

Probing T-branes with 3d mirror symmetry

Andrés Collinucci

ULB & Solvay Institutes

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ongoing work with Raffaele Savelli & Roberto Valandro

Gauge theory vs Geometry

F-theory: Gauge theory + SUGRA \leftrightarrow Geometry

$(D7, \phi_{DBI})$ \xrightarrow{T} $(D6's, \phi_{DBI}) \longrightarrow$ M-theory geometry

$$u v = \det(\mathbb{I}z - \Phi_{DBI})$$

Only sees Casimirs \rightsquigarrow blind to T-branes.

[Sharpe, Gómez, Donagi, Katz; Cecotti, Córdova, Heckman, Tachikawa, Vafa, Wecht;
Donagi, Wijnholt]

Yet, drastic consequences.

E.g. 4 coincident D7-branes, i.e. $G = U(4)$. M-theory lift is

$$u v = z^4 \quad \text{the } A_3 \text{ singularity.}$$

Nilpotents

Switch on a nilpotent worldvolume Φ_{DBI} :

$$\Phi = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightsquigarrow U(1) \times U(2) ,$$

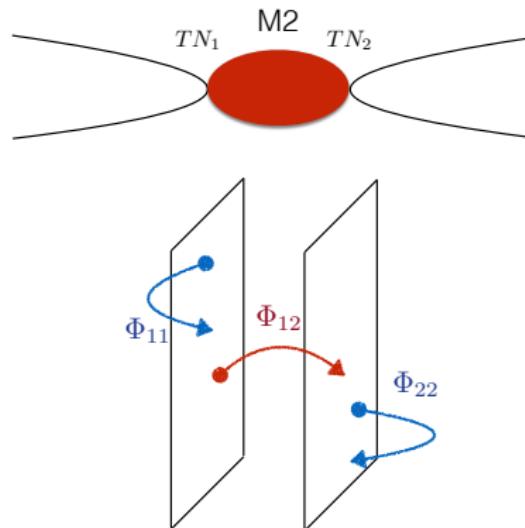
$$\Phi = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightsquigarrow U(1)^2 ,$$

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Geometry still $uv = z^4$. Singularity **believes** true effective field theory.
Where is this information in M-theory?

M2 coherent state



Φ_{11} & $\Phi_{22} \rightarrow$ 11d SUGRA $\Phi_{12} \rightarrow$ coherent M2-states

Goal: Define a framework that houses this extra data.

Two proposals [Anderson, Heckman and Katz], [Savelli, AC]

3d theory “A”

D2 probing stack of N D6's $\rightsquigarrow \mathcal{N} = 4, d = 3 U(1)$ theory.

Field content*:

$$\underbrace{\phi_1, \phi_2, \phi_3}_{\text{3 'transverse scalars'}}, \quad \underbrace{A^\mu}_{\text{photon}}, \quad \underbrace{(Q^i, \tilde{Q}_i)}_{\text{N hypers}}$$

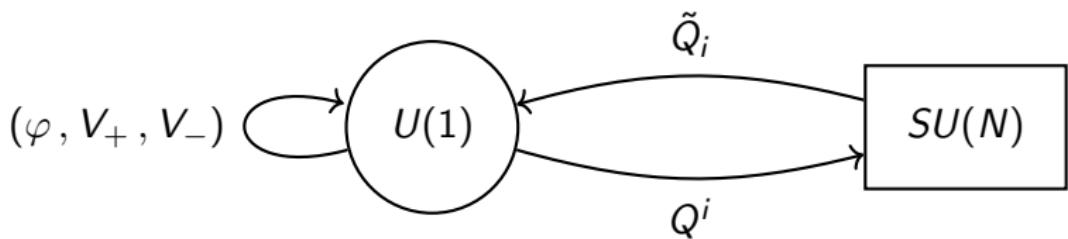
$$\varphi \equiv \phi_1 + i \phi_2, \quad V_\pm \sim \underbrace{e^{\pm \frac{1}{g^2} (\phi_3 + i \gamma_A)}}_{\text{monopole operators}}, \quad Q^i, \tilde{Q}_i,$$

with $d\gamma_A = \star dA$.

Classical equations of motion:

Given by superpotential $W = \varphi \sum_{i=1}^N Q^i \tilde{Q}_i$

A-quiver



$$\sum_i Q^i \tilde{Q}_i = 0, \quad V_+ V_- = \varphi^N$$

Higgs branch

Higgs branch coords = mesons: $M^i{}_j \equiv Q^i \tilde{Q}_j$, subject to:

$$\mathrm{Tr}(M) = 0 \quad , \quad \mathrm{rk}(M) = 1$$

Deformations:

- ▶ Complex masses: $\delta W \sim m_c Q^1 \tilde{Q}_1$.
- ▶ Real masses: $\int d\theta^2 Q^{1\dagger} e^{m_r} Q_1 \sim m_r^2 |Q^1|^2$

$(m_c, m_r) \leftrightarrow$ 3 worldvolume scalars on D6-stack

switching on masses \leftrightarrow moving D6-branes away from stack

Coulomb branch

Coulomb branch is parametrized by φ , $V_{\pm} \sim e^{\pm \frac{1}{g^2} (\phi_3 + i \gamma_A)}$

Quantum:

Monopole equation

$$V_+ V_- = \varphi^N$$

A_{N-1} singularity. Why?

M-theory: Dual photon \cong M-theory circle.

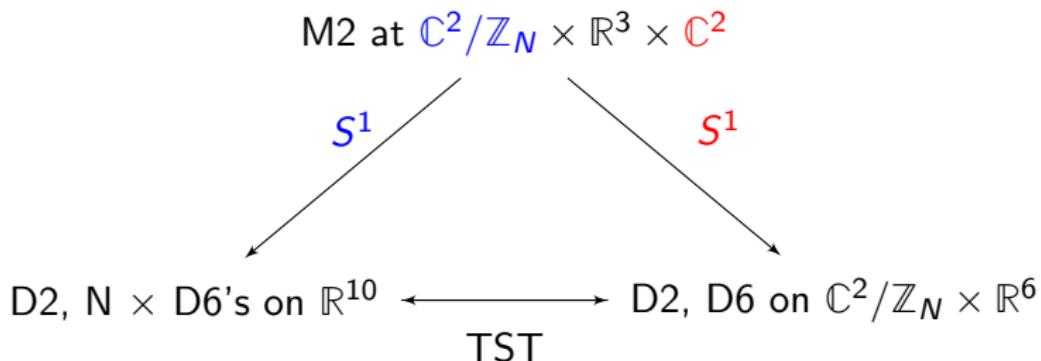
CB is moduli space of M2 probing uplift of N D6's

- ▶ Complex masses $m_c \longrightarrow$ deformations
- ▶ Real masses $m_r \longrightarrow$ resolutions

9-11 flip

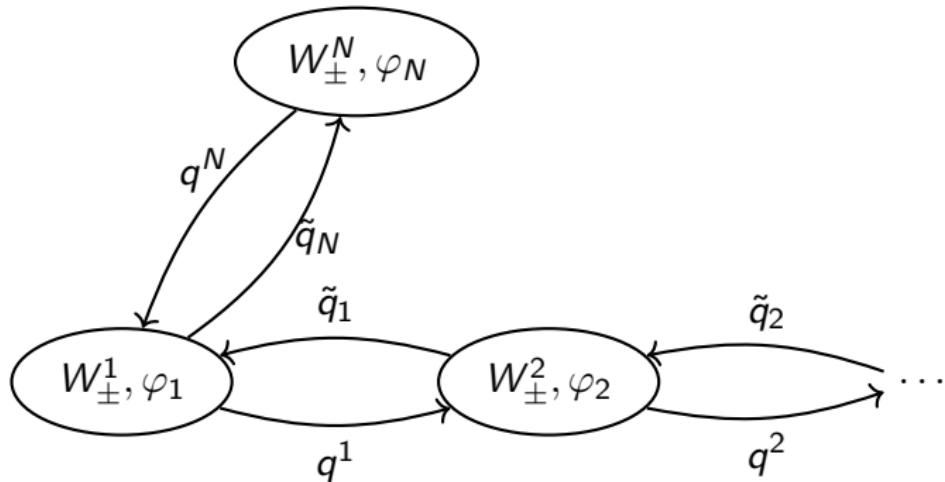
Intriligator & Seiberg: 3d 'mirror theory'.

TST composition, or better yet through '9-11' flip



\Rightarrow theory 'B': same singular geometry as M-theory,
+ power of string theory at singularities \leadsto quiver technology

B-quiver



$$q^i \tilde{q}_i = q^j \tilde{q}_j \quad \forall (i, j), \quad W_+^i W_-^i = (\varphi_{i-1} - \varphi_i)(\varphi_i - \varphi_{i+1})$$

3d mirror symmetry: theory “B”

Intriligator & Seiberg: D2 probing $\mathbb{C}^2/\mathbb{Z}_N$ + one D6

$\rightsquigarrow \mathcal{N} = 4, d = 3, U(1)^{N-1}$ theory.

Mirror map:

$$\varphi \mapsto \lambda, \quad M^i{}_i \mapsto \varphi_i - \varphi_{i+1}, \quad M^{i-1}{}_i \mapsto W^i_+, \quad M^i{}_{i-1} \mapsto W^i_-$$

Classical EoM:

$$W = \sum_i (\varphi_i - \varphi_{i+1}) q^i \tilde{q}_i \quad \Rightarrow \quad q^i \tilde{q}_i = \lambda \quad \forall i$$

Quantum EoM:

$$W^i_+ W^i_- = (\varphi_{i-1} - \varphi_i) (\varphi_i - \varphi_{i+1})$$

Symmetries

Theory 'A' has obvious $SU(N)$ flavor symmetry:

$$Q \mapsto \mathcal{M}_{SU(N)} Q, \quad \tilde{Q}^T \mapsto \tilde{Q}^T \mathcal{M}_{SU(N)}^{-1}$$

where is it in theory 'B'?

Theory 'B'

- ▶ $U(1)^{N-1}$: $\gamma_i \mapsto \gamma_i + c \Rightarrow U(1)$ -currents $J_i = \star F_i = d\gamma_i$
- ▶ Roots: $W_\pm^i \sim e^{\pm(\varphi_i + i\gamma_i)}$, $q_{U(1)_i} = \pm 1$
Spin-1 component \cong current
 \longrightarrow quantum enhancement to $SU(N)$.

T-brane from the probe

A-side: Off-diagonal mass $\sim m_T Q^1 \tilde{Q}_2 \neq$ moving D6-branes

$$W = \varphi \sum_i Q^i \tilde{Q}_i + (m_T)_a{}^b Q^a \tilde{Q}_b \quad \text{for} \quad (m_T)^k = 0$$

F-terms: $(\varphi \cdot \mathbb{1} + m_T) \cdot Q = 0, \quad \tilde{Q} \cdot (\varphi \cdot \mathbb{1} + m_T) = 0$

B-side: Modified quantum relations for CB

$$(\varphi \cdot \mathbb{1} + m_T) \cdot Q = 0$$

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B-side: Modified quantum relations for CB

$$(q\tilde{q} \cdot \mathbb{1} + m_T) \cdot \begin{pmatrix} \varphi_1 - \varphi_2 & W_+^2 \\ W_-^2 & \varphi_2 - \varphi_3 \\ & \ddots \end{pmatrix} = 0$$

m_T breaks $SU(N)$ flavor.

Obstructed blow-ups

A-side:

Real mass $\int d\theta^2 Q^{1\dagger} e^{m_r} Q_1 \sim m_r^2 |Q^1|^2$

$m_r \in$ background v-multiplet

$m Q^1 \tilde{Q}_2$ breaks $U(1)_{1-2} \Rightarrow$ **mass term forbidden**

B-side:

$$q\bar{q}(\varphi_1 - \varphi_2) + m_T W_+^2 = 0 \quad \text{breaks } U(1)_2$$

Blow-ups \leftrightarrow FI terms $\int d^4\theta V_b \Sigma_{U(1)_2} = \int d^4\theta \xi_b V_{U(1)_2}$

But $V_b \in U(1)_2$ gone \Rightarrow blow-up obstructed

[Anderson, Heckman, Katz]

Effective theory

$(\varphi \cdot \mathbb{1} + m_T) \cdot Q = 0$ can be diagonalized. E.g

$$\begin{pmatrix} \varphi & m_T \\ 0 & \varphi \end{pmatrix} \cdot \begin{pmatrix} Q^1 \\ Q^2 \end{pmatrix} = 0 \quad \cong \quad \begin{pmatrix} \varphi^2/m_T & \\ 0 & m_T \end{pmatrix} \begin{pmatrix} Q^{1'} \\ Q^{2'} \end{pmatrix} = 0$$

on B-side:

$$0 = (q \tilde{q} \cdot \mathbb{1} + m_T) \cdot \begin{pmatrix} \varphi_1 - \varphi_2 & W_+^2 \\ W_-^2 & \varphi_2 - \varphi_3 \\ & \ddots \end{pmatrix} \cong$$
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Effective theory

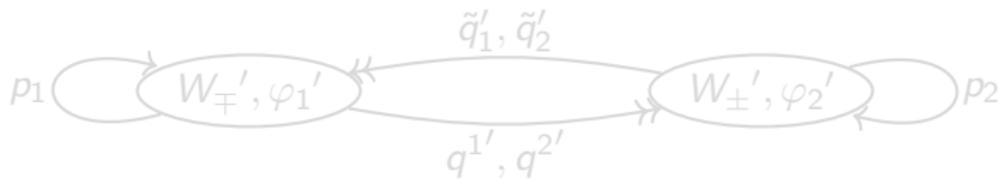
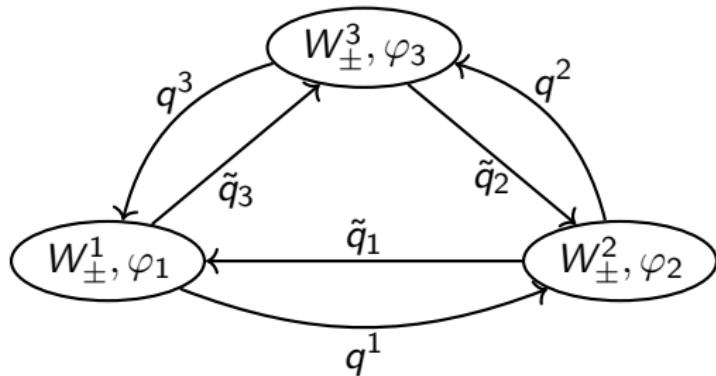
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$$\begin{pmatrix} \varphi & m_T \\ 0 & \varphi \end{pmatrix} \cdot \begin{pmatrix} Q^1 \\ Q^2 \end{pmatrix} = 0 \quad \cong \quad \frac{\varphi^2}{m_T} Q^{1'} = 0$$

on B-side:

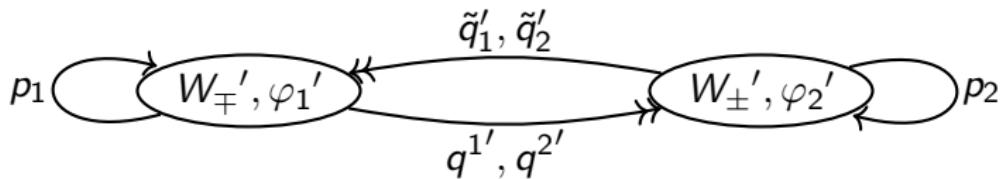
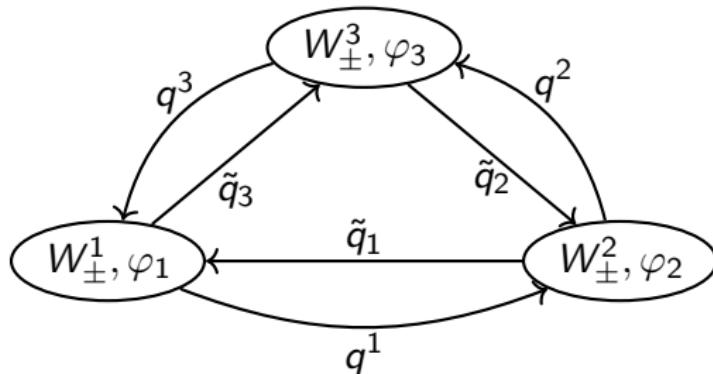
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Effective quiver



$$p_1 = p_2 = q^{1'} \tilde{q}'_1, \quad q^{2'} \tilde{q}'_2 = \frac{(p_1)^2}{m_T}, \quad W_+ W_- = \varphi_1' \varphi_2'$$

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