# On the physical realization of Seiberg dual phases in branes at singularities

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Based on: Ongoing work with Iñaki García-Etxebarria and Ben Heidenreich

String Phenomenology 2015,
Madrid, June 10, 2015

# Introduction

Seiberg duality is an important aspect of $\mathcal{N} = 1$ supersymmetric gauge theory.

In string theory, Seiberg duality arises because a configuration of gauge theory can be realized as a bound state of a collection of branes in more than one way.

In the context of quiver theories, the branes and antibranes involved can be defined algebraically without presupposing that the theory can be realized in terms of D-branes. [Berenstein, Douglas; hep-th/0207027] Therefore, one interesting question is which quiver gauge theories can be realized by D-branes in a CY 3-fold, and which do not.

In particular, we will be interested in the case of D-branes sitting on a singularity inside a Calabi-Yau manifold.

The quiver gauge theory can be thought of as describing a marginal binding of the associated D-branes.

This happens when the periods characterizing the central charges for the associated fractional branes are aligned.[e.g. hep-th/0405134]

Note: we will only consider the Kähler moduli space at weak coupling.

# Phases of $dP_0$

Let us consider $\mathbb{P}^2$ [García-Etxebarria, Heidenreich, Wrase; 1307.1701].

There is a phase (phase I) with $B_2 = 0$ which corresponds to the "orbifold" phase.

The Seiberg dual phase (phase II) corresponds to $B_2 = 1/2$.

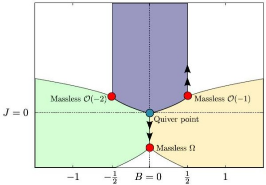


Figure from 1307.1701

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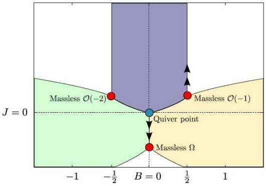


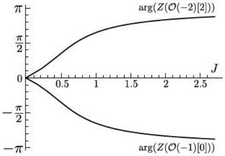


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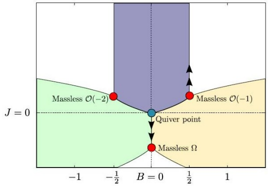


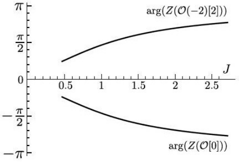


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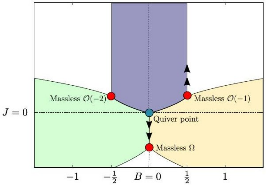
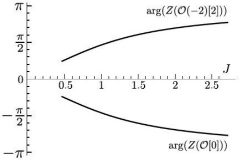
Phase I is supersymmetric at the orbifold point.

Phase II is not supersymmetric.

Therefore, only phase I is physically realized.

Could toric vs. non-toric be the problem?




Figures from 1307.1701

# Generalities of toric varieties

A toric CY 3-fold $X$ is described by a GLSM

$$
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & \cdots & x_n & t \\
\hline
\mathbb{C}^*_1 & q_1^1 & q_1^2 & q_1^3 & \cdots & q_1^n & -d_1 \\
\mathbb{C}^*_2 & q_2^1 & q_2^2 & q_2^3 & \cdots & q_2^n & -d_2 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\mathbb{C}^*_r & q_r^1 & q_r^2 & q_r^3 & \cdots & q_r^n & -d_r
\end{array}
$$

$$n - r = 2, \quad \sum_{i=1}^{d} q_r^i = d_r .$$

The Newton polynomial is constructed by

$$P(x,y) = \sum_{d \in \mathrm{Pic}(X)} c_d x^{a(d)} y^{b(d)}$$

The Kähler moduli of $X$ are related to the $z_i$ variables parametrizing the complex structure moduli of the mirror manifold $Y$

$$t_i = \Phi_i(z) \simeq \frac{1}{2\pi i} \log(z_i) + \dots$$

$$z_i = \prod_{d \in \mathrm{Pic}(X)} c_d^{q_i^d}$$

The periods $\Phi$ satisfy the Picard-Fuchs equations

$$\mathcal{L}_k \Phi = 0$$

$$\mathcal{L}_k = \prod_{q_k^i > 0} \left( \frac{\partial}{\partial c_k} \right)^{q_k^i} - \prod_{q_k^i < 0} \left( \frac{\partial}{\partial c_k} \right)^{-q_k^i}$$

# Generalities of toric varieties

The mirror manifold is given by a hypersurface

$$uv = P(x, y), \qquad u, v \in \mathbb{C}, \quad x, y \in \mathbb{C}$$

It is useful to write it as a double fibration

$$\begin{aligned} W &= uv \\ W &= P(x, y) \end{aligned}$$

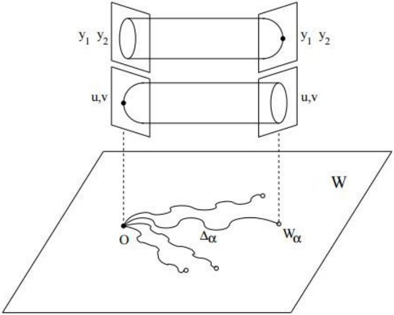
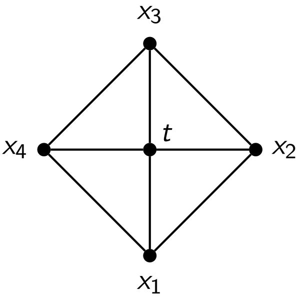



Figure taken from hep-th/0110028.

# $\mathbb{F}_0$ general characteristics

Toric diagram:




Newton polynomial:

$$P(x,y) = \frac{a}{x} + \frac{b}{y} + cx + dy + e$$

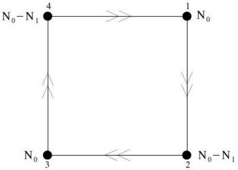
Mori cone:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $t$ |
|---|---|---|---|---|---|
| $\mathcal{C}_1$ | 0 | 1 | 0 | 1 | $-2$ |
| $\mathcal{C}_2$ | 1 | 0 | 1 | 0 | $-2$ |

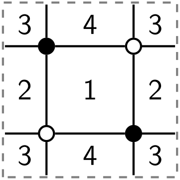
$$z_1 = \frac{ac}{e^2}, \qquad z_2 = \frac{bd}{e^2}$$

# $\mathbb{F}_0$ phase I

## Quiver




## Dimer




## Superpotential

$$\epsilon^{ij}\epsilon_{kl}X_{i12}X_{k23}X_{j34}X_{l41}$$

## Mirror geometry

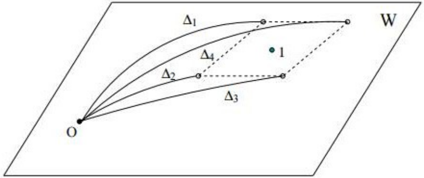
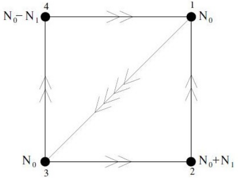



Figure taken from hep-th/0110028.

# $\mathbb{F}_0$ phase II

## Quiver




## Superpotential

$$
\left\{
\begin{array}{l}
M_{11}(Y_1 X_1 - X_2' Y_2') \\
+M_{12}(X_2' Y_1' - Y_2 X_1) \\
+M_{21}(X_1' Y_2' - Y_1 X_2) \\
+M_{22}(Y_2 X_2 - X_1' Y_2')
\end{array}
\right.
$$

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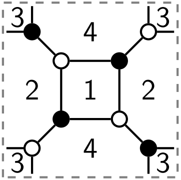
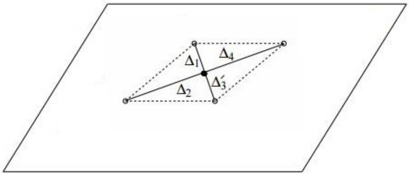



## Mirror geometry

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# $\mathbb{F}_0$ periods and quiver locus

For the Mori cone

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $t$ |
|---|---|---|---|---|---|
| $\mathcal{C}_1$ | 0 | 1 | 0 | 1 | $-2$ |
| $\mathcal{C}_2$ | 1 | 0 | 1 | 0 | $-2$ |

the Picard-Fuchs equations we need to solve are

$$\mathcal{L}_1\Phi(z_1,z_2) = (\theta_1^2 - z_1(2\theta_1+2\theta_2)(2\theta_1+2\theta_2+1))\Phi(z_1,z_2) = 0,$$

$$\mathcal{L}_2\Phi(z_1,z_2) = (\theta_2^2 - z_2(2\theta_1+2\theta_2)(2\theta_1+2\theta_2+1))\Phi(z_1,z_2) = 0,$$

where

$$\theta_i = z_i\frac{\partial}{\partial z_i}, \quad z_1 = \frac{ac}{e^2}, \quad z_2 = \frac{bd}{e^2}.$$

# $\mathbb{F}_0$ periods and quiver locus

The periods are:

$$\begin{aligned}
\Phi(z_1, z_2) &= 1, \\
\Phi_1(z_1, z_2) &= \frac{1}{2\pi i}\log(-z_1) + \frac{1}{\pi i}\sum_{(m,n)\neq(0,0)} \frac{\Gamma(2(m+n))}{\Gamma(m+1)^2\Gamma(n+1)^2} z_1^m z_2^n, \\
\Phi_2(z_1, z_2) &= \frac{1}{2\pi i}\log(-z_2) + \frac{1}{\pi i}\sum_{(m,n)\neq(0,0)} \frac{\Gamma(2(m+n))}{\Gamma(m+1)^2\Gamma(n+1)^2} z_1^m z_2^n, \\
\Phi_{12}(z_1, z_2) &= \frac{1}{(2\pi i)^2}\Bigg[\log(-z_1)\log(-z_2) + 2\sum_{(m,n)\neq(0,0)} \frac{\Gamma(2(m+n))}{\Gamma(m+1)^2\Gamma(n+1)^2} z_1^m z_2^n \\
&\quad \times \big(\log(z_1) + \log(z_2) + 4\psi(2(m+n)) - 2\psi(m+1) - 2\psi(n-1)\big)\Bigg].
\end{aligned}$$

These series converge for $|z_i| < \frac{1}{4}$.

# $\mathbb{F}_0$ periods and quiver locus

The analitic continuation of the periods are:

$$
\begin{aligned}
\Phi &\to 1, \\
\Phi_1 &\to \frac{1}{2\pi i}[\log(-v_1) - \pi i] - \frac{1}{\pi i}(-v_2)^{\frac{1}{2}} \sum_{m,n=0}^{\infty} \frac{\Gamma(m+n+1/2)^2}{2\sqrt{\pi}\Gamma(n+1)^2\Gamma(m+1)\Gamma(m+3/2)} v_2^m v_1^n, \\
\Phi_2 &\to -\frac{1}{\pi i}(-v_2)^{\frac{1}{2}} \sum_{m,n=0}^{\infty} \frac{\Gamma(m+n+1/2)^2}{2\sqrt{\pi}\Gamma(n+1)^2\Gamma(m+1)\Gamma(m+3/2)} v_2^m v_1^n, \\
\Phi_{12} &\to -\frac{1}{12} + \frac{1}{2\pi^2}(-v_2)^{\frac{1}{2}} \sum_{m,n=0}^{\infty} \frac{\Gamma(m+n+1/2)^2}{2\sqrt{\pi}\Gamma(n+1)^2\Gamma(m+1)\Gamma(m+3/2)} v_2^m v_1^n, \\
&\quad \times[\pi i + \log(-v_1) + 2\psi(m+n+1/2) - 2\psi(n+1)],
\end{aligned}
$$

where

$$
v_1 = \frac{z_1}{z_2}, \qquad v_2 = \frac{1}{z_2}.
$$

# $\mathbb{F}_0$ periods and quiver locus

In the plane $v_2 = 0$

$$
\begin{aligned}
\Phi &= 1, \\
\Phi_1 &= \frac{1}{2\pi i} \log v_1, \\
\Phi_2 &= 0. \\
\Phi_{12} &= -\frac{1}{12},
\end{aligned}
$$

up to monodromy.

# $\mathbb{F}_0$ periods and quiver locus

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up to monodromy.

We obtain all real periods for $v_1 = e^{e\pi i\theta}$

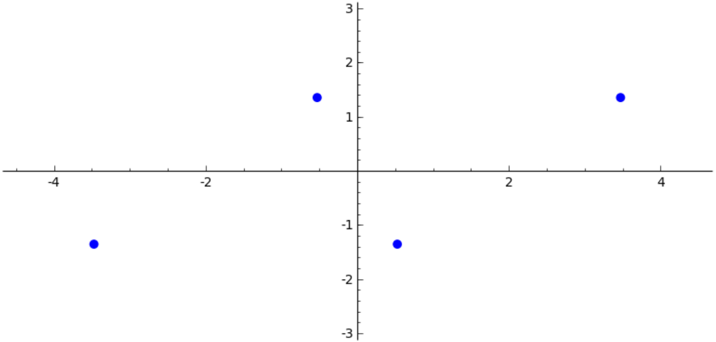
$$\begin{aligned}\Phi &= 1,\\ \Phi_1 &= \theta,\\ \Phi_2 &= 0.\\ \Phi_{12} &= -\frac{1}{12},\end{aligned}$$

## Quiver locus

$$z_1, z_2 \to \infty \quad \text{and} \quad \frac{z_1}{z_2} = e^{2\pi i\theta}$$

# Physical realization of the phases

Phase I is physically realized.
Take e.g. $b = c = d = 1$, $a = i$ and $e = 0$ in the Newton polynomial. The mirror is



# Physical realization of the phases

For the phase II, the situation is different.

We can obtain the phase II if one of the singularities in the mirror goes through the origin.

We need to know the intersection between the quiver locus and the conifold locus.

# Physical realization of the phases

The singularities of the torus

$$W = \frac{a}{x} + \frac{b}{y} + cx + dy + e$$

are given by

$$w = 4(z_1 + z_2) \pm 8\sqrt{z_1 z_2}, \qquad w = \left(1 - \frac{W}{e}\right)^2$$

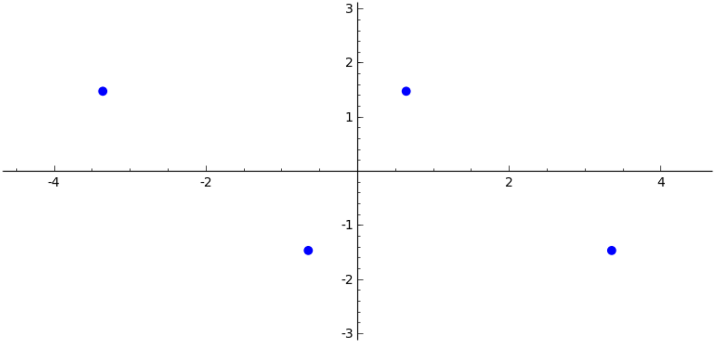
For $z_1 = z_2$ there is a double root at $W = e$.

In the $e \to 0$ limit $(z_1, z_2 \to \infty)$, there is a double root at $W = 0$, so there are precisely *two* branes becomming massless simultaneously.

Therefore, the phase II will never be realized at weak gauge coupling.

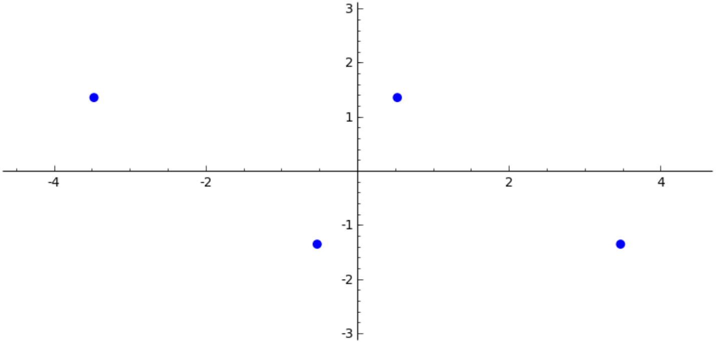
# Physical realization of the phases

Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{-0.5\pi i}$.



# Physical realization of the phases

Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{-0.45\pi i}$.



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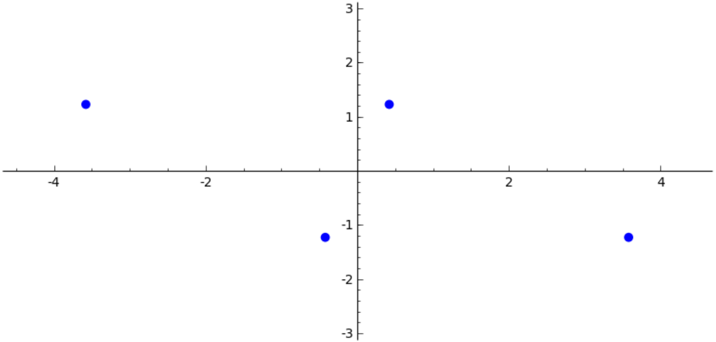
Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{-0.4\pi i}$.



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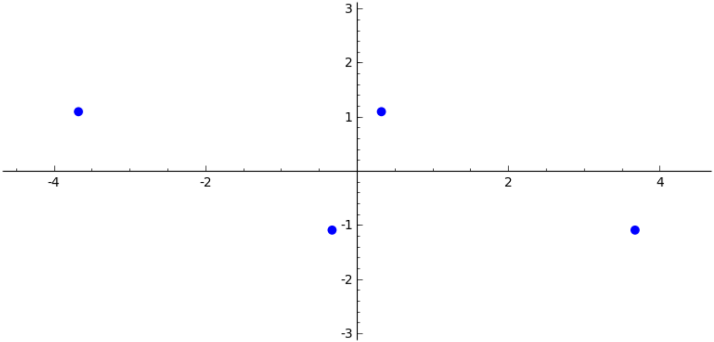
Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{-0.35\pi i}$.



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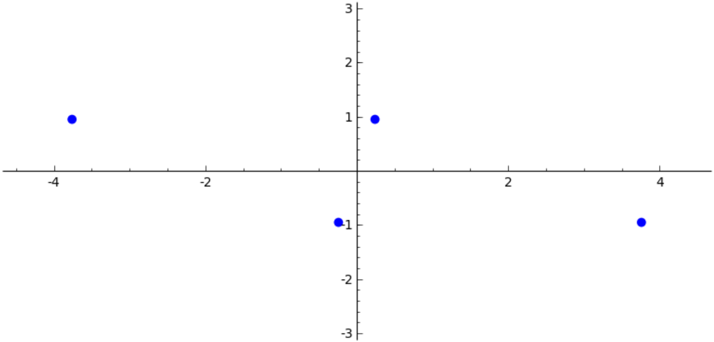
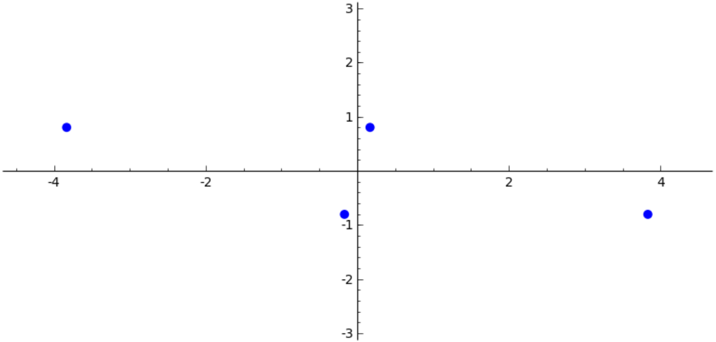


# Physical realization of the phases

Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{-0.25\pi i}$.



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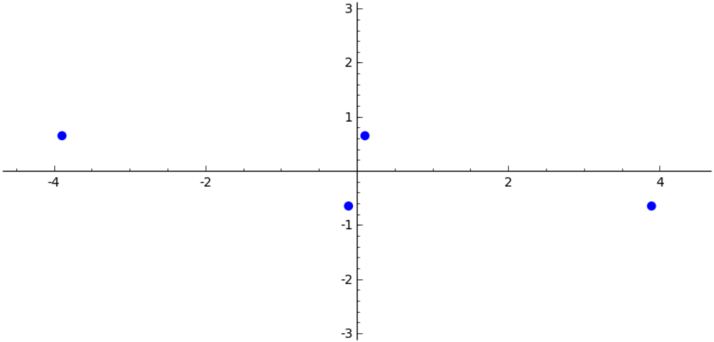

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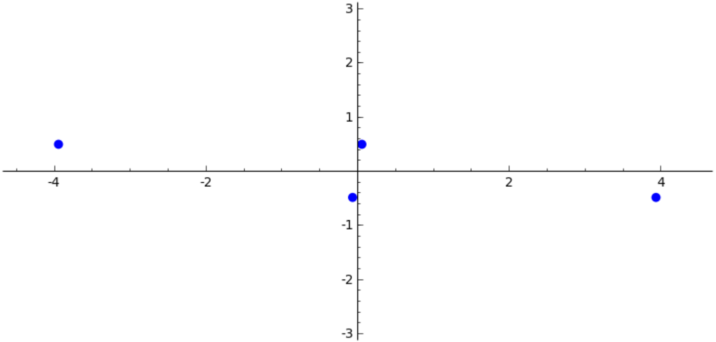

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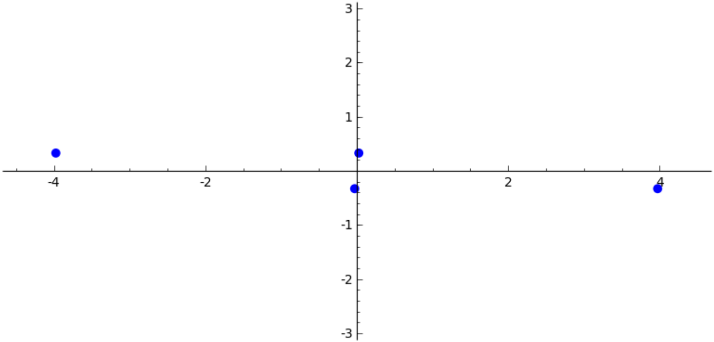
Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{-0.10\pi i}$.



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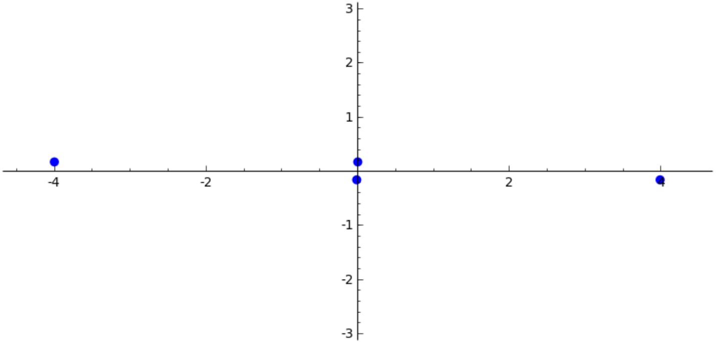
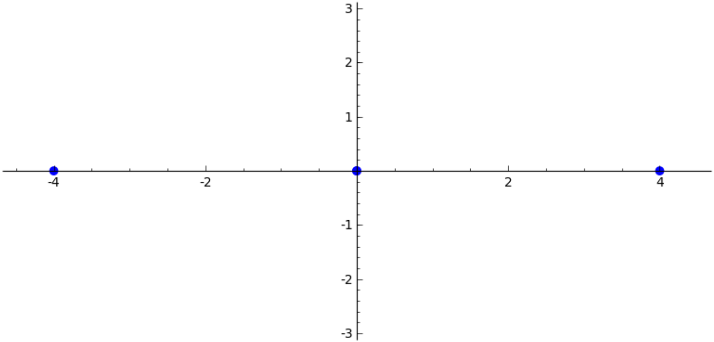
Mirror geometry for $e = 0,\ b = c = d = 1,\ a = e^{-0.05\pi i}$.



# Physical realization of the phases

Mirror geometry for $e = 0$, $b = c = d = 1$, $a = 1$.



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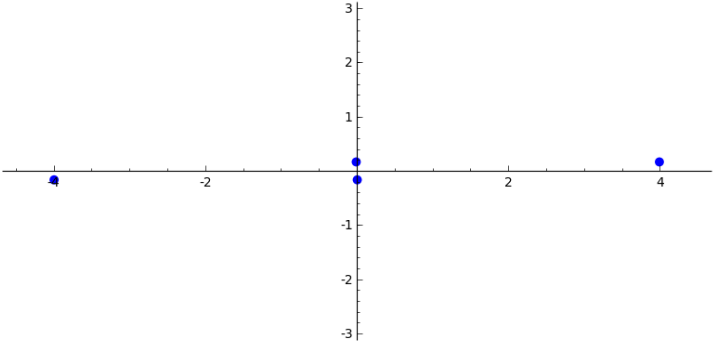

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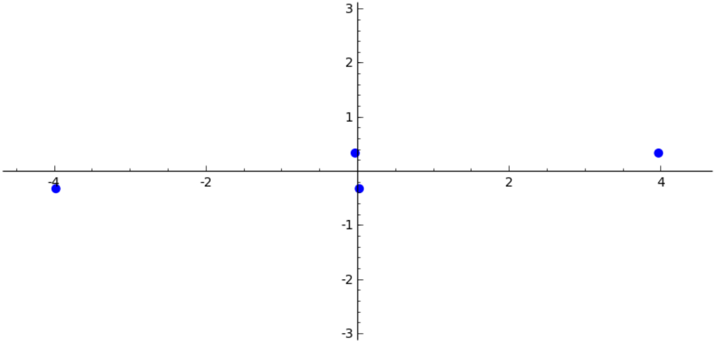

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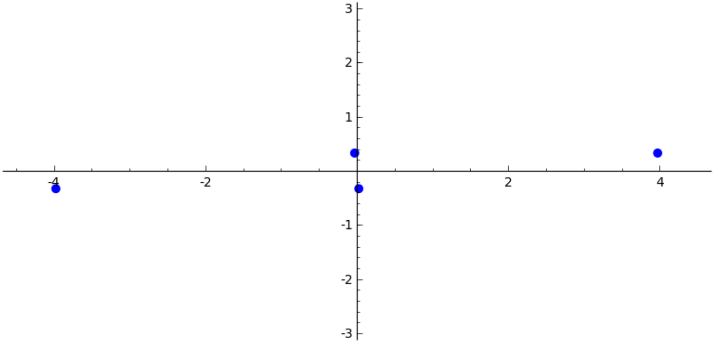

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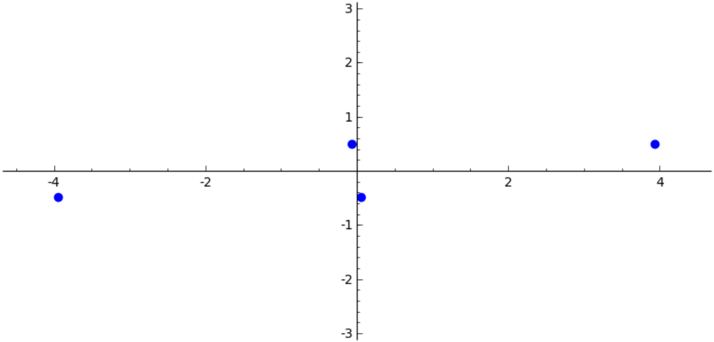

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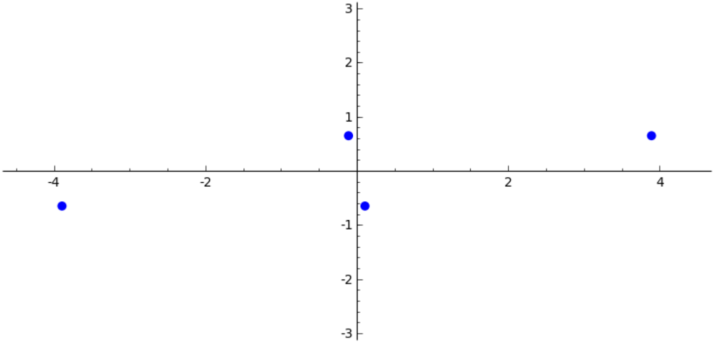

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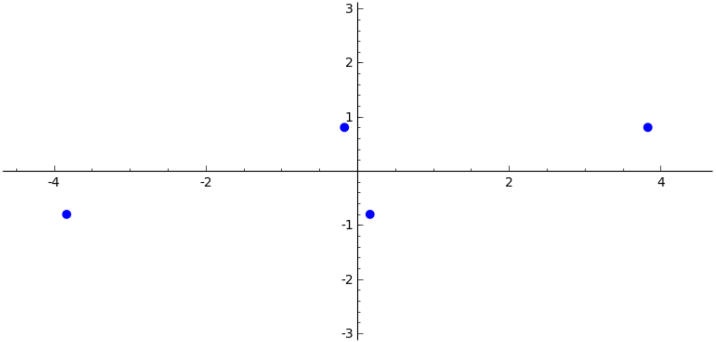

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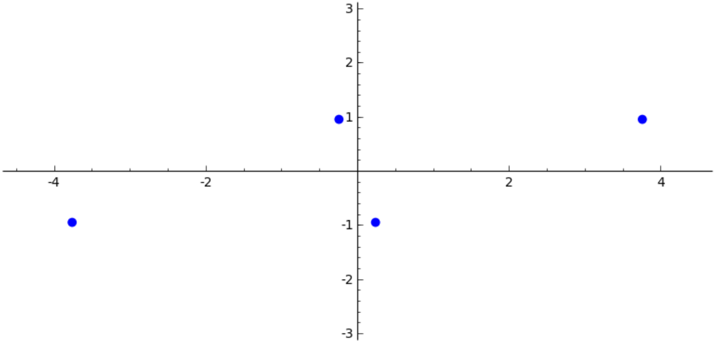
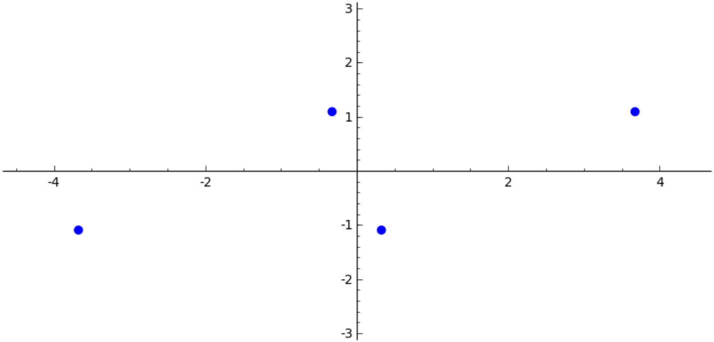


Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{0.4\pi i}$.



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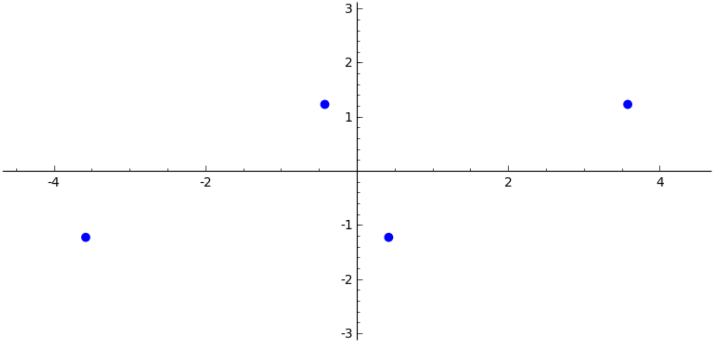
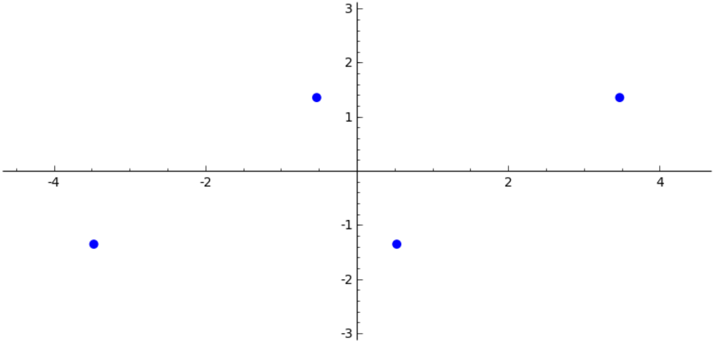


# Physical realization of the phases

Mirror geometry for $e = 0$, $b = c = d = 1$, $a = e^{0.5\pi i}$.



# Conclusions and future directions

## Conclusions

- We have studied the physical realization of Seiberg dual theories in branes at singularities.
- In the $\mathbb{F}_0$ case, only phase I can be realized at weak gauge coupling.

## Future directions

- Extend the analysis to other cases with Seiberg duals phases which are also toric duals like the complex cones over $dP_2$ and $dP_3$.

# Thank you!