Non-Abelian Discrete Gauge Symmetries in F-theory

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based on:

non-Abelian:	1504.06272	with D. Regalado, T. Pugh
Abelian:	1408.6448	with I. García-Etxebarria, J. Keitel
	1406.5180	with L. Anderson, I. García-Etxebarria, J. Keitel

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Some initial struggle

- in the last years we systematically approached: with Savelli, Pugh, Weissenbacher

Can we derive the N=1 effective action of Type IIB flux compactification with warping?

- challenge:

$$\frac{1}{\alpha^2}\int_{Y_4} G^{\mathrm{Flux}} \wedge G^{\mathrm{Flux}} = \frac{\chi(Y_4)}{24}$$

need to include higher-derivative terms in the action

$\rightarrow$ now known

- important steps:
  - find back-reacted solution with higher-derivative terms [Becker,Becker] [TG,Pugh,Weissenbacher]
  - derive effective action by dimensional reduction Part I: arXiv:1412.5073
  - determine Kähler potential and Kähler coordinates Part II: arXiv:1506.nnnn

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$\rightarrow$ Tom Pugh's talk

Goals of this talk

- I like to discuss non-Abelian discrete gauge symmetries in F-theory compactifications on Calabi-Yau fourfolds

- Stepwise introduce:

  (1) Geometrically massive U(1)s and Abelian gauge symmetries

  (2) Discrete non-Abelian gauge symmetries in O7-orientifolds

  (3) Candidate gaugings for discrete non-Abelian gauge symmetries in F-theory $\Rightarrow$ mutually non-local seven-branes

  (4) Derivation via M-theory compactifiactions on manifolds with cohomological torsion

Geometrically massive U(1)s and Abelian discrete symmetries

Geometrically massive U(1)s

- geometrically massive U(1)s arise from D7 - D7′ system with Stückelberg coupling:

$$S_{\mathrm{St}} = \int_{\mathrm{D7}^-} C_6 \wedge F_{\mathrm{U}(1)} \longrightarrow \text{gauging of dual axion:} \quad C_2 = c^a \omega_a$$

$$\mathcal{D}c^a = dc^a + m^a A_{\mathrm{U}(1)}$$

[Jockers,Louis]

- geometrically massive U(1)s in F-theory: [TG,Weigand] [TG,Kerstan,Palti,Weigand]
  - U(1)s arise form M-theory three-form:

$$C_M = c^a \alpha_a + A_{U(1)} \wedge \omega_{U(1)} + \ldots$$

  - non-closed forms induce non-trivial gauging:

$$d\omega_{U(1)} = m^a \alpha_a \longrightarrow \mathcal{D}c^a = dc^a + m^a A_{\mathrm{U}(1)}$$

  - interpretation as non-Kähler geometry or CY fourfolds with cohomological torsion

see also [A. Braun,Collinucci,Valandro]

Abelian discrete symmetries

- geometrically massive U(1)s can leave Abelian discrete gauge symmetry

- so far, main examples: geometries with multi-section (no section)
  → Palti's talk, Kapfer's talk
  - examples with $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$ symmetry studied in
    [Braun,Morrison] [Morrison,Taylor] [Anderson,García-Etxebarria,TG,Keitel] [Klevers etal.] [García-Etxebarria,TG,Keitel] [Mayrhofer,Palti,Till,Weigand] [Braun,TG,Keitel] [Cvetic,Donagi,Klevers,etal.]
  - physical interpretation: mixing of KK-vector with massive U(1)s
    importance of the KK-modes
    [Anderson,García-Etxebarria,TG,Keitel]
    [García-Etxebarria,TG,Keitel] [Mayrhofer,Palti,Till,Weigand]
  - selection rules on Yukawa couplings in F-theory
    [García-Etxebarria,TG,Keitel] [Mayrhofer,Palti,Till,Weigand] [Klevers etal.]
  - connection with cohomological torsion [Mayrhofer,Palti,Till,Weigand]

- alternative suggestions for non-Abelian case [Karozas,King,Leontaris,Meadowcroft]
  → Leontaris' talk

Non-Abelian discrete symmetries in O7-orientifolds

Heisenberg symmetries in CY orientifolds

- recall the symmetries of the N=1 orientifold moduli space

$$G^a = c^a - \tau b^a \longrightarrow$$ from orientifold-odd $C_2, \ B_2$

$$T_\alpha = \rho_\alpha + \frac{1}{2(\tau-\bar\tau)}\mathcal{K}_{\alpha ab}G^a(G-\bar G)^b - \frac{1}{2}i\mathcal{K}_{\alpha\beta\gamma}v^\beta v^\gamma$$ from orientifold-even sector: Kähler + C_4

$$K = -2\log\mathcal{V}$$

- Kähler metric admits Heisenberg symmetry (continuous, non-compact, non-semi-simple)

$$[t_{(1,a)}, t_{(2,b)}] = -\mathcal{K}_{\alpha ab}t^\alpha$$

$$t_{(1,a)} = \partial_{G^a} \qquad t_{(2,a)} = -\tau\partial_{G^a} - \mathcal{K}_{\alpha ab}G^b\partial_{T_\alpha} \qquad t^\alpha = \partial_{T_\alpha}$$

$\Rightarrow$ Can this non-Abelian group be gauged ? (1) by R-R gauge fields
(2) by seven-brane gauge fields

Non-Abelian gaugings with R-R vectors

- non-Abelian discrete gauge symmetries were suggested to arise in Type IIB orientifolds with torsional cohomology [Camara,Ibanez,Marchesano]
[Berasaluce-Gonzalez,Camara,Marchesano,Regalado,Uranga]

$\Rightarrow$ generalization should be simple ✓

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  $\Rightarrow$ generalization should be simple ✓

- details turn out to be tricky:
  - non-Abelian structure due to $Tor(H^2(Y_3))$ and $Tor(H^4(Y_3))$
  - orientifold involution: $Tor(H^2_-(Y_3))$ $\quad d\gamma_i = k_i^a \omega_a$

$$\begin{aligned} B_2 &= A^{1\,i} \wedge \gamma_i + b^a \omega_a \\ C_2 &= A^{2\,i} \wedge \gamma_i + c^a \omega_a \end{aligned} \longrightarrow \quad DG^a = dG^a + k_i^a (A^{2\,i} - \tau A^{1\,i})$$

real + imaginary part gauged with different vectors $\rightarrow$ supersymmetry?

> $\Rightarrow$ N=1 orientifold setting more involved then expected supersymmetrization for negative torsion is a challenge **?!**

Non-Abelian symmetries in the F-theory effective action

Setting in F-theory

- F-theory avoids this problem in an intriguing way:
  - τ is no longer a four-dimensional field
    gets replaced by a holomorphic function $f_{ab}(z)$:

seven-brane positions
complex str. moduli

$$G^a \longrightarrow N^a = -i(a^a + i f^{ab} b_b)$$

also contains Wilson
line moduli of 7-branes

  - symmetry algebra in F-theory

$$[t_a, \tilde{t}^b] = -M_{\alpha a}{}^b\, t^\alpha$$

$$\tilde{t}^b = f^{ab}\partial_{N^a} - N^a(iM_{\alpha a}{}^b + \frac{1}{2} f^{bc} M_{\alpha ca})\partial_{T_\alpha}$$

$$t_a = -i\partial_{N^a} - \frac{i}{2} N^b M_{\alpha ab}\partial_{T_\alpha} \qquad t^\alpha = \partial_{T_\alpha}$$

intersection no. on fourfold: $M_{\alpha ab} = \int \omega_\alpha \wedge \alpha_a \wedge \alpha_b, \qquad M_{\alpha a}{}^b = \int \omega_\alpha \wedge \alpha_a \wedge \beta^b$

Origins of gaugings in F-theory - Part I

- geometric Stückelberg mechanism for D7-branes:

$$DG^a = dG^a - \tilde{k}^a_i A^i$$

D7-brane gauge field

gauges only the axion arising from R-R two-form C_2

- generalization to mutually non-local (p,q)-seven-branes:
$Sl(2,\mathbb{Z})$ acts on (B_2, C_2) : more general gaugings

seven-brane gauge field

$$DN^a = dN^a + i(\tilde{k}^a_i A^i - i f^{ab} k_{ib} A^i)$$

(p,q)-generalization of Stückelberg coupling:

$$S^{(i)} = \int_{\mathbb{M}_4 \times S_i} F^i \wedge (p\, C_6 + q\, B_6)$$

$\Rightarrow$ precisely generalization that is needed to find supersymmetric non-Abelian structure $\Rightarrow$ leave weak string coupling configurations

Origins of gaugings in F-theory - Part II

- possible non-Abelian completion due to two effects
  - (1) fluxes on seven-branes

$$DT_\alpha = dT_\alpha - \Theta_{\alpha i} A^i \qquad \Theta_{\alpha i} = \int_{S_i} \mathcal{F}^i \wedge \omega_\alpha$$

$\Rightarrow$ purely open-string (seven-brane) setup

  - (2) non-trivial torsion in base of F-theory $Tor(H^4(B_3,\mathbb{Z}))$ corresp. to $Tor(H^4_+(Y_3,\mathbb{Z}))$ in orientifold

$$d\omega_\alpha = k_{\alpha\kappa}\beta^\kappa$$

$$DT_\alpha = dT_\alpha - k_{\alpha\kappa} A^\kappa$$

R-R gauge field from C_4

- F-theory settings with mutually non-local (p,q)-seven-branes appear to allow for non-Abelian discrete gauge symmetries

$\Rightarrow$ Checks ?!

Gauge coupling function and kinetic mixing

- gauging a Heisenberg group has profound implications:
  - Heisenberg group has no positive definite Killing form
  - kinetic term for the vectors has to involve the gauged scalars:

$$\mathcal{L} = -\frac{1}{2} f^1_{AB} F^A \wedge * F^B - \frac{1}{2} f^2_{AB} F^A \wedge F^B - g_{ab} D\phi^a \wedge * D\phi^b$$

$$\delta f^i_{AB} = \lambda^C (f_{CA}{}^D f^i_{BD} + f_{CB}{}^D f^i_{AD}) \longrightarrow$$

gauge coupling function should depend on N^a in a very specific way to ensure gauge invariance

- in setting (2) with gaugings arising partly from seven-branes and partly from R-R forms the Heisenberg group dictates the form of the kinetic mixing between brane and bulk gauge fields

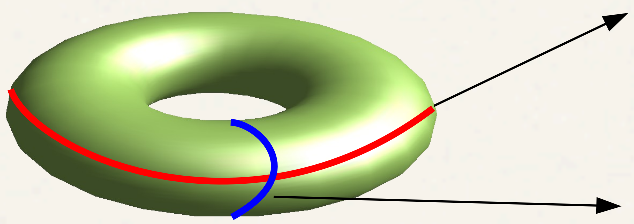
Check: result agrees with expectations at weak coupling for Wilson lines

Computing the F-theory effective action via M-theory

Computing the F-theory effective actions

- No twelve-dimensional low-enegry effective action for F-theory

Analyze and define F-theory via M-theory



(1) A-cycle: if small than M-theory becomes Type IIA

(2) B-cycle: T-duality $\Rightarrow$ Type IIA becomes Type IIB

(3) grow extra dimension: send T^2- volume T-dual $\Rightarrow$ B-cycle becomes large

$\Rightarrow$ M-theory to F-theory limit connects 4d and 3d effective theories

- key insight of recent research:
  - importance of: circle Kaluza-Klein modes $\leftrightarrow$ M2-branes on fiber
  - as of now non-Abelian gauge symmetries in direct lift from 3d to 4d

Reduction of M-theory with torsion

- dimensional reduction of eleven-dimensional supergravity starting with two-derivative action [Cremmer,Julia,Scherk]

- background is taken to be a direct product:

$$d\hat{s}^2 = g_{\mu\nu} dx^\mu dx^\nu + 2(g^0_{m\bar{n}} + i\delta v^\Sigma \omega_{\Sigma m\bar{n}}) dy^m dy^{\bar{n}}$$

$$\hat{C} = A^\Sigma \wedge \omega_\Sigma + \tilde{\xi}^I \alpha_I + \xi_I \beta^I \ .$$

→ talk of T. Pugh (warping + higher-derivatives)

- non-closed forms:

$$d\omega_\Sigma = \tilde{k}^I_\Sigma \alpha_I + k_{\Sigma I} \beta^I$$

can be interpreted as cohomological torsion: $Tor(H^3(Y_4, \mathbb{Z}))$

includes:
(1) M-theory dual of geom. massive U(1)s
(2) non-trivial torsion in the base

$\Rightarrow$ should find the non-Abelian structure suggest by F-theory configuration

Slide of disappointment

- performing dimensional reduction:
  - covariant derivatives found

$$\hat{G} = dA^{\Sigma} \wedge \omega_{\Sigma} + D\tilde{\xi}^{I} \wedge \alpha_{I} + D\xi_{I} \wedge \beta^{I} + \tilde{\xi}^{I} d\alpha_{I} + \xi_{I} d\beta^{I}$$

$$D\tilde{\xi}^{I} = d\tilde{\xi}^{I} - A^{\Sigma}\tilde{k}^{I}_{\Sigma} \qquad D\xi_{I} = d\tilde{\xi}^{I} - A^{\Sigma}k_{\Sigma I}$$

  - checking the gauged symmetry: purely Abelian gauging

Slide of hope

- performing dimensional reduction:
  - covariant derivatives found

$$\hat{G} = dA^\Sigma \wedge \omega_\Sigma + D\tilde{\xi}^I \wedge \alpha_I + D\xi_I \wedge \beta^I + \tilde{\xi}^I d\alpha_I + \xi_I d\beta^I$$

$$D\tilde{\xi}^I = d\tilde{\xi}^I - A^\Sigma \tilde{k}^I_\Sigma \qquad D\xi_I = d\tilde{\xi}^I - A^\Sigma k_{\Sigma I}$$

  - checking the gauged symmetry: purely Abelian gauging

- However: We are not yet in the correct duality frame to perform the F-theory limit !
  - split the 3d fields and dualize:

example: $A^\Sigma \longrightarrow A^i,\ A^\alpha$

has to be dualize into scalar
$\Rightarrow$ axion in 4d complex field

Discovering the Non-Abelian structure

- Dualization of A^α into axion ρ_α, and $\tilde{\xi}_\kappa$ into R-R vector A^κ

$$D\rho_\alpha = d\rho_\alpha - k_{\alpha\kappa}A^\kappa + \frac{1}{2}M_{\alpha a}{}^b(k_{ib}a^a - \tilde{k}_i^a b_b)A^i$$

$$F^\kappa = dA^\kappa + \frac{1}{2}(\tilde{k}_j^a M_{ia}{}^\kappa + k_{ja}M_i{}^{a\kappa})A^i \wedge A^j$$

- dual frame features non-Abelian gauge symmetry as suggested

$$\tilde{k}_j^a M_{ia}{}^\kappa + k_{ja}M_i{}^{a\kappa}$$

control kinetic mixing of
R-R and seven-brane U(1)s
$\rightarrow$ necessary for gauged
Heisenberg group

$$M_{\Sigma I}{}^J = \int_{\hat{Y}_4} \omega_\Sigma \wedge \alpha_I \wedge \beta^J$$

geometrically massive
seven-brane gauge fields
$\rightarrow$ non-closed forms / torsion

Conclusions

- Discrete Abelian gauge symmetries in F-theory
  - much recent progress: analyzing F-theory geometries with multi-sections

- Discrete non-Abelian gauge symmetries at weak string coupling
  - pure R-R gaugings from cohomological torsion in CY orientifolds
    $\Rightarrow$ in apparent conflict with supersymmetry
    $\Rightarrow$ precise reason is mysterious (torsion in orientifold-odd cohomology?)

- Discrete non-Abelian gauge symmetries in F-theory
  - mutually non-local seven-branes required $\Rightarrow$ genuinely F-theoretic
  - gauging of a generalization of a Heisenberg group
    $\Rightarrow$ non-trivial insights about the gauge-coupling function
    $\Rightarrow$ generally true in string theory? insights about kinetic mixing?
  - purely open string settings with fluxes on seven-branes
    $\Rightarrow$ dual description via Higgsing?
    $\Rightarrow$ selection rules on Yukawa couplings
  - discovered Abelian to non-Abelian duality (any dimension) [TG,Regalado,Pugh]