SUPER-WEYL TRANSFORMATIONS IN/FROM MIN SG

$$E^a \longmapsto \hat E^a = e^{W+\bar W} E^a$$

$$E^\alpha \longmapsto \hat E^\alpha = e^{W}\left(E^\alpha + E^a \Xi_a{}^\alpha\right)$$

$$\bar E^{\dot\alpha} \longmapsto \hat{\bar E}^{\dot\alpha} = e^{\bar W}\left(\bar E^{\dot\alpha} + E^a \bar\Xi^{\dot\alpha}{}_a\right)$$

$$\omega^{ab} \longmapsto \hat\omega^{ab} = \omega^{ab} + \Delta\omega^{ab}$$

- $$\hat T^a = \hat{\mathcal D}\hat E^a = -2i\,\hat E^\alpha \wedge \hat{\bar E}^{\dot\alpha}\,\sigma^a_{\alpha\dot\alpha} + \hat E^b \wedge \hat E^\beta\,\hat T_{\beta b}{}^a + \tfrac{1}{i}\hat E^c \wedge \hat E^b\,\hat T_{bc}{}^a$$

$\sim E^\alpha \wedge \bar E^{\dot\alpha}$ ⇐ REQUIRING THE PRESERVATION OF $T_{\alpha\dot\beta}{}^a = -2i\sigma^a_{\alpha\dot\beta}$ = IS SATISFIED IDENTICALLY

$\sim E^b \wedge E^\beta$

$\underline{Ex}$: to obtain →
$$\hat T_{\beta b}{}^a = \delta_b{}^a e^{-W}\nabla_\beta(W+\bar W) - \overset{e^{-W}}{\Delta W_{\beta b}{}^a} - 2i\,\overset{e^{-W}}{\bar\Xi^{\dot\beta}{}_b}\,\sigma^a_{\beta\dot\beta}$$

- $$\hat T^\alpha = \hat{\mathcal D}\hat E^\alpha = \mathcal D\left(e^W(E^\alpha + E^a\Xi_a{}^\alpha)\right) - e^W(E^\beta + E^a\Xi_a{}^\beta)\wedge\Delta w_\beta{}^\alpha =$$
$$= e^W \underset{\sim E^b}{T^\alpha} - 2i e^W E^\beta\wedge\bar E^{\dot\beta}\sigma^a_{\beta\dot\beta}\Xi_a{}^\alpha + E^\beta\wedge E^\gamma\left(\mathcal D_{(\beta}e^W\delta_{\gamma)}{}^\alpha - e^W\Delta W_{(\beta\gamma)}{}^\alpha\right) +$$
$$+ E^\beta \wedge \bar E^{\dot\beta}\left(\bar{\mathcal D}_{\dot\beta}e^W\delta_\beta{}^\alpha - e^W\Delta W_{\dot\beta\beta}{}^\alpha\right) + \sim E^b$$

$\sim E^\beta\wedge\bar E^{\dot\beta}$ ⇐ REQUIREMENT OF PRESERVATION OF $\boxed{T_{\beta\dot\gamma}{}^\alpha = 0}$

RESULTS IN $$0 = \delta_\gamma{}^\alpha \bar{\mathcal D}_{\dot\beta}e^W - e^W\Delta W_{\dot\beta\gamma}{}^\alpha - 2ie^W\sigma^a_{\gamma\dot\beta}\Xi_a{}^\alpha$$

WHICH IS SOLVED BY

$$\boxed{\begin{array}{l} \Xi_a{}^\alpha = -\frac{i}{4}\bar{\mathcal D}_{\dot\alpha}W\cdot\tilde\sigma_a{}^{\dot\alpha\alpha} \\ \Delta W_{\dot\beta\beta\alpha} = 0 \end{array}} \Rightarrow \bar\Xi^{\dot\alpha}{}_a = \frac{i}{4}\mathcal D_\alpha\bar W\,\tilde\sigma_a{}^{\dot\alpha\alpha}$$

$$\Leftrightarrow \boxed{\begin{array}{l} \Delta W_{\dot\gamma}{}^{ab} = +\frac{1}{2}\Delta W_{\dot\gamma}{}^{\dot\alpha\dot\beta}\tilde\sigma^{ab}_{\dot\alpha\dot\beta} \\ \Delta W_\gamma{}^{ab} = \frac{1}{2}\Delta W_\gamma{}^{\alpha\beta}\sigma^{ab}_{\alpha\beta} \end{array}}$$

$$\boxed{\begin{array}{l} \hat E^a = e^{W+\bar W}E^a \\ \hat E^\alpha = e^W\left(E^\alpha - \frac{i}{4}E^a\tilde\sigma_a{}^{\dot\alpha\alpha}\bar{\mathcal D}_{\dot\alpha}W\right) \\ \hat{\bar E}^{\dot\alpha} = e^{\bar W}\left(\bar E^{\dot\alpha} - \frac{i}{4}E^a\tilde\sigma_a{}^{\dot\alpha\alpha}\mathcal D_\alpha\bar W\right) \end{array}}$$

$$\hat\omega^{ab} = \omega^{ab} + \tfrac{1}{2}E^\gamma\Delta W_\gamma{}^{\alpha\beta}\sigma^{ab}_{\alpha\beta} + \tfrac{1}{2}\bar E^{\dot\gamma}\Delta W_{\dot\gamma}{}^{\dot\alpha\dot\beta}\tilde\sigma^{ab}_{\dot\alpha\dot\beta} + E^c\Delta W_c{}^{ab}$$

- $\widetilde{T}^\alpha = \widehat{D}\widetilde{E}^\alpha = \ldots$

2) $\sim E^\beta \wedge E^\gamma$, i.e. $0 = \widetilde{T}_{\beta\gamma}{}^\alpha = 2\left(\delta_{(\beta}{}^\alpha \mathcal{D}_{\gamma)} e^W - e^W \Delta w_{(\beta\gamma)}{}^\alpha\right)$

$\Rightarrow \Delta w_{(\beta\gamma)}{}^\alpha = \delta_{(\beta}{}^\alpha \mathcal{D}_{\gamma)} W \underset{\Delta w_{\alpha\beta}{}^\beta = 0}{\Rightarrow}$ $\boxed{\Delta w_\gamma{}^{\alpha\beta} = 2\delta_\gamma{}^{(\alpha}\mathcal{D}^{\beta)} W}$

- $\widetilde{R}^{\alpha\beta} - R^{\alpha\beta} = \mathcal{D}\Delta w^{\alpha\beta} - \Delta w^{\alpha\varepsilon}\wedge\Delta w_\varepsilon{}^\beta = \underbrace{-\frac{1}{2}\widehat{E}^\alpha\wedge\widehat{E}^\beta\widetilde{\bar{R}} + \frac{1}{2}E^\alpha\wedge E^\beta \bar{R}}_{-\frac{1}{2}e^{2W}E^\alpha\wedge E^\beta\left(\widetilde{\bar{R}} - e^{-2W}\bar{R}\right)} + \sim E^c$ $+ \sim E^c$

$\sim E^\gamma \wedge \bar{E}^{\dot\gamma}$: $\boxed{\Delta w_{\gamma\dot\gamma}{}^{\alpha\beta} = -\frac{i}{2}\bar{\mathcal{D}}_{\dot\gamma}\Delta w_\gamma{}^{\alpha\beta} = -i\bar{\mathcal{D}}_{\dot\gamma}\mathcal{D}^{(\alpha}W\,\delta_\gamma^{\beta)}}$

$\Delta w_c{}^{\alpha\beta} = -\frac{i}{2}\bar{\sigma}_c{}^{\dot\gamma(\alpha}\bar{\mathcal{D}}_{\dot\gamma}\mathcal{D}^{\beta)}W$

$\sim E^\gamma \wedge E^\delta$ $\boxed{\widetilde{\bar{R}} = e^{-2W}\bar{R} - \bar{\mathcal{D}}\bar{\mathcal{D}}e^{-2W} = -(\bar{\mathcal{D}}\bar{\mathcal{D}} - \bar{R})e^{-2W}}$

$\Uparrow$ $\boxed{= e^{-2W}(\bar{R} + 2\bar{\mathcal{D}}\bar{\mathcal{D}}W - 4\bar{\mathcal{D}}^\alpha W \cdot \bar{\mathcal{D}}_\alpha W)}$

This GUARANTEES THAT $\widetilde{\mathcal{D}}_\alpha \widetilde{\bar{R}} = 0$

REASON: FOR SCALAR SUPERFIELD $\widehat{\mathcal{D}}_\alpha = \widehat{\nabla}_\alpha \sim \nabla_\alpha = \mathcal{D}_\alpha$

Ex:
TO SHOW THAT
$\boxed{\widehat{\nabla}_\alpha = e^{-W}\nabla_\alpha}$, $\boxed{\widehat{\bar{\nabla}}_{\dot\alpha} = e^{-\bar{W}}\bar{\nabla}_{\dot\alpha}}$, $\widehat{\nabla}_a = e^{-W-\bar{W}}\nabla_a + \frac{i}{4}\hat{\sigma}_a{}^{\dot\alpha\alpha}(\bar{\nabla}_{\dot\alpha}W\,\nabla_\alpha + \nabla_\alpha\bar{W}\,\bar{\nabla}_{\dot\alpha})$

- Back to $\widetilde{T}_{\beta b}{}^a = \delta_b{}^a e^{-W}\nabla_\beta(W+\bar{W}) - e^{-W}\overbrace{\Delta w_{\beta b}{}^a}^{\Gamma(\sigma_b{}^a)_\beta{}^\alpha\nabla_\alpha W} - 2ie^{-W}\overbrace{\hat{\xi}_b{}^{\dot\beta'}}^{=-\frac{i}{4}\hat{\sigma}_b{}^{\dot\beta'\beta}\nabla_\beta\bar{W}}\sigma^a_{\beta\dot\beta'} =$

$= \frac{1}{4}(\sigma_b\hat{\sigma}^a)_\beta{}^\alpha\widetilde{T}_\alpha$

$\boxed{\widetilde{T}_\alpha := \widetilde{T}_{\alpha b}{}^b = 2e^{-W}\nabla_\alpha(2W+\bar{W})}$

$\boxed{= 2\widetilde{\nabla}_\alpha(2W+\bar{W})}$

$\boxed{W = \frac{2}{3}\bar{\Sigma} - \frac{1}{3}\Sigma}$
$\boxed{\bar{W} = \frac{2}{3}\Sigma - \frac{1}{3}\bar{\Sigma}}$

MIN SUGRA CONSTRAINTS ARE INV. UNDER superWeyl transf-s WITH CHIRAL "PARAMS"
$\nabla_\alpha\bar{\Sigma} = 0$, $\bar{\nabla}_{\dot\alpha}\Sigma = 0$
$2W + \bar{W} = \bar{\Sigma}$
$W + 2\bar{W} = \Sigma$

Remember: WHAT WE ARE DOING IS TO RESCALE $E, \bar{E}$ (see solution

$$\tilde{T}_\alpha := \widehat{T}_{\alpha b}{}^b = 2\widehat{\nabla}_\alpha(2W+\bar{W}) \qquad \boxed{\tilde{F} = e^{-W}F} \qquad \tilde{\nabla}_\alpha = e^{-W}\nabla_\alpha$$

$$\Updownarrow \qquad\qquad\qquad\qquad \nabla_\alpha \leftarrow F\hat{\Delta}_\alpha$$

$$\tilde{T}_\alpha = -\underbrace{\tilde{\nabla}_\alpha}_{=\tilde{F}\hat{\Delta}_\alpha} \ln \tilde{F}^4\tilde{\bar{F}}^2\hat{z} \underset{\text{Generically}}{\neq} 0$$

Remember:

$$T_\alpha = 0 \Rightarrow \quad \hat{z}F^4\bar{F}^2 = \bar{\varphi}^{-3}, \quad \nabla_\alpha\bar{\varphi} = 0$$

Omitting $\sim$ for shortness ($\tilde{F} \to F$ etc.)
we can search for other type of constraints ($\mathrm{II}^{\mathrm{nd}}$ class constr.)

$$\bar{R} = -2F\hat{\Delta}^\alpha\hat{\Delta}_\alpha F - 2\hat{\Delta}^\alpha F\cdot\hat{\Delta}_\alpha F = -F^2\left(\hat{\Delta}^\alpha\hat{\Delta}_\alpha \ln F^2 + \hat{\Delta}^\alpha \ln F^2\,\hat{\Delta}_\alpha \ln F^2\right) = -\hat{\Delta}^\alpha\hat{\Delta}_\alpha F^2$$

suggests to search for some constraints
expressing $F, \bar{F}$ in terms of complex linear multiplet

as
$$\mathcal{D}^\alpha T_\alpha = -F^2\hat{\Delta}^\alpha\hat{\Delta}_\alpha \ln(\hat{z}F^4\bar{F}^2) \underbrace{+2F\hat{\Delta}^\alpha \ln F^2\cdot T_\alpha}_{=-2F^2\hat{\Delta}^\alpha \ln F^2\cdot\hat{\Delta}_\alpha \ln(\hat{z}F^4\bar{F}^2)}$$

$$T^\alpha T_\alpha = F^2\hat{\Delta}^\alpha \ln(\hat{z}F^4\bar{F}^2)\,\hat{\Delta}_\alpha \ln(\hat{z}F^4\bar{F}^2)$$

$$\bar{R} - \bar{\zeta}\mathcal{D}^\alpha T_\alpha - \bar{\zeta}^2 T^\alpha T_\alpha =$$
$$= -F^2\Big(\hat{\Delta}^\alpha\hat{\Delta}_\alpha \ln F^2 + \hat{\Delta}^\alpha\hat{\Delta}_\alpha \ln(\hat{z}F^4\bar{F}^2)^{-\bar{\zeta}} +$$
$$+\underbrace{(\hat{\Delta}\ln F^2)^2 + 2\hat{\Delta}^\alpha \ln F^2\cdot\hat{\Delta}_\alpha \ln(\hat{z}F^4\bar{F}^2)^{-\bar{\zeta}} + (\hat{\Delta}\ln(\hat{z}F^4\bar{F}^2))^2}_{\hat{\Delta}^\alpha \ln F^2(\hat{z}F^4\bar{F}^2)^{-\zeta}\cdot\hat{\Delta}_\alpha \ln F^2(\hat{z}F^4\bar{F}^2)^{-\zeta}}\Big) =$$

$$= -\frac{F^2}{\bar{\Upsilon}}\hat{\Delta}^\alpha\hat{\Delta}_\alpha\bar{\Upsilon} \qquad \boxed{\bar{\Upsilon} = F^2(\hat{z}F^4\bar{F}^2)^{-\zeta}}$$

$$\boxed{\bar{R} - \bar{\zeta}\mathcal{D}^\alpha T_\alpha - \bar{\zeta}^2 T^\alpha T_\alpha = -\frac{F^2}{\bar{\Upsilon}}\hat{\Delta}^\alpha\hat{\Delta}_\alpha\bar{\Upsilon}}$$

Thus imposing $\bar{R} - \bar{\zeta}\mathcal{D}^\alpha T_\alpha - \bar{\zeta}^2 T^\alpha T_\alpha = 0 \iff \boxed{\hat{\Delta}^\alpha\hat{\Delta}_\alpha\bar{\Upsilon} = 0}$

MIN, NON-MIN AND "NEW MIN" SG

$T_{\alpha b}{}^{b} = 0 \quad (\Rightarrow T_{\alpha b}{}^{a}=0)$ min SG $\quad n=-1/3 \quad (\xi=\infty)$

Ferrara + Nieuwenhuizen 78, Stelle + West 78

$$\left\{ \begin{array}{l} T_{\alpha b}{}^{b} = T_\alpha = -F\Delta_\alpha \ln \hat{\varepsilon} F^4 \bar F^2 \neq 0 \\ \bar R - \xi \mathcal{D}^\alpha T_\alpha - \xi T^\alpha T_\alpha = 0 \end{array}\right. \qquad \xi = \frac{n+1}{3n+1}$$

(n) = Siegel's parameter

$$\Downarrow \; -\frac{F^2}{\bar\Upsilon}\hat\Delta^\alpha\hat\Delta_\alpha \bar\Upsilon$$

$$\bar\Upsilon = F^2(\hat\varepsilon F^4\bar F^2)^{-\xi}, \qquad \hat\Delta^\alpha\hat\Delta_\alpha\bar\Upsilon=0$$

$$\Upsilon = \bar F^2(\hat\ell F^2\bar F^4)^{-\xi}, \qquad \hat{\bar\Delta}_{\dot\alpha}\hat{\bar\Delta}^{\dot\alpha}\Upsilon = 0$$

Generic ξ (n) (except for $n+n^*=0$)

$\Rightarrow$ NON-MINIMAL SUGRA

Breitenlohner + Sohnius
Siegel + Gates 79

$$F = F(\hat\ell, \hat\varepsilon, \Upsilon, \bar\Upsilon)$$ (Ex: calculate explicit form)

SINGULAR POINTS:

- $n=-1/3$ min SG $(\xi=\infty)$
- $n=0$ "new MINIMAL" OR "ALTERNATIVE MIN." SG
  Sohnius + West 81
  $\xi = 1$
  actually
  "New" min $\Leftarrow$ [Akulov + Volkov + Soroka 1975, 77]

To see that this point ($n=0$) is singular:

$$0 = \bar R - \mathcal{D}^\alpha T_\alpha - T^\alpha T_\alpha = -\frac{F^2}{\bar\Upsilon}\hat\Delta^\alpha\hat\Delta_\alpha\bar\Upsilon = -\hat\varepsilon F^4\bar F^2\hat\Delta^\alpha\hat\Delta_\alpha\left(\frac{1}{\hat\varepsilon F^2\bar F^2}\right)$$

$$\bar\Upsilon = F^2(\hat\varepsilon F^4\bar F^2)^{-1} = \frac{1}{\hat\varepsilon F^2\bar F^2}, \qquad \hat\Delta^\alpha\hat\Delta_\alpha\Upsilon=0$$

$$\Upsilon = \bar F^2(\hat\ell F^2\bar F^4)^{-1} = \frac{1}{\hat\ell F^2\bar F^2}, \qquad \hat{\bar\Delta}_{\dot\alpha}\hat{\bar\Delta}^{\dot\alpha}\bar\Upsilon = 0$$

1) $F/\bar F$ is NOT DETERMINED (U(1) gauge symm.!)
Guess: $A_\mu \in H^P$ is a U(1) connection!

2) $\hat\varepsilon\bar\Upsilon = \hat\ell\Upsilon = G$ complex linear superfield ~ its c.c.

$$\boxed{\hat\Delta^\alpha\hat\Delta_\alpha(\hat\varepsilon^{-1}G)=0}$$

IN THE FLAT SSP: real linear

IN FLAT SSP REAL LINEAR MULTIPLET OBEYS

$$D^\alpha D_\alpha G = 0 = \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha} G$$

Non-vanishing components

$$G|_0\,,\quad D_\alpha G|_0\,,\quad \bar{D}_{\dot\alpha} G|_0\,,\quad \tilde{\sigma}_a^{\dot\alpha\alpha}[D_\alpha,\bar{D}_{\dot\alpha}]G|_0 = V_a$$

$$0 = \bar{D}\bar{D}DDG = \bar{D}_{\dot\alpha}\{\bar{D}^{\dot\alpha},D^\alpha\}D_\alpha G - \bar{D}^{\dot\alpha}D^\alpha\bar{D}_{\dot\alpha}D_\alpha G = \ldots = \sim \partial_a V^a$$

$$\boxed{\partial_a V^a = 0} \quad \Rightarrow \quad V^a = \epsilon^{abcd}\partial_b B_{cd}$$

Thus one of the components is "notoph" –
– antisymmetric tensor gauge field (dual to a pseudo-scalar in $D=4$)

$\Rightarrow$ second name: "tensor multiplet"

COMPLEX LINEAR MULTIPLET IN FLAT SSP

$$\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}\Upsilon = 0 \qquad (D^\alpha D_\alpha \Upsilon \neq 0)$$

$$\Upsilon|_0\,,\ D_\alpha\Upsilon|_0\,,\ \bar{D}_{\dot\alpha}\Upsilon|_0\,,\ [D_\alpha,\bar{D}_{\dot\alpha}]\Upsilon|_0 = \sigma^a_{\alpha\dot\alpha}\Upsilon_a\,,\ DD\Upsilon|_0\,,\ \bar{D}_{\dot\alpha}DD\Upsilon|_0$$

Its CONSTRAINTS CAN BE SOLVED BY $\Upsilon = \bar{D}_{\dot\alpha}\bar{\Xi}^{\dot\alpha}$ with unrestricted $\bar{\Xi}^{\dot\alpha}$.

DYNAMICAL EQS. OF FREE COMPLEX LINEAR MULTIPLET ARE $\boxed{D_\alpha\Upsilon = 0}$

THUS (DYNAMICAL FREE) Υ IS DUAL TO (ANTI-) CHIRAL SUPERFIELD $\bar\Phi$ FOR WHICH $D_\alpha\bar\Phi = 0$ CONSTRAINTS $\bar{D}\bar{D}\bar\Phi = 0$ EQS OF MOTION

BUT WHAT # OF D.O.F.S BRINGS Υ WHEN USED AS A COMPENSATOR IN THE OFF-SHELL FORMULATION OF SG?