

Ex-5:

SV. 1 b - (1)
SWeyl - (10)



INVERSE SUPER-WEYL TRANSFORMATIONS

Ex: TO OBTAIN

$$\tilde{E}^a = e^{W+\bar{W}} E^a$$

$$E^a = e^{-W-\bar{W}} \tilde{E}^a$$

$$\tilde{E}^\alpha = e^W (E^\alpha - \frac{1}{4} E^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \nabla_a W)$$

$$E^\alpha = e^{-W} (\tilde{E}^\alpha + \frac{1}{4} \tilde{E}^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \nabla_a W)$$

$$\tilde{E}^{\dot{\alpha}} = e^{\bar{W}} (\bar{E}^{\dot{\alpha}} - \frac{1}{4} E^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \nabla_a \bar{W})$$

$$\bar{E}^{\dot{\alpha}} = e^{-\bar{W}} (\tilde{E}^{\dot{\alpha}} + \frac{1}{4} \tilde{E}^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \nabla_a \bar{W})$$

$$\begin{aligned} \tilde{W}^{\alpha\beta} &= W^{\alpha\beta} + 2E^{(\alpha} \mathcal{D}^{\beta)} W - \frac{1}{2} E^a \hat{\sigma}_a^{\dot{\beta}\beta} \mathcal{D}_p \nabla^{\alpha)} W = \\ &= W^{\alpha\beta} + 2\tilde{E}^{(\alpha} \tilde{\mathcal{D}}^{\beta)} W - \frac{1}{2} \tilde{E}^a \hat{\sigma}_a^{\dot{\beta}\beta} \tilde{\mathcal{D}}_p \nabla^{\alpha)} W \end{aligned}$$

$$\tilde{W}^{\dot{\alpha}\dot{\beta}} = W^{\dot{\alpha}\dot{\beta}} - 2\bar{E}^{(\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\beta})} \bar{W} + \frac{1}{2} E^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \mathcal{D}_p \nabla^{\dot{\beta})} \bar{W} =$$

$$= W^{\dot{\alpha}\dot{\beta}} - 2\tilde{E}^{(\dot{\alpha}} \tilde{\mathcal{D}}^{\dot{\beta})} \bar{W} + \frac{1}{2} \tilde{E}^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \tilde{\mathcal{D}}_p \nabla^{\dot{\beta})} \bar{W}$$

$$\leftarrow \frac{1}{4} W^{\alpha\beta} \hat{\sigma}_a^{\dot{\alpha}\dot{\beta}} = (W^{\alpha\beta})^*$$

$$\tilde{W}^{ab} = W^{ab} + \frac{1}{2} \Delta W^{\alpha\beta} \sigma_{\alpha\beta}^{ab} + \frac{1}{2} \Delta W^{\dot{\alpha}\dot{\beta}} \hat{\sigma}_{\dot{\alpha}\dot{\beta}}^{ab} =$$

$$= W^{ab} - E^\alpha \sigma_{\alpha\beta}^{ab} \mathcal{D}_p W + \bar{E}^{\dot{\alpha}} \hat{\sigma}_{\dot{\alpha}\dot{\beta}}^{ab} \bar{\mathcal{D}}_p \bar{W} - \frac{1}{2} E^c \hat{\sigma}_c^{\dot{\alpha}\alpha} (\mathcal{D}_\alpha \mathcal{D}_\beta W + \mathcal{D}_\alpha \bar{\mathcal{D}}_\beta \bar{W}) + \frac{1}{2} E^c \hat{\sigma}_c^{\dot{\alpha}\alpha} \hat{\sigma}_c^{\dot{\beta}\beta} (\bar{\mathcal{D}}_\alpha \bar{\mathcal{D}}_\beta \bar{W} - \bar{\mathcal{D}}_\alpha \mathcal{D}_\beta W)$$

$$= (E \rightarrow \tilde{E}, \mathcal{D} \rightarrow \tilde{\mathcal{D}})$$

⇒ Variation of the super-Weyl-transformed supervielbein and connection:

(FOR SHORTNESS $\tilde{E}^A \xrightarrow{\text{IS DERIVED}} E^A$)

$$\delta_W E^a = E^a (\Lambda(\delta) + \bar{\Lambda}(\delta))$$

$$W' = W + \Lambda(\delta)$$

$$\delta_W E^\alpha = E^\alpha \Lambda(\delta) - \frac{1}{4} E^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \nabla_a \Lambda(\delta)$$

$$\delta_W \bar{E}^{\dot{\alpha}} = \bar{E}^{\dot{\alpha}} \bar{\Lambda}(\delta) - \frac{1}{4} E^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \nabla_a \bar{\Lambda}(\delta)$$

$$\delta_W W^{\alpha\beta} = 2E^{(\alpha} \nabla^{\beta)} \Lambda(\delta) - \frac{1}{2} E^a \hat{\sigma}_a^{\dot{\beta}\beta} \mathcal{D}_p \nabla^{\alpha)} \Lambda(\delta)$$

$$\delta_W W^{\dot{\alpha}\dot{\beta}} = -2\bar{E}^{(\dot{\alpha}} \bar{\nabla}^{\dot{\beta})} \bar{\Lambda}(\delta) - \frac{1}{2} E^a \hat{\sigma}_a^{\dot{\alpha}\alpha} \bar{\mathcal{D}}_p \nabla^{\dot{\beta})} \bar{\Lambda}(\delta)$$

$$\delta_W W^{ab} = -E^\alpha \sigma_{\alpha\beta}^{ab} \mathcal{D}_p \Lambda(\delta) + \bar{E}^{\dot{\alpha}} \hat{\sigma}_{\dot{\alpha}\dot{\beta}}^{ab} \bar{\mathcal{D}}_p \bar{\Lambda}(\delta) +$$

$$+ E^c (\frac{1}{4} \hat{\sigma}_c^{\dot{\alpha}\alpha} \hat{\sigma}_c^{\dot{\beta}\beta} (\bar{\mathcal{D}}_\alpha \mathcal{D}_\beta \Lambda(\delta) - \mathcal{D}_\alpha \bar{\mathcal{D}}_\beta \bar{\Lambda}(\delta)) - \frac{1}{2} \delta_c^{\dot{\alpha}\alpha} \hat{\sigma}_c^{\dot{\beta}\beta} (\mathcal{D}_\alpha \mathcal{D}_\beta \Lambda(\delta) + \mathcal{D}_\alpha \bar{\mathcal{D}}_\beta \bar{\Lambda}(\delta)))$$

Super-Weyl Transformations of Torsion 2-Forms
AND MAIN SUPERFIELDS, (AND CURVATURE)

S.U. 1b - (2)
S.Weyl - (11) - (2)

CONSTRAINTS OF SUPRA
WITHOUT 3rd CLASS CONSTRAINTS
AND THEIR CONSEQUENCES.

EX: TO CALCULATE:

$$\tilde{T}^a = -2i \hat{E}^\alpha \wedge \hat{E}^i + \hat{E}^b \wedge \hat{E}^\alpha \frac{1}{2} (\hat{G}_b \hat{\sigma}^a)_\alpha^\beta \hat{\nabla}_\beta (2W + \bar{W}) + \hat{E}^b \wedge \hat{E}^\alpha \frac{1}{2} (\hat{G}^a \hat{\sigma}_b)_\alpha^\beta \hat{\nabla}_\beta (2\bar{W} + W) + \frac{1}{2} \hat{E}^c \wedge \hat{E}^b T_{bc}^a$$

$$\tilde{T}_{bc}^a = \frac{1}{8} \epsilon^{abcd} (G^d e^{W+\bar{W}} + 4 \hat{\sigma}^{dijk} (\hat{\mathcal{D}}_i \hat{\mathcal{D}}_j W - \hat{\mathcal{D}}_i \hat{\mathcal{D}}_j \bar{W})) + \delta_b^a \hat{\sigma}_{cd}^{ijk} (\frac{1}{2} \hat{\mathcal{D}}_i \hat{\mathcal{D}}_j W + \hat{\mathcal{D}}_i \hat{\mathcal{D}}_j \bar{W} - \hat{\mathcal{D}}_j (W + \bar{W})) + \sim \hat{\nabla}^i \bar{W} \hat{\nabla}^j W$$

EX: TO CALCULATE UP TO FERMION BILINEARS

$$\tilde{T}^a = \frac{i}{8} \hat{E}^b \wedge \hat{E}^c (\hat{G}_b \hat{\sigma}_c)_\alpha^\beta \hat{\nabla}_\beta \tilde{\gamma}^a - \frac{i}{8} \hat{E}^b \wedge \hat{E}^c \hat{\sigma}_{bc}^a \hat{R} + \frac{1}{2} \hat{E}^c \wedge \hat{E}^b T_{bc}^a$$

AN ADVANTAGE OF OUR CHOICE OF CONSTRAINTS:

THE FORM OF FERMIONIC/TORSION IS INVARIANT UNDER SUPER-WEYL TRANSF.S BUT TRANSFORMED G_a IS GENERICALLY NOT REAL

$$\begin{aligned} \hat{\mathcal{D}}_a \tilde{g}_a &= G_a e^{-W-\bar{W}} + \hat{\sigma}_a^{ijk} \hat{\mathcal{D}}_i \hat{\nabla}_j W + 4i \hat{\nabla}_a W + \hat{\nabla}^i \bar{W} \hat{\sigma}_a^i \hat{\nabla} W \\ &= e^{-W-\bar{W}} (G_a + \hat{\mathcal{D}}_i \hat{\nabla}_a W \hat{\sigma}_a^{ijk} + 4i \hat{\nabla}_a W) \neq (\tilde{g}_a)^* \end{aligned}$$

↑ GENERICALLY

$$\begin{aligned} \hat{R} &= R e^{-2\bar{W}} - \hat{\mathcal{D}} \hat{\nabla} W - \hat{\nabla}_i W \cdot \hat{\nabla}^i W \\ &= e^{-2\bar{W}} (R - \hat{\mathcal{D}} \hat{\nabla} W - \hat{\nabla}_i W \hat{\nabla}^i W - 2 \hat{\nabla}_i \bar{W} \cdot \hat{\nabla}^i W) \end{aligned}$$

$$\tilde{R}^{\alpha\beta} = -\frac{1}{2} \hat{E}^\alpha \wedge \hat{E}^\beta \tilde{R} + \hat{E}^b \wedge \hat{E}^\gamma \hat{R}_{\gamma b}^{\alpha\beta} + \hat{E}^b \wedge \hat{E}^c \hat{R}_{bc}^{\alpha\beta} + \frac{1}{2} \hat{E}^d \wedge \hat{E}^c \hat{R}_{cd}^{\alpha\beta}$$

ALSO FORM INV. IN THIS SENSE

$$\begin{aligned} \hat{R} &= e^{-2W} (\bar{R} + 2 \hat{\mathcal{D}} \hat{\nabla} W - 4 \hat{\nabla}^i W \hat{\nabla}_i W) = \\ &= -(\hat{\mathcal{D}} \hat{\mathcal{D}} - \bar{R}) e^{-2W} \neq \tilde{R} \end{aligned}$$

↑ GENERICALLY

$$\hat{\mathcal{D}}_a \tilde{R} = 0 \Rightarrow \hat{\mathcal{D}}_a \hat{R} = 0$$

Super-Meij transformation from with SG:

Torsion and covariant 3d class constraints

Solution of first and 2nd class

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$$E^a = e^{W+\bar{W}} E^a$$

$$E^{\dot{a}} = e^W (E^{\dot{a}} - \frac{1}{4} E^{\dot{a}} \tilde{\partial}_a^{\dot{a} a} D_a W)$$

$$E^{\dot{a}} = e^{\bar{W}} (E^{\dot{a}} - \frac{1}{4} E^{\dot{a}} \tilde{\partial}_a^{\dot{a} a} D_a \bar{W})$$

$$E^a = e^{-W-\bar{W}} E^a$$

$$E^{\dot{a}} = e^{-W} (E^{\dot{a}} + \frac{1}{4} E^{\dot{a}} \tilde{\partial}_a^{\dot{a} a} \nabla_a W)$$

$$E^{\dot{a}} = e^{-\bar{W}} (E^{\dot{a}} + \frac{1}{4} E^{\dot{a}} \tilde{\partial}_a^{\dot{a} a} \nabla_a \bar{W})$$

$$W^\mu = W^\mu + 2 E^{\dot{a}} \partial_a W - \frac{1}{2} E^{\dot{a}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{a} b} W$$

$$\bar{W}^\mu = -(\bar{W}^\mu)^\mu + 2 E^{\dot{a}} \partial_a \bar{W} + \frac{1}{2} E^{\dot{a}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{a} b} \bar{W} \Rightarrow$$

$$\bar{W}^{\dot{a} b} = W^{\dot{a} b} + \frac{1}{2} \Delta W^{\dot{a} b} \tilde{\partial}_c^{\dot{a} c} + \frac{1}{2} \Delta \bar{W}^{\dot{a} b} \tilde{\partial}_c^{\dot{a} c} = W^{\dot{a} b} - E^{\dot{a}} \tilde{\partial}_c^{\dot{a} c} D_c W + E^{\dot{a}} \tilde{\partial}_c^{\dot{a} c} D_c \bar{W} + E^{\dot{a}} \tilde{\partial}_c^{\dot{a} c} \tilde{\partial}_d^{\dot{a} d} \tilde{\partial}_e^{\dot{a} e} W$$

$$+ E^{\dot{a}} \cdot (\frac{1}{2} E^{\dot{a}} \tilde{\partial}_c^{\dot{a} c} \tilde{\partial}_d^{\dot{a} d} (\tilde{\partial}_e^{\dot{a} e} D_e W - D_e \tilde{\partial}_e^{\dot{a} e} W) - \frac{1}{2} \tilde{\partial}_c^{\dot{a} c} \tilde{\partial}_d^{\dot{a} d} \tilde{\partial}_e^{\dot{a} e} W) + \tilde{\partial}_c^{\dot{a} c} \Delta W_{\dot{a} b}$$

$$\tilde{\nabla}^a = -2 E^{\dot{a}} E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} + E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}_c^{\dot{a} c} + E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_c^{\dot{a} c} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}_c^{\dot{a} c} + \frac{1}{2} E^{\dot{c}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}_c^{\dot{a} c}$$

$$\tilde{\nabla}^{\dot{a}} = \frac{1}{2} E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}_c^{\dot{a} c} - \frac{1}{2} E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_c^{\dot{a} c} \tilde{\partial}_b^{\dot{b} b} + \frac{1}{2} E^{\dot{c}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}_c^{\dot{a} c}$$

BIG ADVANTAGE OF OUR CHOICE OF TORSION CONSTRAINTS IS THAT THE FORM OF $T^{\dot{a}}$ IS SUPER-WEIGHT 1/2!

$$\tilde{\partial}_a^{\dot{a} a} = G_a e^{-W-\bar{W}} + \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}_c^{\dot{a} c} W + \tilde{\nabla}_c^{\dot{a} c} \tilde{\partial}_a^{\dot{a} a} \tilde{\nabla}^{\dot{b} b} W \neq \tilde{\partial}_a^{\dot{a} a}$$

$$\tilde{\nabla}^{\dot{a}} = \tilde{\nabla}^{\dot{a}} e^{-2W} - \tilde{\nabla}^{\dot{a}} W - \tilde{\nabla}^{\dot{a}} \bar{W} \neq \tilde{\nabla}^{\dot{a}}$$

$$\tilde{\nabla}^{\dot{a}} = e^{-2W} (\tilde{\nabla}^{\dot{a}} + 2 \partial W - \nabla^{\dot{a}} W \cdot \nabla^{\dot{a}} W) = -(\tilde{\partial} \partial - \tilde{\nabla}) e^{-W}$$

$$\tilde{\nabla}^{\dot{a}} = -\frac{1}{2} E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}^{\dot{c} c} + E^{\dot{b}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_c^{\dot{a} c} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}^{\dot{d} d} + \frac{1}{2} E^{\dot{c}} \tilde{\partial}_a^{\dot{a} a} \tilde{\partial}_b^{\dot{b} b} \tilde{\nabla}^{\dot{c} c}$$

$$\tilde{\mathbb{R}} = \tilde{\mathbb{R}} e^{-2W} - \tilde{\mathbb{R}} \tilde{\nabla}^{\dot{a}} W - \tilde{\nabla}^{\dot{a}} W \cdot \tilde{\nabla}^{\dot{a}} \bar{W} \neq \tilde{\mathbb{R}}$$

$$\tilde{\mathbb{R}} = 0 ; \tilde{\mathbb{R}} = 0 \Leftarrow \tilde{\mathbb{R}} = 0$$

$$\begin{aligned} &= \frac{1}{2} E^{\dot{a}} \tilde{\partial}_c^{\dot{a} c} (\tilde{\partial}_d^{\dot{a} d} e^{dW+\bar{d}\bar{W}} + 4 \tilde{\partial}_d^{\dot{a} d} \tilde{\partial}_e^{\dot{a} e} (\tilde{\partial}_f^{\dot{a} f} W - D_f \tilde{\partial}_f^{\dot{a} f} W)) + \\ &+ \tilde{\partial}_c^{\dot{a} c} (\tilde{\partial}_d^{\dot{a} d} \cdot \frac{1}{2} (\tilde{\partial}_e^{\dot{a} e} D_e W + D_e \tilde{\partial}_e^{\dot{a} e} W) - 2 D_e (\bar{W} + W)) + \dots \end{aligned}$$

Complex

$$E^a = e^{w+\bar{w}} E^a$$

$$E^a = e^w (E^a - \frac{1}{2} E^a \tilde{\sigma}_a^{\alpha\beta} \bar{D}_\alpha \cdot W)$$

$$E^{\alpha 2} = e^w (E^{\alpha 1} - \frac{1}{2} E^a \tilde{\sigma}_a^{\alpha\beta} D_\alpha \bar{w})$$

$$\tilde{\nabla}_\alpha = e^{-w-\bar{w}} \nabla_\alpha + \frac{1}{2} \tilde{\sigma}_a^{\alpha\beta} (\tilde{\nabla}_\alpha W \cdot \nabla_\beta + \nabla_\alpha \bar{w} - \tilde{\nabla}_\alpha \bar{w})$$

$$\tilde{\nabla}_\alpha = e^{-w} \nabla_\alpha$$

$$\tilde{\nabla}_\alpha = e^{-\bar{w}} \tilde{\nabla}_\alpha$$

$$\Delta w^{\alpha\beta} = \tilde{\nabla}_\alpha \tilde{\nabla}_\beta - w^{\alpha\beta} = 2 E^a \tilde{\sigma}_a^{\beta\gamma} W - \frac{1}{2} E^a \tilde{\sigma}_a^{\beta\gamma} \tilde{\nabla}_\alpha \tilde{\nabla}_\beta W$$

$$\Delta w^{\alpha\beta} = \tilde{\nabla}_\alpha \tilde{\nabla}_\beta - w^{\alpha\beta} = 2 E^a \tilde{\sigma}_a^{\beta\gamma} W - \frac{1}{2} E^a \tilde{\sigma}_a^{\beta\gamma} \tilde{\nabla}_\alpha \tilde{\nabla}_\beta W$$

$$\tilde{R} = e^{-2w} \bar{R} - \tilde{\sigma}_a^{\alpha\beta} e^{-2w} = -(\tilde{\sigma}_a^{\alpha\beta} \bar{R}) e^{-2w} = e^{-2w} (\bar{R} + 2\tilde{\sigma}_a^{\alpha\beta} W - \frac{1}{2} \tilde{\sigma}_a^{\alpha\beta} \tilde{\nabla}_\alpha W)$$

$$\tilde{R} = -F^2 (\tilde{\Delta}_\alpha^{\alpha\beta} \tilde{\Delta}_\beta F^2 + \tilde{\Delta}_\alpha^{\alpha\beta} F^2 \tilde{\Delta}_\alpha F^2)$$

$$\tilde{T}_{\alpha\beta}^a = \frac{1}{4} (\tilde{\sigma}_a^{\alpha\beta} \tilde{\sigma}_\alpha^{\gamma\delta}) \tilde{T}_{\beta\gamma}^a$$

$$\tilde{T}_\alpha^i = \tilde{T}_{\alpha 1}^i = 2 \tilde{\nabla}_\alpha (2w + \bar{w})$$

$$\tilde{R} - \tilde{\sigma}_a^{\alpha\beta} \tilde{T}_\alpha^a - \tilde{\sigma}_a^{\alpha\beta} \tilde{T}_\alpha^a = -\frac{F^2}{\tilde{R}} \tilde{\Delta}_\alpha^{\alpha\beta} \tilde{\nabla}_\beta$$

$$\tilde{\nabla}_\alpha = F^2 (\tilde{\sigma}_a^{\alpha\beta} \tilde{\nabla}_\beta \tilde{\nabla}_\alpha) - \tilde{\sigma}_a^{\alpha\beta}$$

$$\tilde{\nabla}_\alpha = F^2 (\tilde{\sigma}_a^{\alpha\beta} \tilde{\nabla}_\beta \tilde{\nabla}_\alpha) - \tilde{\sigma}_a^{\alpha\beta}$$



$$S(\tilde{R} - \tilde{\sigma}_a^{\alpha\beta} \tilde{T}_\alpha^a - \tilde{\sigma}_a^{\alpha\beta} \tilde{T}_\alpha^a) = 0 \iff \tilde{R} - \tilde{\sigma}_a^{\alpha\beta} \tilde{\nabla}_\alpha \tilde{\nabla}_\beta = 0$$

$$(\tilde{\sigma}_a^{\alpha\beta} \tilde{\nabla}_\alpha \tilde{\nabla}_\beta) \tilde{K} = 0$$

$$\tilde{K} := -(1 - 2\tilde{\sigma}_a^{\alpha\beta}) W + \tilde{\sigma}_a^{\alpha\beta} \bar{w} = (2\tilde{\sigma}_a^{\alpha\beta} - 1) W + \tilde{\sigma}_a^{\alpha\beta} \bar{w}$$

$$w=0, \tilde{\sigma}_a^{\alpha\beta} = 1 \quad \tilde{K} = w + \bar{w} = K$$

$$(\tilde{\sigma}_a^{\alpha\beta} - 2\tilde{T}_\alpha^a) \tilde{D}_\alpha K = 0 = (\tilde{\sigma}_a^{\alpha\beta} - 2\tilde{T}_\alpha^a) \tilde{\nabla}_\alpha K$$

$$(\tilde{\sigma}_a^{\alpha\beta} \tilde{R} - \tilde{R}) \tilde{\nabla}_\alpha = \tilde{\Delta}_\alpha^{\alpha\beta} \tilde{\nabla}_\beta (F^2 \tilde{\nabla}_\alpha)$$