ab})_{\alpha\beta} \bar{\mathcal{L}} + \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot\alpha} \wedge \bar{E}^{\dot\beta} (\bar\sigma_{ab})_{\dot\alpha\dot\beta} \mathcal{L} +$$
$$+ \frac{1}{3!} E^c \wedge E^b \wedge E^a \wedge \left(E^\alpha \mathcal{L}_{\alpha abc} + \bar{E}^{\dot\alpha} \mathcal{L}_{\dot\alpha abc} \right) +$$
$$+ \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a \mathcal{L}_{abcd}$$
$$= \mathcal{L}_{(2|2,0)} + \mathcal{L}_{(2|0,2)} + \mathcal{L}_{(3|1,0)} + \mathcal{L}_{(3|0,1)} + \mathcal{L}_{(4|0,0)}$$

IN OTHER WORDS
CONSTRAINTS: $\boxed{\mathcal{L}_{\alpha\beta ab} = (\sigma_{ab})_{\alpha\beta}\bar{\mathcal{L}}}$, $\mathcal{L}_{\alpha\beta\gamma D} = 0$, $\mathcal{L}_{\alpha\dot\beta CD} = 0$
and c.c.

BIs ⇐ conditions to be closed

$$0 \equiv d\mathcal{L}_4 = \frac{1}{5!}E^{C_5}\wedge\ldots\wedge E^{C_1}\, I_{C_1\ldots C_5} =$$

$$= I_{(1|3,1)} + I_{(1|1,3)} + \quad (\text{OTHERS} = 0)$$

$$+ I_{(2|3,0)} + I_{(2|0,3)} + I_{(2|2,1)} + I_{(2|1,2)} +$$

$$+ I_{(3|2,0)} + I_{(3|0,2)} + I_{(3|1,1)} + I_{(4|1,0)} + I_{(4|0,1)} + I_{(5|0,0)}$$

EXS: TO SHOW THAT:
(USING MIN SUGRA CONSTRAINTS)

$I_{(2|3,0)} = 0 \Rightarrow \boxed{\mathcal{D}_\alpha \bar{\mathcal{L}} = 0}$

$I_{(2|0,3)} = 0 \Rightarrow \bar{\mathcal{D}}_{\dot\alpha}\mathcal{L} = 0$

$I_{(2|2,1)} = 0 \Rightarrow \boxed{\mathcal{L}_{\alpha abc} = \frac{1}{4}\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{\mathcal{L}}}$

$I_{(2|1,2)} = 0 \Rightarrow \mathcal{L}_{\dot\alpha abc} = \frac{1}{4}\epsilon_{abcd}\,\mathcal{D}^\alpha\mathcal{L}\,\sigma^d_{\alpha\dot\alpha}$

$I_{(3|1,1)} = 0 \Rightarrow \boxed{\mathcal{L}_{abcd} = -\frac{i}{16}\epsilon_{abcd}\left[(\mathcal{D}\mathcal{D} - 3\bar{R})\mathcal{L} - (\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{\mathcal{L}}\right]}$

EX: TO SHOW THAT

$I_{(3|2,0)} = 0 \Rightarrow 0 = 0$

$I_{(3|0,2)} = 0 \Rightarrow 0 = 0$

$$\boxed{\begin{aligned}\mathcal{L}_4 = {} & \frac{1}{4}E^b\wedge E^a\wedge E^\alpha\wedge E^\beta(\sigma_{ab})_{\alpha\beta}\bar{\mathcal{L}} + \frac{1}{4}E^b\wedge E^a\wedge E^{\dot\alpha}\wedge E^{\dot\beta}\,\bar\sigma_{ab\,\dot\alpha\dot\beta}\mathcal{L} + \\ & + \frac{1}{4!}E^c\wedge E^b\wedge E^a\wedge\epsilon_{abcd}\left(E^\alpha\sigma^d_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{\mathcal{L}} + E^{\dot\alpha}\sigma^d_{\alpha\dot\alpha}\mathcal{D}^\alpha\mathcal{L}\right) \\ & - \frac{i}{16\cdot 4!}E^d\wedge E^c\wedge E^b\wedge E^a\epsilon_{abcd}\left((\mathcal{D}\mathcal{D}-3\bar{R})\mathcal{L} - (\bar{\mathcal{D}}\bar{\mathcal{D}}-3R)\bar{\mathcal{L}}\right)\end{aligned}}$$

FOR MINIMAL SG

$$S = \int d^8 z\, E = \frac{1}{4}\int d^6 \varsigma_L\, \mathcal{E} R + \frac{1}{4}\int d^6 \varsigma_R\, \bar{\mathcal{E}} \bar{R} =$$

$$= \sim \int_{M^4} \mathcal{L}_4(x,0) = \frac{1}{4!}\int d^4x\, \epsilon^{\mu\nu\rho\sigma} E_\mu^{\ A} \dots E_\sigma^{\ D}\, \mathcal{L}_{DCAB}\Big|_{\theta=0=\bar\theta}$$

$$\bar{\mathcal{L}} = -3i\bar{R} \quad , \quad \mathcal{L} = +3iR$$

$$\mathcal{L}_{abcd} = \frac{3}{16}\epsilon_{abcd}\left((\mathcal{D}\mathcal{D} - 3\bar{R})R + (\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{R}\right) = \frac{3}{16}\epsilon_{abcd}\big(\underbrace{\mathcal{D}\mathcal{D}R - \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R}}_{\leftarrow -\frac{16}{3}R_{ab}^{\ \ ab} + 4R\bar{R}} - 6R\bar{R}\big) =$$

$$= -\,\epsilon_{abcd}\left(R_{ef}^{\ \ ef} + \frac{3}{8}R\bar{R}\right)$$

$$\mathcal{L}_{\alpha abc} = -\frac{3i}{4}\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R} \quad , \quad \mathcal{L}_{\dot\alpha abc} = +\frac{3i}{4}\epsilon_{abcd}\,\mathcal{D}^\alpha R\,\sigma^d_{\alpha\dot\alpha}$$

$$\mathcal{L}_4\Big| = -\frac{1}{24} e^d\wedge e^c\wedge e^b\wedge e^a\,\epsilon_{abcd}\left(R_{ef}^{\ \ ef}\Big| + \frac{3}{8}R\bar{R}\right) +$$

$$\underbrace{-\frac{i}{8} e^c\wedge e^b\wedge e^a\wedge\left(\epsilon_{abcd}(\psi\sigma^d)_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R} - \epsilon_{abcd}\,\mathcal{D}^\alpha R(\sigma^d\bar\psi)_\alpha\right)}_{\leftarrow \frac{1}{6}e^c\wedge e^b\wedge e^a\wedge\,\epsilon_{abcd}\left(\psi\sigma^d\tilde\sigma^e\bar\Psi_e + \Psi_e\tilde\sigma^e\sigma^d\bar\psi\right)}$$

$$-\frac{3i}{4}e^b\wedge e^a\wedge\psi\wedge\sigma_{ab}\psi\,\bar{R}\Big| + \frac{3i}{4}e^b\wedge e^a\wedge\bar\psi\wedge\tilde\sigma_{ab}\bar\psi\cdot R\Big|$$

$$\boxed{\mathcal{L}_4\Big| = d^4x\cdot e\,\underbrace{\left(R_{ab}^{\ \ ab}\Big| + \frac{3}{8}R\cdot\bar{R}\right)}_{\leftarrow -\frac{3}{16}(\mathcal{D}\mathcal{D}R + \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R} - 6R\bar{R})} - \underbrace{\frac{3i}{4}\mathcal{D}^\alpha R(\sigma^b\bar\psi_b)_\alpha\Big| - \frac{3i}{4}(\psi_b\sigma^b)_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R}}_{-\frac{3}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R} - \frac{3}{2}\bar\psi_a\tilde\sigma^{ab}\bar\psi_b\cdot R}}$$

$$\mathcal{L}_4\Big| = d^4x\cdot e\left[R_{ef}^{\ \ ef}\Big| - \psi_b\sigma^b\tilde\sigma^a\bar\Psi_a\Big| - \Psi_a\tilde\sigma^a\sigma^b\bar\psi_b\Big| + \right.$$

$$\left. + \frac{3}{8}R\cdot\bar{R}\Big| - \frac{3}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R}\Big| - \frac{3}{2}\bar\psi_a\tilde\sigma^{ab}\bar\psi_b R\right]$$

$$\boxed{\mathcal{L}_4\Big| = d^4x\, e\left[e_a^{\ \mu}e_b^{\ \nu}\mathring{R}_{\mu\nu}^{\ \ \ ab} - 2\mathring{\Psi}_{\dot\alpha}^{\ a}\bar\psi_a^{\ \dot\alpha} - 2\psi_a^{\ \alpha}\bar\Psi_\alpha^{\ a} + \frac{3}{8}R\bar{R} + \frac{3}{32}f^a G_a\right]}$$

MIN SG ACTION

Ectoplasm 5.I.2.-61-(3)-
Local 24/03/2014

$$S=\int_{\Sigma^{(4|4)}} d^8z\, E = \frac{1}{4}\int d^6\zeta_L\, ER + \frac{1}{4}\int d^6\zeta_R\, E\bar{R} = \int_{M^4} \mathcal{L}_4(x,0) = \frac{1}{4!}\int d^4x\, \epsilon^{\mu\nu\rho\sigma} E_\mu{}^A \ldots E_\sigma{}^D \mathcal{L}_{DCBA}\Big|_{\theta=\bar\theta=0} = \int \mathcal{L}_4\Big|_{\substack{E^\alpha\to\psi^\alpha \\ \theta=0}}$$

$$\bar{\mathcal{L}} = -3i\bar{R}\,,\qquad \mathcal{L}=+3iR$$

$$\mathcal{L}_{abcd} = \frac{3}{16}\epsilon_{abcd}\left((\mathcal{D}\mathcal{D}-3\bar{R})R + (\bar{\mathcal{D}}\bar{\mathcal{D}}-3R)\bar{R}\right) = \frac{3}{16}\epsilon_{abcd}\Big(\underbrace{\mathcal{D}\mathcal{D}R+\bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R}}_{-\frac{16}{3}R_{ab}{}^{ab}+4R\bar{R}} - 6R\bar{R}\Big) = -\frac{1}{4}\epsilon_{abcd}\left(R_{ef}{}^{ef}+\frac{3}{8}R\bar{R}\right)\Big|$$

$$\mathcal{L}_{\dot\alpha abc} = -\frac{3i}{4}\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R}\,,\qquad \mathcal{L}_{\alpha abc} = +\frac{3i}{4}\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\mathcal{D}^\alpha R$$

$$-\frac{4i}{3}(\bar\psi_e\bar\sigma^e)^\alpha$$

$$\mathcal{L}_4\Big| = +\frac{3i}{4}e^b\wedge e^a\wedge\psi\wedge\sigma_{ab}\bar\psi\cdot\bar{R}\Big| + \frac{3i}{4}e^b\wedge e^a\wedge\bar\psi\wedge\bar\sigma_{ab}\bar\psi\cdot R\Big| - \frac{i}{8}e^c\wedge e^b\wedge e^a\wedge\left(\epsilon_{abcd}(\psi\sigma^d)_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R} + \epsilon_{abcd}\,\mathcal{D}^\alpha R(\sigma^d\bar\psi)_\alpha\right)$$

$$-\frac{1}{24}e^d\wedge e^c\wedge e^b\wedge e^a\,\epsilon_{abcd}\left(R_{ef}{}^{ef}\Big| + \frac{3}{8}R\bar{R}\Big|\right)$$

$$\to -\frac{1}{6}e^c\wedge e^b\wedge e^a\wedge\epsilon_{abcd}\left(\psi\sigma^d\bar\sigma^e\bar\psi_e + \psi_e\bar\sigma^e\sigma^d\bar\psi\right)$$

$$e^{d'}\psi_{d'}\qquad -d^4x\cdot e\,\delta^{d'}_d\,3!$$

$$+d^4x\cdot e\left(R_{ef}{}^{ef}\Big| + \frac{3}{8}R\bar{R}\Big|\right)$$

$$-d^4x\, e\left(\psi_b\sigma^b\bar\sigma^a\bar\psi_a + \psi_a\bar\sigma^a\sigma^b\bar\psi_b\right)$$

see p. 5.I.26.2 -(2')-

$$\mathcal{D}_\alpha R = -\frac{8}{3}(\sigma^{ab})_\alpha{}^\beta T_{ab\beta} = \frac{4i}{3}\sigma_{a\alpha\dot\alpha}\bar\psi^{a\dot\alpha}\,;\qquad \mathcal{D}^\alpha R = \frac{8}{3}T_{ab}{}^\beta\sigma^{ab}{}_\beta{}^\alpha = -\frac{4i}{3}(\psi_a\sigma^a)^\alpha$$

$$\bar{\mathcal{D}}_{\dot\alpha}\bar{R} = -\frac{8}{3}(\bar\sigma^{ab})^{\dot\beta}{}_{\dot\alpha}T_{ab\dot\beta} = \frac{4i}{3}\bar\psi^{a\alpha}\sigma_{a\alpha\dot\alpha}\,;\qquad \bar{\mathcal{D}}^{\dot\alpha}\bar{R} = \frac{8}{3}\bar\sigma_{ab}{}^{\dot\alpha}{}_{\dot\beta}T^{ab\dot\beta} = -\frac{4i}{3}(\bar\sigma^a\bar\psi_a)^{\dot\alpha}$$

$$\mathcal{L}_4\Big| = d^4x\cdot e\left(R_{ef}{}^{ef} - \psi_b\sigma^b\bar\sigma^a\bar\psi_a - \psi_a\bar\sigma^a\sigma^b\bar\psi_b + \frac{3}{8}R\cdot\bar{R}\right.$$
$$\left. - \frac{3}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R} - \frac{3}{2}\bar\psi_a\bar\sigma^{ab}\bar\psi_b\cdot R\right)$$

$$-\frac{3i}{4}\underbrace{e^b\wedge e^a\wedge e^d\wedge e^c}_{d^4x\,e\,\epsilon^{abcd}}\left(\psi_c\sigma_{ab}\psi_d\bar{R}\Big| - \bar\psi_c\bar\sigma_{ab}\bar\psi_d R\Big|\right) = -2i\left(\psi_a\sigma^{ab}\psi_b\cdot\bar{R} + \bar\psi_a\bar\sigma^{ab}\bar\psi_b R\Big|\right)$$

$$= -\frac{3}{2}d^4x\, e\left(\psi_a\sigma^{ab}\psi_b\bar{R}\Big| + \bar\psi_a\bar\sigma^{ab}\bar\psi_b R\Big|\right)$$

S. I. 2. -41- (4'')

recalculate of (4')

SUPERGRAVITY ACTION:

$$S' = \int d^8 z\, E = \frac{1}{4}\int d^6 \zeta_L\, \mathcal{E} R + \frac{1}{4}\int d^6 \zeta_R\, \bar{\mathcal{E}}\bar{R} = \sim \int_{M^4} \mathcal{L}_4(x,0) = \frac{1}{4!}\int d^4x\, \epsilon^{\mu\nu\rho\sigma} E_\mu{}^A E_\nu{}^B E_\rho{}^C E_\sigma{}^D \mathcal{L}_{DCBA}\Big|_{\theta=U=\bar\theta} = \int \mathcal{L}_4 \Big|_{\substack{E=\to\psi=\\ \theta=0}} = \frac{1}{\kappa^2}\int d^4x\, e\, R_{ab}{}^{ab} \ldots$$

$$\mathcal{L} = +3iR\,, \quad \bar{\mathcal{L}} = -3i\bar{R}\,, \quad \mathcal{L}_{\alpha abc} = -\frac{3i}{4}\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R}\,, \quad \mathcal{L}_{\dot\alpha abc} = +\frac{3i}{4}\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\mathcal{D}^\alpha R\,, \quad \mathcal{L}_{abcd} = +\frac{3}{16}\left((\mathcal{D}\mathcal{D} - 3\bar{R})R + (\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{R}\right)$$

NO

$$\mathcal{L}_4\Big| = \pm\frac{3i}{4}\, e^b \wedge e^a \wedge \psi^\alpha \wedge \sigma_{ab\,\alpha}{}^\beta \psi_\beta \cdot \bar{R} \pm \frac{3i}{4}\, e^b \wedge e^a \wedge \bar\psi_{\dot\alpha} \wedge \tilde\sigma_{ab}{}^{\dot\alpha}{}_{\dot\beta}\bar\psi^{\dot\beta} R \pm \frac{i}{8}\, e^c \wedge e^b \wedge e^a \wedge e^f \epsilon_{abcd}\left((\psi_f \sigma^d)_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R} + \mathcal{D}^\alpha R(\sigma^d \bar\psi_f)_\alpha\right) +$$

$$= \pm\frac{3i}{4}\, e^b \wedge e^a \wedge \left(\psi \wedge \sigma_{ab}\psi \bar{R} - \bar\psi \wedge \tilde\sigma_{ab}\bar\psi R\right)$$

$$= \pm\frac{3i}{4}\, e^d \wedge e^c \wedge e^b \wedge e^a \left(\psi_c \sigma_{ab}\psi_d \cdot \bar{R}| - \bar\psi_c \tilde\sigma_{ab}\bar\psi_d \cdot R|\right)$$

$$= -d^4x\, e\, \left(+\frac{3i}{2}\frac{i}{4}\epsilon^{abcd}\psi_a\sigma_{cd}\psi_b \bar{R} - \frac{i}{2}\epsilon^{abcd}\bar\psi_a\tilde\sigma_{cd}\bar\psi_b \cdot R|\right)$$

$-4 - 2$

$$= \psi_a\sigma^{ab}\psi_b \cdot \bar{R} + \bar\psi_a\tilde\sigma^{ab}\bar\psi_b \cdot R| \Big) = d^4x \cdot e\left(\pm\frac{3}{2}\psi_a\sigma^{ab}\psi_b \cdot \bar{R}| \pm \frac{3}{2}\bar\psi_a\tilde\sigma^{ab}\bar\psi_b \cdot R|\right)$$

$$d^4x \cdot e\,(\epsilon^{abcf} + 3!\,\delta_d^f)$$

$$= \pm\frac{3i}{4}\, d^4x\, e\left(-\frac{4i}{3}\right)\left(\psi_b\sigma^b\tilde\sigma_a\bar\psi^a + \psi^a\tilde\sigma_a\sigma_b\bar\psi^b\right)$$

$$= \pm d^4x \cdot e\left(\psi^a\tilde\sigma_a\sigma_b\bar\psi^b + \psi_b\sigma^b\tilde\sigma^a\bar\psi_a\right)$$

$$+ \frac{1}{4!}\, e^d \wedge e^c \wedge e^b \wedge e^a \epsilon_{abcd}\left(+\frac{3}{16}\left(\mathcal{D}\mathcal{D}R + \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R} - 6R\bar{R}\right)\right)\Big|$$

$$= \cancel{4}\, d^4x \cdot e \cdot \cancel{4!} \qquad = \cancel{4}\, R_{ab}{}^{ab} \cancel{\pm}\frac{3}{8}R\bar{R}$$

$$\sigma^{ab}{}_\alpha{}^\beta T_{ab\beta}{}^{(\cdot)} = -\frac{3}{8}\mathcal{D}_\alpha R$$

$$\mathcal{D}^\alpha R = +\frac{8}{3}\, T_{ab}{}^\beta \sigma^{ab}{}_\beta{}^\alpha = -\frac{4i}{3}(\psi^a\sigma_a)^\alpha$$

$$\bar{\mathcal{D}}^{\dot\alpha}\bar{R} = \frac{8}{3}\tilde\sigma^{ab\dot\alpha}{}_{\dot\beta} T_{ab}{}^{\dot\beta} = -\frac{4i}{3}(\tilde\sigma_a\bar\psi^a)^{\dot\alpha}$$

$$\boxed{\mathcal{L}_4\Big| = d^4x \cdot e\left(R_{ab}{}^{ab}\Big| + \frac{3}{8}R\bar{R}\Big| \pm \psi^a\tilde\sigma_a\sigma_b\bar\psi^b \pm \psi_b\sigma^b\tilde\sigma^a\bar\psi_a\Big| \pm \frac{3}{2}\psi_a\sigma^{ab}\psi_b \cdot \bar{R}\Big| \pm \frac{3}{2}\bar\psi_a\tilde\sigma^{ab}\bar\psi_b \cdot R\Big|\right)}$$

OR, EQUIVALENTLY

$$\boxed{\mathcal{L}_4\Big| = d^4x \cdot e\left(R_{ab}{}^{ab}\Big| + \frac{3}{8}R\bar{R}\Big| \pm \frac{3i}{4}\mathcal{D}^\alpha R\Big|(\sigma_a\bar\psi^a)_\alpha \pm \frac{3i}{4}(\psi^a\sigma_a)_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R}\Big| \pm \frac{3}{2}\psi_a\sigma^{ab}\psi_b \cdot \bar{R}\Big| \pm \frac{3}{2}\bar\psi_a\tilde\sigma^{ab}\bar\psi_b \cdot R\Big|\right)}$$

$$R_{ab}{}^{ab} = e_b{}^\nu e_a{}^\mu \overset{\circ}{R}_{\mu\nu}{}^{ab} + \frac{3}{32}G_a G^a\Big| + \frac{i}{2}\epsilon^{abcd}\psi_a\sigma_b\bar\psi_c G_d$$

S.I.2-4/-(4'')
a-calcul

11/05/2014

$$\mathcal{L}_4 = \frac{i}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \, \sigma_{ab\,\alpha\beta}\, \bar{\mathcal{L}} + \frac{i}{4} E^b \wedge E^a \wedge \bar{E}^{\dot\alpha} \wedge \bar{E}^{\dot\beta}\, \bar\sigma_{ab\,\dot\alpha\dot\beta}\, \mathcal{L} + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge E^\alpha \,\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\,\bar{\mathcal{D}}^{\dot\alpha}\bar{\mathcal{L}} + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge \bar{E}^{\dot\alpha}\,\epsilon_{abcd}\,\mathcal{D}^\alpha\mathcal{L}\,\sigma^d_{\alpha\dot\alpha} +$$

$$+\frac{i}{16\cdot 4!} E^d \wedge E^c \wedge E^b \wedge E^a \,\epsilon_{abcd}\left((\mathcal{D}\mathcal{D} - 3\bar{R})\mathcal{L} - (\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{\mathcal{L}}\right)$$

$3i R \qquad -3i\bar{R}$

$-d^4x\, e\, 4!$

$d^4x\, e\, R_{ab}{}^{ab}|$

$= -\frac{3}{16}\left(\mathcal{D}\mathcal{D}R + \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R} - 4R\bar{R}\right)$

we would like to have

$$+\frac{i}{16}\mathcal{D}\mathcal{D}\mathcal{L} = -\frac{3}{16}\mathcal{D}\mathcal{D}R$$

$\Rightarrow$ $\mathcal{L} = 3iR$, $\bar{\mathcal{L}} = -3i\bar{R}$

$$\mathcal{L}_4 = -\frac{3i}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta\, \sigma_{ab\,\alpha\beta}\,\bar{R} + \frac{3i}{4} E^b \wedge E^a \wedge \bar{E}^{\dot\alpha} \wedge \bar{E}^{\dot\beta}\,\bar\sigma_{ab\,\dot\alpha\dot\beta}\, R + \frac{i}{8} E^c \wedge E^b \wedge E^a \wedge E^\alpha\,\epsilon_{abcd}\,\sigma^d_{\alpha\dot\alpha}\,\bar{\mathcal{D}}^{\dot\alpha}\bar{R} - \frac{i}{8} E^c \wedge E^b \wedge E^a \wedge \bar{E}^{\dot\alpha}\,\epsilon_{abcd}\,\mathcal{D}^\alpha R\,\sigma^d_{\alpha\dot\alpha} + \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a\,\epsilon_{abcd}\,\frac{3}{16}\left(\mathcal{D}\mathcal{D}R + \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R} - 6\ldots\right)$$

$\downarrow$

$$\hat{\mathcal{L}}_4 = +\frac{3i}{4}\underbrace{e^b \wedge e^a \wedge \psi_c\,\sigma_{ab}\,\psi_d}\cdot\bar{R} = \frac{3i}{4}\underbrace{e^b \wedge e^a \wedge \bar\psi_c \wedge \bar\sigma_{ab}\bar\psi_d}\cdot R - \frac{i}{8}\underbrace{e^c \wedge e^b \wedge e^a \wedge e^{d'}}\epsilon_{abcd}\,(\psi_{d'}\sigma^d)_{\dot\alpha}\,\bar{\mathcal{D}}^{\dot\alpha}\bar{R} - \frac{i}{8}\underbrace{e^c \wedge e^b \wedge e^a \wedge e^{d'}}\epsilon_{abcd}\,\mathcal{D}^\alpha R\,\sigma^d_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}_{d'} + d^4x\cdot e\left(-\frac{3}{16}(\mathcal{D}\mathcal{D}R + \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{R} - 4R\bar{R}) + \frac{3}{8}R\bar{R}\right)$$

$= -e^a e^b e^c e^d\,\psi_c\sigma_{ab}\psi_d = -d^4x\, e\,\epsilon^{abcd} = 2i\, d^4x\, e\,\psi_c\sigma^{cd}\psi_d = -\frac{3}{2}d^4x\cdot e\,\psi_a\sigma^{ab}\psi_b\cdot\bar{R}$

$= +\frac{3i}{4}\epsilon^{abcd}\,\bar\psi_c\bar\sigma_{ab}\bar\psi_d = -\frac{3}{2}d^4x\cdot e\,\bar\psi_a\bar\sigma^{ab}\bar\psi_b\cdot R$

$= +d^4x\, e\,\epsilon^{d'abc} = 6\,\delta^{d'}_d$

$= \epsilon^{d'abc}\, d^4x\cdot e = 6\,\delta^{d'}_d$

$= -\frac{3i}{4}d^4x\cdot e\;\mathcal{D}^\alpha R\,(\sigma^b\bar\psi_b)_\alpha$

$= d^4x\, e\left(R_{ab}{}^{ab}| + \frac{3}{8}R\cdot\bar{R}|\right)$

$$\hat{\mathcal{L}}_4 = d^4x\cdot e\left(R_{ab}{}^{ab}| + \frac{3}{8}R\cdot\bar{R}| - \frac{3i}{4}(\psi_b\sigma^b)_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar{R}| - \frac{3i}{4}\mathcal{D}^\alpha R\,(\sigma^b\bar\psi_b)_\alpha| - \frac{3}{2}\psi_a\sigma^{ab}\psi_b\bar{R} - \frac{3}{2}\bar\psi_a\bar\sigma^{ab}\bar\psi_b R\right)$$

$= -\psi_b\sigma^b\bar\sigma^a\bar\psi_a \qquad = -\psi_a\sigma^a\bar\sigma^b\bar\psi_b$

$-\frac{3}{2}\psi_a\sigma^{ab}\psi_b\bar{R}$

$\mathcal{D}_\alpha R| = \frac{4i}{3}\sigma_{a\alpha\dot\alpha}\bar\psi^{\dot\alpha a}|$

$\mathcal{D}^\alpha R| = -\frac{4i}{3}\psi^a_{\dot\alpha}\bar\sigma_a^{\dot\alpha\alpha}$

$\bar{\mathcal{D}}^{\dot\alpha}\bar{R}| = -\frac{4i}{3}\bar\sigma_a^{\dot\alpha\alpha}\psi_\alpha{}^a$

$$\hat{\mathcal{L}}_4 = d^4x\cdot e\left(R_{ab}{}^{ab} - \psi_b\sigma^b\bar\sigma^a\bar\psi_a - \psi^a\bar\sigma_a\sigma^b\bar\psi_b + \frac{3}{8}R\cdot\bar{R}| - \frac{3}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R} - \frac{3}{2}\bar\psi_a\bar\sigma^{ab}\bar\psi_b\cdot R\right)$$

$2i\psi_a\bar\psi^a - 4i\psi_a\sigma^a\bar\psi^b$

$$\mathcal{L}_4\Big| = d^4x\, e\Big(\underbrace{R_{ab}{}^{ab}\Big|}_{} + \frac{3}{8} R\bar{R} - \cancel{\psi^a \bar{\sigma}_a \sigma_b \bar{\psi}^b} - \cancel{\psi_b \sigma^b \bar{\sigma}^a \bar{\psi}_a} \mp \frac{1}{2}\psi_a \sigma^{ab} \cancel{\psi_b \bar{R}} - \frac{1}{2}\bar{\psi}_a \bar{\sigma}^{ab} \cancel{\bar{\psi}_b \cdot R} \Big)$$

2 1 3 5 4 6

$$\Big[e_a{}^\mu e_b{}^\nu \overset{0}{R}_{\mu\nu}{}^{ab} + \frac{i}{2}\cancel{\epsilon^{abcd} \psi_a \sigma_b \bar{\psi}_c \cdot G_d} + \frac{3}{32} G^a G_a + \frac{2}{2}\psi_a \sigma^{ab} \psi_b \cdot \bar{R} + \frac{2}{2}\bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b \cdot R -$$

5 3 4

$$\underbrace{-2\bar{\psi}_{\dot\alpha}{}^a \psi_a{}^{\dot\alpha} - 2\psi_a{}^\alpha \bar{\psi}_\alpha{}^a}_{} + \cancel{\psi^a \bar{\sigma}_a \sigma_b \bar{\psi}^b} + \cancel{\psi^b \sigma_b \bar{\sigma}_a \bar{\psi}^a}$$

2 C

$$\Big(\Big(-2\overset{0}{\bar{\psi}}_{\dot\alpha}{}^{\dot\alpha} \bar{\psi}_a{}^{\dot\alpha} - 2\psi_a{}^\alpha \overset{0}{\psi}_\alpha{}^a + R\, \cancel{\bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b} + \bar{R}\, \cancel{\psi_a \sigma^{ab} \psi_b} + \phi + \underbrace{\frac{i}{2}\epsilon^{abcd} G_a \cancel{\psi_b \sigma_c \bar{\psi}_d}}_{}$$

6 5

$$\| \qquad -\frac{i}{2}\epsilon^{abcd} \cancel{\psi_a \sigma_b \bar{\psi}_c G_d}$$

C

$$\mathcal{L}_4\Big| = d^4x \cdot e \Big(e_a{}^\mu e_b{}^\nu \overset{0}{R}_{\mu\nu}{}^{ab} \mp 2\overset{0}{\bar{\psi}}_{\dot\alpha} \bar{\psi}_a{}^{\dot\alpha} - 2\psi_a{}^\alpha \overset{0}{\psi}_\alpha{}^a + \frac{3}{8} R\bar{R} + \frac{3}{32} G^a G_a \Big)$$

Remember that (p.p. 5.I-2b-2 - (1÷4) ?)

$$T_{ab}{}^{\alpha}\Big| = \overset{o}{\psi}_{ab}{}^{\alpha} - \frac{i}{8}(\psi_{[a}\sigma_{b]}\hat{\sigma}_c)^{\alpha}G^c\Big| - \frac{i}{8}\psi_{[a}{}^{\alpha}G_{b]}\Big| - \frac{i}{4}(\psi_{[a}\hat{\sigma}_{b]})^{\alpha}R\Big|$$

$$\Leftarrow e_a{}^{\mu}e_b{}^{\nu}\, 2\overset{o}{D}_{[\mu}\psi_{\nu]}{}^{\alpha}\Big|$$

$$\psi_{\dot\alpha}{}^{a}\Big| = \overset{o}{\psi}_{\dot\alpha}{}^{a} + \frac{i}{8}\epsilon^{abcd}(\psi_b\sigma_c)_{\dot\alpha}G_d\Big| + \frac{i}{4}(\psi_b\sigma^a)_{\dot\alpha}G^b\Big| - \frac{i}{4}(\psi_b\sigma^b)_{\dot\alpha}G^a\Big| + $$
$$+ \frac{i}{2}(\psi_b\hat{\sigma}^{ab})_{\dot\alpha}\cdot R\Big| \quad (*)$$

$$R_{ab}{}^{ab}\Big| = \underbrace{e_b{}^{\nu}e_a{}^{\mu}R_{\mu\nu}{}^{ab}\Big|}_{} + \frac{1}{2}\psi_a\sigma^{ab}\psi_b\cdot\bar{R}\Big| + \frac{1}{2}\bar{\psi}_a\hat{\sigma}^{ab}\bar{\psi}_b\cdot R\Big| +$$

5.I.2b.2-(1) $\Leftarrow e_b{}^{\nu}e_a{}^{\mu}\overset{o}{R}_{\mu\nu}{}^{ab}\Big| + \frac{i}{2}\epsilon^{abcd}\psi_a\sigma_b\bar{\psi}_c G_d\Big| + \frac{3}{32}G_aG^a$

$$+ \cancel{\frac{3i}{8}(\psi^b\sigma_b)_{\dot\alpha}\bar{D}^{\dot\alpha}\bar{R}\Big|} - \cancel{\frac{3i}{8}(\bar{\psi}_a\bar{\sigma}^a)^{\alpha}D_\alpha R\Big|} +$$

$$+ \frac{i}{2}\left(\psi_a{}^{\alpha} - \frac{i}{2}(\psi_b\sigma^b\tilde{\sigma}_a)^{\alpha}\right)D_\alpha G^a\Big| + \frac{i}{2}\bar{D}_{\dot\alpha}G^a\left(\bar{\psi}_a{}^{\dot\alpha} + \frac{i}{2}(\tilde{\sigma}_a\sigma^b\bar{\psi}_b)^{\dot\alpha}\right)$$

THIS IMPLIES

$$\boxed{\mathcal{L}_4\Big| = d^4x\, e\left(e_b{}^{\nu}e_a{}^{\mu}\overset{o}{R}_{\mu\nu}{}^{ab} + \frac{3}{8}R\cdot\bar{R}\Big| + \frac{3}{32}G_aG^a\Big| + \text{fermions}^2\right)}$$

Instead of performing exhausting calculation of the fermionic contributions we can use a shortcut: restore them from local susy of the action.

Restoration of component SUGRA action:

- $$\mathcal{L}_4\Big| = d^4x\, e\left(e_b{}^\nu e_a{}^\mu \overset{\circ}{R}_{\mu\nu}{}^{ab} + \frac{3}{8} R\cdot\bar{R}\Big| + \frac{3}{32} G_a\cdot G^a\Big| + \text{fermions}^2\right)$$
  $\mathcal{L}$ from ectoplasm method.

- SUSY – from SSP:

$$\delta_\epsilon e_\mu^a = -2i\left(\psi_\mu \sigma^a\bar{\epsilon} - \epsilon\sigma^a\bar{\psi}_\mu\right)$$

$$\delta_\epsilon \psi_\mu{}^\alpha = D\epsilon^\alpha\Big| + i_\epsilon T^\alpha\Big| = \underbrace{D_\mu\epsilon^\alpha}_{\overset{\circ}{D}_\mu\epsilon^\alpha - \frac{i}{16} e_\mu^b(\epsilon\sigma_{bc})^\alpha G^c|} + \underbrace{e_\mu^b\, \epsilon^\beta T_{\beta b}{}^\alpha\Big|}_{e_\mu^b \frac{i}{8}(\epsilon\tilde{\sigma}_b)^\alpha R| + \frac{i}{8} e_\mu^b(\epsilon\sigma_b\hat{\sigma}_c)^\alpha G^c} =$$

$$= \overset{\circ}{D}_\mu\epsilon^\alpha + \frac{i}{8}(\epsilon\tilde{\sigma}_\mu)^\alpha R + \frac{3i}{16}\left(\epsilon^\alpha G_\mu - \frac{1}{3}(\epsilon\sigma_a\tilde{\sigma}_\mu)^\alpha G^a\right)$$

- Observe that $\int_{M^4} \mathcal{L}_4^{EH} + \mathcal{L}_4^{RS} = \int_{M^4} \underbrace{\frac{1}{2}\epsilon_{abcd}\overset{\circ}{R}{}^{ab}\wedge e^c\wedge e^d}_{d^4x\, e\, \overset{\circ}{R}_{\mu\nu}{}^{ab} e_a^\mu e_b^\nu} - 4\overset{\circ}{D}\psi\wedge\sigma^{(1)}\bar{\psi} + 4\psi\wedge\sigma^{(1)}\wedge\overset{\circ}{D}\bar{\psi}$

  is inv under the "on-shell" SUSY: $\delta_\epsilon e^a = -2i(\psi\sigma^a\bar{\epsilon} - \epsilon\sigma^a\bar{\psi})$

  the same as for off-shell SUSY!

  $\delta_\epsilon\psi^\alpha = D\epsilon^\alpha$

- $\Rightarrow$ under "off-shell" susy

see component SUGRA

$$\delta_\epsilon\left(\mathcal{L}_4^{EH} + \mathcal{L}_4^{RS}\right) = -\mathcal{E}_{a3}\wedge\delta_\epsilon e^a + 8\,\delta_\epsilon\psi^\alpha\wedge\underbrace{\mathcal{E}_{3\alpha}}_{\frac{1}{2}e_a^{\wedge 3}\overset{\circ}{\Psi}{}_\alpha^a} - 8\underbrace{\mathcal{E}_{3\dot{\alpha}}}_{\frac{1}{2}e_a^{\wedge 3}\overset{\circ}{\bar{\Psi}}{}_{\dot{\alpha}}^a}\wedge\delta_\epsilon\bar{\psi}^{\dot{\alpha}}$$

$$= 4\left(\delta_\epsilon\psi^\alpha - D\epsilon^\alpha\right)\wedge e_a^{\wedge 3}\overset{\circ}{\Psi} - 4\, e_a^{\wedge 3}\overset{\circ}{\bar{\Psi}}{}_{\dot{\alpha}}^a\wedge\left(\delta_\epsilon\bar{\psi}^{\dot{\alpha}} - D\bar{\epsilon}^{\dot{\alpha}}\right)$$

$$= \frac{i}{2} d^4x\, e\left(\bar{R}\,\overset{\circ}{\Psi}{}^a\tilde{\sigma}_a\epsilon - \bar{\epsilon}\tilde{\sigma}_a\overset{\circ}{\bar{\Psi}}{}^a R\right) +$$

$$+ \frac{3i}{4} d^4x\, e\left(\overset{\circ}{\bar{\Psi}}{}_{\dot{\alpha}}^a\bar{\epsilon}^{\dot{\alpha}} - \frac{1}{3}\overset{\circ}{\Psi}{}^b\tilde{\sigma}_b\sigma^a\bar{\epsilon}\right)\cdot G_a\Big| - \frac{3i}{4} d^4x\, e\left(\epsilon^\alpha\overset{\circ}{\Psi}{}_\alpha^a - \frac{1}{3}\epsilon\sigma^a\tilde{\sigma}_b\overset{\circ}{\bar{\Psi}}{}^b\right)\cdot G_a\Big|$$

? Can this be compensated by some $\delta_\epsilon R|$, $\delta_\epsilon\bar{R}|$ and $\delta_\epsilon G_a|$ transformations of the quadratic Lagrangian term ? :

$$\delta_\epsilon\, d^4x\, e\left(\frac{3}{8}R\bar{R} + \frac{3}{32}G_aG^a\right) = d^4x\cdot e\left(2i(\epsilon\sigma^a\bar{\psi}_a - \psi_a\sigma^a\bar{\epsilon})\left(\frac{3}{8}R\bar{R} + \frac{3}{32}G_aG^a\right) + \frac{3}{8}R\,\delta_\epsilon\bar{R} + \frac{3}{8}\delta_\epsilon R\cdot\bar{R} + \frac{3}{16}G^a\delta_\epsilon G_a\right)$$

$$\delta_\epsilon d^4x\cdot e\left(\tfrac{3}{8}R\bar{R}+\tfrac{3}{32}G_aG^a\right)=d^4x\cdot e\left(2i(\epsilon\sigma^a\bar\psi_a-\psi_a\sigma^a\bar\epsilon)\left(\tfrac{3}{8}R\cdot\bar R+\tfrac{3}{32}G_aG^a\right)+\tfrac{3}{16}G_a\delta_\epsilon G^a+\tfrac{3}{8}R\delta_\epsilon\bar R+\tfrac{3}{8}\bar R\delta_\epsilon R\right)=$$

$$=d^4x\cdot e\left(\tfrac{i}{2}\underbrace{\epsilon\sigma_a\overset{\circ}{\bar\Psi}{}^a}_{=-\overset{\circ}{\Psi}{}^a\sigma_a\epsilon}\bar R-\tfrac{i}{2}\underbrace{\overset{\circ}{\bar\Psi}{}^a\sigma_a\bar\epsilon}_{=-\bar\epsilon\hat\sigma_a\overset{\circ}{\Psi}{}^a}\cdot R+\tfrac{3i}{4}\left(\left(\epsilon\overset{\circ}{\Psi}_a-\tfrac{1}{3}\epsilon\sigma_a\tilde\sigma_b\overset{\circ}{\Psi}{}^b\right)-\left(\overset{\circ}{\Psi}_a\bar\epsilon-\tfrac{1}{3}\overset{\circ}{\Psi}{}^b\tilde\sigma_b\sigma_a\bar\epsilon\right)\right)G^a\right)$$

$\Downarrow$ The SUSY transformations of auxiliary fields should be

$$\delta_\epsilon R=-\tfrac{4i}{3}\overset{\circ}{\Psi}{}^a\tilde\sigma_a\epsilon-i(\epsilon\sigma^a\bar\psi_a-\psi_a\sigma^a\bar\epsilon)R+i\alpha(\epsilon)R+h_a(\epsilon)G^a$$

$$\delta_\epsilon\bar R=+\tfrac{4i}{3}\bar\epsilon\hat\sigma_a\overset{\circ}{\bar\Psi}{}^a-i(\epsilon\sigma^a\bar\psi_a-\psi_a\sigma^a\bar\epsilon)\cdot\bar R-i\alpha(\epsilon)\bar R+\bar h_a(\epsilon)G^a$$

$$\delta_\epsilon G_a=-4i\left(\overset{\circ}{\Psi}_a\bar\epsilon-\tfrac{1}{3}\overset{\circ}{\Psi}{}^b\tilde\sigma_b\sigma_a\bar\epsilon\right)+4i\left(\epsilon^\alpha\overset{\circ}{\Psi}_{\alpha a}-\tfrac{1}{3}\epsilon\sigma_a\tilde\sigma_b\overset{\circ}{\Psi}{}^b\right)-$$
$$-i(\epsilon\sigma^b\bar\psi_b-\psi_b\sigma^b\bar\epsilon)G_a+a_{[ab]}(\epsilon)G^b-2h_a(\epsilon)\bar R-2\bar h_a(\epsilon)R$$

These coincide with the transformation rules following from superspace formalism . (see p. b2-⑥ attached as b1-7')
if we set

$$a_{ab}(\epsilon)=\tfrac{i}{2}\epsilon_{abcd}\left(\psi^c\sigma^d\bar\epsilon+\epsilon\sigma^c\bar\psi^d\right)$$
$$h_a(\epsilon)=-i\epsilon\psi_a\,,\qquad \bar h_a(\bar\epsilon)=i\bar\psi^a\bar\epsilon$$
$$i\alpha(\epsilon)=-i\epsilon\sigma_b\bar\psi^b-i\psi_b\sigma^b\bar\epsilon$$

Thus $\int\mathcal{L}_4\Big|=\int\mathcal{L}^{EH}+\mathcal{L}^{RS}+d^4x\cdot e\left(\tfrac{3}{8}R\cdot\bar R\Big|+\tfrac{3}{16}G_aG^a\Big|\right)$

is inv. under SUSY:

$$\boxed{\begin{aligned}
&\delta_\epsilon e^a=-2i(\psi\sigma^a\bar\epsilon-\epsilon\sigma^a\bar\psi)\\
&\delta_\epsilon\psi_\mu^\alpha=\overset{\circ}{\mathcal{D}}_\mu\epsilon^\alpha+\tfrac{i}{8}(\bar\epsilon\tilde\sigma_\mu)^\alpha R+\tfrac{3i}{16}\left(\epsilon^\alpha G_\mu-\tfrac{1}{3}(\epsilon\sigma_a\tilde\sigma_\mu)^\alpha G^a\right)\\
&\delta_\epsilon G_a\Big|=-4i\left(\overset{\circ}{\Psi}_a\bar\epsilon-\tfrac{1}{3}\overset{\circ}{\Psi}{}^b\tilde\sigma_b\sigma_a\bar\epsilon\right)+4i\left(\epsilon\overset{\circ}{\Psi}_a-\tfrac{1}{3}\epsilon\sigma_a\hat\sigma_b\overset{\circ}{\Psi}{}^b\right)+\\
&\qquad+i(\psi_b\sigma^b\bar\epsilon-\epsilon\sigma^b\bar\psi_b)G_a+\tfrac{i}{2}\epsilon_{abcd}(\psi^c\sigma^d\bar\epsilon+\epsilon\sigma^c\bar\psi^d)G^b+\\
&\qquad+2i\epsilon\psi^a\cdot\bar R-2i\bar\psi^a\bar\epsilon\cdot R\\
&\delta_\epsilon R\Big|=-\tfrac{4i}{3}\overset{\circ}{\Psi}{}^a\tilde\sigma_a\epsilon-2i\epsilon\sigma_b\bar\psi^b\cdot R-i\epsilon\psi^a\cdot G_a\\
&\delta_\epsilon\bar R\Big|=\tfrac{4i}{3}\bar\epsilon\hat\sigma_a\overset{\circ}{\bar\Psi}{}^a+2i\psi^b\sigma_b\bar\epsilon\cdot\bar R+i\bar\psi^a\bar\epsilon\cdot G_a
\end{aligned}}$$

To obtain the result, we have used

. COPY OF: TASK: TO ATTACH S.I.2b-1-4''' as 6' and a copy of 6''

Furthermore, taking into account that (see attached)

$$T_{ab}{}^{\alpha}\Big| = 2\, e_a{}^{\mu} e_b{}^{\nu}\, \overset{\circ}{\mathcal{D}}_{[\mu}\psi_{\nu]}{}^{\alpha} - \frac{i}{8}(\psi_{[a}\sigma_{b]}\hat{\bar\sigma}_c)^{\alpha} \bar G^{c} - \frac{i}{8}\psi_{[a}{}^{\alpha} G_{b]}\Big| - \frac{i}{4}(\bar\psi_{[a}\hat{\bar\sigma}_{b]})^{\alpha} R\Big|$$

$$\Downarrow$$

$$\bar\Psi^{a}_{\dot\alpha}\Big| = \overset{\circ}{\bar\Psi}{}^{a}_{\dot\alpha} + \frac{i}{8}\epsilon^{abcd}(\psi_b\sigma_c)_{\dot\alpha} G_d + \frac{1}{4}(\psi_b\sigma^{b})_{\dot\alpha}\bar G^{b} - \frac{1}{4}(\psi_b\sigma^{b})_{\dot\alpha} G^{a}\Big| + \frac{1}{2}(\bar\psi_b\hat{\bar\sigma}^{ab})_{\dot\alpha} R\Big|$$

$$\Leftrightarrow (\Psi^{b}\tilde\sigma_b)^{\alpha}\Big| = (\overset{\circ}{\Psi}{}^{b}\tilde\sigma_b)^{\alpha} + \frac{3}{4}\psi_b{}^{\alpha}G^{b} - \frac{3}{2}(\bar\psi_b\bar\sigma^{b})^{\alpha} R$$

<u>We find:</u>

$$(\sigma_b\bar\Psi^{b})_{\alpha}\Big| = (\sigma_b\overset{\circ}{\bar\Psi}{}^{b})_{\alpha} - \frac{3}{4}\psi_{\alpha b}G^{b} - \frac{3}{2}(\sigma^{b}\bar\psi_b)_{\alpha} R$$

$$\bar{\mathcal{D}}_{\dot\alpha} G^{a}\Big| = 4i\left(\overset{\circ}{\bar\Psi}{}_{\dot\alpha}{}^{a} - \frac{1}{3}(\overset{\circ}{\bar\Psi}{}^{b}\tilde\sigma_b\sigma^{a})_{\dot\alpha}\right) - i(\bar\psi_b\bar\sigma^{b})_{\dot\alpha}G^{a} - \frac{i}{2}\epsilon^{abcd}(\bar\psi_b\bar\sigma_c)_{\dot\alpha}G_d + 2i\bar\psi_{\dot\alpha}{}^{a}R$$

$$\Downarrow$$

$$\mathcal{D}_{\alpha} G^{a}\Big| = 4i\left(\overset{\circ}{\Psi}{}_{\alpha}{}^{a} - \frac{1}{3}(\sigma^{a}\tilde\sigma_b\bar\Psi^{b})_{\alpha}\right) - i(\sigma^{b}\bar\psi_b)_{\alpha}G^{a} - \frac{i}{2}\epsilon^{abcd}(\sigma_b\bar\psi_c)_{\alpha}G_d + 2i\psi_{\alpha}{}^{a}\bar R$$

$$\mathcal{D}_{\alpha} R\Big| = \frac{4i}{3}\epsilon\sigma_b\bar\Psi^{b}\Big| = \frac{4i}{3}(\sigma_b\overset{\circ}{\bar\Psi}{}^{b})_{\alpha} - i\psi_{\alpha b}G^{b} - 2i(\sigma_b\bar\psi^{b})\cdot R$$

$$\bar{\mathcal{D}}_{\dot\alpha}\bar R\Big| = \frac{4i}{3}(\Psi^{b}\sigma_b)_{\dot\alpha}\Big| = \frac{4i}{3}(\overset{\circ}{\Psi}{}^{b}\sigma_b)_{\dot\alpha} - i\bar\psi_{\dot\alpha b}G^{b} - 2i(\psi^{b}\sigma_b)_{\dot\alpha}\cdot\bar R$$

THIS ALLOWS US TO OBTAIN THE LOCAL SUSY TRANSFORMATIONS OF THE AUXILIARY FIELDS

$$\delta_\epsilon R\Big| = \epsilon^{\alpha}\mathcal{D}_{\alpha}R\Big| = \frac{4i}{3}\epsilon\sigma_b\overset{\circ}{\bar\Psi}{}^{b} - i\epsilon\psi_b\cdot G^{b} - 2i\epsilon\sigma_b\bar\psi^{b}\cdot R$$

$$\delta_\epsilon\bar R\Big| = \bar\epsilon^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\bar R\Big| = -\frac{4i}{3}\overset{\circ}{\Psi}{}^{b}\sigma_b\bar\epsilon + i\bar\psi_b\bar\epsilon\, G^{b} + 2i\psi^{b}\sigma_b\bar\epsilon\cdot\bar R$$

$$\delta_\epsilon G_a\Big| = \epsilon^{\alpha}\mathcal{D}_{\alpha}G^{a}\Big| + \bar\epsilon^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}G^{a}\Big| =$$

$$= -4i\left(\overset{\circ}{\Psi}{}_a\bar\epsilon - \frac{1}{3}\overset{\circ}{\Psi}{}^{b}\hat\sigma_b\sigma_a\bar\epsilon\right) + 4i\left(\epsilon\overset{\circ}{\bar\Psi}{}_a - \frac{1}{3}\epsilon\sigma_a\hat\sigma_b\overset{\circ}{\bar\Psi}{}^{b}\right)$$

$$+ i(\psi_b\sigma^{b}\bar\epsilon - \epsilon\sigma^{b}\bar\psi_b)G^{a}\Big| + \frac{i}{2}\epsilon^{abcd}(\psi_b\sigma_c\bar\epsilon + \epsilon\sigma_b\bar\psi_c)G_d\Big| +$$

$$+ 2i\epsilon\psi^{a}\cdot R\Big| - 2i\bar\psi^{a}\bar\epsilon\cdot R\Big|$$

Calculations for p 7

(!)

Instead of performing explicit calculations up to the very end, let us

- fix the bosonic part of the actions

$$\mathcal{L}_4\Big| = d^4x\cdot e\cdot\left(e_b^\nu e_a^\mu \hat R_{\mu\nu}{}^{ab} + \frac{3}{8}R\cdot\bar R + \frac{3}{32}G_aG^a + \text{fermions}\right)$$

- Recall that $\mathcal{L}_4^{EH} + \mathcal{L}_4^{RS}(e,\psi) = \frac{1}{2}\varepsilon_{abcd}R^{ab}\wedge e^c\wedge e^d - 4D\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\bar\psi^{\dot\alpha} + 4\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge D\bar\psi^{\dot\alpha}$ is inv. under "off-shell" SUSY $\begin{cases}\delta_\epsilon e^a = -2i\psi\sigma^a\bar\epsilon + 2i\epsilon\sigma^a\bar\psi\\ \delta\psi^\alpha = D\epsilon^\alpha,\end{cases}$

- Then, under the superspace induced susy of GRAVITON and GRAVITINO,

$$\delta e^a = -2i\psi\sigma^a\bar\epsilon + 2i\epsilon\sigma^a\bar\psi$$

$$\delta\psi^\alpha = D\epsilon^\alpha + i_\epsilon T^\alpha\Big| = D\epsilon^\alpha + \underbrace{e^b\wedge\epsilon^\beta T_{\beta b}{}^\alpha\Big|}_{} + \underbrace{e^b\bar\epsilon^{\dot\beta}T_{\dot\beta b}{}^\alpha\Big|}_{=e^b\frac{i}{8}(\bar\epsilon\tilde\sigma_b)^\alpha R|} = D\epsilon^\alpha + \frac{i}{8}e^b((\epsilon\sigma_b\tilde\sigma_c)^\alpha G^c(\bar\epsilon\tilde\sigma_b\dots$$

$$\Big\llcorner e^b\cdot\frac{i}{8}(\epsilon\sigma_b\tilde\sigma_c)^\alpha G^c$$

$$\Big\llcorner \mathring{D}\epsilon^\alpha - \frac{1}{4}\omega^{cb}(\epsilon\sigma_{cb})^\alpha \quad \text{"}-\frac{1}{8}e_c\,\epsilon^{abcd}G_d\text{"} = +\frac{1}{32}e_aG_b\,\epsilon^{abcd}(\epsilon\sigma_{cd})^\alpha \quad \text{"}-2i(\epsilon\sigma^{ab})^\alpha\text{"} = -\frac{i}{16}e^b(\epsilon\sigma_{bc})^\alpha G^c$$

$$\Big\llcorner \frac{i}{8}e^b\epsilon^\alpha G_b + \frac{i}{16}e^b(\epsilon\sigma_{bc})^\alpha G^c$$

$$\frac{3i}{16}e^b\left(\epsilon^\alpha G_b - \frac{1}{3}(\epsilon\sigma_c\tilde\sigma_b)^\alpha G^c\right) \qquad \text{"}-\epsilon\sigma_c\tilde\sigma_b + \eta_{bc}\text{"}$$

$$\delta_\epsilon\psi^\alpha = \mathring{D}\epsilon^\alpha + e^b\,\frac{i}{8}(\bar\epsilon\tilde\sigma_b)^\alpha R + e^b\,\frac{3i}{16}\left(\epsilon^\alpha G_b - \frac{1}{3}(\epsilon\sigma_c\tilde\sigma_b)^\alpha G^c\right)$$

$$\delta_\epsilon\bar\psi^{\dot\alpha} = \mathring{D}\bar\epsilon^{\dot\alpha} + e^b\left(-\frac{i}{8}\right)(\tilde\sigma_b\epsilon)^{\dot\alpha}\bar R + e^b\left(-\frac{3i}{16}\right)\left(\bar\epsilon^{\dot\alpha}G_b - \frac{1}{3}(\tilde\sigma_b\sigma_c\bar\epsilon)^{\dot\alpha}G^c\right)$$

"on-shell susy" ⇒ only "new" terms contribute

$$\delta_\epsilon\left(\mathcal{L}_4^{EH} + \mathcal{L}_4^{RS}\right) \overset{\text{mod div}}{=\!=\!=} -\mathcal{E}_{a3}\wedge\delta_\epsilon e^a + 8\,\delta_\epsilon\psi^\alpha\wedge\underbrace{\mathcal{E}_{3\alpha}}_{\frac{1}{2}e_a^{\wedge 3}\bar\psi^a_{\dot\alpha}} - 8\,\underbrace{\mathcal{E}_{3\dot\alpha}}_{\frac{1}{2}e_a^{\wedge3}\psi^a_\alpha}\wedge\delta_\epsilon\bar\psi^{\dot\alpha} =$$

$$= 0 + 4\,(\delta_\epsilon\psi^\alpha - \mathring{D}\epsilon^\alpha)\wedge e_a^{\wedge3}\bar\psi^a_\alpha - 4\,e_a^{\wedge3}\psi^a_\alpha\wedge(\delta_\epsilon\bar\psi^{\dot\alpha} - \mathring{D}\bar\epsilon^{\dot\alpha}) =$$

$$= -4\,\underbrace{e_a{}^{\wedge3}\wedge e^b}_{d^4x\cdot e\cdot\delta_a^b}\left(\frac{i}{8}(\bar\epsilon\tilde\sigma_b\bar\psi^a)\cdot R + \frac{3i}{16}\left(\epsilon^\alpha\bar\psi^a_\alpha G_b - \frac{1}{3}\epsilon\sigma_c\tilde\sigma_b\bar\psi^a\cdot G^c\right)\right.$$

$$\left. - \frac{i}{8}(\psi^a\tilde\sigma_b\epsilon)\cdot\bar R - \frac{3i}{16}\left(\psi^a_{\dot\alpha}\bar\epsilon^{\dot\alpha}G_b - \frac{1}{3}\psi^a\tilde\sigma_b\sigma_c\bar\epsilon\cdot G^c\right)\right) = \frac{i}{2}d^4x\,e\left(\psi^a\tilde\sigma_a\epsilon\cdot\bar R - \bar\epsilon\tilde\sigma_a\bar\psi^a R\right) + \frac{3i}{4}d^4x\,e\left(\psi^a_{\dot\alpha}\bar\epsilon^{\dot\alpha}G_a - \epsilon\bar\psi^a G_a - \frac{1}{3}\psi^a\tilde\sigma_a\sigma_b\bar\epsilon\,G^b + \frac{1}{3}\epsilon\sigma_b\tilde\sigma_a\bar\psi^a G^b\right)$$

$$\delta\, d^4x\,e\left(\frac{3}{8}R\bar R + \frac{3}{32}G_aG^a\right) = d^4x\,e\left(2i(\epsilon\sigma^a\bar\psi_a - \psi_a\sigma^a\bar\epsilon)\left(\frac{3}{8}R\bar R + \frac{3}{32}G_aG^a\right) + \frac{3}{8}R\,\delta\bar R + \frac{3}{8}\bar R\,\delta R + \frac{3}{16}G_a\,\delta G^a\right)$$

$\delta e = e\,e_a^\mu\delta e_\mu^a = -2ie(\psi_a\sigma^a\bar\epsilon - \epsilon\sigma^a\bar\psi_a)$

$$\Rightarrow \begin{cases}\delta_\epsilon G_a = -4i\left(\mathring{\bar\psi}_a{}^{\dot\alpha}\bar\epsilon - \frac{1}{3}\mathring{\bar\psi}^b\tilde\sigma_b\sigma_a\bar\epsilon\right) + 4i\left(\epsilon^\alpha\mathring\psi_{\alpha a} - \frac{1}{3}\epsilon\sigma_a\tilde\sigma_b\mathring\psi^b\right) - i(\epsilon\sigma^b\bar\psi_b - \psi_b\sigma^b\bar\epsilon)\,G_a + a_{ab}(\epsilon)G^b - 2h_a(\epsilon)\bar R - 2\bar h_a(\epsilon)R\\[4pt] \delta_\epsilon R = \frac{4i}{3}\mathring{\psi}^a\tilde\sigma_a\epsilon - i(\epsilon\sigma^a\bar\psi_a - \psi_a\sigma^a\bar\epsilon)\,R + i\alpha(\epsilon)R + h_a(\epsilon)G^a \quad \to -2i\epsilon\sigma_b\bar\psi^b\cdot R\\[4pt] \delta_\epsilon\bar R = +\frac{4i}{3}\bar\epsilon\tilde\sigma_a\mathring{\bar\psi}^a - i(\epsilon\sigma^a\bar\psi_a - \psi_a\sigma^a\bar\epsilon)\bar R - i\alpha(\epsilon)\bar R + \bar h_a(\epsilon)G^a\end{cases}$$

This coincides with what we obtain for SSP

$\delta_\epsilon G^a| = \epsilon^\alpha\mathcal{D}_\alpha G^a| + \bar\epsilon_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}G^a|$

$\delta_\epsilon R| = \epsilon^\alpha\mathcal{D}_\alpha R| = \frac{4i}{3}\epsilon\sigma_b\bar\psi^b| = \frac{4i}{3}\epsilon\sigma_b\mathring{\bar\psi}^b - i\epsilon\psi_b G^b - 2i\epsilon\sigma_b\bar\psi^b R$

$\delta_\epsilon\bar R| = \bar\epsilon^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\bar R| = -\frac{4i}{3}\psi^b\sigma_b\bar\epsilon| =$

the true: to get $\delta_\epsilon R$ we have to set

$$\boxed{i\alpha(\epsilon) = -i\epsilon\sigma_b\bar\psi^b - i\psi_b\sigma^b\bar\epsilon}$$

$$\boxed{h_a = -i\epsilon\psi_a}$$

$$\boxed{\bar h_a = i\bar\psi_a\bar\epsilon}$$

to get true $\delta_\epsilon G_a$ we set

$$\boxed{a_{ab}(\epsilon) = \frac{i}{2}\epsilon_{abcd}\left(\psi^b\sigma^c\bar\epsilon + \epsilon\sigma^b\bar\psi^c\right)}$$

$h_a(\epsilon) = -i\epsilon\psi^a$

$\bar h_a(\epsilon) = i\bar\psi^a\bar\epsilon$

FROM SUPERSPACE TO COMPONENT ACTION. ECTOPLASM METHOD.

S.I.2.-61—

$$S = \int_{M^4} \frac{1}{4!} d^4x \, \epsilon^{\mu\nu\rho\sigma} E_\mu{}^A E_\nu{}^B E_\rho{}^C E_\sigma{}^D \mathcal{L}_{DCBA} = \int_{M^4} \hat{\mathcal{L}}_4 \quad ; \quad \frac{\partial}{\partial\theta^\alpha} S = 0 \text{ if } \boxed{d\mathcal{L}_4 = 0}$$

$$d\theta^\alpha \frac{\partial}{\partial\theta^\alpha} \int_{M^4} \mathcal{L}_4 = \int_{M^4} d\mathcal{L}_4 - \cancel{\int_{\partial M^4} \mathcal{L}_4}$$

$\underline{d\mathcal{L}_4 = 0}$:

$$\mathcal{L}_4 = \mathcal{L}_{(2|2,0)} + \mathcal{L}_{(2|0,2)} + \mathcal{L}_{(3|1,0)} + \mathcal{L}_{(3|0,1)} + \mathcal{L}_{(4|0,0)} =$$

$$= \frac{1}{4} E^b \wedge E^a \wedge E^{\dot\alpha} \wedge E^\beta (\sigma_{ab})_{\alpha\beta} \bar{\mathcal{L}} + \frac{1}{4} E^b \wedge E^a \wedge E^{\dot\alpha} \wedge E^{\dot\beta} (\bar\sigma_{ab})_{\dot\alpha\dot\beta} \mathcal{L} + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge \epsilon_{abcd} \left(E^\alpha \sigma^d_{\alpha\dot\alpha} \bar{\mathcal{D}}^{\dot\alpha} \bar{\mathcal{L}} + E^{\dot\alpha} \sigma^d_{\alpha\dot\alpha} \mathcal{D}^\alpha \mathcal{L} \right) -$$

$$- \frac{i}{16 \cdot 4!} E^d \wedge E^c \wedge E^b \wedge E^a \epsilon_{abcd} \left[(\mathcal{D}\mathcal{D} - 3\bar{R})\mathcal{L} - (\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{\mathcal{L}} \right]$$

$$\mathcal{D}_\alpha \bar{\mathcal{L}} = 0, \quad \bar{\mathcal{D}}_{\dot\alpha} \mathcal{L} = 0$$

MIN SUGRA ACTION

$$S' = \sim \int d^8z \, E = \sim \left(\frac{1}{4} \int d^6 z_L \, \mathcal{E} R + \frac{1}{4} \int d^6 z_R \, \bar{\mathcal{E}} \bar{R} \right) = \int_{M^4} \overset{= \hat{\mathcal{L}}_4}{\mathcal{L}_4(x,0)} = \frac{1}{4!} \int d^4x \, \epsilon^{\mu\nu\rho\sigma} E_\mu{}^A \ldots E_\sigma{}^D \mathcal{L}_{DCAB} \Big|_{\theta=0=\bar\theta}$$

Clearly $\bar{\mathcal{L}} \sim \bar{R}$, $\mathcal{L} \sim R$. Correct coeff. is fixed by requiring $\hat{\mathcal{L}}_4 = d^4x \, e \, R_{ab}{}^{ab} | + \ldots$

$$\boxed{\mathcal{L} = 3iR} \, , \quad \boxed{\bar{\mathcal{L}} = -3i\bar{R}} \qquad \boxed{\hat{\mathcal{L}}_4 = d^4x \, e \, L^{SG}}$$

$$L^{\min SG} = \underbrace{R_{ab}{}^{ab} + \frac{3}{8} R \bar{R}}_{L} - \frac{3i}{4} \mathcal{D}^\alpha R \cdot (\sigma_a \bar\psi)_\alpha - \frac{3i}{4} (\psi^a \sigma_a)_{\dot\alpha} \bar{\mathcal{D}}^{\dot\alpha} \bar{R} - \frac{3}{2} \psi_a \sigma^{ab} \psi_b \cdot \bar{R} - \frac{3}{2} \bar\psi_a \bar\sigma^{ab} \bar\psi_b \cdot R$$

$$- \frac{3}{16} (\mathcal{D}\mathcal{D} R + \bar{\mathcal{D}}\bar{\mathcal{D}} \bar{R} - 6 R \bar{R})$$

$$= \underbrace{R_{ab}{}^{ab}}_{L} + \frac{3}{8} R\bar{R} - \psi_b \sigma^b \bar\sigma^a \bar\Psi_a - \Psi_a \bar\sigma^a \sigma^b \bar\psi_b - \frac{3}{2} \psi_a \sigma^{ab} \psi_b \cdot \bar{R} - \frac{3}{2} \bar\psi_a \bar\sigma^{ab} \bar\psi_b \cdot R$$

$$\rightarrow L: \; e_a{}^\mu e_b{}^\nu \overset{\circ}{R}_{\mu\nu}{}^{ab} - 2 \overset{\circ}{\Psi}{}_\alpha^{\;a} \bar\psi_a{}^{\dot\alpha} - 2 \psi_a{}^\alpha \overset{\circ}{\bar\Psi}{}_{\dot\alpha}{}^a + \frac{3}{32} G^a G_a + \Psi^a \bar\sigma_a \sigma_b \bar\psi^b + \psi^b \sigma_b \bar\sigma_a \bar\Psi_a + \frac{3}{2} \psi_a \sigma^{ab} \psi_b \cdot \bar{R} + \frac{3}{2} \bar\psi_a \bar\sigma^{ab} \bar\psi_b R$$

see

$$\boxed{L^{\min SG} = e_a{}^\mu e_b{}^\nu \overset{\circ}{R}_{\mu\nu}{}^{ab} - 2 \overset{\circ}{\Psi}{}_\alpha^{\;a} \bar\psi_a{}^{\dot\alpha} - 2 \psi_a{}^\alpha \overset{\circ}{\bar\Psi}{}_{\dot\alpha}{}^a + \frac{3}{32} G^a G_a + \frac{3}{8} R\bar{R}}$$