Type IIB vacua from G-theory

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Introduction

1. Turning on fluxes implies that the internal manifold is of a generalized type.
2. A supersymmetric solution of the motion equations implies that the KSE are satisfied.
3. By using the pure spinor formalism we can extract topological information about the compact space.
4. The fluxes of the global solutions are entirely codified in the geometry of a auxiliary K3 fibration over $\mathbb{CP}^1$.

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Framework

Low energy limit of Type IIB string theory

$$S = \frac{1}{2\kappa_{10}} \int d^{10}x\sqrt{-g}\Big(e^{-2\phi}\left[R + (\nabla\phi)^2 - \frac{1}{6}H^2\right] - \\ - \frac{1}{2}F_1^2 - \frac{1}{12}F_3^2 - \frac{1}{120}F_5^2\Big) - \frac{1}{4\kappa_{10}}\int C_4 \wedge H_3 \wedge F_3$$

where $H = dB$, $F_n = d_H C = dC - H \wedge C$ and the Bianchi identities are

$$dH = 0\,, \quad d_H F = 0 \tag{1}$$

Framework

A particularly elegant form of the KSE is given by

$$d_H(e^{3A-\phi}\Phi_1) = 0 \tag{2}$$

$$d_H(e^{2A-\phi}\mathrm{Re}\Phi_2) = 0 \tag{3}$$

$$d_H(e^{4A-\phi}\mathrm{Im}\Phi_2) = \frac{e^{4A}}{8} * \lambda(F)\,, \tag{4}$$

which specify two nowhere vanishing polyforms [1]

$$\begin{aligned} \Phi_1 &= -\frac{1}{8}K \wedge \left(\sin\alpha\ e^{-\mathrm{i}j} + \mathrm{i}\cos\alpha\ \Omega_2\right) \\ \Phi_2 &= \frac{e^{-\mathrm{i}\theta}}{8} e^{\frac{1}{2}K\wedge\bar{K}} \left(\cos\alpha\ e^{-\mathrm{i}j} - \mathrm{i}\sin\alpha\ \Omega_2\right) \end{aligned} \tag{5}$$

[1] The SU(2) structure can be embedded into SU(3) via $J = j + \frac{\mathrm{i}}{2}K\wedge\bar{K}$, $\Omega_3 = K\wedge\Omega_2$

Solution class A

$\alpha = 0,\ \theta = \pi/2$

$$\Phi_1 = -\frac{\mathrm{i}}{8}\Omega_3 \quad \Phi_2 = -\frac{\mathrm{i}}{8}e^{-\mathrm{i}\mathrm{J}} \tag{6}$$

The Eqs. (2-4) leads to

$$\mathrm{d}(\mathrm{e}^{3\mathrm{A}-\phi}\Omega_3) = \mathrm{d}(\mathrm{e}^{2\mathrm{A}-\phi}\mathrm{J}) = \mathrm{H}\wedge\mathrm{J} = \mathrm{H}\wedge\Omega = 0 \tag{7}$$

$$dC_4 - dB\wedge C_2 = \frac{1}{2}d^c[e^{\phi-4A}\,\hat{J}\wedge\hat{J}]$$
$$dC_2 - C_0\,dB = e^{-\phi}d^cB$$
$$dC_0 = -d^ce^{-\phi}$$

Example A

5 Holomorphic functions

$$\Omega_3 = e^{\phi-3A}\, h\, dz \wedge (dy^1 + \mathrm{i}\, dy^4) \wedge (dy^2 + \mathrm{i} dy^3)$$
$$J = e^{\phi-2A}\left[dy^1 \wedge dy^4 + dy^2 \wedge dy^3\right] + \frac{\mathrm{i}}{2}\, e^{2D}\, |h|^2\, dz \wedge d\bar{z}\ .$$

Which can be written as

$$\begin{aligned} ds^2 &= e^{2A}\sum_{\mu=0}^{3} dx^\mu dx_\mu + e^{\phi-2A}\sum_{m,n=1}^{4}\delta_{mn}dy^m dy^n + e^{-2A}|h(z)|^2 dz d\bar{z} \\ e^{-\phi} &= \tau_2\,, \qquad C_0 = \tau_1\,, \\ B &= -\frac{1}{\tau_2}\beta_2^{(a)}\,\chi_a^- \qquad C_2 = \left(\beta_1^{(a)} - \frac{\tau_1}{\tau_2}\beta_2^{(a)}\right)\chi_a^-\,, \\ C_4 &= \left(-\sigma_1 + \frac{2}{\tau_2}\,\vec{\beta}_1\cdot\vec{\beta}_2 - \frac{\tau_1}{\tau_2^2}\,\vec{\beta}_2\cdot\vec{\beta}_2\right) dy^1\wedge dy^2\wedge dy^3\wedge dy^4\,, \end{aligned} \tag{8}$$

Solution class B

$\alpha = \pi/2,\ \theta = 0$

$$\Phi_1 = -\frac{1}{8} K\, e^{-\mathrm{ij}} \qquad \Phi_2 = -\frac{\mathrm{i}}{8}\, \Omega_2\, e^{\frac{1}{2}K\wedge\bar{K}} \tag{9}$$

The Eqs. (2-4) leads to

$$d(e^{3A-\phi}K) = K\wedge d(B+\mathrm{ij}) = \mathrm{d}(\mathrm{e}^{2\mathrm{A}-\phi}\mathrm{Im}\ \Omega_2) = \mathrm{H}\wedge\mathrm{Im}\ \Omega_2 = 0$$

$$dC_0 = 0 \tag{10}$$

$$dC_2 - C_0 dB = d^c(e^{-2A} *_4 \mathrm{Re}\hat{\Omega}_2) \tag{11}$$

$$dC_4 - dB\wedge C_2 = 0\ . \tag{12}$$

Example B

5 Holomorphic functions

$$\Omega_2 = e^{\phi}(dy^1 + \mathrm{i}\, dy^4) \wedge (dy^2 + \mathrm{i}\, dy^3)$$
$$j = \tau_2 dy^1 \wedge dy^4 + \sigma_2 dy^2 \wedge dy^3 - \beta_2^{(1)} \chi_2^- + \beta_2^{(2)} \chi_1^- \,. \tag{13}$$

Which can be written as

$$g_{mn} = \begin{pmatrix} \tau_2 & -\beta_2^{(1)} & -\beta_2^{(2)} & 0 \\ -\beta_2^{(1)} & \sigma_2 & 0 & \beta_2^{(2)} \\ -\beta_2^{(2)} & 0 & \sigma_2 & -\beta_2^{(1)} \\ 0 & \beta_2^{(2)} & -\beta_2^{(1)} & \tau_2 \end{pmatrix} \tag{14}$$

$$ds^2 = \sum_{\mu=0}^{3} dx^\mu dx_\mu + \sum_{m,n=1}^{4} g_{mn} \mathrm{d}y^m \mathrm{d}y^n + e^{2\phi} |h(z)|^2 dz d\bar{z}$$
$$B = \tau_1 dy^1 \wedge dy^4 + \sigma_1 dy^2 \wedge dy^3 - \beta_1^{(1)} \chi_2^- + \beta_1^{(2)} \chi_1^- \,,$$
$$C_0 = C_2 = C_4 = 0 \tag{15}$$

Solution class C

$\alpha = 0,\ \theta = \pi$

$$\Phi_1 = \frac{1}{8}\Omega_3 \quad , \quad \Phi_2 = -\frac{1}{8}e^{-\mathrm{i}J} \ . \tag{16}$$

The Eqs. (2-4) leads to

$$d(e^{3A-\phi}\Omega_3) = d(e^{2A-\phi}) = H = dJ \wedge J = 0$$

$$\begin{aligned} dC_0 &= 0, \\ dC_2 &= -d^c(e^{-2A}j), \\ dC_4 &= 0 \ . \end{aligned} \tag{17}$$

Example C

4 Holomorphic functions

$$\Omega_2 = (dy^1 + \mathrm{i}\, dy^4) \wedge (dy^2 + \mathrm{i}\, dy^3) \tag{18}$$
$$j = e^{2A}\left[\tau_2 dy^1 \wedge dy^4 + \sigma_2 dy^2 \wedge dy^3 - \beta_2^{(1)}\chi_2^- + \beta_2^{(2)}\chi_1^-\right]$$

Which can be written as

$$g_{mn} = e^{2A}\begin{pmatrix} \tau_2 & -\beta_2^{(1)} & -\beta_2^{(2)} & 0 \\ -\beta_2^{(1)} & \sigma_2 & 0 & \beta_2^{(2)} \\ -\beta_2^{(2)} & 0 & \sigma_2 & -\beta_2^{(1)} \\ 0 & \beta_2^{(2)} & -\beta_2^{(1)} & \tau_2 \end{pmatrix} \tag{19}$$

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Conclusions

1. We presented explicit solutions where the ten-dimensional spacetime takes the local form $\mathbb{R}^{1,3} \times M_4 \times \Sigma$, with M_4 a generalized complex manifold with $SU(2)$ structure and Σ an open subset of $\mathbb{C}$.
2. We display explicit examples for $M_4 = T^4$ specified by up to four holomorphic functions. These solutions can be viewed as supersymmetric solutions of $\mathcal{N} = (2, 2)$ maximal supergravity in six dimensions with a set of scalar fields varying over the z-plane.

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