Spacetime, component action of "new" min SG and its SUSY

We know (from SSP approach) the field content of the "new" min SG: $e_\mu^a, \psi_\mu^\alpha, A_\mu, B_{\mu\nu} \leftrightarrow H_{\mu\nu\rho} \leftrightarrow \epsilon_{abcd} G^d$

A_μ: $\mathcal{D}$ - U(1) covariant; $\mathcal{D}_a G^a = 0$

and SUSY, including

$$\delta e^a = -2i\,\psi\sigma^a\bar\epsilon + c.c.$$
$$\delta\psi^\alpha = \mathcal{D}\epsilon^\alpha + \tfrac{i}{8}\left(e^b(\epsilon\sigma_b\bar\sigma_a)^\alpha G^a + \gamma\, G_b\,\epsilon\right)$$

like in "on-shell" SUGRA, but with (R-symm.) U(1) - covariant derivative

To find the complete "off-shell" action, let us start from

$\mathcal{L}_{EH} + \mathcal{L}_{RS}\big|_{D\to\mathcal{D}}$ with U(1) covariant derivatives:

$$\mathcal{D}\mathcal{D}\psi^\alpha = -\psi^\beta\wedge R_\beta{}^\alpha - \psi^\alpha\wedge F$$
$$\mathcal{D}\mathcal{D}\bar\psi^{\dot\alpha} = -\bar\psi^{\dot\beta}\wedge R_{\dot\beta}{}^{\dot\alpha} + \bar\psi^{\dot\alpha}\wedge F$$

$$\mathcal{L}_{EH} = \tfrac{1}{2}\epsilon_{abcd}R^{ab}\wedge e^c\wedge e^d \longmapsto d^4x\, e\, R(e,\omega)$$

$$\mathcal{L}_4'^{RS} = -4\mathcal{D}\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\bar\psi^{\dot\alpha} + 4\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\mathcal{D}\bar\psi^{\dot\alpha} \longmapsto 8\,d^4x\,e\,\epsilon^{\mu\nu\rho\sigma}e_\mu^a\,\psi_\nu\sigma_a\overleftrightarrow{\mathcal{D}}_\rho\bar\psi_\sigma$$

$$d\left(\mathcal{L}_4^{EH} + \mathcal{L}_4'^{RS}\right) = -\mathcal{E}_{a3}\wedge\left(T^a + 2i\,\psi\sigma^a\bar\psi\right) - 8\,\mathcal{D}\psi\wedge\sigma^{(1)}\wedge\mathcal{D}\bar\psi + 8\,\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\bar\psi^{\dot\alpha}\wedge F$$

($\mathcal{E}_{a3} = \epsilon_{abcd}R^{bc}\wedge e^d - 4\mathcal{D}\psi\wedge\sigma_a\bar\psi + 4\psi\sigma_a\wedge\mathcal{D}\bar\psi$) $\rightsquigarrow$ 1,5 formalism

new term ($F = dA = -F^*$)

which produces the variation $8\,\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\bar\psi^{\dot\alpha}\wedge\delta A$

Comes from superspace $\rightsquigarrow -E^a\wedge E^\alpha\wedge\bar E^{\dot\alpha}\,\sigma_{a\alpha\dot\alpha}$

which was present in the constraint for H_3, the spacetime restriction of which reads

$$dB_2 + i\,\psi\wedge\sigma^{(1)}\wedge\bar\psi = -\frac{1}{4!}e^c\wedge e^b\wedge e^a\,\epsilon_{abcd}G^d = -\tfrac{1}{4}e^{\wedge 3}_a\,G^a \qquad (*)$$

TO REPRODUCE THE EQUATION $G_a = 0$ FROM δA WE THUS NEED TO ADD TO THE LAGRANGIAN FORM

$$\boxed{\mathcal{L}_4^{BF} = -8i\,B_2\wedge dA}$$ (TIPICAL B-F TERM)

THEN $\delta A \Rightarrow dB_2 + i\psi\wedge\sigma^{(1)}\wedge\bar\psi = 0 \Leftrightarrow G_a = 0.$

One can also guess that we can add a kind of fake kinetic term for B_2: $\sim H_{abc}H^{abc} = c\, d^4x\, e\, G_a G^a$

[Why? δ_ϵ(SUSY) of the sum of the previous terms give contributions $\sim G_a$

Thus

$$\mathcal{L}_4^{new\ min} = \tfrac{1}{2}\epsilon_{abcd}R^{ab}\wedge e^c\wedge e^d - 4\mathcal{D}\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\bar\psi^{\dot\alpha} + 4\psi\wedge\sigma^{(1)}\wedge\mathcal{D}\bar\psi - 8i\,B_2\wedge dA + c\, d^4x\, e\, G_a G^a$$

USING SSP RULES FOR SUSY: $c = \frac{3}{8}$ for $\gamma = 1$

c: constant to be fixed by SUSY OR:

Indeed, using the formulae of "on-shell" SUSY action + $D \to \mathcal{D}$ $\Rightarrow$ $\mathcal{D}\mathcal{D}\psi^\alpha = -\psi^\beta \wedge R_\beta{}^\alpha - \psi^\alpha \wedge F$

$$\delta_\epsilon \mathcal{L}_4^{\text{new min}} = -8\,\mathcal{D}\psi^\alpha \wedge \sigma^{(1)}_{\alpha\dot\alpha} \wedge (\delta_\epsilon\bar\psi^{\dot\alpha} - \mathcal{D}\bar\epsilon^{\dot\alpha}) + 8(\delta_\epsilon\psi^\alpha - \mathcal{D}\epsilon^\alpha)\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\mathcal{D}\bar\psi^{\dot\alpha} +$$

($\mathcal{D}\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}$: $\frac{1}{2} e_c^{\wedge 3}\overset{\circ}{\psi}{}^c_{\alpha}$; $\delta_\epsilon\bar\psi^{\dot\alpha}-\mathcal{D}\bar\epsilon^{\dot\alpha}$: $-\frac{i}{8}e^b\big((\tilde\sigma_a\sigma_b\bar\epsilon)^{\dot\alpha}G^a + \gamma G_b\bar\epsilon^{\dot\alpha}\big)$)

$\Leftarrow \frac{i}{2}d^4x\, e\,\big((\overset{\circ}{\bar\psi}{}^b\tilde\sigma_a\sigma_b\bar\epsilon - \epsilon\sigma_b\tilde\sigma_a\overset{\circ}{\bar\psi}{}^b)G^a + \gamma G^a(\overset{\circ}{\psi}_a\bar\epsilon - \epsilon\overset{\circ}{\bar\psi}_a)\big)$

$$+\,8(\psi\wedge\sigma^{(1)}\bar\epsilon + \epsilon\wedge\sigma^{(1)}\bar\psi)\wedge i\,dA + 8\,\psi\wedge\sigma^{(1)}\wedge\bar\psi\wedge\delta_\epsilon A - 8i\,B_2\wedge d\delta_\epsilon A -$$

(1)

$-8i\,B_2\wedge d\delta_\epsilon A$: $\Leftarrow -8i\,dB_2\wedge\delta_\epsilon A - d(\dots)$

$\Leftarrow -8i(dB_2 + i\psi\wedge\sigma^{(1)}\wedge\bar\psi)\wedge\delta_\epsilon A$

$\Leftarrow : -\frac{1}{4}e_a^{\wedge 3}G^a$

$e_a^{\wedge 3} := \frac{1}{3!}\epsilon_{abcd}\,e^b\wedge e^c\wedge e^d$

$e_a^{\wedge 3}\wedge e^b = \delta_a{}^b d^4x\, e$

$\Leftarrow 2i\, e_a^{\wedge 3}\wedge G^a\delta_\epsilon A$

$\Leftarrow 2i\,d^4x\cdot e\;G^a\, i_a\delta_\epsilon A$

$$-8i\,\delta_\epsilon B_2\wedge dA + c\,d^4x\big(\delta_\epsilon e\,G_aG^a + 2e\,G^a\delta_\epsilon G_a\big)$$

from SSP → $\delta_\epsilon B_2$: $\Leftarrow -i\psi\wedge\sigma^{(1)}\bar\epsilon - i\epsilon\sigma^{(1)}\wedge\bar\psi$ (1)

$\delta_\epsilon e\,G_aG^a$: $\Leftarrow -2ie(\psi^b\sigma_b\bar\epsilon - \epsilon\sigma^b\bar\psi_b)G_aG^a$

$2eG^a\delta_\epsilon G_a$: ← from SSP or tanh? $-4ie(\overset{\circ}{\psi}_a\bar\epsilon - \epsilon\overset{\circ}{\bar\psi}_a)G^a +$
$+4ie\,G_a G^{[a}(\psi_b\sigma^{b]}\bar\epsilon - \epsilon\sigma^{b]}\bar\psi_b)$

$\Leftarrow -4ic\,d^4x\cdot e\,(\overset{\circ}{\psi}\bar\epsilon - \epsilon\overset{\circ}{\bar\psi}) -$
$-2ic\,d^4x\cdot e\;G^aG^b(\psi_a\sigma_b\bar\epsilon - \epsilon\sigma_b\bar\psi_a)$

FOR SIMPLEST CASE $\gamma = 1$

$\frac{i}{2}d^4x\, e\,(\overset{\circ}{\psi}{}^b\sigma_b\tilde\sigma_a\bar\epsilon - \epsilon\sigma_a\tilde\sigma_b\overset{\circ}{\bar\psi}{}^b)$ (2)
$+\frac{3i}{4}G^aG^b(\psi_{(a}\sigma_{b)}\bar\epsilon - \epsilon\sigma_{(a}\bar\psi_{b)})$

$$\Leftarrow \frac{i}{2}d^4x\cdot e\,\big((\epsilon\sigma_a\tilde\sigma_b\overset{\circ}{\bar\psi}{}^b - \overset{\circ}{\psi}{}^b\sigma_b\tilde\sigma_a\bar\epsilon)G^a + (\gamma+2)G^a(\overset{\circ}{\psi}_a\bar\epsilon - \epsilon\overset{\circ}{\bar\psi}_a)\big)$$

(2)

FOR THE SIMPLEST CASE OF $\gamma = 1$ $\exists$ ONLY 2 TYPES OF CONTRIBUTIONS, SETTING COEFFS FOR THEM $= 0$:

$\sim G^a(\overset{\circ}{\psi}_a\bar\epsilon - \epsilon\overset{\circ}{\bar\psi}_a)$: $\quad 0 = \frac{i}{2}(\underset{=1}{\gamma}+2) - 4ic \Rightarrow$

$\sim G^aG^b(\psi_{(a}\sigma_{b)}\bar\epsilon - \epsilon\sigma_{(a}\bar\psi_{b)})$: $\quad 0 = \frac{3i}{4} - 2ic \Rightarrow$

$$\boxed{c = \frac{3}{8}}$$

$$\boxed{\mathcal{L}_4^{\text{new min}} = \frac{1}{2}\epsilon_{abcd}R^{ab}\wedge e^c\wedge e^d - 4\,\mathcal{D}\psi^\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\bar\psi^{\dot\alpha} + 4\,\psi_\alpha\wedge\sigma^{(1)}_{\alpha\dot\alpha}\wedge\mathcal{D}\bar\psi^{\dot\alpha} - 8i\,B_2\wedge dA + \frac{3}{8}d^4x\, e\,G_aG^a}$$

New min SG action and Ectoplasm method

In the U(1) SSP of new min SG, we can also define a closed 4 form.

$$T^a = -2i E^\alpha \wedge \bar{E}^{\dot\alpha} \sigma^a_{\alpha\dot\alpha}$$
$$T^\alpha = \frac{i}{8} E^c \wedge E^\beta \left((\sigma_c \tilde\sigma_d)_\beta{}^\alpha G^d + G_c \delta_\beta^\alpha \right) + \frac{1}{2} E^c \wedge E^b T_{bc}{}^\alpha$$
$$\bar{T}^{\dot\alpha} = -\frac{i}{8} E^c \wedge \bar{E}^{\dot\beta} \left((\tilde\sigma_d \sigma_c)^{\dot\alpha}{}_{\dot\beta} G^d + G_c \delta_{\dot\beta}^{\dot\alpha} \right) + \frac{1}{2} E^c \wedge E^b T_{bc}{}^{\dot\alpha}$$

$d\mathcal{L}_4 = 0$

$$\mathcal{L}_4 = \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta (\sigma_{ab})_{\alpha\beta} \bar{\mathcal{L}} + \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot\alpha} \wedge \bar{E}^{\dot\beta} (\tilde\sigma_{ab})_{\dot\alpha\dot\beta} \mathcal{L} + \frac{1}{3!} E^c \wedge E^b \wedge E^a \wedge \left(E^\alpha \mathcal{L}_{\alpha abc} + \bar{E}^{\dot\alpha} \mathcal{L}_{\dot\alpha abc} \right) + \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a \mathcal{L}_{abcd}$$

with $\mathcal{D}_\alpha \bar{\mathcal{L}} = 0$, $\bar{\mathcal{D}}_{\dot\alpha} \mathcal{L} = 0$,

$$\mathcal{L}_{\alpha abc} = \frac{1}{4} \epsilon_{abcd} \sigma^d_{\alpha\dot\alpha} \bar{\mathcal{D}}^{\dot\alpha} \bar{\mathcal{L}} \qquad \mathcal{L}_{\dot\alpha abc} = \frac{1}{4} \epsilon_{abcd} \sigma^d_{\alpha\dot\alpha} \mathcal{D}^\alpha \mathcal{L}$$

$$\mathcal{L}_{abcd} = -\frac{i}{16} \epsilon_{abcd} \sigma^d_{\alpha\dot\alpha} \left(\mathcal{D}\mathcal{D}\mathcal{L} - \bar{\mathcal{D}}\bar{\mathcal{D}}\bar{\mathcal{L}} \right)$$

(see min SG case p 5.I.2 - (3) -)

As in min SG, but with U(1) cov. derivatives & R=0.

Now the question is what should be chiral superfields $\mathcal{L}$; $\bar{\mathcal{D}}_{\dot\alpha}\mathcal{L} = 0$, and $\bar{\mathcal{L}}$, $\mathcal{D}_\alpha \bar{\mathcal{L}} = 0$, for the case of min SG?

In this case we have U(1) (R-symm) SSP connection.

$$A = E^a A_a + E^\alpha A_\alpha + \bar{E}^{\dot\alpha} \bar{A}_{\dot\alpha} = -(A)^*$$

with (super)field strength 2-form $\quad F = dA = i E^b \wedge E^\alpha (\sigma_b \bar{W})_\alpha - i E^b \wedge \bar{E}^{\dot\alpha} (W \sigma_b)_{\dot\alpha} + \frac{1}{2} E^b \wedge E^a F_{ab}$

$$\mathcal{D}_{(\alpha} A_{\beta)} = 0 \quad , \quad \bar{\mathcal{D}}_{(\dot\alpha} \bar{A}_{\dot\beta)} = 0$$

Ex: to show →

$$\Downarrow \qquad\qquad \Downarrow$$

$$\mathcal{D}_\alpha \mathcal{D}^\beta A_\beta = 0 \quad , \quad \bar{\mathcal{D}}_{\dot\alpha} \bar{\mathcal{D}}^{\dot\beta} \bar{A}_{\dot\beta} = 0$$

Thus the 4-form of the new min SG action is defined by chiral superfields

$$\bar{\mathcal{L}} = a \mathcal{D}^\alpha A_\alpha \quad , \quad \mathcal{L} = \bar{a} \bar{\mathcal{D}}^{\dot\alpha} \bar{A}_{\dot\alpha}$$

$a = \text{const} = (\bar{a})^*$

As $\mathcal{D}_{(\alpha} A_{\beta)} = 0 \Rightarrow A_\alpha = \mathcal{D}_\alpha T \quad , \quad \bar{A}_{\dot\alpha} = -\bar{\mathcal{D}}_{\dot\alpha} T$

so that $\quad \bar{\mathcal{L}} = a \mathcal{D}\mathcal{D} T \quad , \quad \mathcal{L} = \bar{a} \bar{\mathcal{D}}\bar{\mathcal{D}} T$

This is in agreement with

$$S^{\text{new min}} = \frac{1}{2}\int d^8z\, E\,(T+\bar{T}) - \frac{1}{4}\int d^6_{\zeta_L}\,\mathcal{E}\,\bar{\mathcal{D}}\bar{\mathcal{D}}T + \frac{1}{4}\int d^6_{\zeta_R}\,\bar{\mathcal{E}}\,\mathcal{D}\mathcal{D}\bar{T}$$

↑ pre-potential of $A_\alpha = \sim D_\alpha T$, $\bar{A}_{\dot\alpha} = \sim(-\bar{D}_{\dot\alpha}\bar{T})$

[Gauge transf.s can be used to reach $T=\bar{T}$]

pre-gauge inv: $\begin{cases} T \to T + \bar{\alpha}(\zeta_R) \\ \bar{T} \to \bar{T} + \alpha(\zeta_L) \end{cases}$

fixes the correct chiral measure

ALSO COMPARISION with $\hat{\mathcal{L}}_4$ ALLOWS (without working out $\mathcal{D}\mathcal{D}\mathcal{L}$ and $\bar{\mathcal{D}}\bar{\mathcal{D}}\bar{\mathcal{L}}$, but just using $\mathcal{L} = \mathcal{L}_0 + \Theta^\alpha \mathcal{D}_\alpha \mathcal{L} + \frac{1}{2}\Theta\Theta(-\frac{1}{4}\mathcal{D}\mathcal{D}\mathcal{L})$)

to find

$$\boxed{\mathcal{E} = e\left(1 + 2i\Theta^\alpha(\sigma_a\bar{\psi})_\alpha - 2\Theta\Theta\, \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\right)}$$

EVEN SIMPLER THAN IN MIN SG (DUE TO R=0)