The potential terms

$$\int d^6\varsigma_L\, \mathcal{E} W(\varphi) + c.c. \longrightarrow \int d^6\varsigma\, \mathcal{E} + c.c. = \int d^6\varsigma \cdot e(1+\tfrac{3}{4}\Theta\Theta R) + c.c. = \overset{+\ \text{fermions}}{}$$

$$= \int d^4x\ \tfrac{3}{2}(R_0 + \bar R_0) + \text{fermions}$$

$$-3\int d^8z\, E\, e^{-\frac{1}{3}\mathcal{K}(\varphi,\bar\varphi)} + \int d^6\varsigma_L\, \mathcal{E} + \int d^6\varsigma_R\, \bar{\mathcal{E}} =$$

$$= \int d^4x \Big(\tfrac{2}{3} e\Omega R + \boxed{\tfrac{1}{16} e\Omega \mathcal{G}^a\mathcal{G}_a} + \boxed{4e\hat{\mathcal{D}}^a\varphi^i\hat{\mathcal{D}}_a\bar\varphi^{\bar i}\Omega''_{i\bar i}} +$$

$$+ \tfrac{3}{2}R_0 + \tfrac{3}{2}\bar R_0 \quad + \tfrac{i}{2} e\,\mathcal{G}^a(\mathcal{D}_a\bar\varphi^{\bar i}\Omega'_{\bar i} - \mathcal{D}_a\varphi^i\Omega'_i) +$$

$$+ \underbrace{\tfrac{1}{4}e\Omega\cdot R_0\bar R_0}_{\ni\ \frac{1}{4}e\Omega\,|R_0 - \frac{2}{3}\bar F^{\bar i}K_{\bar i}|^2} + \underbrace{\tfrac{e}{2}R_0 F^i\Omega'_i + \tfrac{e}{2}\bar R_0\bar F^{\bar i}\Omega'_{\bar i} + eF^i\bar F^{\bar i}\Omega''_{i\bar i}}_{\ni\ \frac{e\Omega}{9}F^iK_i\bar F^{\bar i}K_{\bar i} - \frac{e\Omega}{6}R_0F^iK_i - \frac{e\Omega}{6}\bar R_0\bar F^{\bar i}K_{\bar i} - \frac{1}{3}e\Omega F^i\bar F^{\bar i}\underbrace{K_{i\bar i}}_{=g_{i\bar i}}} + \text{fermions}\Big)$$

In our notation: $\boxed{\Omega = -3e^{-\frac{1}{3}\mathcal{K}(\varphi,\bar\varphi)}}$, $\Rightarrow$ $\boxed{\begin{aligned}\Omega'_i &= \mathcal{K}_i e^{-\frac{1}{3}\mathcal{K}} = -\tfrac{1}{3}\mathcal{K}_i\Omega \\ \Omega'_{\bar i} &= \mathcal{K}_{\bar i}e^{-\frac{1}{3}\mathcal{K}} = -\tfrac{1}{3}\mathcal{K}_{\bar i}\Omega\end{aligned}}$

$$\boxed{\Omega''_{i\bar i} = -\tfrac{1}{3}(\underbrace{\mathcal{K}_{i\bar i}}_{=:\ g_{i\bar i}} - \tfrac{1}{3}\mathcal{K}_i\mathcal{K}_{\bar i})\cdot\Omega}$$

By algebraic transf-s and defining: $\begin{cases}\hat R_0 := \bar R_0 - \tfrac{2}{3}F^i\mathcal{K}_i \\ \tilde R_0 := R_0 - \tfrac{2}{3}\bar F^{\bar i}\mathcal{K}_{\bar i}\end{cases}$

$$-3\int d^8z\, E\, e^{-1/3\,\mathcal{K}(\varphi,\bar\varphi)} + \int d^6\varsigma_L\,\mathcal{E} + \int d^6\varsigma_R\,\bar{\mathcal{E}} =$$

$$= \int d^4x\Big(\tfrac{2}{3}e\Omega R + 4e\hat{\mathcal{D}}^a\bar\varphi^{\bar i}\hat{\mathcal{D}}_a\varphi^i\Omega''_{i\bar i} + \tfrac{i}{2}e\,\mathcal{G}^a(\mathcal{D}_a\bar\varphi^{\bar i}\Omega'_{\bar i} - \mathcal{D}_a\varphi^i\Omega'_i) +$$

$$+ \tfrac{1}{16}e\Omega\,\mathcal{G}^a\mathcal{G}_a - \tfrac{1}{3}e\Omega\,F^i\bar F^{\bar i}\underbrace{\mathcal{K}_{i\bar i}}_{=g_{i\bar i}} + e\bar F^{\bar i}\mathcal{K}_{\bar i} + eF^i\mathcal{K}_i +$$

$$+ \tfrac{1}{4}e\Omega\,\tilde R_0\cdot\hat R_0 + \tfrac{3}{2}e(\tilde R_0 + \hat R_0) + \text{fermions}\Big)$$

AUXILIARY FIELD EQUATIONS

$\delta \sigma^a: \Rightarrow$ $$\boxed{\sigma_a = -\frac{4i}{\Omega}\left(\mathcal{D}_a\varphi^i\Omega'_i - \mathcal{D}_a\bar{\varphi}^{\bar{i}}\Omega'_{\bar{i}}\right)}$$

$$\Rightarrow \frac{1}{16}e\Omega\sigma^a\sigma_a + \frac{i}{2}e\sigma^a\left(\hat{\mathcal{D}}_a\bar{\varphi}^{\bar{i}}\Omega'_{\bar{i}} - \hat{\mathcal{D}}_a\varphi^i\Omega'_i\right) = \frac{1}{\Omega}\left(\mathcal{D}_a\varphi^i\Omega'_i - \mathcal{D}_a\bar{\varphi}^{\bar{i}}\Omega'_{\bar{i}}\right)^2$$

$\delta\tilde{\hat{R}}_0: \Rightarrow$ $$\boxed{\tilde{R}_0 = -2\cdot\frac{3}{\Omega}}, \quad \boxed{\hat{R}_0 = -2\frac{3}{\Omega}} \Leftarrow \delta\tilde{R}_0$$

$$\Rightarrow +\frac{1}{4}e\Omega\hat{R}_0\tilde{\hat{R}}_0 + \frac{3}{2}e\left(\hat{R}_0 + \tilde{\hat{R}}_0\right) = -\frac{9}{\Omega}$$

$\delta\bar{F}^{\bar{i}}: \Rightarrow$ $$0 = -\frac{1}{3}\Omega e F^i \underbrace{K_{i\bar{i}}}_{[\,]^{-1} =: g^{i\bar{i}}} + e K_{\bar{i}}$$

$$\Leftrightarrow \boxed{F^i = \frac{3}{\Omega}g^{i\bar{i}}K_{\bar{i}}} \quad \text{c.c.} \quad \boxed{\bar{F}^{\bar{i}} = \frac{3}{2}g^{\bar{i}i}K_i} \quad \Leftarrow \delta F^i$$

$$-\frac{1}{3}e\Omega F^i\bar{F}^{\bar{i}}K_{i\bar{i}} + e\bar{F}^{\bar{i}}K_{\bar{i}} + eF^iK_i = \frac{3}{\Omega}K_i g^{i\bar{i}}K_{\bar{i}}$$

SUBSTITUTING THIS BACK TO THE ACTION, WE FIND

$$-3\int d^8z\, E\, e^{-\frac{1}{6}K(\varphi,\bar{\varphi})} + \int d^6\varsigma_L\, \mathcal{E} + \int d^6\varsigma_R\, \bar{\mathcal{E}} =$$

$$= \int d^4x\left(\underline{\frac{2}{3}e\Omega R} + 4e\hat{\mathcal{D}}^a\varphi^i\hat{\mathcal{D}}_a\bar{\varphi}^{\bar{i}}\Omega''_{i\bar{i}} + \frac{1}{\Omega}\left(\mathcal{D}_a\varphi^i\Omega'_i - \mathcal{D}_a\bar{\varphi}^{\bar{i}}\Omega'_{\bar{i}}\right)^2 + \right.$$

$\ldots R e^{-\frac{1}{3}K(\varphi,\bar{\varphi})}$

Brans-Dicke type

But we can redefine e^a_r (and φ^α_r) to come to the Kanonical form

$$+ \mathcal{V}(\varphi,\bar{\varphi}) + \text{fermions}$$

Potential $$\boxed{\mathcal{V}(\varphi,\bar{\varphi}) = e^{\frac{1}{3}K(\varphi,\bar{\varphi})}\left(K_i g^{i\bar{i}}K_{\bar{i}} - 3\right)}$$

min $\mathcal{N}=1$

SYM in SUGRA background

(S)U(N) SYM (FOR SIMPLICITY)

$$\mathbb{F} = d\mathbb{A} - \mathbb{A}\wedge\mathbb{A} = \frac{1}{2} dZ^N \wedge dZ^M \underbrace{\mathbb{F}_{MN}}_{\partial_M \mathbb{A}_N - (-)^{NM}\partial_N \mathbb{A}_M + [\mathbb{A}_M, \mathbb{A}_N\}} = \frac{1}{2} E^B \wedge E^A \underbrace{\mathbb{F}_{AB}}_{\mathcal{D}_A \mathbb{A}_B - (-)^{AB} D_B \mathbb{A}_A + [\mathbb{A}_A, \mathbb{A}_B\} + T_{AB}{}^C \mathbb{A}_C} = -\mathbb{F}^\dagger = \mathbb{F}^I T_{I(N)}^{(1)}$$

- OFF-SHELL CONSTRAINTS:

$$\mathbb{F}_{\alpha\beta} = 0 = \mathbb{F}_{\dot\alpha\dot\beta}, \quad \mathbb{F}_{\alpha\dot\beta} = 0$$

- Solution in the "chiral" basis

THIS IS A GAUGE, IT IS INV. UNDER CHIRAL : $\bar{D}_{\dot\alpha} U_I^J = 0$

$$\mathcal{D}_\alpha = e^{-V} D_\alpha e^V, \quad \bar{\mathcal{D}}_{\dot\alpha} = \bar{D}_{\dot\alpha}$$

$$\mathbb{A}_\alpha = \left(e^{-V} D_\alpha e^V\right), \quad \mathbb{A}_{\dot\alpha} = 0$$

- BIs: $\quad \mathcal{D}\mathbb{F} = 0$

for $\mathbb{F} = E^b \wedge E^\alpha \wedge \mathbb{F}_{\alpha b} + E^b \wedge \bar{E}^{\dot\alpha} \wedge \mathbb{F}_{\dot\alpha b} + \frac{1}{2} E^b \wedge E^a \mathbb{F}_{ab}$

dim 3/2

$\sim E^\alpha \wedge E^\beta \wedge \bar{E}^{\dot\beta} \quad \Rightarrow \mathbb{F}_{(\alpha\,\beta)\dot\beta} = 0 \Rightarrow \boxed{\mathbb{F}_{\alpha b} = i\sigma_{b\alpha\dot\alpha} \bar{\mathbb{W}}^{\dot\alpha}}$ c.c. $\boxed{\mathbb{F}_{\dot\alpha b} = -i\sigma_{b\alpha\dot\alpha} \mathbb{W}^\alpha}$

$$\mathbb{F} = i E^b \wedge E^\alpha (\sigma_b \bar{\mathbb{W}})_\alpha - i E^b \wedge \bar{E}^{\dot\alpha} (\mathbb{W}\sigma_b)_{\dot\alpha} + \frac{1}{2} E^b \wedge E^a \mathbb{F}_{ab} = -\mathbb{F}^\dagger$$

dim 2:

$\sim E^b \wedge E^\alpha \wedge E^\beta \quad \boxed{\mathcal{D}_\beta \bar{\mathbb{W}}^{\dot\alpha} = 0} \quad \sim E^b \wedge \bar{E}^{\dot\alpha} \wedge \bar{E}^{\dot\beta} \quad \boxed{\bar{\mathcal{D}}_{\dot\beta} \mathbb{W}^\alpha = 0}$

THIS CAN BE SEEN EASILY ALSO FROM THE SOLUTION OF THE CONSTRAINTS AS

$$\boxed{\mathbb{W}_\alpha = -\frac{1}{8}(\bar{D}\bar{D} - R)\left(e^{-V} D_\alpha e^V\right)}$$

and $\quad \bar{D}_{\dot\alpha}(\bar{D}\bar{D} - R)\, U_\alpha \equiv 0$

$\sim E^b \wedge E^\alpha \wedge \bar{E}^{\dot\alpha} \quad 0 = -i\mathcal{D}_\alpha \mathbb{W}^\beta \sigma_{b\beta\dot\alpha} + i\sigma_{b\,\alpha\dot\beta} \bar{\mathcal{D}}_{\dot\alpha} \bar{\mathbb{W}}^{\dot\beta} - 2i\sigma^a_{\alpha\dot\alpha} \mathbb{F}_{ab} \Rightarrow$

$$\mathbb{F}_{ab} = \frac{1}{4} \mathcal{D}_\alpha \mathbb{W}^\beta (\sigma_{ab})_\beta{}^\alpha + \frac{1}{4} \bar{\sigma}_{ab}{}^{\dot\alpha}{}_{\dot\beta} \bar{\mathcal{D}}_{\dot\alpha} \bar{\mathbb{W}}^{\dot\beta} \iff \mathbb{F}_{\alpha\dot\alpha\,\beta\dot\beta} = \epsilon_{\dot\alpha\dot\beta} \mathcal{D}_{(\alpha} \mathbb{W}_{\beta)} - \epsilon_{\alpha\beta} \bar{\mathcal{D}}_{(\dot\alpha} \bar{\mathbb{W}}_{\dot\beta)}$$

AND $\quad \boxed{\mathcal{D}^\alpha \mathbb{W}_\alpha - \bar{\mathcal{D}}^{\dot\alpha} \bar{\mathbb{W}}_{\dot\alpha} = 0}$

$$\boxed{\mathcal{D}^\alpha \mathbb{W}_\alpha + \bar{\mathcal{D}}^{\dot\alpha} \bar{\mathbb{W}}_{\dot\alpha} =: 2\mathbb{D} = 2\mathbb{D}^I T_I} \quad \leftarrow \text{AUXILIARY SYM FIELD}$$

$\boxed{\mathcal{D}_\alpha \mathbb{W}_\beta = \frac{1}{2}\epsilon_{\alpha\beta} \mathbb{D} - \frac{1}{2} \mathbb{F}_{ab} \sigma^{ab}_{\alpha\beta}}$ c.c. $\boxed{\bar{\mathcal{D}}_{\dot\alpha} \bar{\mathbb{W}}_{\dot\beta} = +\frac{1}{2}\epsilon_{\dot\alpha\dot\beta} \mathbb{D} + \frac{1}{2} \mathbb{F}_{ab} \bar{\sigma}^{ab}_{\dot\alpha\dot\beta}}$

dim $5/2$, $\sim E^b \wedge E^\gamma \wedge E^{\dot\alpha}$

$$\mathcal{D}_\alpha \mathbb{F}_{ab} = -2i\, \sigma_{[a|\alpha\dot\alpha}\, \mathcal{D}_{|b]} \overline{\mathbb{W}}^{\dot\alpha} - R (\sigma_{ab})_\alpha{}^\beta\, \mathbb{W}_\beta$$

$$\bar{\mathcal{D}}_{\dot\alpha} \mathbb{F}_{ab} = -2i\, \mathcal{D}_{[a|} \mathbb{W}^\alpha \sigma_{|b]\alpha\dot\alpha} + \bar{R}\, \overline{\mathbb{W}}_{\dot\beta} (\hat\sigma_{ab})^{\dot\beta}{}_{\dot\alpha}$$

$$\Downarrow$$

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} \mathbb{F}_{ab} = 2i\, \sigma^c_{\alpha\dot\alpha} \mathcal{D}_c \mathbb{F}_{ab} = -2i\, \sigma_{[a|\alpha\dot\alpha}\, \bar{\mathcal{D}}_{\dot\alpha} \mathcal{D}_{|b]} \overline{\mathbb{W}}^{\dot\alpha} + 2i\, \sigma_{[a|\beta\dot\alpha}\, \mathcal{D}_\alpha \mathcal{D}_{|b]} \mathbb{W}^\beta$$

$\circ\ \circ\ \circ$

$\circ\ \circ\ \circ$

FOR SIMPLICITY WE WILL DISCUSS THE CASE OF GAUGE GROUP $\subset$ (S)U(N) AND SCALAR MULTIPLETS DESCRIBED BY CHIRAL SUPERFIELDS IN FUNDAMENTAL OF (S)U(N).

$$U^+U = I$$

$$\Phi \longmapsto \Phi U, \qquad \mathcal{D}\Phi = d\Phi - \Phi \mathbb{A}, \qquad \mathcal{D}\mathcal{D}\Phi = -\Phi \mathbb{F}$$

$$\bar\Phi \longmapsto U^+\bar\Phi, \qquad \mathcal{D}\bar\Phi = d\bar\Phi + \mathbb{A}\bar\Phi, \qquad \mathcal{D}\mathcal{D}\bar\Phi = \mathbb{F}\bar\Phi$$

$$\mathbb{A}^+ = -\mathbb{A}, \qquad \mathbb{A} \longmapsto \mathbb{A}' = U^{-1}dU + U^{-1}\mathbb{A}U, \qquad U^{-1} = U^+$$

$$\mathbb{F} = d\mathbb{A} - \mathbb{A}\wedge\mathbb{A}, \qquad \mathbb{F} = -\mathbb{F}^+, \qquad \mathbb{F} \to \mathbb{F}' = U^+\mathbb{F}U$$

OUR CONSTRAINTS $\mathbb{F}_{\alpha\beta} = 0$, $\mathbb{F}_{\dot\alpha\dot\beta} = 0$ AND $\mathbb{F}_{\alpha\dot\beta} = 0$

$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{D_\alpha, D_\beta\}$ ARE SOLVED BY $\{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = \{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\}$

$$\boxed{\mathbb{A}_\alpha = e^{-\bar{\mathcal{V}}} D_\alpha e^{\bar{\mathcal{V}}}}, \qquad \boxed{\mathbb{A}_{\dot\alpha} = e^{\mathcal{V}} \bar D_{\dot\alpha} e^{-\mathcal{V}}}$$

notice: $\mathbb{A}^+ = -\mathbb{A} \Rightarrow \mathbb{A}_\alpha = +(\mathbb{A}_{\dot\alpha})^+$, $(D_\alpha)^* = -\bar D_{\dot\alpha}$

$$e^{\mathcal{V}} \longmapsto e^{\mathcal{V}'} = U^+(z)e^{\mathcal{V}(z)}\mathcal{U}(\varsigma_L),$$
$$e^{\bar{\mathcal{V}}} \longmapsto e^{\bar{\mathcal{V}}'} = \mathcal{U}^+(\varsigma_R)e^{\bar{\mathcal{V}}}U(z)$$

← complexified (S)U(N) transf with chiral superfield parameters (actually GL(N) (ς_L))

Thus "$\mathcal{D}_\alpha = e^{-\bar{\mathcal{V}}}\mathcal{D}_\alpha e^{\bar{\mathcal{V}}}$", "$\bar{\mathcal{D}}_{\dot\alpha} = e^{\mathcal{V}}\bar D_{\dot\alpha} e^{-\mathcal{V}}$"

Physical d.o.f.s are in (not pure gauge) hermitian PREPOTENTIAL $\mathbb{V}(z) = \mathbb{V}^+(z)$.

$$\boxed{e^{\mathbb{V}} = e^{\bar{\mathcal{V}}}e^{\mathcal{V}}} \qquad \boxed{e^{\mathbb{V}} \mapsto e^{\mathbb{V}'} = \mathcal{U}^+(\varsigma_R)e^{\mathbb{V}}\mathcal{U}(\varsigma_L)}$$

Covariant chirality:

$0 = \mathcal{D}_\alpha\bar\Phi \iff D_\alpha(e^{\bar{\mathcal{V}}}\bar\Phi) = 0$, $e^{\bar{\mathcal{V}}}\bar\Phi \mapsto \mathcal{U}^+(\varsigma_R)(e^{\bar{\mathcal{V}}}\bar\Phi)$, $\iff \partial^R_\alpha(e^{\bar{\mathcal{V}}}\bar\Phi) = 0$,

$0 = \bar{\mathcal{D}}_{\dot\alpha}\Phi \iff \bar D_{\dot\alpha}(\Phi e^{\mathcal{V}}) = 0$, $\Phi e^{\mathcal{V}} \mapsto (\Phi e^{\mathcal{V}})\mathcal{U}(\varsigma_L)$, $\iff \bar\partial^L_{\dot\alpha}(\Phi e^{\mathcal{V}}) = 0$.

THUS THE NATURAL PRESCRIPTION TO SWITCH ON THE INTERACTION WITH SYM FIELD ((S)U(N) SYM) IS TO START FROM THE GENERIC SG ⊕ SCALAR MATTER COUPLING AND TO SUBSTITUTE:

$\bar{\Phi}$ by $e^{\bar{V}}\bar{\Phi} = (e^{\bar{V}}\bar{\Phi})(\zeta_R)$

Φ by $\Phi e^{V} = (\Phi e^{V})(\zeta_L)$

IN SUCH A WAY WE ARRIVE AT

$$\boxed{-3\int d^8z\, E\, e^{-\frac{1}{3}K(\Phi e^{V}, e^{\bar{V}}\bar{\Phi})} + \int d^6\zeta_L\, \mathcal{E}\, W(\Phi e^{V}) + \int d^6\zeta_R\, \bar{\mathcal{E}}\, \bar{W}(e^{\bar{V}}\bar{\Phi})}$$

WE SHOULD ALSO ADD THE KINETIC TERM(S) FOR GAUGE FIELD(S) WHICH ARE WRITTEN IN TERMS OF THEIR FERMIONIC FIELD STRENGTH(S) $\mathbb{W}_\alpha = \mathbb{W}_\alpha{}^r \underbrace{T_r}_{\text{gauge group generators}}$ and $\bar{\mathbb{W}}_{\dot\alpha} = \bar{\mathbb{W}}_{\dot\alpha}{}^r T_r$.

THESE ARE COVARIANTLY CHIRAL: $\bar{\mathcal{D}}_{\dot\alpha}\mathbb{W}_\alpha = 0$, $\mathcal{D}_\alpha \bar{\mathbb{W}}_{\dot\alpha} = 0$

$$\boxed{\mathcal{D}\mathbb{W}_\alpha := D\mathbb{W}_\alpha + [\mathbb{A}, \mathbb{W}_\alpha]} \qquad \mathbb{W}_\alpha \mapsto \mathbb{W}'_\alpha = U^{+}\mathbb{W}_\alpha U(z)$$

$$0 = \bar{\mathcal{D}}_{\dot\alpha}\mathbb{W}_\alpha \iff \boxed{\bar{D}_{\dot\alpha}(e^{-V}\mathbb{W}_\alpha e^{V}) = 0}$$

$$0 = \mathcal{D}_\alpha \bar{\mathbb{W}}_{\dot\alpha} \iff \boxed{D_\alpha(e^{\bar{V}}\bar{\mathbb{W}}_{\dot\alpha} e^{-\bar{V}}) = 0}$$

NOTICE THAT $0 = \bar{D}_{\dot\alpha}(e^{-V}\mathbb{W}^\alpha \mathbb{W}_\alpha e^{V}) = \bar{\partial}^L_{\dot\alpha}(e^{-V}(\mathbb{W}\mathbb{W})e^{V})$

AND, HENCE, $0 = \bar{D}_{\dot\alpha}(tr(\mathbb{W}^\alpha \mathbb{W}_\alpha)) = \bar{\partial}^L_{\dot\alpha}\, tr(\mathbb{W}^\alpha\mathbb{W}_\alpha)$

$$tr\, \mathbb{W}^\alpha \mathbb{W}_\alpha = W^{\alpha r} W_\alpha{}^s f_{rs} \quad \leftarrow \sim \delta_{rs} \text{ for } SU(N)$$

A GENERIC KIN. TERM

$$\boxed{\int d^6\zeta_L\, \mathcal{E}\, \underbrace{f_{rs}(\Phi e^{V})}_{\text{dependence on chiral superfield!!!}}\, W^{\alpha r} W_\alpha{}^s + c.c.}$$

EX: TO OBTAIN EQUATIONS OF MOTION FOR (SELF-)INTERACTING CHIRAL (SCALAR) SUPERMULTIPLET(S) IN SUGRA background

$$S = \int d^8z\, E\, \overbrace{\Omega(\Phi,\bar\Phi)}^{-3e^{-\frac{1}{3}K(\Phi,\bar\Phi)}} + \int d^6\rho_L\, \mathcal{E} W(\Phi) + \int d^6\rho_R\, \bar{\mathcal{E}} \bar W(\bar\Phi)$$

$$\boxed{\delta\Phi^i = (\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\,\delta P^i} \qquad \delta\bar\Phi^{\bar i} = (\mathcal{D}\mathcal{D} - \bar R)\,\delta\bar P^{\bar i}$$

prepotential

$$\delta S = \int d^8z\, E\left(\Omega'_i (\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\delta P^i + \Omega'_{\bar i}(\mathcal{D}\mathcal{D} - \bar R)\delta\bar P^{\bar i}\right) +$$

int. by parts $\to$ $(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\Omega'_i \cdot \delta P^i$

$$+ \int d^6\rho_L\, \mathcal{E}\, W'_i (\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\delta P^i + \text{c.c.}$$

$(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)(\delta P^i W'_i)$

$\to 2\int d^8z\, E\left(\delta P^i W'_i + \delta\bar P^{\bar i} \bar W'_{\bar i}\right)$

δP^i: $\boxed{(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\Omega'_i - 2W'_i = 0}$

$$\Updownarrow$$

$$\boxed{(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\bar\Phi^{\bar i}\,\Omega''_{i\bar i} - 2W'_i + \bar{\mathcal{D}}_{\dot\alpha}\bar\Phi^{\bar i}\,\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi^{\bar j}\,\Omega'''_{i\bar i\bar j} = 0}$$

ON MATTER COUPLING OF NEW MIN SG

$n=0$. As in this case $R=0$, hence

$$\int d^8z\, E = -\frac{1}{2}\int d^2\varsigma_L\, \mathcal{E}\, \bar{\mathcal{D}}\bar{\mathcal{D}}\cdot 0 = 0$$

and even $\int d^8z\, E\, \Phi(\varsigma_L) = -\frac{1}{2}\int d^2\varsigma_L\, \mathcal{E}\, \bar{\mathcal{D}}\bar{\mathcal{D}}\Phi(\varsigma_L) = 0$

THUS IT IS EASY TO WRITE THE COUPLING TO kähler potential

$$\int d^8z\, E\, K(\varphi,\bar\varphi)$$

and this coupling is inv UNDER kähler transformations

$$K(\varphi,\bar\varphi) \longrightarrow K(\varphi,\bar\varphi) + F(\varphi) + \bar{F}(\bar\varphi).$$

REMEMBER THAT THE "KIN." TERM FOR SUGRA

$$\int d^8z\, E\,(T+\bar{T})$$

INCLUDES THE SUM OF PREPOTENTIALS (THE PREPOTENTIAL) FOR U(1) CONNECTION

$$A_\alpha = \mathcal{D}_\alpha T, \quad \bar{A}_{\dot\alpha} = -\bar{\mathcal{D}}_{\dot\alpha}\bar{T}$$

and is inv. UNDER THE U(1) GAUGE SYMM & PRE-GAUGE SYM-S

$$\delta T = i\beta(z) + \bar\alpha(\varsigma_R), \quad \delta\bar{T} = -i\beta(z) + \alpha(\varsigma_L)$$

ON MATTER COUPLING OF NON-MIN. SG. S.IV

[Butter & Kuzenko 2012]

"SUPERCONF. VERSION" OF MATTER COUPLING in MIN SG

$$-3\int d^8z\, E\, e^{-K/3} + \int d^6\zeta_L\, \mathcal{E}\, W(\Phi) + \int d^6\zeta_R\, \bar{\mathcal{E}}\, \bar{W}(\bar{\Phi})$$

LET US PERFORM A SUPER-WEYL TRANSF-S WHICH PRESERVE: MIN SG CONSTRAINTS

$$E \to e^{2w+2\bar{w}} E = e^{\frac{2}{3}(\Sigma+\bar{\Sigma})} E$$

$$\mathcal{E} \to e^{2\Sigma}\mathcal{E} \qquad \bar{\mathcal{D}}_{\dot{\alpha}}\Sigma = 0 = \mathcal{D}_\alpha\bar{\Sigma}$$

(No requirements to be compensated by Kähler transf-s)

THEN, DENOTING

$$\phi = e^{\frac{2}{3}\Sigma}, \quad \bar{\phi} = e^{\frac{2}{3}\bar{\Sigma}}$$

we get

$$-3\int d^8z\, E\, \phi\bar{\phi}\, e^{-K(\Phi,\bar{\Phi})/3} + \int d^6\zeta_L\, \mathcal{E}\, \phi^3 W(\Phi) + \int d^6\zeta_R\, \bar{\mathcal{E}}\, \bar{W}(\bar{\Phi})$$

Notice

ALTOUGH THIS IS NOT THE COMPENSATOR Φ_0 OF MIN SG

$$E = \mathrm{sdet}\frac{\partial z_c}{\partial z}\cdot \hat{\rho}^{-1}(\hat{\rho}_2^1)^{1/3}\; \Phi_0\cdot\bar{\Phi}_0$$

the compensator of min SG enters now in the product with ϕ

$$E\phi\bar{\phi} = \mathrm{sdet}\frac{\partial z_c}{\partial z}\, \hat{\rho}^{-1}(\hat{\rho}_2^1)^{1/3}\, \underbrace{\Phi_0\phi}\cdot\underbrace{\bar{\Phi}_0\bar{\phi}}$$

THIS ACTION IS ALSO INV. UNDER KÄHLER SYMM.

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + F(\Phi) + \bar{F}(\bar{\Phi})$$

$$W(\Phi) \to e^{-F(\Phi)}W(\Phi)$$

$$\phi \mapsto e^{1/3 F(\Phi)}\phi$$

[Butler, Kuzenko 2012]:

WRITE THE FOLLOWING ACTION WITH UNCONSTRAINED $\varphi, \bar\varphi$

($\bar{\mathcal D}_{\dot\alpha}\varphi \neq 0$ YET!)

$$S = \int d^8z\, E\left(-3\varphi\bar\varphi\, e^{-K(\varphi,\bar\varphi)/3} + Y\varphi^3 + \bar Y\bar\varphi^3\right)$$

WHERE Y OBEYS IMPROVED LINEAR SUPERFIELD CONSTRAINTS [in flat SSP Deo & Gates 85]

$$\boxed{(\bar{\mathcal D}\bar{\mathcal D} - R)\,Y = -4W(\varphi)}$$

• No superpotential: $(\bar{\mathcal D}\bar{\mathcal D} - R)\,Y = 0$

STANDARD COMPLEX LINEAR MULTIPLET.

(Then $\int d^8z\, E\, Y\varphi^3 = -\frac{1}{2}\int d^6z_L\, \mathcal E(\bar{\mathcal D}\bar{\mathcal D} - R)\,Y\varphi^3 = 0$

if $\bar{\mathcal D}_{\dot\alpha}\varphi^3 = 0$)

• $W(\varphi) = \mu = \text{const} \neq 0$.

$$(\bar{\mathcal D}\bar{\mathcal D} - R)\,Y = -4\mu$$

$$\int d^8z\, E\, Y\varphi^3 = -\frac{1}{2}\int d^6z_L\, \mathcal E(\bar{\mathcal D}\bar{\mathcal D} - R)\,Y\varphi^3$$

after imposing $\bar{\mathcal D}_{\dot\alpha}\varphi^3 = 0$ $\Rightarrow \int d^6z_L\, \mathcal E\varphi^3$

Cosmological term.

The variation δY preserving the constraint obeys

$$(\bar{\mathcal D}\bar{\mathcal D} - R)\,\delta Y = 0$$

$$\Rightarrow \quad \delta Y = \bar{\mathcal D}_{\dot\alpha}\,\delta\bar X^{\dot\alpha}$$

THE CORRESPONDING EQUATION READS $\bar{\mathcal D}_{\dot\alpha}\varphi^3 = 0 \Rightarrow$

$$\Rightarrow \boxed{\bar{\mathcal D}_{\dot\alpha}\varphi^3 = 0}.$$

AND WE REPRODUCE $\quad -3\int d^8z\, E\,\varphi\bar\varphi\, e^{-K(\varphi,\bar\varphi)/3} + \int d^6z_L\,\mathcal E\varphi^3 W$ + c.c.

with chiral φ

$$\delta\bar{\varphi} \text{ (unconstrained!)} \Rightarrow \quad \bar{\Upsilon}\bar{\varphi}^2 = \varphi\, e^{-K/3}$$

$$\delta\varphi \Rightarrow \quad \Upsilon\varphi^2 = \bar{\varphi}\, e^{-K/3}$$

$$\varphi\bar{\varphi} = \Upsilon^{-1}\bar{\Upsilon}^{-1}e^{-K} \qquad \varphi^3 = \frac{1}{\Upsilon^2\bar{\Upsilon}e^K}$$

$$\boxed{S_{dual} = -\int d^8z\ E\,\Upsilon\bar{\Upsilon}^{-1}e^{-K(\Phi,\bar{\Phi})}}$$

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\Upsilon = -2W(\Phi)$$

THIS GIVES THE GENERAL MATTER COUPLING OF $N=1$ non-MINIMAL SUGRA

BUT INCLUSION OF THE SUPERPOTENTIAL IS QUITE IMPLICIT.