6. Recent applications & advanced topics

6.1. SUSY IN RIGID CURVED SSPs

IS OF INTEREST, IN PARTICULAR, FOR APPLICATIONS OR LOCALIZATION TECHNIQUE.

[G. Festuccia & N. Seiberg, JHEP 2011] – USE OFF-SHELL SG TO FIND IT.

- SCALAR MULTIPLET IN CURVED SPACE FROM ITS COUPLING TO (MIN) SUGRA.

  THEY DO NOT USE SG EQS., BUT CONSIDER (THE OFF-SHELL) SG AS A FIXED BACKGROUND; AUX. FIELDS CAN HAVE ARBITRARY VALUES.

- CONSIDER GENERIC SG + SCALAR MULTIPLET COUPLING

$$\underbrace{-\frac{3}{K^2}\int d^8z\, E\, e^{-\frac{1}{3}K^2 K(\Phi,\bar\Phi)}} + \int d^6\zeta_L\, W(\Phi) + \int d^6\bar\zeta_R\, \bar W(\bar\Phi)$$

$$\Downarrow \quad -\frac{3}{K^2}\int d^8z\, E + \int d^8z\, E\; K(\Phi,\bar\Phi) + \sim K^2$$

blows up at $K^2 \to 0$.
But if SUGRA is A BACKGROUND, THIS is JUST (AN ∞) CONSTANT.

Disappear as $K^2 \to 0$

Does not affect the dynamics of matter in a fixed background.

Thus $$\int d^8z\, E\, K(\Phi,\bar\Phi) + \int d^6\zeta_L\, W(\Phi) + \int d^6\bar\zeta_R\, \bar W(\bar\Phi)$$

THE SAME AS GENERIC SUGRA + matter action, but with $\Omega(\Phi,\bar\Phi) := -3e^{-\frac{1}{3}K(\Phi,\bar\Phi)} \longmapsto K(\Phi,\bar\Phi)$

IN TERMS OF SPACETIME COMPONENT FIELDS

AND with gravitino set to zero, $\boxed{\psi_\mu{}^\alpha = 0}$,

so that $\mathcal{E} = e(1 + \frac{3}{4}\Theta\Theta\bar{R}_0)$, $R = R_0 + \frac{1}{2}\Theta\Theta(\frac{4}{3}\mathcal{R} - \frac{3}{2}R\bar{R} + \frac{1}{8}b_a b^a + i e_a{}^\mu \mathcal{D}_\mu b^a)$

$$\underbrace{\int d^8 z\, E\, \Omega(\varphi,\bar\varphi)} + \int d^6 \varrho_L\, \underbrace{\mathcal{E}}\, \underbrace{W(\varphi)} + \int d^6 \varrho_R \mathcal{E}\, \bar W(\bar\varphi) =$$

$e(1 + \frac{3}{4}\Theta\Theta\bar{R}_0)$ $(W(\varphi) + \Theta^\alpha \chi_\alpha^i W_i'(\varphi) + \frac{1}{2}\Theta\Theta(F^i W_i' - \frac{1}{2}\chi^i\chi^j W''_{ij})$

$\int d^6\varrho_L \cdot \frac{1}{2}\Theta\Theta\, e\,(\frac{3}{2}\bar{R}_0 W + F^i W_i' - \frac{1}{2}\chi^i\chi^j W''_{ij}(\varphi)) + \text{eff. } 0$

$-\frac{1}{2}\int d^6\varrho_L \mathcal{E}(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\Omega(\varphi,\bar\varphi)$

$$\int d^6\varrho_L\, \overbrace{e(1+\tfrac{3}{4}\Theta\Theta\bar{R}_0)}^{R\cdot\mathcal{E}}\Big(\underbrace{-\tfrac{1}{2}(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\bar\varphi^{\bar i}}_{=-\frac{1}{2}\Box\bar\varphi^{\bar i}}\,\Omega'_{\bar i}(\varphi,\bar\varphi) - \tfrac{1}{2}R\,(\bar\varphi^{\bar i}\Omega'_{\bar i}(\varphi,\bar\varphi) - \Omega(\varphi,\bar\varphi)) - \tfrac{1}{2}\underbrace{\bar{\mathcal{D}}_{\dot\alpha}\bar\varphi^{\bar i}}\,\bar{\mathcal{D}}^{\dot\alpha}\bar\varphi^{\bar j}\,\Omega''_{\bar i\bar j}(\varphi,\bar\varphi)\Big)$$

see above: $-16 e_a{}^\mu \mathcal{D}_\mu \hat{\mathcal{D}}^a \bar\varphi^{\bar i} + 4 i\, b^a \hat{\mathcal{D}}_a \bar\varphi^{\bar i} - 4 R_0 \cdot F^{\bar i} - \bar\varphi^{\bar i}\mathcal{D}\mathcal{D}R + \sim\text{gravitino}$.

see above (p. 5, $\boxed{IV}$-10): $(-\bar\chi_{\dot\alpha}^{\bar i} + 2i(\Theta\sigma^a)_{\dot\alpha}\hat{\mathcal{D}}_a\bar\varphi^{\bar i} + \frac{1}{2}\Theta\Theta\bar\varphi^{\bar i}_{\dot\alpha} R_0)$

To make an economic calculation we can use (see above)

$$\mathcal{E}W(\varphi) = eW(\varphi) + e\,\Theta^\alpha\chi_\alpha^i W_i'(\varphi) + \tfrac{1}{2}\Theta\Theta e\left(\tfrac{3}{2}\bar{R}_0 W + F^i \Omega''_{i\bar i}(\varphi,\bar\varphi) - \tfrac{1}{2}\chi^i\chi^j W''_{ij}\right)$$

with $W(\varphi) \longrightarrow \Omega'_{\bar i}(\varphi,\bar\varphi),\ (\bar\varphi^{\bar i}\Omega'_{\bar i}(\varphi,\bar\varphi) - \Omega(\varphi,\bar\varphi)),\ \Omega''_{\bar i\bar j}(\varphi,\bar\varphi)$

up to $\int\partial_\mu(\dots)$

$$\begin{aligned}
= \int d^4x\, e\Big(&\tfrac{2}{3}\mathcal{R}\Omega + 4\mathcal{D}^a\bar\varphi^{\bar i}\mathcal{D}_a\varphi^i\,\Omega_{i\bar i}(\varphi,\bar\varphi) - i(\chi^i\sigma^\mu\mathcal{D}_\mu\bar\chi^{\bar i} - \mathcal{D}_\mu\chi^i\sigma^\mu\bar\chi^{\bar i})\Omega''_{i\bar i}(\varphi,\bar\varphi) \\
&+ \tfrac{1}{16}b^a b_a\Omega - \tfrac{i}{2}b^a(\mathcal{D}_a\varphi^i\Omega_i' - \mathcal{D}_a\bar\varphi^{\bar i}\Omega'_{\bar i} + i\chi^i\sigma_a\bar\chi^{\bar j}\Omega''_{i\bar j}) - i e\,\chi^i\sigma^a\bar\chi^{\bar i}(\mathcal{D}_a\bar\varphi^{\bar j}\Omega'''_{i\bar i\bar j} - \mathcal{D}_a\varphi^j\Omega'''_{i\bar i j}) + \\
&+ \tfrac{1}{4}R_0\bar{R}_0\Omega + \tfrac{3}{2}R_0\bar W + \tfrac{1}{2}R_0 F^i\Omega_i' + \tfrac{1}{2}\bar{R}_0\bar F^{\bar i}\Omega'_{\bar i} + \tfrac{3}{2}\bar{R}_0 W \\
&+ F^i\bar F^{\bar i}\Omega''_{i\bar i} + F^i W_i' + \bar F^{\bar i}\bar W_{\bar i} - \\
&- \tfrac{1}{2}\chi^i\chi^j(W''_{ij} + \tfrac{1}{2}R_0^2\Omega''_{ij} + F^{\bar i}\Omega'''_{ij\bar i}) - \tfrac{1}{2}\bar\chi^{\bar i}\bar\chi^{\bar j}(\bar W''_{\bar i\bar j} + \tfrac{1}{2}\bar R_0^2\Omega''_{\bar i\bar j} + F^i\Omega'''_{i\bar i\bar j}) + \\
&+ \tfrac{1}{4}\chi^i\chi^j\,\bar\chi^{\bar i}\bar\chi^{\bar j}\,\Omega''''_{ij\bar i\bar j}\,.
\end{aligned}$$

can be included as Kähler connection in $\mathcal{D}_a\bar\chi^{\bar i}$ and $\mathcal{D}_a\chi^i$)

The limit by Festuccia & Seiberg:

$$\delta\Omega(\varphi,\bar\varphi) = -3\, e^{-\frac{1}{3}K(\varphi,\bar\varphi)} \longmapsto K(\varphi,\bar\varphi)$$

Then $\Omega_{i\bar i} \mapsto K_{i\bar i} = g_{i\bar i}$ etc.

$$S = \int d^8z\, E\, K(\varphi,\bar\varphi) + \int d^6\zeta_L\, \mathcal{E}\, \mathcal{W}(\varphi) + \int d^6\zeta_R\, \bar{\mathcal{E}}\, \bar{\mathcal{W}}(\bar\varphi) = \int d^4x \left(\mathcal{L}^{bosonic} + \mathcal{L}^{fermionic} \right)$$

$$\mathcal{L}^{bosonic} = \left(\frac{2}{3}R + \frac{1}{4}R_0\cdot\bar R_0 + \frac{1}{16}G^a G_a\right) K(\varphi,\bar\varphi) + \underbrace{K_{i\bar j}}_{=g_{i\bar j}(\varphi,\bar\varphi)} \left(F^i \bar F^{\bar j} + 4\partial^\mu\varphi^i \partial_\mu \bar\varphi^{\bar j}\right) +$$
$$+ F^i \mathcal{W}'_i + F^{\bar j}\bar{\mathcal{W}}'_{\bar j} + \frac{1}{2}K'_i R_0 F^i + \frac{1}{2}K'_{\bar i}\bar R_0 \bar F^{\bar i} +$$
$$+ \frac{3}{2}R_0 \bar{\mathcal{W}} + \frac{3}{2}\bar R_0 \mathcal{W} - \frac{i}{2} G^\mu\left(\partial_\mu\varphi^i K'_i - \partial_\mu\bar\varphi^{\bar i} K'_{\bar i}\right)$$

$$\mathcal{L}^{fermionic} = -i\chi^i\sigma^\mu \hat\partial_\mu \bar\chi^{\bar i} g_{i\bar i} + i\hat\partial_\mu \chi^i \sigma^\mu \bar\chi^{\bar i} g_{i\bar i} + \frac{1}{2} G^a \chi^i \sigma_a \bar\chi^{\bar j} g_{i\bar j} -$$
$$-\frac{1}{2}\chi^i\chi^j\left(\mathcal{W}''_{ij} + \frac{1}{2}R_0 K''_{ij} + \bar F^{\bar i} g_{i\bar i,j}\right) - \frac{1}{2}\bar\chi^{\bar i}\bar\chi^{\bar j}\left(\bar{\mathcal{W}}''_{\bar i\bar j} + \frac{1}{2}\bar R_0 K''_{\bar i\bar j} + F^i g_{i\bar i,\bar j}\right)$$
$$+ \frac{1}{4}\chi^i\chi^j\bar\chi^{\bar i}\bar\chi^{\bar j}\, K_{ij\bar i\bar j}$$

Kähler inv. is not guaranteed as we are in SG BACKGROUND

$$\boxed{K(\varphi,\bar\varphi) \longmapsto K(\varphi,\bar\varphi) + Y(\varphi) + \bar Y(\bar\varphi)}$$
$$\boxed{\mathcal{W}(\varphi) \longmapsto \mathcal{W}(\varphi) + \delta\mathcal{W}(\varphi)}$$

$$\mathcal{L}^{bosonic} \mapsto \mathcal{L}^{bosonic} + \left(\frac{2}{3}R + \frac{1}{4}R_0\cdot\bar R_0 + \frac{1}{16}G^a G_a\right)\left(Y(\varphi) + \bar Y(\bar\varphi)\right) + \frac{3}{2}R_0 \underbrace{\delta\bar{\mathcal{W}}}_{=-\frac{1}{2}\bar R_0\bar Y} + \frac{3}{2}\bar R_0 \underbrace{\delta\mathcal{W}}_{=-\frac{1}{2}R_0 Y}$$
$$+ \frac{1}{2}R_0 F^i Y'_i + F^i(\delta\mathcal{W})'_i + \bar F^{\bar i}(\delta\bar{\mathcal{W}})'_{\bar i} + \frac{1}{2}\bar R_0 \bar F^{\bar i}\bar Y'_{\bar i} - \frac{i}{2}G^\mu\partial_\mu Y + \frac{i}{2}G^\mu\partial_\mu\bar Y$$

cancell if $\boxed{\delta\mathcal{W} = -\frac{1}{2}R_0 Y(\varphi)}$ $\boxed{\delta\bar{\mathcal{W}} = -\frac{1}{2}\bar R_0 \bar Y(\bar\varphi)}$

Vanishes if $\boxed{\frac{2}{3}\underbrace{R}_{e_a^\mu e_b^\nu \mathring{R}_{\mu\nu}{}^{ab}} - \frac{1}{2}R_0\cdot\bar R_0 + \frac{1}{16}G^a G_a = 0}$

Vanishes if $\boxed{D_\mu G^\mu = 0}$

CONDITIONS FOR THE OFF-SHELL SG BACKGROUND

CONDITIONS OF PRESERVATION OF SUSY by a solution with $\psi_\mu^\alpha = 0$ are usually obtained by solving the Killing spinor eq. :

$$\delta_\epsilon \psi_\mu^\alpha = 0$$

As $\delta_\epsilon \psi_\mu^\alpha = \mathcal{D}\epsilon^\alpha + i_\epsilon T^\alpha \big|_0$, we can use also the superspace approach. $\psi_\mu^\alpha = 0 \iff T_{ab}{}^\alpha = 0$

$$0 = T_{\alpha\dot\alpha\,\beta\dot\beta\,\gamma} = -\frac{1}{8}\epsilon_{\alpha\beta}\,\bar{\mathcal{D}}_{(\dot\alpha|}G_{\gamma|\dot\beta)} - \frac{1}{8}\epsilon_{\dot\alpha\dot\beta}W_{\alpha\beta\gamma} - \frac{1}{8}\epsilon_{\dot\alpha\dot\beta}\,\epsilon_{\gamma(\alpha}\mathcal{D}_{\beta)}R$$

$\Rightarrow$ • $W_{\alpha\beta\gamma} = 0$, $\bar W_{\dot\alpha\dot\beta\dot\gamma} = 0$

• $\mathcal{D}_\alpha R = 0$, as also $\bar{\mathcal{D}}_{\dot\alpha} R = 0$ $\} \Rightarrow \mathcal{D}_a R = 0$ $\boxed{dR = 0}$

in particular $\partial_\mu R = 0$

$\bar{\mathcal{D}}_{\dot\alpha}\bar R = 0$, $\mathcal{D}_\alpha \bar R = 0$ $\} \Rightarrow \mathcal{D}_a \bar R = 0 \Rightarrow \boxed{d\bar R = 0}$, $\partial \bar R = 0$

• $\bar{\mathcal{D}}_{(\dot\alpha|}G_{\gamma|\dot\beta)} = 0$
AS ALSO $\bar{\mathcal{D}}^{\dot\alpha}G_{\gamma\dot\alpha} = -\mathcal{D}_\gamma R = 0$ $\} \Rightarrow \bar{\mathcal{D}}_{\dot\alpha}G_a = 0$ c.c. $\mathcal{D}_\alpha G_a = 0 \Rightarrow \boxed{\mathcal{D}G_a = 0}$

in particular $\mathcal{D}_\mu G_a = 0$

THUS FOR SOLUTION PRESERVING ALL 4 SUSY

$\boxed{dR = 0}$, $\boxed{d\bar R = 0}$, $\boxed{\mathcal{D}G_a = 0}$, $\boxed{W_{\alpha\beta\gamma} = 0 = \bar W_{\dot\alpha\dot\beta\dot\gamma}}$

$\Rightarrow$ $\partial_\mu R_0 = 0$, $\partial_\mu \bar R_0 = 0$, $\mathcal{D}_\mu G_a| = 0$ $\Rightarrow R\cdot G_a = 0$

$$R_{bc}{}^{ac} = \overset{\circ}{R}_{bc}{}^{ac} + \frac{1}{8}\epsilon_b{}^{acd}\underbrace{\mathcal{D}_c G_d}_{\to 0} - \frac{1}{32}\left(G_b G^a - \delta_b{}^a G_c G^c\right) =$$

$$= \frac{3}{16}R\bar R + \sim \underbrace{\mathcal{D}\bar{\mathcal{D}}G_a}_{\to 0} + \sim \underbrace{\mathcal{D}\mathcal{D}R}_{\to 0}, \underbrace{\bar{\mathcal{D}}\bar{\mathcal{D}}\bar R}_{\to 0}$$

$$\boxed{R_{ab} := \overset{\circ}{R}_{acb}{}^c = \frac{1}{32}\left(G_a G_b - \eta_{ab}G^c G_c\right) + \frac{3}{16}\eta_{ab}R\cdot\bar R}$$

Kähler cur cond-s are satisfied: $\not\Leftrightarrow$ $R - \frac{3}{4}R_0\cdot\bar R_0 + \frac{3}{32}G^a G_a = 0$

Thus we have obtained the superfield generalizations (almost all) (see next page) $\to R\cdot G_a| = 0$ of the conditions of 4 SUSY preservation in [Festuccia + Seiberg JHEP2011]

COMMENT ON CONVENTIONAL CONSTRAINTS AND w^{ab} REDEFINITION

IN OUR SET OF CONSTRAINTS INCLUDES $R_{\alpha\dot\beta}{}^{ab}=0$, AND, AS A RESULT, $T_{ab}{}^{c}\neq 0$ $\quad T_{bc}{}^{a}=\frac{1}{8}\epsilon^{a}{}_{bcd}G^{d}$.

OUR LORENTZ CONNECTION w^{ab} IS RELATED TO THE CONNECTION $\overset{o}{w}{}^{ab}$ OBEYING $T_{bc}{}^{a}=0$ constraints BY "SHIFT":

$$w^{ab}=\overset{o}{w}{}^{ab}+\Delta w^{ab}\,, \qquad \boxed{\Delta w^{ab}=-\frac{1}{8}E^{c}\epsilon^{ab}{}_{cd}G^{d}}$$

Ex: TO FIND THE RELATION OF R^{ab} TO $\overset{o}{R}{}^{ab}=d\overset{o}{w}{}^{ab}-\overset{o}{w}{}^{ac}\wedge\overset{o}{w}{}_{c}{}^{b}$

$$R^{ab}=\overset{o}{R}{}^{ab}+D\Delta w^{ab}-\Delta w^{ac}\wedge\Delta w_{c}{}^{b}=$$

$$=\overset{o}{R}{}^{ab}+\frac{i}{4}E^{\alpha}\wedge\bar{E}^{\dot\alpha}\epsilon^{ab}{}_{cd}\sigma^{c}_{\alpha\dot\alpha}G^{d}-\frac{1}{8}E^{c}\wedge\overset{o}{D}G^{d}\epsilon^{ab}{}_{cd}+$$
$$-\frac{1}{32}E^{d}\wedge E^{c}\delta_{c}^{[a}G^{b]}G_{d}-\frac{1}{64}E^{a}\wedge E^{b}G^{c}G_{c}$$

$$\boxed{R_{\alpha\dot\alpha}{}^{ab}=0=\overset{o}{R}_{\alpha\dot\alpha}{}^{ab}+\frac{i}{4}\epsilon^{abcd}\sigma_{c\alpha\dot\alpha}G_{d}}$$

THUS $\overset{o}{R}_{\alpha\dot\alpha}{}^{ab}=-\frac{i}{8}\epsilon^{abcd}\sigma_{c\alpha\dot\alpha}G_{d}\neq 0$!

$$\boxed{R_{\gamma c}{}^{ab}=\overset{o}{R}_{\gamma c}{}^{ab}-\frac{1}{8}\epsilon^{ab}{}_{cd}D_{\gamma}G^{d}}$$

$$\boxed{R_{cd}{}^{ab}=\overset{o}{R}_{cd}{}^{ab}-\frac{1}{4}\epsilon^{abe}{}_{[c}D_{d]}G_{e}+\frac{1}{16}G^{[a}\delta_{[c}^{b]}G_{d]}+\frac{1}{32}\delta_{[c}^{a}\delta_{d]}^{b}G^{f}G_{f}}$$

$$\boxed{R_{cb}{}^{ab}=\overset{o}{R}_{cb}{}^{ab}+\frac{1}{8}\epsilon_{c}{}^{aef}\overset{(o)}{D}_{e}G_{f}-\frac{1}{32}\left(G_{c}G^{a}-\delta_{c}^{a}G_{b}G^{b}\right)}$$

$$\boxed{R_{ab}{}^{ab}=\overset{o}{R}_{ab}{}^{ab}+\frac{3}{32}G_{a}G^{a}}$$

SUPERSPACE DESCRIPTION OF THE SOLUTIONS PRESERVING SUSY is GIVEN BY THE /STRUCTURE EQS/CONSTRAINTS

$$\boxed{\begin{aligned} T^a &= -2i\,\sigma^a_{\alpha\dot\alpha}\,E^\alpha\wedge\bar E^{\dot\alpha} + \tfrac{1}{8}\,E^c\wedge E^b\,\epsilon^a{}_{bcd}\,G^d \\ T^\alpha &= \tfrac{i}{8}\,E^c\wedge E^\beta(\sigma_c\tilde\sigma_d)_\beta{}^\alpha G^d - \tfrac{i}{8}\,E^c\wedge\bar E^{\dot\beta}\,\sigma_{c\dot\beta}{}^\alpha R \\ \bar T^{\dot\alpha} &= \tfrac{i}{8}\,E^c\wedge E^\beta\,\sigma_{c\beta}{}^{\dot\alpha}\bar R - \tfrac{i}{8}\,E^c\wedge\bar E^{\dot\beta}(\tilde\sigma_d\sigma_c)^{\dot\alpha}{}_{\dot\beta}\,G^d \end{aligned}}$$

$$\boxed{R^{\alpha\beta} = -\tfrac{1}{2}\,E^\alpha\wedge E^\beta\,\bar R + \tfrac{1}{2}\,E^d\wedge E^c\,R_{cd}{}^{\alpha\beta}}$$

$$\boxed{R^{\dot\alpha\dot\beta} = +\tfrac{1}{2}\,\bar E^{\dot\alpha}\wedge\bar E^{\dot\beta}\cdot R + \tfrac{1}{2}\,E^d\wedge E^c\,R_{cd}{}^{\dot\alpha\dot\beta}}$$

with const R and cov. constant G_a

$$dR = 0 = d\bar R$$

$$\mathcal{D}G_a = 0$$

$$\boxed{R\cdot G_a = 0}$$

$$R_{\gamma\dot\gamma\,\delta\dot\delta}{}^{\alpha\beta} := \sigma^c_{\gamma\dot\gamma}\,\sigma^d_{\delta\dot\delta}\,R_{cd}{}^{\alpha\beta} = -\tfrac{1}{8}\,\epsilon_{\dot\gamma\dot\delta}\,\delta_{(\gamma}{}^\alpha\delta_{\delta)}{}^\beta\cdot R\cdot\bar R$$

$\frac{1}{4}\tilde\sigma_c^{\dot\gamma\gamma}\tilde\sigma_d^{\dot\delta\delta}$

$$\Rightarrow\quad R_{cd}{}^{\alpha\beta} = \tfrac{1}{32}\,\sigma_{cd}{}^{\alpha\beta}\,R\cdot\bar R\,,\quad\Rightarrow\quad R_{cd}{}^{\dot\alpha\dot\beta} = -\tfrac{1}{32}\,\tilde\sigma_{cd}{}^{\dot\alpha\dot\beta}\cdot R\cdot\bar R$$

$$R_{cd}{}^{ab} = \tfrac{1}{2}R_{cd}{}^{\alpha\beta}\sigma^{ab}{}_{\alpha\beta} - \tfrac{1}{2}R_{cd}{}^{\dot\alpha\dot\beta}\tilde\sigma^{ab}{}_{\dot\alpha\dot\beta} = -\tfrac{1}{2}R_{cd\alpha}{}^\beta\sigma^{ab\alpha}{}_\beta + \tfrac{1}{2}R_{cd}{}^{\dot\alpha}{}_{\dot\beta}\tilde\sigma^{ab\dot\beta}{}_{\dot\alpha} =$$

$$= \tfrac{1}{32}R\cdot\bar R\cdot\underbrace{\left(\mathrm{Sp}\,\sigma_{cd}\sigma^{ab} + \mathrm{Sp}\,\tilde\sigma_{cd}\tilde\sigma^{ab}\right)}_{=-8\,\delta_{[c}{}^a\delta_{d]}{}^b} = -\tfrac{1}{4}\cdot R\cdot\bar R\;\delta_{[c}{}^a\delta_{d]}{}^b$$

$$R_{cd}{}^{ab} = -\tfrac{1}{4}\,R\cdot\bar R\cdot\delta_{[c}{}^a\delta_{d]}{}^b$$

$$\Rightarrow\quad \mathring{R}_{cd}{}^{ab} + \tfrac{1}{32}\,\delta_{[c}{}^a\delta_{d]}{}^b\,G^fG_f + \tfrac{1}{16}\,G^{[a}\delta_{[c}{}^{b]}G_{d]} - \cancel{\tfrac{1}{4}\,\epsilon^{abe}{}_{[c}\underbrace{\mathcal{D}_{d]}G_e}_{=0}} \to 0$$

$$\boxed{R^{ab} = -\tfrac{1}{4}\,E^\alpha\wedge E^\beta\,\sigma^{ab}_{\alpha\beta}\,\bar R - \tfrac{1}{4}\,\bar E^{\dot\alpha}\wedge\bar E^{\dot\beta}\,\tilde\sigma^{ab}_{\dot\alpha\dot\beta}\cdot R + \tfrac{1}{8}\,E^a\wedge E^b\cdot R\cdot\bar R}$$

$$\Rightarrow\quad \mathring{R}^{ab} + \tfrac{i}{4}\,E^\alpha\wedge\bar E^{\dot\alpha}\,\epsilon^{abcd}\,\sigma_{c\alpha\dot\alpha}G_d - \tfrac{1}{32}\,E^d\wedge E^c\,\delta_c^{[a}G^{b]}G_d - \tfrac{1}{64}\,E^a\wedge E^b\cdot G^cG_c$$

$$-\cancel{\tfrac{1}{8}\,E^c\wedge\underbrace{\mathring{\mathcal{D}}G^d}_{=0}\,\epsilon^{ab}{}_{cd}} \to 0$$

Festuccia & Seiberg [JHEP 2011] also stated that, to preserve 4 SUSYs, one needs to impose

$$\boxed{R\,G_a = 0 = \bar R\,G_a}$$

and

This FOLLOW FROM THE SELFCONSISTENCY OF $\mathcal{D}G_a = 0 \Rightarrow \boxed{R_a{}^bG_b = 0}$

THUS THE CONDITIONS TO PRESERVE 4 SUSY'S ARE.

$$\boxed{dR=0},\quad \boxed{d\bar{R}=0},\quad \boxed{W_{\alpha\beta\gamma}=0\ ;\ \bar{W}_{\dot\alpha\dot\beta\dot\gamma}=0}$$

$$\boxed{\mathcal{D}G_a=0} \underset{\Uparrow}{\Longrightarrow} \boxed{G_a R=0=G_a\bar{R}}$$

$$R_a{}^b G_b=0$$

$$R^{ab}=-\frac{1}{4}E^\alpha\wedge E^\beta\sigma^{ab}_{\alpha\beta}\bar{R}-\frac{1}{4}\bar{E}^{\dot\alpha}\wedge\bar{E}^{\dot\beta}\hat\sigma^{ab}_{\dot\alpha\dot\beta}R+\frac{1}{8}E^a\wedge E^b R\bar{R}$$

$$\supset \overset{\circ}{R}{}^{ab}+\frac{i}{4}E^\alpha\wedge\bar{E}^{\dot\alpha}\epsilon^{abcd}\sigma_{c\alpha\dot\alpha}G_d+\frac{1}{32}E^{[a|}\wedge E^c G_c G^{|b]}-\frac{1}{64}E^a\wedge E^b G^c G_c+\sim\underbrace{\mathcal{D}G}_{0}$$

$$\boxed{R_{bca}{}^c-\frac{3}{16}\eta_{ab}R\cdot\bar{R}=\underbrace{\overset{\circ}{R}_{bca}{}^c}_{=R_{ba}}-\frac{1}{32}(G_aG_b-\eta_{ab}G^cG_c)-\frac{3}{16}\eta_{ab}R\cdot\bar{R}=0}$$

- $W_{\alpha\beta\gamma}=0 \Rightarrow \mathcal{D}_{(\alpha}W_{\beta\gamma\delta)}=0$ (Weyl tensor) $\Rightarrow$ METRIC IS CONF. FLAT
- $G_a R=0=G_a\bar{R}$

$\exists$ 2 branches
- * $G_a=0$ WITH constant R and $\bar{R}$
- ** $R=\bar{R}=0$, $\mathcal{D}G_a=0$, $G_a\neq0$ cov. constant vector.

*1 AdS$_4$ SUPERSPACE $\quad R=\bar{R}=-\frac{2}{\rho}\quad$ $\boxed{\rho=\text{AdS}_4 \text{ radius}}$

$$R^{ab}=\frac{1}{2\rho}E^\alpha\wedge E^\beta\sigma^{ab}_{\alpha\beta}+\frac{1}{2\rho}\bar{E}^{\dot\alpha}\wedge\bar{E}^{\dot\beta}\hat\sigma^{ab}_{\dot\alpha\dot\beta}+\frac{1}{2\rho^2}E^a\wedge E^b$$

$$\supset \overset{\circ}{R}{}^{ab}$$

$$\frac{2}{3}R+\frac{1}{4}R_0\bar{R}_0=\frac{3}{\rho^2}$$

$$\boxed{\begin{aligned}L^{\text{bosonic}}_{\text{AdS}}&=K_{i\bar{j}}(F^iF^{\bar{j}}+4\partial_\mu\varphi^i\partial^\mu\bar\varphi^{\bar{j}})+\frac{3}{\rho^2}K(\varphi,\bar\varphi)-\frac{1}{\rho}K_i'F^i-\frac{1}{\rho}K_{\bar{i}}'\bar{F}^{\bar{i}}\\&-\frac{3}{\rho}(\bar{W}+W)+F^iW_i'+\bar{F}^{\bar{j}}\bar{W}_{\bar{j}}'\end{aligned}}$$

inv. under $\quad K\to K+Y(\varphi)+\bar{Y}(\bar\varphi)$

$$W\to W+\frac{1}{\rho}Y(\varphi),\qquad \bar{W}\to\bar{W}+\frac{1}{\rho}\bar{Y}(\bar\varphi)$$

$*^2$ $\mathbb{S}^4$ SSP. We take the theory to be Euclidean,

4-sphere and $R \neq (\bar{R})^*$

$\rho = \mathbb{S}^4$ radius

$$R = \bar{R} = -\frac{2i}{\rho}$$

also $L \to -L$

$$-L^{\text{bosonic}}_{\mathbb{S}^4} = K_{i\bar{j}}''(F^i \bar{F}^{\bar{j}} - 4\partial_\mu \varphi^i \partial^\mu \bar{\varphi}^{\bar{j}}) - \frac{3}{\rho^2} K(\varphi,\bar{\varphi}) - \frac{3i}{\rho}(W + \bar{W}) - \frac{i}{\rho} K_i' F^i - \frac{i}{\rho} K_{\bar{i}}' \bar{F}^{\bar{i}} + F^i W_i' + \bar{F}^{\bar{i}} \bar{W}_{\bar{i}}'$$

$$L^{\text{bosonic}}_{\mathbb{S}^4} = \underbrace{\mathcal{K}_{i\bar{j}}''}_{= g_{i\bar{j}}}(4\partial_\mu \varphi^i \partial^\mu \bar{\varphi}^{\bar{j}} - F^i \bar{F}^{\bar{j}}) + \frac{3}{\rho^2} \mathcal{K}(\varphi,\bar{\varphi}) - \frac{i}{\rho}\mathcal{K}_i' F^i - \frac{i}{\rho}\mathcal{K}_{\bar{i}}' \bar{F}^{\bar{i}}$$

aux. field eqs. $F^i = -\frac{i}{\rho} g^{i\bar{j}} \mathcal{K}_{\bar{j}}'$, $\bar{F}^{\bar{j}} = -\frac{i}{\rho} g^{i\bar{j}} \mathcal{K}_i'$

$(\neq (F^j)^*)$

$$\boxed{\mathcal{K}(\varphi,\bar{\varphi}) \equiv K(\varphi,\bar{\varphi}) - \rho W(\varphi) - \rho \bar{W}(\bar{\varphi})}$$

$$L^{\text{bosonic}}_{\mathbb{S}^4}\Big|_{\text{F-shell}} = 4 g_{i\bar{j}} \partial_\mu \varphi^i \partial^\mu \bar{\varphi}^{\bar{j}} + \frac{1}{\rho^2}\left(3\mathcal{K}(\varphi,\bar{\varphi}) - \mathcal{K}_i' g^{i\bar{j}} \mathcal{K}_{\bar{j}}'\right)$$

$$\boxed{V = \frac{1}{\rho^2}\left(3\mathcal{K}(\varphi,\bar{\varphi}) - \mathcal{K}_i' g^{i\bar{j}} \mathcal{K}_{\bar{j}}'\right)}$$

SOLUTIONS ARE SUSY if $0 = F^i = -\frac{i}{\rho} g^{i\bar{j}}(K_{\bar{j}}' - \rho \bar{W}_{\bar{j}}') = -\frac{i}{\rho} g^{i\bar{j}} \mathcal{K}_{\bar{j}}'$

[IF $F^i \neq 0$; then $\chi_\alpha^i = \partial_\alpha \varphi^i|_0$ TRANSFORMS AS GOLDSTONE: $\delta\chi_\alpha^i = \sim \epsilon_\alpha F^i_{\ldots} \Rightarrow$ SPONTANEOUS BREAKING OF SUSY]

$\boxed{\mathcal{K}_j' = 0}$ We can search for SUSY solution in a power series in $\frac{1}{\rho}$

$\Leftarrow$ $\boxed{W_j' = \frac{1}{\rho} K_j'}$; $\boxed{\varphi = \varphi_0 + \frac{1}{\rho}\varphi_1 + \ldots}$

$W_j'(\varphi_0) = 0$, i.e. φ_0 is a stationary point

$$W_j'(\varphi_0 + \tfrac{1}{\rho}\varphi_1 + \ldots) = \frac{1}{\rho} K_j'(\varphi_0 + \tfrac{1}{\rho}\varphi_1 + \ldots)$$

$$W_{ji}''(\varphi_0) \cancel{\tfrac{1}{\rho}} \varphi_1^i = \cancel{\tfrac{1}{\rho}} K_j'(\varphi_0) \Rightarrow \boxed{\varphi_1^i = W^{-1\,ij}_{(\varphi_0)} K_j'(\varphi_0)}$$

Potential at saddle point $V_s := V(\varphi_0 + \frac{1}{\rho}\varphi_1 + \ldots)$

$$\boxed{V\big|_s = \frac{3}{\rho^2}\mathcal{K}(\varphi_0,\varphi_0) + \sim \mathcal{K}_i'(\varphi)\Big|_{\to 0} = \frac{3}{\rho^2}\left(K(\varphi_0,\bar{\varphi}_0) - i\rho W(\varphi_0) - i\rho \bar{W}(\bar{\varphi}_0)\right)}$$

$\mathbb{R}^1 \otimes \mathbb{S}^3$ SSP $\qquad R=0,\ \bar{R}=0$

$$\boxed{G_a = \frac{8}{\rho}\,\delta_a^0}$$

$$\overset{\circ}{R}{}^{ab} = \frac{2i}{\rho} E^\alpha \wedge \bar{E}^{\dot\alpha}\,\epsilon^{0abc}\,\sigma_{c\alpha\dot\alpha} + \frac{2}{\rho^2} E^0 \wedge E^{[a}\delta_0^{b]} + \frac{1}{\rho^2} E^a \wedge E^b$$

$$R_{ab} = \frac{2}{\rho^2}\left(\delta_a^0\delta_b^0 - \eta_{ab}\right) = \frac{2}{\rho^2}\left(\delta_a^I\delta_b^I\right)$$

$$R = \eta^{ab}R_{ab} = \frac{2}{\rho^2}\eta^{II} = -\frac{6}{\rho^2}$$

$$\Rightarrow \quad \frac{2}{3}R + \frac{1}{16}G_a G^a = 0$$

no $\frac{1}{\rho^2}$ contribution to $\mathcal{L}$

$$\mathcal{L}^{bosonic}_{\mathbb{R}^1\times\mathbb{S}^3} = g_{i\bar{j}}\left(F^i\bar{F}^{\bar{j}} + 4\partial_\mu\varphi^i\partial^\mu\bar{\varphi}^{\bar{j}}\right) + F^i\mathcal{W}_i' + \bar{F}^{\bar{i}}\bar{\mathcal{W}}_{\bar{i}}' - \frac{4i}{\rho}\left(\partial_0\varphi^i K_i' - \partial_0\bar{\varphi}^{\bar{i}}K_{\bar{i}}'\right)$$

$$\mathcal{L}^{fermionic}_{\mathbb{R}^1\times\mathbb{S}^3} = -i\chi^i\sigma^\mu\mathcal{D}_\mu\bar{\chi}^{\bar{i}}g_{i\bar{i}} - \frac{1}{2}\chi^i\chi^j\left(\mathcal{W}_{ij}'' + \bar{F}^{\bar{i}}g_{i\bar{i},j}\right) + c.c. + \frac{1}{4}\chi^i\chi^j\bar{\chi}^{\bar{i}}\bar{\chi}^{\bar{j}}K_{ij\bar{i}\bar{j}} + \frac{4}{\rho}\chi^i\sigma_0\bar{\chi}^{\bar{j}}g_{i\bar{j}}$$

inv. under $\qquad K(\varphi,\bar{\varphi}) \to K(\varphi,\bar{\varphi}) + Y(\varphi) + \bar{Y}(\bar{\varphi})$

without transforming $\mathcal{W}$, $\bar{\mathcal{W}}$

Thus we have a separation of holomorphic data in $\mathcal{W}$ from non-holomorphic in $K(\varphi,\bar{\varphi})$ (like in flat SSP)

6.2. SUPERGRAVITY WITH HIGHER DERIVATIVE CONTRIBUTIONS

ARE OF INTEREST AS

- THEY SHOULD APPEAR AT LOW ENERGY LIMITS OF STRING THEORY
- SUSY GENERALIZATIONS OF THE Starobinsky model THE RENEWED INTEREST TO WHICH APPEARED AFTER RECENT PLANK & BICEP2 RESULTS.

1ST SUGRA GENERALIZATION OF STAROBINSKY MODEL WAS DEVELOPED BY CECOTTI IN 87 USING CONFORMAL TENSOR CALCULUS.

SUPERFIELD VERSION – E.G. IN [Ketov 1309.0293]

STAROBINSKY MODEL:

$$\boxed{\int d^4x \sqrt{|g|}\left(R + \alpha R^2 - 2\Lambda\right)}$$

equiv. form:

massive scalar ⊕ gravity

$$\int d^4x \sqrt{|g|}\left[(1+2\alpha\varphi)R - \alpha\varphi^2 - 2\Lambda\right]$$

$R_{ab}{}^{cd}$

$$\Downarrow \quad g_{\mu\nu} \to g_{\mu\nu}' = (1+2\alpha\varphi(x))\, g_{\mu\nu}$$

$$\int d^4x \sqrt{|g|}\left(R - \frac{6\alpha^2\partial^\mu\varphi\,\partial_\mu\varphi}{(1+2\alpha\varphi)^2} * \frac{\alpha\varphi^2 + 2\Lambda}{(1+2\alpha\varphi)^2}\right)$$

field redef

$\sim \partial_\mu\varphi\,\partial^\mu\varphi$

A PARTICULAR CASE OF SO-CALLED $f(R)$ gravity

$$\boxed{\int d^4x \sqrt{|g|}\, f(R)}$$

SUPERSYMM. VERSION IN THE FRAME OF MIN. SG

CECOTTI 87 $\leftrightarrow$

$$S = \int d^4x\, d^4\theta\; E\; \mathcal{N}(R, \bar{R})$$

where R, $\bar{R} = (R)^*$ $\quad \bar{\mathcal{D}}_{\dot\alpha} R = 0$ is THE CHIRAL MAIN SUPERFIELD OF MIN. SG.

TO SEE THAT WE INDEED HAVE $R + \sim R^2$
IN THE ACTION, LET US REMEMBER THAT THE MATTER
COUPLING STUDY RESULTED IN

$$\int d^8z\, E\, \Omega(\varphi,\bar\varphi) = \int d^4x \left[\tfrac{2}{3} e \Omega R + \tfrac{1}{4} DD\varphi| \cdot \bar D\bar D\bar\varphi|\, \Omega''_{\varphi\bar\varphi} + \right.$$

$$\left. + 4e\, D^a\varphi\, D_a\bar\varphi\, \Omega''_{\varphi\bar\varphi} + \ldots \right]$$

AND, HENCE, AS

$$DDR| = \sim R + \ldots \quad (R_{ab}{}^{ab})$$
$$\bar D\bar D R| = \sim R + \ldots$$

$$\int d^8z\, E\, \mathcal{N}(R,\bar R) = \int d^4x \left[\tfrac{2}{3} e\, \mathcal{N}(R_0,\bar R_0) \cdot R + \sim R^2 \cdot \mathcal{N}''_{R\bar R} + \right.$$

$$\left. + 4e\, D^a R_0\, D_a \bar R_0\, \mathcal{N}''_{R\bar R} + \ldots \right]$$

FOR $\mathcal{N}(R,\bar R) = 1 + \tilde\alpha R \cdot \bar R$ we reproduce the terms of the STAROBINSKY ACTION $e(R + \alpha R^2)$.

BUT WE ALSO SEE THAT IN THIS MODEL THE AUXILIARY FIELDS OF MIN SG
R_0, $\bar R_0$
BECOME DYNAMICAL (GET KIN TERM $D^a R_0 D_a \bar R_0$)

ONE CAN ALSO WRITE A GENERALIZATION OF "CHIRAL" FORM OF THE SG ACTION : $\int d^8z\, E = \sim \int d^6 z_L\, \mathcal{E} R$

HIGHER ORDER GENERALIZATION: [GATES & KETOV 2009]

$$\int d^6 z_L\, \mathcal{E}\, F(R) + \text{c.c.}$$

ONE CAN COMBINE TWO TERMS

[see e.g. Ketov & Terada JHEP 2013]

$$S_{N+F} = \int d^8z\ E\, N(R,\bar{R}) + \left[\int d^6 z_L\, \mathcal{E}\, F(R) + c.c.\right]$$

$$= \int d^4x\, d^2\Theta\, \mathcal{E}\left[-\tfrac{1}{2}(\bar{\mathcal{D}}\bar{\mathcal{D}} - R)N(R,\bar{R}) + F(R)\right] + c.c.$$

CAN BE REWRITTEN IN THE MATTER COUPLED STANDARD min SUGRA

$$S' = \int d^8z\ E\, N(J,\bar{J}) + \left\{\int d^6 z_L\, \mathcal{E}\,[F(J) + 2\Lambda(J-R)] + c.c.\right\}$$

$$\bar{\mathcal{D}}_{\dot\alpha} J = 0\ ,\quad \mathcal{D}_\alpha \bar{J} = 0$$

$$\bar{\mathcal{D}}_{\dot\alpha}\Lambda = 0$$

$\delta\Lambda$: $\qquad J = R$

and we can come back to S_{N+F}

ACTIONS WITH HIGHER POWERS OF R
(E.G. RADIATIVE CORRECTIONS TO STAROBINSKI ACTION)

E.G.

$$S' = \int d^6 z_L\, \mathcal{E}\, F(R, \Sigma(\bar{R})) + c.c.$$

$$\Sigma(\bar{R}) = (\bar{\mathcal{D}}\bar{\mathcal{D}} - R)\bar{R}$$