New Branches of Heterotic/F-theory Duality

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Work done in collaboration with:

(W. Taylor) - arXiv:1405.2074

(J. Gray, N. Raghuram, W. Taylor) - arXiv:1506.xxxxx

(J. Gray, X. Gao, S. J. Lee) - arXiv:150x.xxxxx

String Phenomenology 2015

Insituto de Fisica Teorica UAM-CSIC

June 11th, 2015

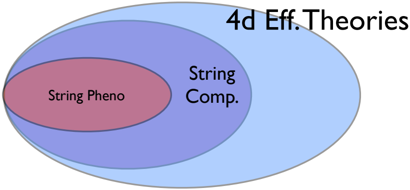
Motivation: A rich framework for string phenomenology

F-theory is a versatile/flexible arena to address questions in string phenomenology

- However – At present our primary windows into the theory consist of dualities
  - Systematic study of these dualities

$$\boxed{\text{F-theory}}$$
$$\Updownarrow$$
$$\boxed{\text{M-theory/Heterotic/Type IIB}}$$

leads to improved tools



$$\boxed{\text{What possible EFTs?}}$$
$$\Updownarrow$$
$$\boxed{\text{Which geometries?}}$$

Today's talk: Heterotic/F-theory duality

Heterotic/F-theory: Geometric ingredients:

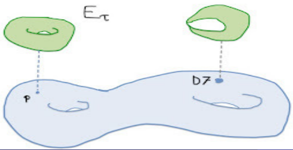
F-theory

- An elliptically fibered Calabi-Yau $n+1$-fold, $\pi : Y_{n+1} \xrightarrow{\mathbb{E}} \mathcal{B}_n$
  - For fibrations w/ section,

$$y^2 = x^3 + f(u_i)x + g(u_i)$$

u_i coords on B_2, $f \in H^0(B_2, K_{B_2}^{-4})$, $g \in H^0(B_2, K_{B_3}^{-6})$

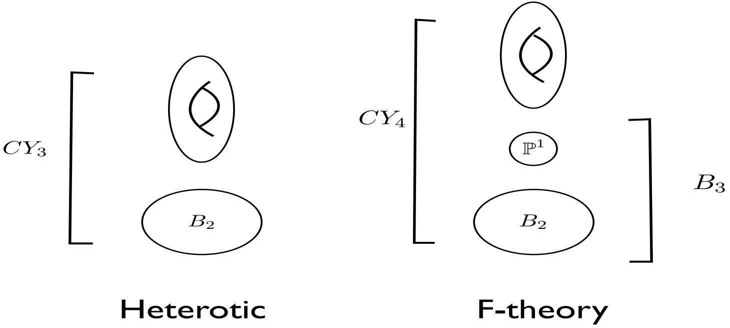
- Degenerations: $\Delta = 4f^3 + 27g^2 = 0$



Heterotic

- A Calabi-Yau n-fold, X_n
- Two principle H_i-bundles, $(V_2, \mathcal{V}_2)$ on X (with structure group $H_i \subset E_8$). Leading to a collection of holomorphic, Mumford (poly)-stable vector bundles: $V_i, V_i^\vee, \wedge^2 V_i, \ldots$ etc.
- Bundles, V_i satisfy the Hermitian-Yang-Mills Eq.s:

$$F_{ab} = F_{\bar{a}\bar{b}} = 0 \qquad g^{a\bar{b}} F_{a\bar{b}} = 0$$



Heterotic on $\pi_h : X_n \xrightarrow{\mathbb{E}} B_{n-1}$ $\Leftrightarrow$ F-theory on $\pi_f : Y_{n+1} \xrightarrow{K3} B_{n-1}$

(with $\pi : Y_{n+1} \xrightarrow{\mathbb{E}} \mathcal{B}_n$ and $\rho : \mathcal{B}_n \xrightarrow{\mathbb{P}^1} B_{n-1}$)

Descends from 8-dim: Het on $T^2 \leftrightarrow$ F-theory on $\pi : K3 \xrightarrow{\mathbb{E}} \mathbb{P}^1$ (Vafa)

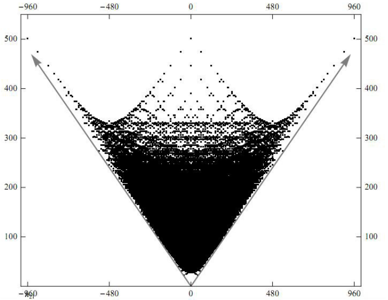
(Rich history: Vafa, Morrison, Friedman, Morgan, Witten, Donagi, Curio, Aspinwall, Katz, Plesser, Andreas, Watari, Hayashi, Toda, Yamazaki, Schafer-Nameki, Saulina, Marsano, Cvetic...)

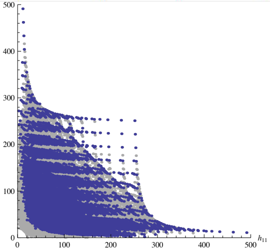
$\boxed{\text{Where these two theories are dual, there is a finite set of geometries to study}}$

Goals for string phenomenology

- A good arena for *exhaustive/systematic* study
- Heterotic/F-theory duality linked to much progress in F-theory. Still, global dual geometries still contain much new physics.
- Heterotic $\Rightarrow$ F-theory Understand the structure of the effective theory $\rightarrow$ spectra, eff. potential (D-terms + F-terms) and flux.
- F-theory $\Rightarrow$ Heterotic F-theory is very good at linking *generic* structure of the effective theory to geometry. Powerful tool for understanding which heterotic geometries are viable for string pheno.
  - Heterotic Standard Model Building $\rightarrow$ Exhaustive scans underway, but hugely labor intensive. At present, we do not understand the rules/patterns.
- In both directions, important tools for string pheno.

Genericity of fibrations for known datasets





(from Taylor 1205.0952)

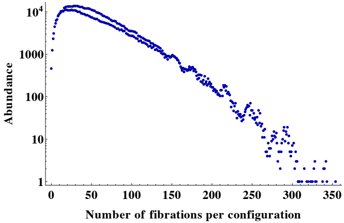
- There are a finite number of elliptically fibered CY threefolds
- 99% of known manifolds elliptically (and $K3$) fibered (see talks of Gray, Gao)
- This extends to newest constructions of CYs (see talks of Lee, Gray)
- 6D statement: Of known bases B_2 of elliptically fibered 3-folds, majority are rationally fibered $\Rightarrow$ heterotic duals (generically non-pert.).

Multiplicity of fibrations

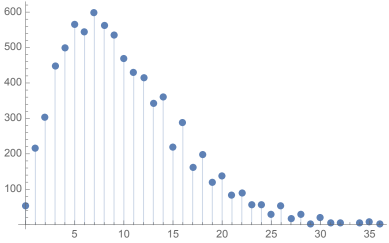
- Point of interest: When CY manifolds have fibrations, they generically do not have just one...
- In fact there can be **many**
- Relevant here: CYs with multiple $K3$-fibrations:

$$\begin{tikzcd} & Y_n \arrow[dl, "\mathbb{E}"', "\pi_1"] \arrow[dr, "\mathbb{E}", "\pi_2"'] & \\ \mathcal{B}^{(1)}_{n-1} & & \mathcal{B}^{(2)}_{n-1} \end{tikzcd}$$

Duality "Cartography" in progress (w/ J. Gray, X. Gao and S. J. Lee)



CICY 4-folds (Gray, et al 1405.2073)($\sim 10^8$ fibrations)



CICY 3-folds ($\sim 78,000$ fibrations)

Organization: What do we hope to learn?

- Understand geometry $\leftrightarrow$ EFT link in both theories
- Systematic study of
  1. Gauge Symmetries + chiral index (most work so far)
  2. Matter content (full)
  3. Structure of the potential
- Today: I will quickly review work with W. Taylor on 1) and highlight two aspects of recent work on points 2) and 3) (with J. Gray and N. Raghuram) ...

η: Building bundles and $\mathcal{B}_3$

- Idea: $\boxed{\text{Choose topology of bundles } (V_1, V_2) \Leftrightarrow \text{Build } \rho : B_3 \xrightarrow{\mathbb{P}^1} B_2}$

Heterotic:

- Can expand:

  $c_2(V_i) = \eta_i \wedge \omega_0 + \zeta_i,$

  w/ η_i (resp. ζ_i) $\{1,1\}$ (resp. $\{2,2\}$) forms on B_2 and ω_0 dual to the zero section.

- Anomaly Cancellation $\Rightarrow$

$$\eta_{1,2} = 6c_1(B_2) \pm t$$

- Can build $\mathcal{B}_3$ over B_2 by "twisting" the $\mathbb{P}^1$ fibration (analog of $\mathbb{F}_n$ surfaces in $6D$)

  $\mathcal{B}_3 = \mathbb{P}(\mathcal{O} \oplus \mathcal{L})$

- $c_1(\mathcal{B}_3) = c_1(B_2) + 2\Sigma + t$

  where Σ is dual to the zero-section of the $\mathbb{P}^1$-fibration

In Het/F-dual pairs, two t's are the same (FMW), (Grimm + Taylor)

$\boxed{\text{Next: Bounds on twists} \Rightarrow \text{finite \# } \mathcal{B}_3 \text{ sol'ns/enumeration}}$

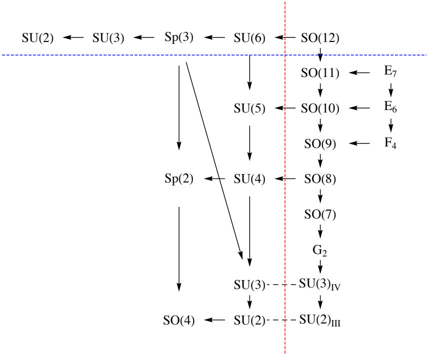
Bounds on the structure group, H

(w/ W. Taylor)

- "Generic" symmetries on Y_4 provide rank(V)-dependent vanishing criteria for $\mathcal{M}(c(V))$. (First studied by Rajesh and Berglund & Myer)
- Also constraints on which symmetries can be enhanced
- non-Higgsable $SU(2), SU(3) \not\to SU(5)$
- Can be pinned at exactly one symmetry (or a sparse set)
- Intriguing for string pheno...

H	$\eta \geq Nc_1(B_2)$ $N =$
$SU(n)$	$n \quad (n \geq 2)$
$SO(7)$	4
$SO(m)$	$\frac{m}{2} \quad (m \geq 8)$
$Sp(k)$	$2k \quad (k \geq 2)$
F_4	$\frac{13}{3}$
G_2	$\frac{7}{2}$
E_6	$\frac{9}{2}$
E_7	$\frac{14}{3}$
E_8	5

Higgsing Chains



- Understanding these Higgsing Chains (and where you get stuck!) has been of interest since the beginning of F-theory (Morrison, Vafa, Bershadsky, Intrilligator, Kachru, Sadov, etc.)

- The plot at left is the simplest chain. There are many others (e.g. Aldazabal, Ibanez, Font, Quevedo, Uranga)
- Transitions in chain → geometric transitions in CYs
- To explore matter and the potential, look at the first row: $SO(12) \to SU(6) \to Sp(3) \to \ldots$
- Let's look at matter spectra and more...

Matter in Transition

- Geometric transitions in F-theory characterized by their effect on the spectrum. Three main types:
  1. Blowing-up/down the base $\Rightarrow$ tensionless string transitions ($6D$: change n_T and n_H)
  2. Higgsing/unHiggsing transitions $\Rightarrow$ ($6D$: n_T unchanged, change in n_V, n_H)
  3. More exotic: Matter multiplicities change without changing gauge symmetry ($6D$: n_T and n_V unchanged. Only representation content of matter fields change.) (Morrison, Taylor)
- Let's look at $SU(N)$ examples...

Consistent Transitions:

Rep.	N	Dimension	A_R	B_R	C_R	g
Adjoint	N	$N^2 - 1$	$2N$	$2N$	6	1
	6, 7, 8	**35**, **48**, **63**	12, 14, 18	12, 14, 18	6	1
$\square$	N	N	1	1	0	0
$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	N	$\frac{N(N-1)}{2}$	$N - 2$	$N - 8$	3	0
	6, 7, 8	**15**, **21**, **28**	4, 5, 6	-2, -1, 0	3	0
$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	N	$\frac{N(N-1)(N-2)}{6}$	$\frac{N^2-5N+6}{2}$	$\frac{N^2-17N+54}{2}$	$3N - 12$	0
	6, 7, 8	**20**[**10**], **35**, **56**	6[3], 10, 15	-6[-3], -8, -9	6, 9, 12	0
$\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}$	N	$\frac{N(N-1)(N-2)(N-3)}{24}$	$\frac{(N-2)(N-3)(N-4)}{6}$	$\frac{(N-4)(N^2-23N+96)}{6}$	$\frac{3(N^2-9N+20)}{2}$	0
	8	**70**[**35**]	20[10]	-16[-8]	18[9]	0

Anomalies:

$$-a \cdot b = -\frac{1}{6}\left(A_{\mathrm{adj}} - \sum_R x_R A_R\right)$$

$$0 = B^i_{\mathrm{adj}} - \sum_R x_R B_R$$

$$b \cdot b = -\frac{1}{3}\left(C_{\mathrm{adj}} - \sum_R x_R C_R\right)$$

Where a, b are the coefficients of BR^2, BF^2 Green-Schwarz terms. (Morrison, Taylor. . .)

Examples

- $SU(6)$

$$\mathbf{10}\ \left(\frac{1}{2}\begin{array}{|c|}\hline \\\hline \\\hline \\\hline\end{array}\right) + \mathbf{6}\,(\Box)\,. \quad \leftrightarrow \quad \mathbf{15}\,\left(\begin{array}{|c|}\hline \\\hline \\\hline\end{array}\right) + \mathbf{1}\,.$$

- $SU(7)$

$$\mathbf{35}\ \left(\begin{array}{|c|}\hline \\\hline \\\hline \\\hline\end{array}\right) + 5\times\mathbf{7}\,(5\times\Box)\,. \quad \leftrightarrow \quad 3\times\mathbf{21}\,\left(\begin{array}{|c|}\hline \\\hline \\\hline\end{array}\right) + 7\times\mathbf{1}\,.$$

- $SU(8)$

$$\mathbf{56}\ \left(\begin{array}{|c|}\hline \\\hline \\\hline \\\hline\end{array}\right) + 9\times\mathbf{8}\,(\times\Box)\,. \quad \leftrightarrow \quad 4\times\mathbf{28}\,\left(\begin{array}{|c|}\hline \\\hline \\\hline\end{array}\right) + 16\times\mathbf{1}\,.$$

and

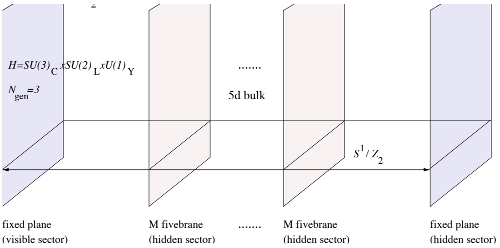
$$\mathbf{35}\ \left(\frac{1}{2}\begin{array}{|c|}\hline \\\hline \\\hline \\\hline \\\hline\end{array}\right) + 8\times\mathbf{8}\,(\Box) \quad \leftrightarrow \quad 3\times\mathbf{28}\,\left(\begin{array}{|c|}\hline \\\hline \\\hline\end{array}\right) + 15\times\mathbf{1}\,.$$

Heterotic Geometry: $SU(6)$

- The commutant of $SU(6) \subset E_8$ is $SU(3) \times SU(2) \to V = V_2 \oplus V_3$ with $c_1(V_2) = c_1(V_3) = 0$
- $c_2(V_2) + c_2(V_3) + c_2(V_{hidden}) = c_2(TX_3)$
- Transition moves "pieces" of $c_2(V_2) \leftrightarrow c_2(V_3)$ (within bounds)
- $6D$ illustration (Bershadsky, et al): $c_2(V) = 12 + n$, $c_2(V_2) = 4 + r$, $c_2(V_3) = 16 + 2n + r$
- Spectra a function of integers (r, n):

$$\frac{r}{2}\mathbf{20} + (16 + r + 2n)\mathbf{6} + (2 + n - r)\mathbf{15}$$

Dual Interpretation: Heterotic M-theory



- Higgsing $\rightarrow$ Deforming V/Y_{n+1}
- Blowing-up/down the base $\rightarrow$ small instanton transitions across $S^1/\mathbb{Z}_2$
- Exotic transitions $\rightarrow$ Small instanton transitions on the *same* fixed plane.

- Deformation/Resolution of superconformal loci
- Straightforward to classify which symmetries admit these matter transitions. Almost all coupled to superconformal loci (exceptions: Duals of $SO(32)$ heterotic theories)
- Relevant to recent developments in superconformal matter (Vafa, Heckman, Morrison, Park)

$SU(N)$ Matter Transitions

- What about the heterotic duals of the $SU(7)$ and $SU(8)$ F-theory models?
- Here, unlike other cases, the commutant inside of E_8 takes the generic (and special form): $S[U(m_1) \times U(m_2)]$
- These bundles do **not** generically satisfy HYM eqns. Polystability $\Rightarrow$ non-trivial D-term conditions constraining the EFT and restricting moduli.
- If $V = \bigoplus V_i$ with $c_1(V_i) \neq 0$ and $c(V) = 0$, polystability $\rightarrow$ $\mu(V_i) = c_1(V_i) \wedge (\omega^{1,1})^{n-1} = 0$
- Split ($U(n)$) spectral covers have been studied in many e.g.s (Hayashi, Choi, Watari, Braun, Mayrhofer, Palti, Weigand...)
- Here the special feature is that this splitting is required/generic in the complete moduli space.

Illustration: $SU(7)$

- The commutant of $SU(7) \subset E_8$ is $SU(2) \times U(1)$ of a very particular form:

$$V = L \otimes V_2 \oplus (L^{\vee})^{\otimes 2}$$

with $c_1(V)$, $c_1(V_2) = 0$, but $c_1(L) \neq 0$

- Non-trivial constraint, $\mu(L) = 0$
- $U(1)$ factor is self-commuting in E_8 and Green-Schwarz massive

Representation	Cohomology	$6D$ Multiplicity
1	$H^1(\mathrm{End}_0(V_2))$	$4c_2(V_2) - 6$
7	$H^1(V_2 \otimes L^3) \oplus H^1(L^{\vee 4})$	$(c_2(V_2) - 9c_1(L)^2 - 4) + (-8c_1(L)^2 - 2)$
$\bar{7}$	$H^1(V_2 \otimes L^{\vee 3}) \oplus H^1(L^4)$	$(c_2(V_2) - 9c_1(L)^2 - 4) + (-8c_1(L)^2 - 2)$
35	$H^1(L^2)$	$-2c_1(L)^2 - 2$
$\overline{35}$	$H^1(L^{\vee 2})$	$-2c_1(L)^2 - 2$
21	$H^1(V_2 \otimes L) \oplus H^1(V_2 \otimes L^{\vee})$	$(c_2(V_2) - c_1(L)^2 - 4) + (c_2(V_2) - c_1(L)^2 - 4)$

Green-Schwarz Massive $U(1)$s in heterotic theories

- The Kähler axions transform under $U(1)$ via shifts

$$\delta\chi^i = -c_1^i(L)\eta^a$$

- Kähler moduli transverse to a locus Higgs the $U(1)$ symmetries.
- All matter charged under anomalous $U(1)$ $\Rightarrow$ charges restrict the theory
- D-terms: (Lukas, Stelle, Blumenhagen, Weigand, Honecker,etc ...)

$$D \sim FI - \sum_{b>a}(|C_{ab}^+|^2 - |C_{ab}^-|^2) \stackrel{!}{=} 0$$

$$FI \sim \frac{\epsilon_s \epsilon_R^2}{k_4^2}\frac{\mu(L)}{Vol} + \frac{\epsilon_s^2\epsilon_R^2}{k_4^4}\frac{\beta_i c_1^i(L)}{S}$$

- Restrictions from $U(1)$ and D-terms significantly constrain the EFT, its pheno and branch structure (Watari, LA, Gray, Ovrut, Lukas).
- Global/Discrete remnants produce effects which persist even into "neighboring" regions of moduli space.

GS Anomalous $U(1)$s and discrete symmetries in F-theory

- We have found new Higgsing chains of symmetries which will carry the global/discrete remnants anomalous $U(1)$ throughout their *entire* moduli space..
- All of these *must* have non-trivial D-terms. Better yet: effect on EFT computable/known!
- Of interest in F-theory (See talks of Palti, Weigand, Klevers, Grimm, Leontaris, ...)!
- Note: D-terms have significantly different features in $6D$ vs. $4D$:
  - In $6D$ all moduli in $h^{1,1}(K3)$, $\mu(V) = c_1(V) \wedge J^{ij} = 0 \overset{\text{rotated}}{\leftrightarrow} SU(2)$ R-symmetry
  - In $4D$, Kähler moduli only constrained by D-term. But, may involve both F-theory Kähler and C.S. moduli.
- Interesting features: new forms of the stable degeneration limit, link to G-flux in $4D$, etc.

Summary and Conclusions

- Heterotic/F-theory duality continues to provide important tools to better determine the structure of both theories
- Tools are being developed to study this duality across a wide range of known geometries
- In dual pairs, exotic matter transitions in F-theory are linked to heterotic small instanton transitions and can be classified.
- There are new Higgsing chains of geometries which provide an explicit/calculable arena to explore GS massive $U(1)$s and their generic effects on the potential/G-flux.

Thanks!