§ 5.III. MATTER COUPLING TO SUGRA

5.III.1. SCALAR MULTIPLETS

– a – GENERAL SCALAR MULTIPLET(S) (SELF-)INTERACTION IN FLAT SSP

– b – Coupling to SUGRA

5.III.2. VECTOR MULTIPLET

SCALAR MULTIPLET SELF INTERACTION IN FLAT SSP

How we pass to the spacetime component action in flat SSP

FIRST, LET US BEGIN FROM (SIMPLEST) KIN. TERM:

$$S = \int d^8z\, \Phi\bar{\Phi} := \int d^8z\, \underbrace{\Phi(x+i\theta\sigma\bar{\theta},\theta)}_{=e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}\Phi(x,\theta)}\; \bar{\Phi}(\underbrace{x-i\theta\sigma\bar{\theta}}_{x_R},\bar{\theta}) =$$

$$\Phi(x+i\theta\sigma\bar\theta,\theta) = \varphi(x)+\theta\chi+\tfrac{1}{2}\theta\theta F(x)+i\theta\sigma^\mu\bar\theta\partial_\mu\varphi - \tfrac{i}{2}\theta\theta\,\partial_\mu\chi^\alpha(\sigma^\mu\bar\theta)_\alpha - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\varphi$$

$$\bar\Phi = \bar\varphi(x_R)+\bar\theta_{\dot\alpha}\bar\chi^{\dot\alpha}(x_R)+\tfrac{1}{2}\bar\theta\bar\theta\bar F(x_R)$$

$$= \bar\varphi(x)+\bar\theta_{\dot\alpha}\bar\chi^{\dot\alpha}(x)+\tfrac{1}{2}\bar\theta\bar\theta\bar F(x)-i\theta\sigma^\mu\bar\theta\partial_\mu\bar\varphi + \tfrac{i}{2}\bar\theta\bar\theta(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\bar\chi^{\dot\alpha} - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\bar\varphi$$

$$= \varnothing + \int d^8z\, \tfrac{1}{4}\theta\theta\cdot\bar\theta\bar\theta\,\Big[2\partial^\mu\varphi\partial_\mu\bar\varphi - \varphi\partial_\mu\partial^\mu\bar\varphi - \partial_\mu\partial^\mu\varphi\cdot\bar\varphi - i\chi\sigma^\mu\partial_\mu\bar\chi + i\partial_\mu\chi\sigma^\mu\bar\chi + F\bar F\Big] =$$

Ex: TO OBTAIN!

$$= \int d^4x\,\left(4\partial^\mu\varphi\partial_\mu\bar\varphi - i\chi\sigma^\mu\partial_\mu\bar\chi + i\partial_\mu\chi\sigma^\mu\bar\chi + F\cdot\bar F\right)$$

OTHER WAY TO CALCULATE:

$$S = \int d^8z\,\Phi\bar\Phi = \int d^6\zeta_L\, d^2\bar\theta\; \Phi(\zeta_L)\,\underbrace{\bar\Phi(\bar\zeta_R)}_{=e^{-2i\theta\sigma^\mu\bar\theta\partial^L_\mu}\bar\Phi(x_L,\bar\theta)} =$$

$$\bar\Phi(\bar\zeta_R) = \bar\varphi(x_L) + \bar\theta_{\dot\alpha}\left(\bar\chi^{\dot\alpha}(x_L)+2i(\theta\sigma^\mu)^{\dot\alpha}\partial^L_\mu\bar\varphi\right) + \tfrac{1}{2}\bar\theta\bar\theta\left(\bar F(x) + 2i(\theta\sigma^\mu)_{\dot\alpha}\partial^L_\mu\bar\chi^{\dot\alpha}(x_L) - 2\theta\theta\partial^L_\mu\partial^{\mu L}\bar\varphi\right)$$

$$= \int d^6\zeta_L\,\Phi(\zeta_L)\left(\bar F(x_L) + 2i(\theta\sigma^\mu)_{\dot\alpha}\partial^L_\mu\bar\chi^{\dot\alpha}(x_L) - 2\theta\theta\partial^L_\mu\partial^{\mu L}\bar\varphi(x_L)\right)$$

$$= \int d^4x\,\left(-4\varphi\partial_\mu\partial^\mu\bar\varphi - 2i\chi\sigma^\mu\partial_\mu\bar\chi + F\bar F\right)$$

THE SAME AS ABOVE UP TO A TOTAL DERIVATIVE

Ex: TO OBTAIN:

$$\boxed{\bar\Phi(\bar\zeta_R) = \bar\varphi(x_L) + \bar\theta_{\dot\alpha}\bar\chi^{\dot\alpha}(x_L) + \tfrac{1}{2}\bar\theta\bar\theta\bar F - 2i\theta\sigma^\mu\bar\theta\partial_\mu\bar\varphi + i\bar\theta\bar\theta(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\bar\chi^{\dot\alpha} - \theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\bar\varphi}$$

MOST GENERAL (SELF-) INTERACTION OF SCALAR MULTIPLET IN FLAT SSP

CHIRAL SUPERSPACE FORM

$$S = \int d^8 z\, K(\varphi, \bar{\varphi}) + \int d^6 \zeta_L\, W(\varphi) + \int d^6 \bar{\zeta}_R\, \bar{W}(\bar{\varphi})$$

1 WAY:

$$\int d^8 z\, K(\varphi,\bar{\varphi}) = \int d^6 \zeta_L\, d^2\bar{\theta}\, K(\varphi^i(\zeta_L), \underbrace{\bar{\varphi}^{\bar{i}}(\zeta_R)}) =$$

$$= \bar{\varphi}^{\bar{i}}(x_L) + \bar{\theta}_{\dot{\alpha}}(\bar{\chi}^{\dot{\alpha}\bar{i}}(x_L) + 2i(\theta\sigma^\mu)^{\dot{\alpha}}\partial_\mu\bar{\varphi}^{\bar{i}}) + \frac{1}{2}\bar{\theta}\bar{\theta}(\bar{F}^{\bar{i}} + 2i\theta\sigma^\mu\partial_\mu\bar{\chi} - 2\theta\theta\partial_\mu\partial^\mu\bar{\varphi}^{\bar{i}})$$

$$= \int d^6\zeta_L \Big[(\bar{F}^{\bar{i}} + 2i\theta\sigma^\mu\partial_\mu\bar{\chi}^{\bar{i}} - 2\theta\theta\partial_\mu\partial^\mu\bar{\varphi}^{\bar{i}})K'_{\bar{i}}(\varphi,\bar{\varphi}) +$$

$$+ K''_{\bar{i}\bar{j}}(\varphi,\bar{\varphi})(-\frac{1}{2}\bar{\chi}_{\dot{\alpha}}^{\bar{i}}\bar{\chi}^{\dot{\alpha}\bar{j}} + 2i\theta\sigma^\mu\bar{\chi}^{\bar{i}}\partial_\mu\bar{\varphi}^{\bar{j}} + 2\theta\theta\partial_\mu\bar{\varphi}^{\bar{i}}\partial^\mu\bar{\varphi}^{\bar{j}})\Big].$$

Ex: TO OBTAIN.

ALTERNATIVE WAY:

$$\int d^8 z\, K(\varphi,\bar{\varphi}) = -\frac{1}{2}\int d^6\zeta_L\, \bar{D}\bar{D}K(\varphi,\bar{\varphi}) =$$

$$= -\frac{1}{2}\int d^6\zeta_L\, \bar{D}\bar{D}\bar{\varphi}^{\bar{i}}\cdot K'_{\bar{i}}(\varphi,\bar{\varphi}) + \bar{D}_{\dot{\alpha}}\bar{\varphi}^{\bar{i}}\,\bar{D}^{\dot{\alpha}}\bar{\varphi}^{\bar{j}}K''_{\bar{i}\bar{j}}(\varphi,\bar{\varphi})$$

$$\bar{D}_{\dot{\alpha}}\bar{\varphi}^{\bar{i}} = \bar{\partial}^L_{\dot{\alpha}}\bar{\varphi}^{\bar{i}}(\zeta_R) = \bar{\partial}^L_{\dot{\alpha}}\bar{\varphi}^{\bar{i}}(x_L - 2i\theta\sigma\bar{\theta}, \bar{\theta}) =$$

$$= -\bar{\chi}_{\dot{\alpha}}(x_L) - \bar{\theta}_{\dot{\alpha}}\bar{F}(x_L) + 2i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\bar{\varphi} - 2\bar{\theta}_{\dot{\alpha}}(i(\theta\sigma^\mu)_\beta\partial_\mu\bar{\chi}^\beta) - 2\bar{\theta}_{\dot{\alpha}}\theta\theta\partial_\mu\partial^\mu\bar{\varphi}$$

!!!

$$\bar{D}_{\dot{\alpha}}\bar{\varphi}^{(\bar{i}}\bar{D}^{\dot{\alpha}}\bar{\varphi}^{\bar{j})} = \bar{\chi}_{\dot{\alpha}}^{\bar{i}}\bar{\chi}^{\dot{\alpha}\bar{j}} - 4i(\theta\sigma\bar{\chi}^{(\bar{i}}, \partial\bar{\varphi}^{\bar{j})}) + 4\theta\theta\partial_\mu\bar{\varphi}^{\bar{i}}\partial^\mu\bar{\varphi}^{\bar{j}} + \sim\bar{\theta}$$

$$\bar{D}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}} = -2$$

!!

$$\bar{D}\bar{D}\bar{\varphi}^{\bar{i}} = -2(\bar{F}^{\bar{i}}(x_L) + 2i\theta\sigma^\mu\partial_\mu\bar{\chi} + 2\theta\theta\partial_\mu\partial^\mu\bar{\varphi})$$

$$\int d^8 z\, K(\varphi,\bar{\varphi}) = \int d^6\zeta_L(\bar{F}^{\bar{i}}(x_L) + 2i\theta\sigma^\mu\partial_\mu\bar{\chi}^{\bar{i}} - 2\theta\theta\partial_\mu\partial^\mu\bar{\varphi}^{\bar{i}})K'_{\bar{i}}(\varphi,\bar{\varphi}) +$$
$$+ \int d^6\zeta_L(-\frac{1}{2}\bar{\chi}_{\dot{\alpha}}^{\bar{i}}\bar{\chi}^{\dot{\alpha}\bar{j}} + 2i(\theta\sigma^\mu\bar{\chi}^{\bar{i}})\partial_\mu\bar{\varphi}^{\bar{j}} - 2\theta\theta\partial_\mu\bar{\varphi}^{\bar{i}}\partial^\mu\bar{\varphi}^{\bar{j}})K''_{\bar{i}\bar{j}}(\varphi,\bar{\varphi})$$

$$S = \int d^8 z\, K(\varphi,\bar\varphi) + \int d^6 \varrho_L\, W(\varphi) + \int d^6 \bar\varrho_R\, \bar W(\bar\varphi) =$$

$$= \Big(\underbrace{-\tfrac{1}{4}\int d^6\varrho_L\, \bar D \bar D K(\varphi,\bar\varphi)}_{\text{L } \frac{1}{2}(\ldots \text{ see previous page})} + \underbrace{\int d^6 \varrho_L\, W(\varphi)}_{\text{L } \int d^4x \left(F^i W'_i(\varphi) - \frac{1}{2}\chi^i\chi^j W''_{ij}(\varphi)\right)} \Big) + \text{c.c.} =$$

total deriv: $\partial_\mu(\partial^\mu\bar\varphi^{\bar i} K'_{\bar i} + \partial^\mu\varphi^i K'_i) - 2\,\partial_\mu\partial^\mu\bar\varphi^{\bar i} K'_{\bar i} = 2\partial_\mu\partial^\mu\varphi^i K'_i - 2\left(\partial_\mu\bar\varphi^{\bar i}\partial^\mu\varphi^j K''_{\bar i j} + \text{c.c.}\right)$

this is what really appears.

Ex: TO OBTAIN:

$$= \int d^4x \Big[4\,\partial_\mu\varphi^i\partial^\mu\bar\varphi^{\bar j} K''_{i\bar j}(\varphi,\bar\varphi) +$$

$$+ F^i \bar F^{\bar i} K''_{i\bar i}(\varphi,\bar\varphi) + F^i\left(W'_i - \tfrac{1}{2}\bar\chi^{\bar i}\bar\chi^{\bar j} K'''_{i\bar i\bar j}\right) + \bar F^{\bar i}\left(\bar W'_{\bar i} - \tfrac{1}{2}\chi^i\chi^j K'''_{\bar i ij}\right)$$

$$- i\left(\chi^i\sigma^\mu\partial_\mu\bar\chi^{\bar i} - \partial_\mu\chi^i\sigma^\mu\bar\chi^{\bar i}\right) K''_{i\bar i}(\varphi,\bar\varphi) -$$

$$- i\,\chi^i\sigma^\mu\bar\chi^{\bar j}\left(\partial_\mu\varphi^j K'''_{ij\bar i} + \partial_\mu\bar\varphi^{\bar j} K'''_{i\bar i\bar j}\right) - \tfrac{1}{2}\chi^i\chi^j W''_{ij} - \tfrac{1}{2}\bar\chi^{\bar i}\bar\chi^{\bar j}\bar W''_{\bar i\bar j}$$

$$+ \tfrac{1}{4}\chi^i\chi^j\bar\chi^{\bar i}\bar\chi^{\bar j} K''''_{ij\bar i\bar j}(\varphi,\bar\varphi) \Big]$$

$$-i\,\chi^i\sigma^\mu D_\mu\bar\chi^{\bar i} + i\, D_\mu\chi^i\sigma^\mu\bar\chi^{\bar i}$$

<u>Kin terms:</u> $\left(4\partial_\mu\varphi^i\partial^\mu\bar\varphi^{\bar j} g_{i\bar j}(\varphi,\bar\varphi) - i\left(\chi^i\sigma^\mu\partial_\mu\bar\chi^{\bar j} - \partial_\mu\chi^i\sigma^\mu\bar\chi^{\bar j}\right) g_{i\bar j}(\varphi,\bar\varphi)\right)$

$$\boxed{g_{i\bar j} = \frac{\partial}{\partial\varphi^i}\frac{\partial}{\partial\bar\varphi^{\bar j}} K(\varphi,\bar\varphi) \equiv K_{i\bar j} \;\Big\|\; \text{Kähler geometry}}$$

SUSY OF NONLINEAR σ-MODEL REQUIRES THE TARGET SPACE TO BE KÄHLER MANIFOLD

[B. ZUMINO]

$g_{i\bar j} = K''_{i\bar j}$, $\boxed{K'''_{ij\bar j} = g_{k\bar j}\Gamma^k_{ij}}$ (L $g_{i\bar j, j}$), $\boxed{K'''_{i\bar i\bar j} = g_{i\bar k}\Gamma^{\bar k}_{\bar i\bar j}} = g_{i\bar i,\bar j}$

$$D_\mu\chi^i = \partial_\mu\chi^i + \Gamma^i_{jk}\partial_\mu\varphi^k\chi^j = \partial_\mu\chi^i + g^{i\bar i} K'''_{jk\bar i}\partial_\mu\varphi^k\chi^j$$

$$D_\mu\bar\chi^{\bar i} = \partial_\mu\bar\chi^{\bar i} + \Gamma^{\bar i}_{\bar j\bar k}\partial_\mu\bar\varphi^{\bar k}\bar\chi^{\bar j} = \partial_\mu\chi^i + g^{i\bar i} K'''_{i\bar j\bar k}\partial_\mu\bar\varphi^{\bar k}\cdot\bar\chi^{\bar j}$$

THE AUXILIARY FIELD CONTRIBUTE TO THE LAGRANGIAN THE FOLLOWING TERMS:

$$g_{i\bar{j}} = K''_{i\bar{j}}(\varphi,\bar\varphi)$$

$$\boxed{\mathcal{L}_{aux} = F^i \bar{F}^{\bar{j}} g_{i\bar{j}}(\varphi,\bar\varphi) + F^i\left(\bar{W}'_{i} - \tfrac{1}{2}\bar\chi^{\bar{i}}\bar\chi^{\bar{j}} g_{i\bar{i},\bar{j}}\right) + \bar{F}^{\bar{i}}\left(W'_{i} - \tfrac{1}{2}\chi^i\chi^j g_{i\bar{i},j}\right)}$$

$$\begin{aligned}\mathcal{L} &= \int d^4\theta\, K(\varphi,\bar\varphi) + \int d^2\theta\, W(\varphi) + \int d^2\bar\theta\, \bar{W}(\bar\varphi) = \\ &= 4\partial_\mu\varphi^i\partial^\mu\bar\varphi^{\bar{j}} g_{i\bar{j}}(\varphi,\bar\varphi) + i\left(D_\mu\chi^i\sigma^\mu\bar\chi^{\bar{j}} - i\chi^i \not{D}\bar\chi^{\bar{j}}\right) g_{i\bar{j}}(\varphi,\bar\varphi) - \\ &\quad - \tfrac{1}{2}\chi^i\chi^j W''_{ij} - \tfrac{1}{2}\bar\chi^{\bar{i}}\bar\chi^{\bar{j}}\bar{W}''_{\bar{i}\bar{j}} + \tfrac{1}{4}\chi^i\chi^j\bar\chi^{\bar{i}}\bar\chi^{\bar{j}} g_{i\bar{i},j\bar{j}} \\ &\quad + \mathcal{L}_{aux}\end{aligned}$$

$$\delta_{F,\bar{F}}\mathcal{L}_{aux} = 0 \Rightarrow \boxed{\begin{aligned} F^i &= -g^{i\bar{i}}\left(\bar{W}'_{\bar{i}} - \tfrac{1}{2} g_{i\bar{i},j}\chi^i\chi^j\right) \\ \bar{F}^{\bar{i}} &= -g^{i\bar{i}}\left(W'_{i} - \tfrac{1}{2} g_{i\bar{i},\bar{j}}\bar\chi^{\bar{i}}\bar\chi^{\bar{j}}\right)\end{aligned}}$$

$$\begin{aligned}\mathcal{L}_{aux}\Big|_{\text{on F-shell}} &= -\left(W'_{i} - \tfrac{1}{2}\bar\chi^{\bar{i}}\bar\chi^{\bar{j}} g_{i\bar{i},\bar{j}}\right) g^{i\bar{i}}\left(\bar{W}'_{\bar{i}} - \tfrac{1}{2} g_{\bar{i}i,j}\chi^i\chi^j\right) \\ &= -W'_i g^{i\bar{j}}\bar{W}'_{\bar{j}} - \tfrac{1}{2}\chi^i\chi^j W'_k\Gamma^k_{ij} - \tfrac{1}{2}\bar\chi^{\bar{i}}\bar\chi^{\bar{j}}\bar{W}'_{\bar{k}}\Gamma^{\bar{k}}_{\bar{i}\bar{j}} - \\ &\quad - \tfrac{1}{4}\chi^i\chi^j\bar\chi^{\bar{i}}\bar\chi^{\bar{j}}\end{aligned}$$

Thus, after eliminating the auxiliary fields

$$\boxed{\begin{aligned}\mathcal{L} &= 4\partial_\mu\varphi^i\partial^\mu\bar\varphi^{\bar{j}} g_{i\bar{j}}(\varphi,\bar\varphi) - i\left(\chi^i\not{D}\bar\chi^{\bar{j}} - D_\mu\chi^i\sigma^\mu\bar\chi^{\bar{j}}\right) g_{i\bar{j}}(\varphi,\bar\varphi) \\ &\quad \underbrace{- W'_i g^{i\bar{j}}\bar{W}'_{\bar{j}}}_{-V(\varphi,\bar\varphi)} - \tfrac{1}{2}\chi^i\chi^j\underbrace{\left(W''_{ij} + W'_k\Gamma^k_{ij}\right)}_{D_iD_jW} - \tfrac{1}{2}\bar\chi^{\bar{i}}\bar\chi^{\bar{j}}\underbrace{\left(\bar{W}''_{\bar{i}\bar{j}} + \bar{W}'_{\bar{k}}\Gamma^{\bar{k}}_{\bar{i}\bar{j}}\right)}_{D_{\bar{i}}D_{\bar{j}}\bar{W}} \\ &\quad + \tfrac{1}{4}\chi^i\chi^j\bar\chi^{\bar{i}}\bar\chi^{\bar{j}}\underbrace{\left(g_{i\bar{i},j\bar{j}} - g_{k\bar{i},\bar{j}}\, g^{k\bar{k}}\, g_{\bar{k}i,j}\right)}_{\frac{1}{2}R_{i\bar{i}j\bar{j}}}\end{aligned}}$$

$$\boxed{V(\varphi,\bar\varphi) = W'_i\, g^{i\bar{j}}(\varphi,\bar\varphi)\,\bar{W}'_{\bar{j}}}$$

1 field, $g_{i\bar{j}} = \delta_{i\bar{j}}$

$$\boxed{V = |W'|^2}$$

SCALAR MULTIPLET COUPLING TO MIN. SG.

MINIMAL SG ACTION

$$\int d^8z\, E = \int d^8z_R\, \hat{\bar{\nabla}}^{-1} F^{-2} \bar{F}^{-2}\Big| = \int d^8z_R\, \hat{\bar{\nabla}}^{-1} (\hat{\bar{\nabla}}\bar{\varphi})^{1/3} \varphi\bar{\varphi}$$

$$\hat{\bar{\nabla}} F^4 \bar{F}^{-2} = \bar{\varphi}^{-3}$$

MIN SG CONSTRAINTS ARE INV UNDER THE

SUPER-WEYL TRANSFORMATIONS $\longleftrightarrow$ $F \to e^{-W} F$, $\bar{F} \to e^{-\bar{W}} \bar{F}$

IF $\bar{\nabla}_{\dot\alpha}(2W + \bar{W}) = 0 \iff 2W + \bar{W} = \bar{\Sigma}\,,\quad \nabla_\alpha \bar{\Sigma} = 0$

δT_α

$2\bar{W} + W = \Sigma \qquad \bar{\nabla}_{\dot\alpha} \Sigma = 0$

$$\boxed{W = \tfrac{1}{3}(2\bar{\Sigma} - \Sigma)} \qquad W + \bar{W} = \tfrac{1}{3}(\bar{\Sigma} + \Sigma)$$

$$\boxed{\bar{W} = \tfrac{1}{3}(2\Sigma - \bar{\Sigma})}$$

However the action is NOT INV., BECAUSE

$$E := \mathrm{sdet}\, E_M{}^A \longrightarrow e^{2W + 2\bar{W}} E = e^{\frac{2}{3}(\bar{\Sigma} + \Sigma)} E$$

Keeping this in mind, let us recall that the generic kinetic term of the scalar superfield in FLAT SSP

$$\int d^8z\, K(\varphi, \bar{\varphi})$$

kähler potential

is inv. UNDER KÄHLER TRANSFORMATIONS: $\boxed{K(\varphi,\bar{\varphi}) \mapsto K(\varphi,\bar{\varphi}) + F(\varphi) + \bar{F}(\bar{\varphi})}$

COUPLING OF SCALAR SUPERFIELD TO SUGRA

"KIN. TERM"

$$\boxed{-3\int d^8z\, E\, e^{-\frac{1}{3}K(\varphi,\bar{\varphi})}}$$

PARAMETER OF CHIRAL WEYL TRANSFORMATIONS BECOMES FUNCTION OF CHIRAL MATTER SUPERFIELD

IS INV. UNDER SUPER-WEYL TRANSF SUPPLEMENTED BY THE KÄHLER TRANSF-S PROVIDED: $\boxed{F(\varphi) = 2\Sigma}$ OR BETTER $\boxed{\Sigma = \tfrac{1}{2}F(\varphi)}$

How we can reproduce the flat SSP limit $\int d^8z\, K(\varphi,\bar\varphi)$?

OBSERVATIONS: • CANONICAL DIM. OF SCALAR (SUPER)FIELD IS $[\varphi]=L^{-1}$

$\left(\int d^8z\, \varphi\bar\varphi \text{ – CANONICAL KIN. TERM} \quad \underbrace{[d^8z]}_{d^4x\, DD\cdot\bar D\bar D} = L^{4-2}=L^2 \right)$

• HENCE $[K(\varphi,\bar\varphi)] = L^{-2}$

• AS THE COMPLETE FORM OF SUGRA action is

$$\frac{1}{\kappa^2}\int d^8z\, E \qquad [\kappa^2] = L^{+2} \qquad \kappa \sim \frac{1}{M_{Pl}}$$

$$\left(\frac{c^3}{16\pi G_N} = \left(\frac{M_{Pl}\, c}{\hbar}\right)^2 \right)$$

• RESTORING THE DIM. CONSTANTS, OUR COUPLING IS

$$-\frac{3}{\kappa^2}\int d^8z\, E\, e^{-\frac{\kappa^2}{3}K(\varphi,\bar\varphi)}$$

• SWITCHING OFF THE GRAVITY IMPLIES $\kappa \to 0$ $(M_{Pl}\to\infty)$

$\frac{1}{\kappa^2}$ blows up, but it is coeff. for $\int d^8z\, E \to \int d^8z \cdot 1 = 0$

$$-\frac{3}{\kappa^2}\int d^8z\, E\left(1-\frac{\kappa^2}{3}K(\varphi,\bar\varphi) + \kappa^2\,\mathcal{O}(\kappa^2)\right) \longrightarrow$$

$$\xrightarrow[\substack{\kappa\to 0\\ E=1}]{} \int d^8z\, K(\varphi,\bar\varphi)$$

SUPERPOTENTIAL TERM IN THE PRESENCE OF SG

- IN FLAT SSP: $\sim \left(\int d^6 \varsigma_L \, \mathcal{W}(\varphi) + \int d^6 \varsigma_R \, \overline{\mathcal{W}}(\bar\varphi) \right)$
- IN CURVED SSP $\sim \boxed{\int d^6 \varsigma_L \, \mathcal{E} \, \mathcal{W}(\varphi) + c.c.}$

EX: TO FIND
How the superpotential scales under sWeyl/kähler transformations?

SOLUTION: LET US CONSIDER:

$$\int d^8 z \, E \, \mathbb{L}_\Delta = -\frac{1}{2} \int d^6 \varsigma_L \, \mathcal{E} \left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right) \mathbb{L}_\Delta$$

$$E \mapsto e^{2W + 2\overline{W}} E \underset{\text{min SG}}{\overset{\uparrow}{=}} e^{\frac{2}{3}(\Sigma + \bar\Sigma)} E$$

$$R \mapsto e^{-2\overline{W}} R - \bar{\mathcal{D}}\bar{\mathcal{D}} e^{-\overline{W}}$$

$$\bar{\mathcal{D}}_{\dot\alpha} \mapsto e^{-\overline{W}} \bar{\mathcal{D}}_{\dot\alpha} + \text{possible connection transf-s}$$

$$\Downarrow \qquad \llcorner\!\!= \; w^{\dot\alpha\dot\beta} \mapsto w^{\dot\alpha\dot\beta} + 2 \bar{E}^{(\dot\alpha} \bar{\mathcal{D}}^{\dot\beta)} \overline{W} + \sim E^q$$

$$\bar{\mathcal{D}}\bar{\mathcal{D}} \xmapsto{\text{when acting on a scalar}} e^{-\overline{W}} \bar{\mathcal{D}}_{\dot\alpha} e^{-\overline{W}} \bar{\mathcal{D}}^{\dot\alpha} + 2 e^{-2\overline{W}} \delta_{\dot\alpha}^{(\dot\alpha} \bar{\mathcal{D}}^{\dot\beta)} \overline{W} \cdot \bar{\mathcal{D}}_{\dot\beta} =$$

$$= e^{-2\overline{W}} \bar{\mathcal{D}}\bar{\mathcal{D}} - 4 e^{-2\overline{W}} \bar{\mathcal{D}}_{\dot\alpha} \overline{W} \cdot \bar{\mathcal{D}}^{\dot\alpha}$$

$$\left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right)_{\dots} \xmapsto{\text{when acting on a SCALAR SUPERFIELD}} \left(e^{-2\overline{W}} \bar{\mathcal{D}}\bar{\mathcal{D}} + 2 \bar{\mathcal{D}}_{\dot\alpha} e^{-2\overline{W}} \bar{\mathcal{D}}^{\dot\alpha} + \bar{\mathcal{D}}\bar{\mathcal{D}} e^{-2\overline{W}} - e^{-2\overline{W}} R \right)_{\dots} =$$

$$= \boxed{\left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right) e^{-2\overline{W}} \dots}$$

If $\int d^8 z \, E \, \mathbb{L}$ is inv under the Weyl transf-s,

$$E \to E e^{2W + 2\overline{W}} \Rightarrow \mathbb{L}_\Delta \to e^{-2W - 2\overline{W}} \mathbb{L}_\Delta$$

WITH OUR ABOVE RESULT

$$\left(\left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right) \mathbb{L}_\Delta \right) \longrightarrow \left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right) \underbrace{e^{-2W - 4\overline{W}}}_{= e^{-2\Sigma}} \mathbb{L}_\Delta = \qquad \left(\bar{\mathcal{D}}_{\dot\alpha} \Sigma = 0 \right)$$

$$= e^{-2\Sigma} \left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right) \mathbb{L}_\Delta$$

AS $\int d^8 z \, E \, \mathbb{L}_\Delta = \int d^6 \varsigma_L \, \mathcal{E} \left(\bar{\mathcal{D}}\bar{\mathcal{D}} - R \right) \mathbb{L}_\Delta = \text{inv} \Rightarrow \boxed{\mathcal{E} \mapsto e^{+2\Sigma} \mathcal{E}}$

$$\boxed{\mathcal{W}(\varphi) \longmapsto e^{-2\Sigma} \mathcal{W}(\varphi)} \qquad \llcorner\!\!= \left(\text{if } \int d^6 \varsigma_L \, \mathcal{E} \, \mathcal{W} = \text{inv} \right)$$

$e^{-2\Sigma}$: $\hookrightarrow e^{-2W - 4\overline{W}}$ ← chiral ONLY for min SG!

As $\mathcal{W}(\varphi) \longmapsto e^{-2\Sigma}\mathcal{W}(\varphi)$

IFF $\mathcal{W}(\varphi) \neq 0$ $\to$ $\left[\begin{array}{l} \langle \mathcal{W}(\varphi) \rangle = 0 \\ \mathcal{W}(0) \neq 0 \end{array} \right.$ $\Rightarrow$ NONTRIVIAL CONTRIBUTION TO COSMOLOGICAL CONSTANT

WE CAN "GAUGE AWAY" $\ln \mathcal{W}(\varphi)$

$$\Sigma = \frac{1}{2}\ln \mathcal{W}(\varphi) : \quad \boxed{\mathcal{W}(\varphi) \longrightarrow 1},$$

So that $\int d^6 \zeta_L \, \mathcal{E}\, \mathcal{W}(\varphi) \longmapsto \int d^6 \zeta_L \, \mathcal{E}$,

AND

$$e^{-\frac{1}{3}K(\varphi,\bar\varphi)} \longmapsto e^{-\frac{1}{3}K(\varphi,\bar\varphi) - \frac{2}{3}\Sigma - \frac{2}{3}\bar\Sigma} =$$

$$= e^{-\frac{1}{3}\left(K(\varphi,\bar\varphi) + \ln \mathcal{W}(\varphi) + \ln \bar{\mathcal{W}}(\bar\varphi)\right)}$$

$$= (\mathcal{W}\cdot\bar{\mathcal{W}})^{-1/3} e^{-\frac{1}{3}K(\varphi,\bar\varphi)}$$

Thus the most general coupling of MIN SG to scalar supermultiplet is DESCRIBED BY THE ACTION

$$\boxed{-3\int d^8 z \, E \, e^{-\frac{1}{3}\mathcal{K}(\varphi,\bar\varphi)} + \int d^6 \zeta_L \, \mathcal{E} + \int d^6 \zeta_R \, \bar{\mathcal{E}}}$$

$$\boxed{\mathcal{K}(\varphi,\bar\varphi) = K(\varphi,\bar\varphi) + \ln \mathcal{W}(\varphi) + \ln \bar{\mathcal{W}}(\bar\varphi)}$$

BAD DEFINED WHEN $\mathcal{W}(\varphi) = 0$.

TO PASS TO THE SPACETIME COMPONENT ACTION WE FIRST WRITE (THE SUPERFIELD ACTION IN ITS FORM) IT WITH CHIRAL MEASURE, LIKE $\int d^6\rho_L\, \mathcal{E} + c.c.$

KIN TERM:

$$(\sim(-3))\int d^8z\, E\, e^{-\frac{1}{3}\mathcal{K}(\varphi,\bar\varphi)} = -\frac{1}{2}\int d^6\rho_L\, \mathcal{E}\,(\bar{\mathcal{D}}\bar{\mathcal{D}}-R)\,\Omega(\varphi;\bar\varphi^{\bar i})$$

$$\mathcal{K}(z,\bar z) = -3\ln\Omega(z,\bar z)$$

$$= -\frac{1}{2}\int d^6\rho_L\, \mathcal{E}\Big((\bar{\mathcal{D}}\bar{\mathcal{D}}-R)\bar\varphi^{\bar i}\,\Omega'_{\bar i}(\varphi,\bar\varphi) + R\,(\bar\varphi^{\bar i}\Omega'_{\bar i}(\varphi,\bar\varphi)-\Omega(\varphi,\bar\varphi)) + \bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\cdot\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar j}\,\Omega''_{\bar i\bar j}(\varphi,\bar\varphi)\Big) =$$

here we have a chiral superfield $\boxdot = (\bar{\mathcal{D}}\bar{\mathcal{D}}-R)\,\Omega$:

$\bar{\mathcal{D}}_{\dot\alpha}\boxdot = 0$. We can decompose it on Θ and then perform (GRASSMANN (Berezin)) integration Extracting $[\mathcal{E}\boxdot]_F$ = coeff. for $\frac{1}{2}\Theta\Theta$.

Notice that we can set $\bar\Theta=0$ as the superfield result is chiral

$$= -\frac{1}{2}\int d^6\rho_L\, \mathcal{E}\Big((\bar{\mathcal{D}}\bar{\mathcal{D}}-R)\bar\varphi^{\bar i}\Big|_{\bar\Theta=0,\Theta\neq0}\Omega'_{\bar i}(\varphi,\bar\varphi) + R\,(\bar\varphi^{\bar i}\Omega'_{\bar i}(\varphi,\bar\varphi)-\Omega(\varphi,\bar\varphi)) + \bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\,\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar j}\Big|_{\substack{\bar\Theta=0\\ \Theta\neq0}}\Omega''_{\bar i\bar j}(\varphi,\bar\varphi)\Big)$$

As $\mathcal{D}_\alpha\bar\varphi^{\bar i}=0$ we have substituted $\bar\varphi^{\bar i}\to\bar\varphi^{\bar i}$ $(:=\bar\varphi^{\bar i}|_{\Theta=0})$ and $\mathcal{D}_\alpha\bar\varphi^{\bar i}|_0=0$, $\mathcal{D}\mathcal{D}\bar\varphi^{\bar i}|_0=0$

But $\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}$ does depend on Θ:

$$\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i} = 2i\,\mathcal{D}_{\alpha\dot\alpha}\bar\varphi^{\bar i} \Rightarrow \mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\Big| = 2i\,\sigma^a_{\alpha\dot\alpha}e_a{}^\mu\partial_\mu\bar\varphi^{\bar i}(x) + 2i\,\sigma^a_{\alpha\dot\alpha}\bar\psi_a{}^{\dot\beta}\bar\chi_{\dot\beta}{}^{\bar i}$$

$$\boxed{\bar\chi_{\dot\beta}{}^{\bar i} := -\bar{\mathcal{D}}_{\dot\beta}\bar\varphi^{\bar i}|}$$

(!)

$$(\mathcal{D}\mathcal{D}-R)\,\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i} = 0 \Rightarrow \boxed{\mathcal{D}\mathcal{D}\,\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\Big| = -R(x)\,\bar\chi_{\dot\alpha}{}^{\bar i}(x)}$$

Ex: to obtain

$$[\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}]_F = -\frac{1}{2}\mathcal{D}\mathcal{D}\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\Big|_0 = \frac{1}{2}R\,\bar\chi_{\dot\alpha}{}^{\bar i}(x)$$

Resuming:

$$\boxed{\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i} = -\bar\chi_{\dot\alpha}{}^{\bar i}(x) + 2i(\Theta\sigma^a)_{\dot\alpha}\hat{\mathcal{D}}_a\bar\varphi^{\bar i}(x) + \frac{1}{4}\Theta\Theta\,\bar\chi_{\dot\alpha}{}^{\bar i}(x)R\Big| + \sim\bar\Theta}$$

$$\boxed{\hat{\mathcal{D}}_a\bar\varphi^{\bar i}(x) := e_a{}^\mu\partial_\mu\bar\varphi^{\bar i}(x) - \bar\chi_{\dot\beta}{}^{\bar i}\,\bar\psi_a{}^{\dot\beta}(x)}$$

do not contribute to our integral.

$$\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\cdot\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar j} = \bar\chi_{\dot\alpha}^{\bar i}\bar\chi^{\dot\alpha\bar j} + 4i\bar\Theta\bar\sigma^a\bar\chi^{(\bar i}\hat{\mathcal{D}}_a\bar\varphi^{\bar j)} + \frac{1}{2}\bar\Theta\bar\Theta\left(8\hat{\mathcal{D}}_a\bar\varphi^{\bar i}\hat{\mathcal{D}}^a\bar\varphi^{\bar j} - 2\bar\chi_{\dot\alpha}^{\bar i}\bar\chi^{\dot\alpha\bar j}R(x)\right) + \sim\Theta$$

A BIT MORE COMPLICATED IS THE CALCULATION OF $\boxed{\bar\square^{\bar i} = (\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\bar\varphi^{\bar i}}$

$$\mathcal{D}_\alpha\bar\square^{\bar i} = 4i\mathcal{D}_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar i} - G_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar i} - \mathcal{D}_\alpha R\cdot\bar\varphi^{\bar i}$$

$$\| \quad \mathcal{D}_\alpha(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\bar\varphi^{\bar i} = \mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar i} - \mathcal{D}_\alpha R\cdot\bar\varphi^{\bar i}$$

$$\boxed{\mathcal{D}_\alpha\bar\square^{\bar i}\Big| = -4i\sigma^a_{\alpha\dot\alpha}\hat{\mathcal{D}}_a\bar\chi^{\dot\alpha\bar i} + G_{\alpha\dot\alpha}\bar\chi^{\dot\alpha\bar i} + i\bar\varphi^{\bar i}\left(\sigma^a\bar\psi_{a\alpha} - 2(\sigma_a\bar\psi^a)_\alpha\cdot R\right)}$$

$$\mathcal{D}\mathcal{D}\bar\square^{\bar i} = -16\mathcal{D}_a\mathcal{D}^a\bar\varphi^{\bar i} + 4iG^a\mathcal{D}_a\bar\varphi^{\bar i} + 2\bar R\cdot\bar{\mathcal{D}}\bar{\mathcal{D}}\bar\varphi^{\bar i} - \mathcal{D}\mathcal{D}R\cdot\bar\varphi^{\bar i} + \bar{\mathcal{D}}_{\dot\alpha}\bar R\cdot\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar i}$$

with $\hat{\mathcal{D}}_a\bar\varphi^{\bar i} = \mathcal{D}_a\bar\varphi^{\bar i}\Big|$, $\hat{\mathcal{D}}_a\bar\chi_{\dot\alpha}^{\bar i} = \mathcal{D}_a\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\Big|$

$\bar\chi_{\dot\alpha}^{\bar i} := -\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\Big|$, $\bar F^{\bar i} := -\frac{1}{2}\bar{\mathcal{D}}\bar{\mathcal{D}}\bar\varphi^{\bar i}\Big|$

$$\boxed{\begin{aligned}\mathcal{D}\mathcal{D}\bar\square^{\bar i}\Big| = {}& -16e_a{}^m\mathcal{D}_m\hat{\mathcal{D}}^a\bar\varphi^{\bar i} + 4iG^a\hat{\mathcal{D}}_a\bar\varphi^{\bar i} - 4\bar R\cdot\bar F^{\bar i} - \bar\varphi^{\bar i}\cdot\mathcal{D}\mathcal{D}R\Big| + \\ & \underbrace{-\bar{\mathcal{D}}_{\dot\alpha}\bar R\Big|\cdot\bar\chi^{\dot\alpha\bar i}} - 16\bar\psi^{a\dot\alpha}\hat{\mathcal{D}}_a\bar\chi_{\dot\alpha}^{\bar i} - 2i\psi^b\sigma_b\cancel{\bar\chi^{\bar i}\cdot\bar R}\Big| + 2i\bar\chi^{\bar i}\hat\sigma_a\sigma_b\bar\psi^b\cdot G^a\Big|\end{aligned}}$$

(1)

$$\underbrace{-\frac{4i}{3}(\bar\psi^a\sigma_a)_{\dot\alpha}\Big|\cdot\bar\chi^{\dot\alpha\bar i}} = -\frac{4i}{3}(\overset{\circ}{\bar\psi}{}^a\sigma_a)_{\dot\alpha}\bar\chi^{\dot\alpha\bar i} + 2i\cancel{\bar R\cdot\psi^b\sigma_b\bar\chi^{\bar i}}\bar R + iG^a\bar\chi^{\bar i}\bar\psi_a$$

(1)

Now we have all the ingredients to calculate

$$-3\int d^8z\, E\, e^{-\frac{1}{3}K(\varphi,\bar\varphi)} =: \int d^8z\, E\, \Omega(\varphi,\bar\varphi) =$$

$$= -\frac{1}{2}\int d^6\varsigma_L\, \mathcal{E}\, \Xi^{\bar i}\, \Omega'_{\bar i}(\varphi,\bar\varphi) - \frac{1}{2}\int d^6\varsigma_L\, R(\bar\varphi^{\bar i}\Omega'_{\bar i} - \Omega(\varphi,\bar\varphi)) - \frac{1}{2}\int d^6\varsigma_L\, \bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar j}\Omega''_{\bar i\bar j}$$

$$e\left(1+2i\Theta\sigma_a\bar\psi^a + \tfrac{1}{2}\Theta\Theta\left(\tfrac{3}{2}\bar R - \bar\psi_a\bar\sigma^{ab}\bar\psi_b\right)\right)\left(-2F^{\bar i} - R_0\bar\varphi^{\bar i} + \Theta^\alpha \mathcal{D}_\alpha \Xi^{\bar i} + \tfrac{1}{2}\Theta\Theta(-\tfrac{1}{4}\mathcal{D}\mathcal{D}\Xi^{\bar i})\right)\left(\Omega'_{\bar i}(\varphi,\bar\varphi) + \Theta^\alpha\chi^i_\alpha\Omega''_{i\bar i}(\varphi,\bar\varphi) + \tfrac{1}{2}\Theta\Theta\left(F^i\Omega''_{i\bar i}(\varphi,\bar\varphi) - \tfrac{1}{2}\chi^i\chi^j\Omega'''_{ij\bar i}(\varphi,\bar\varphi)\right)\right)$$

see previous p. 10

Ex. (*OPTIONAL AND VERY INVOLVING*): TO FIND THE COMPLETE EXPRESSION.
(SEE BAGGER & WESS FOR THE COMPLETE EXPRESSION)

WE SIMPLIFY OUR LIFE: FOLLOW ONLY BOSONIC FIELDS: FERMIONIC FIELDS $\mapsto 0$

THEN: $\mathcal{E} = e(1+\frac{3}{4}\Theta\Theta\bar R)$;

$\Omega(\varphi,\bar\varphi) = \Omega(\varphi,\bar\varphi) + \frac{1}{2}\Theta\Theta F^i\Omega'_i(\varphi,\bar\varphi)$;

$R(\varsigma_L) = R(x) + \frac{1}{2}\Theta\Theta\left(\frac{4}{3e}L^{\text{kin}\,SG} - \frac{3}{2}R\cdot\bar R + i e_a{}^m \mathcal{D}_m G^a\right)$

$$= \int d^6\varsigma_L\left(-\frac{1}{2}\mathcal{E}\,\Xi^{\bar i}\,\Omega'_{\bar i}(\varphi,\bar\varphi) - \frac{1}{2}R(\varsigma_L)\left(\bar\varphi^{\bar i}\Omega'_{\bar i}(\varphi,\bar\varphi) - \Omega(\varphi,\bar\varphi)\right) - \frac{1}{2}\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar j}\cdot\Omega''_{\bar i\bar j}(\varphi,\bar\varphi)\right)$$

$\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i} \to 2i(\Theta\sigma^a)_{\dot\alpha}\mathcal{D}_a\bar\varphi^{\bar i}$ + fermions

coeff. $\Theta\Theta$: $\frac{1}{2}\Theta\Theta e\left(\bar F^{\bar i}F^i\Omega''_{i\bar i} + \frac{1}{2}R_0\bar\varphi^{\bar i}F^i\Omega''_{i\bar i}\right)$ (5)

$+\frac{1}{2}\Theta\Theta e\left(\frac{1}{4}\mathcal{D}\mathcal{D}\Xi^{\bar i}\Omega'_{\bar i} + \frac{3}{2}\bar R_0 F^{\bar i}\Omega'_{\bar i} + \frac{3}{4}R_0\bar R_0\bar\varphi^{\bar i}\Omega'_{\bar i}\right)$ (4) (3)

$\to -\frac{1}{4}\bar\varphi^{\bar i}\Omega'_{\bar i}\mathcal{D}\mathcal{D}R$ (1) $- R_0 F^{\bar i}\Omega'_{\bar i}$ (4) $+ iG^a\hat{\mathcal{D}}_a\bar\varphi^{\bar i}\Omega'_{\bar i} - 4e_a{}^m\mathcal{D}_m\hat{\mathcal{D}}^a\bar\varphi^{\bar i}\Omega'_{\bar i}$

$\to 4\hat{\mathcal{D}}^a\bar\varphi^{\bar i}\mathcal{D}_a\varphi^i\Omega''_{i\bar i} + 4\mathcal{D}^a\varphi\,\mathcal{D}_a\bar\varphi^{\bar j}\Omega''_{\bar i\bar j}$ (2) $- 4e_a{}^m\mathcal{D}_m(\hat{\mathcal{D}}^a\bar\varphi^{\bar i}\Omega'_{\bar i})$ + f. — total derivative (+ fermions)

coeff. $\Theta\Theta$: $\frac{1}{2}\Theta\Theta e\left(\frac{1}{4}\mathcal{D}\mathcal{D}R\,\bar\varphi^{\bar i}\Omega'_{\bar i} - \frac{1}{4}\mathcal{D}\mathcal{D}R\cdot\Omega(\varphi,\bar\varphi)\right)$ (1) $+$

$+\frac{1}{2}\Theta\Theta e\left(+\frac{1}{2}R_0F^i\Omega'_i - \frac{1}{2}R_0F^i\bar\varphi^{\bar i}\Omega''_{i\bar i} + \frac{3}{4}R_0\bar R_0\Omega - \frac{3}{4}R_0\bar R_0\bar\varphi^{\bar i}\Omega'_{\bar i}\right)$ (5) (3)

$\frac{2}{3}\cdot\frac{1}{e}L^{\text{kin}\,SG} - \frac{1}{2}R\cdot\bar R + \frac{i}{2}\mathcal{D}_a G^a$

$\frac{1}{2}\Theta\Theta e\left(-4\hat{\mathcal{D}}_a\bar\varphi^{\bar i}\hat{\mathcal{D}}^a\bar\varphi^{\bar j}\Omega''_{\bar i\bar j}\right)$ + fermions (2)

Ex: TO obtain (solution is above)

$$= \int d^4x\Big[\frac{2}{3}e\,\Omega R + \frac{1}{16}e\,\Omega\,G^aG_a + \frac{1}{4}e\,\Omega\cdot R\cdot\bar R + 4e\,\mathcal{D}^a\bar\varphi^{\bar i}\mathcal{D}_a\varphi^i\,\Omega''_{i\bar i}(\varphi,\bar\varphi) +$$

$$+F^i\bar F^{\bar i}\Omega''_{i\bar i} + \frac{1}{2}F^i\bar R_0\Omega'_i + \frac{1}{2}\bar F^{\bar i}R_0\Omega'_{\bar i} -$$

$$-\frac{i}{2}e\,G^a\mathcal{D}_a\varphi^i\,\Omega'_i + \frac{i}{2}e\,G^a\mathcal{D}_a\bar\varphi^{\bar i}\Omega'_{\bar i} + \text{FERMIONS}\Big]$$