On E_8 and F-theory GUTs

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Work in collaboration with

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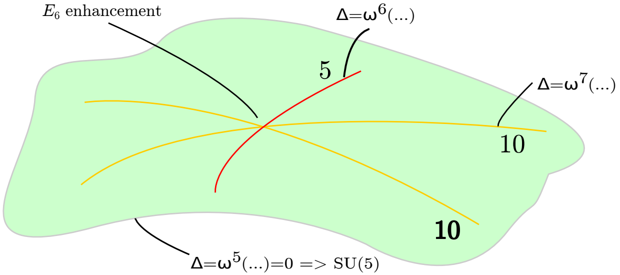
Motivations

- There are hints for Grand Unified Theories (GUT), minimally $SU(5)$.
- Can string theory constrain extensions of GUT symmetries and/or representations thereof ?
- F-theory offers a good framework to try to address gauge theory questions from a string theoretic point of view.
- Specifically : Does E_8, as the maximal exceptional group, play a role in controlling matter and gauge symmetries ?

$SU(5)$ F-theory GUTs

To get $SU(5)$ (SUSY) GUT group in F-theory, one can tune the coefficients of the elliptic fibration so that the discriminant Δ vanishes to order 5.



The existence of a top Yukawa $\mathbf{5\,10\,10}$ implies the existence of a point of E_6.

E_8 in F-theory

- Given the exceptional structure of the top Yukawa, does E_8 — having the highest rank in the exceptional series — have a role in controlling matter and gauge group ?

- Recently, many works studied models with gauge group $SU(5) \times U(1)^n$.

  A nice way to parameterise it is as arising from E_8 :

$$\begin{aligned} E_8 \;&\longrightarrow SU(5)_{\mathsf{GUT}} \times SU(5)_{\perp} \longrightarrow \; SU(5)_{\mathsf{GUT}} \;\times\; U(1)^4 \\ \mathbf{248} \;&\longrightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus \big((\mathbf{10},\mathbf{5}) \oplus (\overline{\mathbf{5}},\mathbf{10}) \oplus \text{h.c}\big) \end{aligned}$$

E_8 tree

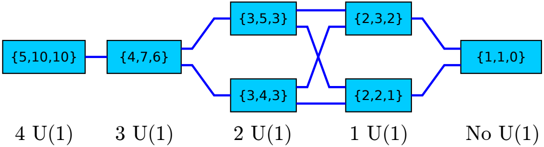
- We can parameterise the $U(1)$'s coming from the Cartan as $U(1)_A = \sum_{i=1}^{5} \alpha_i^A t^i$ with $\sum_{i=1}^{5} \alpha_i^A = 0$. The charges of the matter states are then given by :

$$\mathbf{10}_i : \; t_i \qquad \overline{\mathbf{5}}_{ij} : \; t_i - t_j \qquad \mathbf{1}_{ij} : \; t_i + t_j \qquad i, j = 1, \ldots, 5 \qquad t_i t^j = \delta_i^j$$

- We can then reach the desired number of $U(1)$'s by a Higgs mechanism through $SU(5)$ singlets :

$$\langle \mathbf{1}_{ij} \rangle = \langle \overline{\mathbf{1}}_{ij} \rangle \neq 0$$

- Breaking the $U(1)$'s one by one gives rise to the $\mathbf{E_8}$ **tree** :



(Each model is denoted by it's $SU(5)$ spectrum $\{\#\mathbf{10}, \#\mathbf{5}, \#\mathbf{1}\}$)

Beyond E_8 models

- Consider for example a **one** $U(1)$ **model**. From the E_8 tree, there are two possible spectra :

  $\{2,2,1\}: \ \mathbf{10}_{-4}, \mathbf{10}_{1}, \mathbf{5}_{-3}, \mathbf{5}_{2}, \mathbf{1}_{5}$ $\qquad$ $\{2,3,1\}: \ \mathbf{10}_{-2}, \mathbf{10}_{3}, \mathbf{5}_{-6}, \mathbf{5}_{4}, \mathbf{5}_{-1}, \mathbf{1}_{5}$

  Can every single $U(1)$ model be embedded in these spectra ?

- An example in the literature gives the spectrum [Braun,Grimm,Keitel '13] :

  $$\text{BGK}: \ \mathbf{10}_{-1}, \mathbf{5}_{-8}, \mathbf{5}_{-3}, \mathbf{5}_{2}, \mathbf{5}_{7}, \mathbf{1}_{5}, \mathbf{1}_{10}, \mathbf{1}_{15}$$

  Not embeddable !

A model with an additional singlet

- One can try constructing a global model based on the $\{2,3,1\}$ breaking of E_8. [Mayrhofer, Palti, Weigand '12 ; Marsano, Saulina, Schafer-Nameki '09]
- A possibility is a fibration in $P_{[1,1,2]}$:

$$w^2 + b_0 u^2 w + b_1 uvw + b_2 v^2 w + c_0 u^4 + c_1 u^3 v + c_2 u^2 v^2 + c_3 uv^3 = 0$$

$$b_0 = -\omega\alpha d_3 \qquad b_1 = -e_2 d_3 \qquad b_2 = \delta$$

$$c_0 = \omega^3\alpha\gamma \qquad c_1 = \omega^2(\alpha d_2 + \gamma e_2) \qquad c_2 = \omega e_2 d_2 \qquad c_3 = \omega\beta$$

- One finds the following matter spectrum :

$$\mathbf{10}_{-2} : \ \omega = d_3 = 0 \qquad \mathbf{10}_3 : \ \omega = e_2 = 0$$

$$\mathbf{5}_{-6} : \ \omega = \delta = 0 \qquad \mathbf{5}_4 : \ \omega = \beta d_3 + d_2\delta = 0 \qquad \mathbf{5}_{-1} : \ \omega = f_1(\alpha,\beta,\gamma,\delta,e_i,d_i)$$

$$\mathbf{10}_5 : \ f_2(\alpha,\beta,\gamma,\delta,e_i,d_i) = f_3(\alpha,\beta,\gamma,\delta,e_i,d_i) = 0 \qquad \mathbf{1}_{10} : \ \beta = \delta = 0$$

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$$\mathbf{10}_5:\ f_2(\alpha,\beta,\gamma,\delta,e_i,d_i) = f_3(\alpha,\beta,\gamma,\delta,e_i,d_i) = 0 \qquad \mathbf{1}_{10}:\ \beta = \delta = 0$$

- The singlet $\mathbf{1}_{10}$ makes this model **non-embeddable** in the tree.

- It is also possible to break the $U(1)$ with this extra singlet to get a $SU(5)\times\mathbb{Z}_2$ model by deforming the fibration [Morrison, Taylor, '14] :

$$P_{[1,1,2]} \xrightarrow{U(1)\to\mathbb{Z}_2} P_{[1,1,2]} + c_{4,1}\omega v^4$$

The $\mathbf{E_8}$ tree doesn't have any model with discrete symmetry.

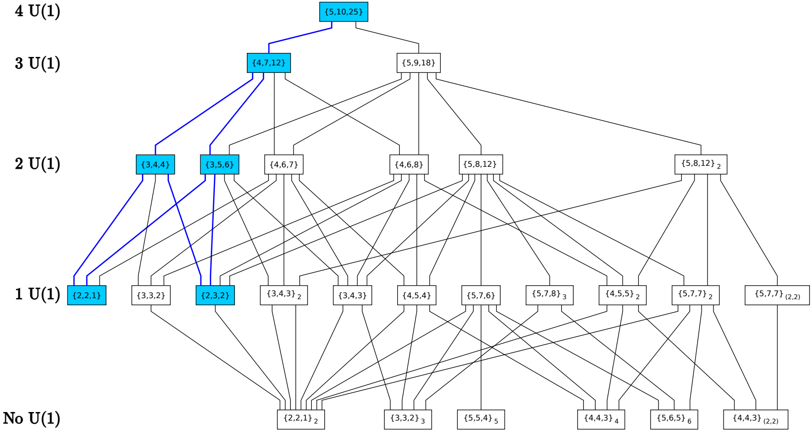
Extending E_8

- The spectrum
$$\mathbf{10}_{-2}, \mathbf{10}_{3}, \mathbf{5}_{-6}, \mathbf{5}_{4}, \mathbf{5}_{-1}, \mathbf{1}_{5}, \mathbf{1}_{10}$$
has the following property : For each pair of $\mathbf{5}$, there is a singlet to form a $U(1)$ neutral $\mathbf{5}\overline{\mathbf{5}}\mathbf{1}$ cubic coupling.

- This is a generic feature of many fibrations : We call these fibrations **complete networks**.

- Extend the E_8 spectrum to allow for **complete networks** by **adding 15 new singlets** :
$$\mathbf{10}_i : \; t_i \qquad \overline{\mathbf{5}}_{ij} : \; t_i - t_j \qquad \mathbf{1}_{ij} : \; t_i + t_j \qquad \mathbf{1}_{ijkl} : \; t_i + t_j - t_k - t_l$$
From there we can construct a **"Beyond $\mathbf{E_8}$ tree"**.

Beyond E_8 tree



Embedding into the beyond E_8 tree

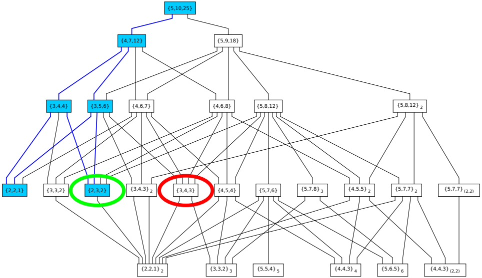
What about the examples ?

- Extended {2,3,1} :

  $\mathbf{10}_{-2}, \mathbf{10}_{3}, \mathbf{5}_{-6}, \mathbf{5}_{4}, \mathbf{5}_{-1}, \mathbf{1}_{5}, \mathbf{1}_{10}$

- BGK :

  $\mathbf{10}_{-1}, \mathbf{5}_{-8}, \mathbf{5}_{-3}, \mathbf{5}_{2}, \mathbf{5}_{7}, \mathbf{1}_{5}, \mathbf{1}_{10}, \mathbf{1}_{15}$



Embedding into the beyond E_8 tree

What about other models ? We looked at 30 fibrations in the literature [Borchmann, Braun, Cvetic, Garca-Etxebarria, Grassi, Grimm, Kapfer, Keitel, Klevers, Kuntzler, Lawrie, Mayorga Pena, Mayrhofer, Oehlmann, Palti, Piragua, Sacco, Schafer-Nameki, Reuter, Till, Weigand].

- 27/30 are embeddable in our extended tree.
- 1 fibration is not consistent (cannot turn off non-flat point).
- 2 fibrations cannot be embedded. These models are the only ones that **do not form a complete network**.

Model	spectrum embedded in
No $U(1)$ models	
[24, 25]	$\{2,2,2\}_2$
[25]	$\{2,2,2\}_2$
One $U(1)$ models	
[12]	$\{3,4,2\}$
[19], [22] fiber type $I_5^{(01)}$	$\{3,3,2\}$
[22] fiber type $I_{5,ncnc}^{(01)}$	$\{3,3,2\}$
[19], [22] fiber type $I_5^{(0\vert 1)}$	$\{4,5,4\}$ or $\{\mathbf{2},\mathbf{3},\mathbf{2}\}$
[19], [22] fiber type $I_{5,nc}^{(0\vert 1)}$	$\{\mathbf{2},\mathbf{3},\mathbf{2}\}$
[19], [22] fiber type $I_{5,nc}^{(0\Vert 1)}$	$\{3,4,3\}$
Two $U(1)$'s models	
[11] $4-1$ split	$\{\mathbf{2},\mathbf{2},\mathbf{1}\}$
[11] $3-2$ split	$\{\mathbf{2},\mathbf{3},\mathbf{2}\}$
Top 1	$\{\mathbf{3},\mathbf{5},\mathbf{6}\}$
Top 2	$\{5,8,12\}$
Top 3	$\{4,6,7\}$
Top 4	$\{4,6,8\}$
[26]	$\{5,8,12\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(2,2,2,0,0,0,0,0)$	$\{\mathbf{3},\mathbf{4},\mathbf{4}\}$, $\{4,6,7\}$, $\{5,8,12\}*$
$I_5^{s(0\vert 1\vert 2)}$ $(2,1,1,1,0,0,1,0)$	$\{\mathbf{3},\mathbf{5},\mathbf{6}\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(2,1,1,1,0,0,1,0)$	$\{5,8,12\}$
$I_5^{s(1\vert 0\vert 2)}$ $(3,2,1,1,0,0,0,0)$	$\{5,8,12\}$
$I_5^{s(01\vert 2)}$ $(3,2,1,1,0,0,0,0)$	$\{4,6,8\}$
$I_5^{s(0\vert 12)}$ $(4,2,0,2,0,0,0,0)$	Not embeddable
$I_5^{s(012)}$ $(5,2,0,2,0,0,0,0)$	Not embeddable
$I_5^{s(01\Vert 2)}$ $(2,2,2,0,0,0,0,0)$	$\{4,6,7\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(2,1,1,1,0,0,0,0)$	$\{\mathbf{3},\mathbf{5},\mathbf{6}\}$ *
$I_5^{s(01\Vert 2)}$ $(2,1,1,1,0,0,0,0)$	$\{4,6,7\}$
$I_5^{s(1\vert 0\vert 2)}$ $(2,1,1,1,0,0,0,0)$	$\{5,8,12\}$
$I_5^{s(0\vert 2\Vert 1)}$ $(1,1,1,1,0,0,1,0)$	$\{5,8,12\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(1,1,1,0,0,0,0,0)$	No consistent way to turn off non-flat points.
[15] 2 Fibrations	Any of the 2 $U(1)$ models

Summary and Conclusion

- We studied the relation between global F-theory GUTs with gauge group $SU(5) \times U(1)^n$ and E_8.
- We presented an extension of the set of theories that can be reached through breaking by introducing additional $SU(5)$ singlets such that complete network can be formed.
- All 27 complete flat fibrations can be embedded in the extended tree.
- 2 fibrations do not have an embedding. Further investigation needed (another extension of E_8 ?)