VANISHING (SUPER)VOLUME OF ALTERNATIVE MIN. SG SSP.

$$\int d^8z\, E = \int d^8z_L\, \hat{\ell}^{-1} F^{-2} \bar{F}^{-2} = \int d^8 z_R\, \hat{z}^{-1} F^{-2} \bar{F}^{-2}$$

(see p. 5. I. 1b. – 8 –)

$$\bar{R} - \bar{\rho}\, \mathcal{D}^{\varepsilon} T_{\varepsilon} - \bar{\rho}^2 T^{\varepsilon} T_{\varepsilon} = 0 \underset{\text{p. 5 II 1 (Sweyl) – 4 –}}{\Longrightarrow} \quad \bar{\Upsilon} = F^2 (\hat{z} F^4 \bar{F}^2)^{-\bar{\rho}}, \quad \hat{\Delta}^{\alpha} \hat{\Delta}_{\alpha} \bar{\Upsilon} = 0$$

$$\Upsilon = \bar{F}^2 (\hat{\ell} F^2 \bar{F}^4)^{-\rho} \quad \hat{\Delta}_{\dot\alpha} \hat{\Delta}^{\dot\alpha} \Upsilon = 0$$

We study real $\rho = \bar{\rho}$

$$\Upsilon \bar{\Upsilon} = (\hat{z} \hat{\ell})^{-\rho} (F^2 \bar{F}^2)^{1-3\rho} = (\hat{z}\hat{\ell})^{-\frac{n+1}{3n+1}} (F^2 \bar{F}^2)^{\frac{2n}{3n+1}}$$

$$F^2 \bar{F}^2 = (\hat{\ell}\hat{z})^{\frac{\rho}{1-3\rho}}$$

$$F^2 \bar{F}^2 = (\hat{\ell} \hat{z})^{\frac{\rho}{1-3\rho}} (\Upsilon \bar{\Upsilon})^{\frac{1}{1-3\rho}} = (\hat{\ell} \hat{z})^{-\frac{n+1}{2}} (\Upsilon \bar{\Upsilon})^{-\frac{3n+1}{2}}$$

ACTION OF NON-MINIMAL SG

$$\boxed{\int d^8z\, E = \int d^8 z_L\, \hat{\ell}^{-1} F^{-2} \bar{F}^{-2} = \int d^8 z_L\, \hat{\ell}^{-\frac{n+3}{2}} \hat{z}^{-\frac{n+1}{2}} (\Upsilon \bar{\Upsilon})^{-\frac{3n+1}{2}}}$$

$$\rho = \frac{n+1}{3n+1} = \bar{\rho}$$

SINGULARITY: $\underline{n = -1/3}$ (MIN SG: $\Upsilon \to \Phi$)

$\underline{n = 0}$ 'NEW' ('ALTERNATIVE') MIN SG

Comment: on-shell $\int d^8_z E = 0$ also for min SG: $R=0$ $\int d^8 z E = \frac{1}{2} \int d^6 z_L\, \mathcal{E} R$

ACTUALLY FOR $n = 0$ $\boxed{\int d^8 z\, E = 0}$

off-shell!

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INDEED: $n = 0$, $\rho = 1$: $\Upsilon = \bar{F}^2 (\hat{\ell} F^2 \bar{F}^4)^{-1} = \hat{\ell}^{-1} F^{-2} \bar{F}^{-2}$

$$\hat{\Delta}_{\dot\alpha} \hat{\Delta}^{\dot\alpha} \Upsilon = 0 \iff \bar{\partial}^L_{\dot\alpha} \bar{\partial}^{\dot\alpha L} \Upsilon = 0$$

$$\boxed{\int d^8 z\, E = \int d^8 z_L\, \hat{\ell}^{-1} F^{-2} \bar{F}^{-2} = \int \underbrace{d^8 z_L}_{= d^6 \zeta_L\, \bar{\partial}^L_{\dot\alpha} \bar{\partial}^{\dot\alpha L}} \Upsilon = 0}$$

$$\bar{\partial}^L_{\dot\alpha} \bar{\partial}^{\dot\alpha L} \Upsilon = 0$$

5 II.2 -(1'')-

$$E_\alpha{}^M \partial_M = \nabla_\alpha = F \hat{\Delta}_\alpha = F \partial_\alpha{}^R = F\left(\partial_\alpha{}^L + 2i \hat{\Delta}_\alpha \mathcal{H}^\mu \partial_\mu{}^L\right) = \mathcal{L}_\alpha{}^M \partial_M{}^L = \mathcal{R}_\alpha{}^M \partial_M{}^R$$

$$\bar{E}_{\dot\alpha}{}^M \partial_M = \bar\nabla_{\dot\alpha} = \bar F \hat{\bar\Delta}_{\dot\alpha} = \bar F\left(\bar\partial_{\dot\alpha}{}^R - 2i \hat{\bar\Delta}_{\dot\alpha} \mathcal{H}^\mu \partial_\mu{}^R\right) = \bar F \bar\partial_{\dot\alpha}{}^L = \mathcal{L}_{\dot\alpha}{}^M \partial_M{}^L = \mathcal{R}_{\dot\alpha}{}^M \partial_M{}^R$$

$$E_a{}^M \partial_M = \nabla_a = \underbrace{F \bar F \hat{\Delta}_a}_{\hat{e}_a{}^\mu \partial_\mu{}^L = \hat{e}_a{}^\mu \partial_\mu{}^R} + \underbrace{F \chi_a{}^\beta \hat{\Delta}_\beta}_{-\frac{i}{4}\bar\sigma_a{}^{\dot\alpha\alpha}(F\omega_{\dot\alpha\alpha}{}^\beta + \bar F \bar\Delta_{\dot\alpha} F \delta_\alpha{}^\beta)} + \underbrace{\bar F \bar\chi_a{}^{\dot\beta} \hat{\bar\Delta}_{\dot\beta}}_{-\frac{i}{4}\bar\sigma_a{}^{\dot\alpha\alpha}(\bar F \omega_{\alpha\dot\alpha}{}^{\dot\beta} + F\Delta_\alpha \bar F \delta_{\dot\alpha}{}^{\dot\beta}} = \mathcal{L}_a{}^M \partial_M{}^L = \mathcal{R}_a{}^M \partial_M{}^R$$

$$E^{-1} = \mathrm{Ber}\, E_A{}^M = \mathrm{Ber}\, \mathcal{L}_A{}^M \Big/ \mathrm{Ber}\, \frac{\partial z_L}{\partial z} = \mathcal{L}^{-1} \big/ \det(\delta - \mathcal{X}h - i\mathcal{H}))$$

$$\mathcal{L}^{-1} = \mathrm{Ber}\, \mathcal{L}_A{}^M = \mathrm{Ber}\begin{pmatrix} F\bar F\, \hat{e}_a{}^\mu + 2iF\chi_a{}^\beta \hat{\Delta}_\beta \mathcal{H}^\mu & F\chi_a{}^\beta & \bar F \chi_a{}^{\dot\beta} \\ 2iF\hat{\Delta}_\alpha \mathcal{H}^\mu & F\delta_\alpha{}^\beta & 0 \\ 0 & 0 & \bar F \chi_{\dot\alpha}{}^{\dot\beta} \end{pmatrix} \begin{array}{l} \\ -\chi_\alpha{}^\beta \\ -\chi_{\dot\alpha}{}^{\dot\beta} \end{array}$$

$$= \mathrm{Ber}\begin{pmatrix} F\bar F\, \hat{e}_a{}^\mu & 0 & 0 \\ 0 & F\delta_\alpha{}^\beta & 0 \\ 0 & 0 & \bar F \delta_{\dot\alpha}{}^{\dot\beta} \end{pmatrix} = \hat{e}\, F^2 \bar F^2$$

$$E = \hat{e}^{-1} F^{-2} \bar F^{-2} \cdot \det \frac{\partial x_L}{\partial x} = \hat{e}^{-1} F^{-2} \bar F^{-2}\, \mathrm{Ber}\, \frac{\partial z_L}{\partial z}$$

Thus the Wess-Zumino action for SUGRA is

$$\boxed{\int d^8 z\, E = \int d^8 z_L\, \hat{e}^{-1} F^{-2} \bar F^{-2} = \int d^8 z_R\, \hat{z}^{-1} F^{-2} \bar F^{-2}}$$

WHERE F^2 AND $\bar F^2$ ARE DETERMINED BY THE 3RD CLASS CONSTRAINTS

IN THE CASE OF MIN SG THESE ARE

$$T_\alpha = -\nabla_\alpha \ln \hat{z} F^4 \bar F^2 = 0$$

$$\Rightarrow \quad F^2 = \hat{e}^{1/3} \hat{z}^{-2/3} \varphi \bar\varphi^{-2}, \quad \bar F^2 = \hat{e}^{-2/3} \hat{z}^{1/3} \varphi^{-2} \bar\varphi$$

$$\Rightarrow \quad F^2 \bar F^2 = \hat{e}^{-1/3} \hat{z}^{-1/3} \varphi^{-1} \bar\varphi^{-1}$$

$$\boxed{\hat{e}^{-1} F^{-2} \bar F^{-2} = \hat{e}^{-2/3} \hat{z}^{1/3} \varphi \bar\varphi}$$

MIN. SG.

$$\boxed{\int d^8 z\, E = \int d^8 z_L\, \hat{e}^{-2/3} \hat{z}^{1/3} \varphi\bar\varphi = \int d^8 z_R\, \hat{e}^{1/3} \hat{z}^{-2/3} \varphi\bar\varphi}$$

CAN WE PROVE THAT IN ALTERNATIVE MIN SG $\int d^8z\, E = 0$ WITHOUT USE OF THE SOLUTION OF THE CONSTRAINTS?

YES:

- START FROM $\int d^8z\, E\, (\mathcal{D}_A \xi^A + \xi^B T_{BA}{}^A)(-)^A \equiv 0$

  FOR $\xi^A = (\xi^\alpha, 0, 0)$ AND WITH OUR CONSTRAINTS ($T_{\alpha\beta}{}^\gamma = 0 = T_{\alpha\beta}{}^{\dot\gamma}$, $T_{\alpha b}{}^b = T_\alpha$) IT READS

$$\boxed{\int d^8z\, E\, (\mathcal{D}_\alpha + T_\alpha)\xi^\alpha \equiv 0} \quad \forall \xi^\alpha$$

- CONSTRAINTS OF NON-MIN. AND ALT. MIN SG

$$\bar R - \bar\varsigma\, \mathcal{D}^\alpha T_\alpha - \bar\varsigma^2 T^\alpha T_\alpha = 0$$

for $\bar R \neq 0$ can be written as ($\mathcal{D}_\alpha \bar R = 0$!)

$$1 = (\mathcal{D}_\alpha + \bar\varsigma T_\alpha)\, \bar\varsigma \frac{T^\alpha}{\bar R}$$

$\frac{1}{\bar R}$

FOR $n=0$, $\varsigma = 1$ THIS READS

$$(\mathcal{D}_\alpha + T_\alpha)\left(\frac{T^\alpha}{\bar R}\right) = 1$$

- THUS CHOOSING $\xi^\alpha = \frac{T^\alpha}{\bar R}$ IN THE ABOVE IDENTITY

$$\boxed{0 \equiv \int d^8z\, E\, (\mathcal{D}_\alpha + T_\alpha)\underset{\frac{T^\alpha}{\bar R}}{\xi^\alpha} = \int d^8z\, E\, (\mathcal{D}_\alpha + T_\alpha)\left(\frac{T^\alpha}{\bar R}\right) = \int d^8z\, E}$$

$n=0$, $\varsigma = 1$ ONLY

ACTION FOR NEW MIN. SG $(n=0)$

IT IS CONVENIENT TO CONSTRUCT IT STARTING FROM THE FIRST ORDER ACTION WITH INDEPENDENT $F, \bar{F}$. [Zima, 1984]

THIS HAS TO REPRODUCE $\hat{\ell} F^2 \bar{F}^2 = \gamma^{-1}$, $\hat{z} F^2 \bar{F}^2 = \bar{\gamma}^{-1}$

as eqs. of motion, so that "of the F-shell" $\int d^8 z_L \hat{\ell}^{-1} F^{-2} \bar{F}^{-2} \in O$

$$\boxed{K^2 S(n=0) = \int d^8 z_L \hat{\ell}^{-1} F^{-2} \bar{F}^{-2} + \int d^8 z_L \gamma \ln \bar{F}^2/\gamma + \int d^8 z_R \bar{\gamma} \ln F^2/\bar{\gamma}}$$

$\int d^8 z E\big|_{n=0}$; on the F-shell $\int d^8 z_L \gamma$

$\ln \bar{F}^2$ is seen from eqs.

The presence of $\gamma \ln \gamma$ can be seen

- from $n \to 0$ limit of the NON-MINIMAL SG ACTION (see below)
- $T_\alpha = -\nabla_\alpha \ln \hat{z} F^4 \bar{F}^2 = \nabla_\alpha \ln F^2/\bar{\gamma} \equiv \nabla_\alpha \ln T$

AS T_α IS TRANSFORMED AS $U(1)$ CONNECTION,

$\ln T$ plays the role of $U(1)$ prepotential

so on the F-shell $\int d^8 z_R \bar{\gamma} \ln F^2/\bar{\gamma} \overset{+c.c.}{=} \int d^8 z\, E(\ln T + \ln \bar{T})$

is A COVARIANT VERSION OF F-I TERM (BUT FOR COMPOSED PREPOTEN.)

$\delta S/\delta F^2 = 0 \Rightarrow \boxed{(\hat{z} F^4 \bar{F}^2)^{-1} = \bar{\gamma}/F^2}$ AS IT SHOULD BE

$(\hat{\ell} F^2 \bar{F}^4)^{-1} = \gamma/\bar{F}^2$

$$\boxed{\begin{aligned} K^2 S(n=0)\Big|_{\frac{\delta S}{\delta F^2}=0} &= \int d^8 z_L \gamma \ln \bar{F}^2/\gamma + \int d^8 z_R \bar{\gamma} \ln F^2/\bar{\gamma} = \\ &= \int d^8 z\, E\, (\ln \bar{F}^2/\gamma + \ln F^2/\bar{\gamma}) = \\ &= \int d^8 z\, E\, (\ln T + \ln \bar{T}) \end{aligned}}$$

$$\boxed{= -3 \int d^8 z_L \hat{\ell}^{-1} G \ln\left(G (\hat{\ell}\hat{z})^{-1/3}\right)}$$

$\frac{(F\bar{F})^2}{\gamma\bar{\gamma}} = \frac{\hat{\ell}\hat{z}}{G^3}$

$\int d^8 z\, E(\ldots) = \int d^8 z_L \underset{\hat{\ell}^{-1} G}{\underset{\shortparallel}{\gamma}} (\ldots)$

NAIVE FLAT SSP LIMIT WOULD ($\hat{\ell} \to 1, \hat{z} \to 1$) BE IMPROVED TENSOR MULTIPLET ACTION $\sim \int d^8 z\, G \ln G$

ACTION FOR NON-MINIMAL SUGRA

FIRST ORDER FORM [ZIMA 1984]

$$2K^2 S(n) = \int d^8z_L\, \hat{\ell}^{-1} \bar{F}^{-2} \hat{F}^{-2} - \frac{n+1}{2n} \int d^8 z_L\, \bar{F}^{-\frac{4n}{n+1}} \Upsilon^{\frac{3n+1}{n+1}} -$$
$$- \frac{n^*+1}{2n^*} \int d^8 z_R\, F^{-\frac{4n^*}{n^*+1}} \bar{\Upsilon}^{\frac{3n^*+1}{n^*+1}}$$

Ex: To show that this produces

$$\bar{\Upsilon} = F^2 (\hat{\Sigma} F^4 \bar{F}^2)^{-\zeta^*} \qquad \zeta = \frac{n+1}{3n+1}$$
$$\Upsilon = \bar{F}^2 (\hat{\ell} F^2 \bar{F}^4)^{-\zeta}$$

Ex: To calculate action on the F-shell

[Siegel & Gates 79 — result]

$n = n^*$

$$\boxed{S(n) = -\frac{1}{2K^2} \cdot \frac{1}{n} \int d^8 z_L\, \hat{\ell}^{-1} (\Sigma \hat{\ell})^{-\frac{n+1}{2n}} (\Upsilon \bar{\Upsilon})^{-\frac{3n+1}{2n}}}$$

Ex: calculate the action for $n^* \neq n$; show that at $n^* + n = 0$ there is a singularity

Notice:

$$- \frac{n+1}{2n} \int d^8 z_L\, \bar{F}^{-\frac{4n}{n+1}} \Upsilon^{\frac{3n+1}{n+1}} = - \frac{n+1}{2n} \int d^8 z_L \underbrace{(\bar{F}^{-2} \Upsilon)^{\frac{2n}{n+1}}}_{= (\bar{F}^2/\Upsilon)^{-\frac{2n}{n+1}}} \Upsilon \longrightarrow$$

$$\xrightarrow[\substack{n \to 0 \\ (\frac{2n}{n+1} \to 0)}]{} + \int d^8 z_L\, \Upsilon \ln \bar{F}^2 / \Upsilon$$

At least in the sense of reproducing $n \to 0$ limit of the F-shell eqs.