

LECTURES ON SUPERSYMMETRY AND SUPERGRAVITY IN CURVED SPACES AND SUPERSPACES.

<http://workshops.ift.uam-csic.es/iftw.php/ws/168>

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Aim of the course: Supergravity, the supersymmetric generalization of Einstein's theory of gravity, is a fundamental tool in the current research in Theoretical Physics. Supergravity can be formulated in spacetimes of $D \leq 11$ dimensions. The maximal $D = 11$ supergravity is the best known ingredient of M-theory. In $D = 10$ spacetime it describes the low energy limit of String Theory and in $D < 10$ of its compactifications. The recent interest in maximally extended $\mathcal{N} = 8$ supergravity in $D = 4$ is due to its exceptional convergence properties and to the recent discovery of new gaugings which may provide new, more realistic, de Sitter-type scenarios. The simplest $\mathcal{N} = 1$ $D = 4$ supergravity, in addition to being the basis for promising phenomenological model building, is also interesting from other points of view: its off-shell formulation provides a basis to construct the supersymmetric gauge theories in curved spacetime, in particular the models on spheres to be solved using localization technique. Also the higher derivative extensions of simple supergravity are actively studied and used as a basis for cosmological models.

The goal of the present course is to provide certain level of familiarity with supergravity, so that, if reached, the researchers, besides understanding the general ideas, will be able to make explicit calculations using spacetime component and superfield methods in simple and extended superspaces. At the end of the course we intend to address some of the recent applications of the supergravity formalism(s) mentioned above. The lectures are prepared mainly for black board presentations. The exercises include some instructive and important calculations.

Venue: Institute of Theoretical Physics (IFT) UAM/CSIC, Madrid.

<http://www.ift.uam-csic.es/>

Duration: 18 hours. 9 2-hour lectures (flexible), in 3 weeks (3 sessions/ week). The lectures will start every day at 15:00 and will be given in different rooms ("aulas") each week (see the timetable with information on the different topics of the program to be studied each day).

Addressed to: PhD students and researchers.

Registration and fees: This course is supported by the *Excellence Centers* program and no fee is required. No support for those attending the course can be provided, though. Registration in

<http://workshops.ift.uam-csic.es/iftw.php/ws/168/registro>

is required to all participants

Preliminary program: ¹

1. Introduction. SUSY in flat space(s) and superspace(s).

2. Supergravity in D=4 spacetime.

- 2.1. General relativity in differential form notation.
- 2.2 Rarita-Schwinger action and its 'pre-supersymmetry'.
- 2.3. N=1 supergravity and its local supersymmetry.

3. Supergravity in superspace. I. D=4, N=1 on the mass shell ('on shell').

- 3.1. Basic notions. Supervielbein, and spin connection of curved superspace, Bianchi identities and constraints. Superspace diffeomorphisms and local spacetime supersymmetry.
- 3.2. On shell constraints in D=4 superspace. Bianchi identities and derivation of equations of motion from superspace constraints. Dragon theorem.
- 3.3. Wess-Zumino gauge and component equations of motion.
- 3.4. Properties of the Wess-Zumino gauge (generic case).

4. Supergravity in superspace II. Higher N and higher D on the mass shell.

- 4.1. Supersymmetric gauge theory in D=4 and D=10 superspaces.
- 4.2. On shell constraints and equations of motion of D=11 supergravity.
- 4.3. On shell constraints and equations of motion of type IIA and type IIB D=10 supergravity.
- 4.4. On shell constraints and equations of motion of D=4 N=8 supergravity.

5. Off-shell formulations of supergravity.

5. I. Minimal off-shell formulation of supergravity.

5.I.1 Minimal supergravity in superspace.

- 5.I.1a** Superspace constraints and solution of Bianchi identities. Auxiliary fields and their supersymmetry transformation. Superspace form of the supergravity equations of motion.
- 5.I.1b.** Wess-Zumino gauge in superspace of minimal. supergravity; (First terms in) decomposition of supervielbein and connection on fermionic coordinates.

¹The order of presentation of different topics may be slightly changed.

- 5.I.1c.** Solution of the constraints. Prepotentials. Ogievetsky-Sokatchev axial vector superfield, Siegel's compensators.
- 5.I.1d.** Supervolume of D=4 N=1 superspace as minimal supergravity action.
- 5.I.1e.** Wess-Zumino approach. Admissible variations of supervielbein and spin connection. Superfield equations of supergravity.
- 5.I.2.** Spacetime component description of minimal supergravity. Auxiliary fields in supergravity action.
- 5.I.2a.** Chiral projectors in superspace. Chiral form of the superfield supergravity action. Chiral superfield in curved superspace. Chiral density.
- 5.I.2b.** Spacetime action with off-shell supersymmetry from the superspace supergravity action.
 - 5.I.2b.1** 'Ectoplasm method'.
 - 5.I.2b.2** Curvature and gravitino field strength in Wess-Zumino gauge.
 - 5.I.2b.3.** Spacetime component action and its local supersymmetry
 - 5.I.2b.4.** Chiral density from 'Ectoplasm'.
- 5.II. 'Alternative minimal' ('new minimal') and other formulations of simple supergravity.**
 - 5.II.1.** Super-Weyl-transformations in superspace and 3rd class constraints of minimal, non-minimal and 'new' minimal SUGRA. Chiral, complex linear and real linear compensator superfields. Vanishing volume of the superspace of 'new minimal' supergravity.
 - 5.II.2.** Superfield actions for alternative minimal and nonminimal supergravity.
 - 5.II.3.** U(1) superspace. Alternative minimal supergravity in U(1) superspace.
 - 5.II.4.** Spacetime component action of alternative minimal supergravity.
 - 5.II.*5.** Special minimal supergravity and generation of cosmological constant.
- 5.III. Matter coupling to supergravity.**
- 6. Recent developments and advanced topics (*).**
 - 6.1.** Supersymmetry in rigid curved (super)spaces.
 - 6.2.** Supergravity with higher derivative contributions: $R + R^2$ and others.
 - * Conformal supergravity and conformal methods in supergravity.
 - * Supergravity interacting with supersymmetric extended objects.
 - * Vanishing volume of N=8 superspace.
 - * Rheonomic approach to supergravity as 'bottom up' version of the 'ectoplasm method'

Timetable: ²

²The items marked by * are optional and will be covered in the case of having time.

Week 1 (in “Aula Gris I”)

Lecture 1. October 14: 1, 2

Lecture 2. October 15: 3.1-3.3(4)

Lecture 3. October 16: 3.4, 4.1-4.4

Week 2 (in “Aula Gris I”)

Lecture 4. October 21: (4.4,) 5.I.1a-c.

Lecture 5. October 22: 5.I.1c,d,e, 5.I.2a (b1-4)

Lecture 6. October 23: (5.I.2a) 5.I.2b1-4 (5.II)

Week 3 (in “Aula Audiovisuales”)

Lecture 7. October 28: (5.I.2b1-4), 5.II.1-4 (*5)

Lecture 8. October 29: (5.II. 1-4) 5.II.*5, 5III.

Lecture 9. October 30: (5.III) 6.1, 6.2, *’s