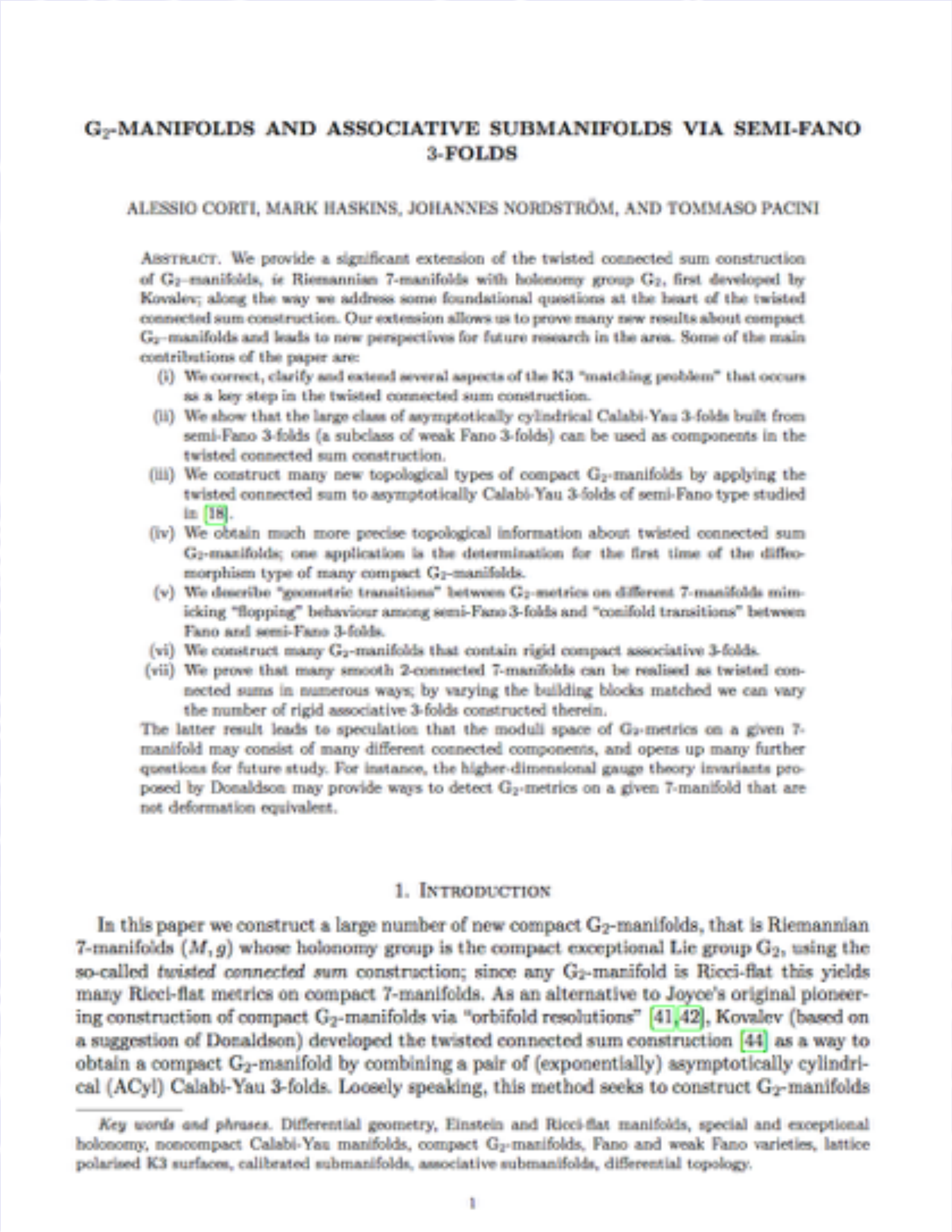
Gauge Enhancement and Landscapes in G_2 Compactifications of M-theory

Jim Halverson
Kavli Institute for Theoretical Physics

based on works with Dave Morrison (1412.4123 and 1507.xxxxx appear.)

A REALLY GREAT G2 MATH PAPER



G_2-MANIFOLDS AND ASSOCIATIVE SUBMANIFOLDS VIA SEMI-FANO 3-FOLDS

ALESSIO CORTI, MARK HASKINS, JOHANNES NORDSTRÖM, AND TOMMASO PACINI

ABSTRACT. We provide a significant extension of the twisted connected sum construction of G_2–manifolds, ie Riemannian 7-manifolds with holonomy group G_2, first developed by Kovalev; along the way we address some foundational questions at the heart of the twisted connected sum construction. Our extension allows us to prove many new results about compact G_2–manifolds and leads to new perspectives for future research in the area. Some of the main contributions of the paper are:

(i) We correct, clarify and extend several aspects of the K3 "matching problem" that occurs as a key step in the twisted connected sum construction.
(ii) We show that the large class of asymptotically cylindrical Calabi-Yau 3-folds built from semi-Fano 3-folds (a subclass of weak Fano 3-folds) can be used as components in the twisted connected sum construction.
(iii) We construct many new topological types of compact G_2-manifolds by applying the twisted connected sum to asymptotically Calabi-Yau 3-folds of semi-Fano type studied in [18].
(iv) We obtain much more precise topological information about twisted connected sum G_2-manifolds; one application is the determination for the first time of the diffeomorphism type of many compact G_2–manifolds.
(v) We describe "geometric transitions" between G_2-metrics on different 7-manifolds mimicking "flopping" behaviour among semi-Fano 3-folds and "conifold transitions" between Fano and semi-Fano 3-folds.
(vi) We construct many G_2-manifolds that contain rigid compact associative 3-folds.
(vii) We prove that many smooth 2-connected 7-manifolds can be realised as twisted connected sums in numerous ways; by varying the building blocks matched we can vary the number of rigid associative 3-folds constructed therein.

The latter result leads to speculation that the moduli space of G_2-metrics on a given 7-manifold may consist of many different connected components, and opens up many further questions for future study. For instance, the higher-dimensional gauge theory invariants proposed by Donaldson may provide ways to detect G_2-metrics on a given 7-manifold that are not deformation equivalent.

1. INTRODUCTION

In this paper we construct a large number of new compact G_2-manifolds, that is Riemannian 7-manifolds (M, g) whose holonomy group is the compact exceptional Lie group G_2, using the so-called *twisted connected sum* construction; since any G_2-manifold is Ricci-flat this yields many Ricci-flat metrics on compact 7-manifolds. As an alternative to Joyce's original pioneering construction of compact G_2-manifolds via "orbifold resolutions" [41, 42], Kovalev (based on a suggestion of Donaldson) developed the twisted connected sum construction [44] as a way to obtain a compact G_2-manifold by combining a pair of (exponentially) asymptotically cylindrical (ACyl) Calabi-Yau 3-folds. Loosely speaking, this method seeks to construct G_2-manifolds

Key words and phrases. Differential geometry, Einstein and Ricci-flat manifolds, special and exceptional holonomy, noncompact Calabi-Yau manifolds, compact G_2-manifolds, Fano and weak Fano varieties, lattice polarised K3 surfaces, calibrated submanifolds, associative submanifolds, differential topology.

1

"CHNP"

1207.4470

KOVALEV'S TWISTED CONNECTED SUMS (TCS)

OVER 50 MILLION EXAMPLES.

CONSTRUCTS SOME ASSOCIATIVE SUBMANIFOLDS.

I cannot recommend this paper enough. Pedagogical and readable!

Many Other Key Works!

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Physics Literature

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- First compact examples: [Joyce]

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and many many others! (please let me know if I acc. didn't include you, I'm still learning this literature!)

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see [Acharya, Gukov] review and references therein!

THIS TALK

neither strings nor pheno, unfortunately!

Still a lot to learn about TCS G_2 before thinking pheno.

Instead, present and discuss physics of TCS G_2
and develop a language / set of techniques for ***global*** models,
specifically as it regards gauge enhancement.

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  what is a G_2 manifold?
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  I'll discuss physics and give an example.

- ***Gauge Enhancement***
  Need singularities. Can't use standard CY trick. What to do?
  Example: Coulomb branches, monopoles, instantons, etc.

G_2 REVIEW

both math and physics

My apologies that this has to be so fast. This is a lightning review.

Two slides on math.
One slide on physics, since you've heard it.

G_2 Manifolds

- G_2 is an exceptional Lie group with rk=2 and dim=14.
- A *G_2 manifold* X is a 7-manifold with hol(G_2). No CAG. No Yau's thm.
- Characterized by a *G_2 structure* that determines a metric g_Φ and G_2 three-form Φ. These give a G_2 action on every tangent space.
- Note well: only *one* type of metric modulus, from Φ.
- The following are equivalent:

$$Hol(g_\Phi) \subseteq G_2$$

Φ TORSION FREE →

$$\nabla\Phi = 0, \text{ and}$$

$$d\Phi = d\star\Phi = 0.$$

- Holonomy is *exactly* G_2 iff above and X has finite fundamental group.

G_2 Calibrations

- G_2 manifolds can have calibrated submanifolds,
  associative 3-mflds, coassociative 4-mflds. **[Harvey, Lawson]**
- They are volume minimizing reps of their hom. classes.
- Can compute their volumes, even though we don't know the metric.

$$Vol(M_3) = \int_{M_3} \Phi \qquad\qquad Vol(M_4) = \int_{M_4} \star\Phi$$

- CHNP: first construction theorem for associatives
  in compact G_2 manifolds.

(rough: rigid one to each rigid holomorphic curve in build. block
non-rigid for each appropriate sLag in building block).

11D SUGRA + M-branes on G_2

- Key point: gives d=4 N=1 theory.
- KK reduction: reduce metric mod **Φ** as well as C_3 and C_6.

  Summary: $b_2(X)$ abelian vector multiplets
  $b_3(X)$ massless uncharged chirals

  [PAPADOPOULOS, TOWNSEND]
- M2 (M5) on two (five) -cycles give charged particles (monopoles).
- Also from wrapped M-branes: instantons, domain walls, strings, etc.
- **Key:** at most abelian gauge symmetry!
  so we need singularities for non-abelian theories (NAGS)
  or massless charged matter (MCM)
  Smoothing is the Higgs mechanism. How do we reliably un-Higgs?
- Local work: NAGS, vector matter, chiral matter from codim 4, 6, 7.

A LANDSCAPE

from twisted connected sum G_2 manifolds.

Classical, on smooth G_2 at large volume.

There's been huge math progress recently,
and there are now over 50 million smooth, compact G_2 manifolds.

What does M-theory on them yield?
What can we learn about this landscape?
What is still to be done?

Twisted Connected Sums

How do we use the TCS construction to get a G_2 manifold? [Kovalev]

- Idea: "appropriately" glue two "appropriate" building blocks M_{+-}
  X inherits G_2 structure from blocks. Show torsion free.

- $M_\pm = V_\pm \times S^1_\pm$ where $V_\pm$ is an ACyl Calabi-Yau threefold.
- i.e. $V_\pm$ asymptotes to "Calabi-Yau cylinder" $\mathbb{C}^* \times S$, S a K3.

$$z = e^{t+i\theta} \text{ on } \mathbb{C}^*$$

- This gives a natural G_2 structure at each asymptotic end:

$$\Phi = d\varphi \wedge dt \wedge d\theta + d\varphi \wedge \omega_S + d\theta \wedge \mathrm{Re}(\Omega_S) + dt \wedge \mathrm{Im}(\Omega_S)$$

- Change CS of S to get K3 Σ such that $\omega_\Sigma = \mathrm{Re}(\Omega_S),\ \mathrm{Re}(\Omega_\Sigma) = \omega_S$, $\mathrm{Im}(\Omega_\Sigma) = -\mathrm{Im}(\Omega_S)$
- Key: then (φ, t, θ, S) to $(\theta, -t, \varphi, \Sigma)$ leaves Φ invariant!

Kovalev's Theorem [KOVALEV] (DUH!)

- This is the key observation for the gluing. It means we can glue G_2 structures on M_{+-} to get G_2 structure on X.
- K3 diffeomorphism r satisfying:

$$r^*(\omega_{S_-}) = \mathrm{Re}(\Omega_{S_+})$$
$$r^*(\mathrm{Re}(\Omega_{S_-})) = \omega_{S_+}$$
$$r^*(\mathrm{Im}(\Omega_{S_-})) = -\,\mathrm{Im}(\Omega_{S_+})$$

- Gluing map:

$$\begin{array}{llcl} F: & M_+ \cong S^1_- \times \mathbb{R}^+ \times S^1_+ \times S_+ & \longrightarrow & S^1_+ \times \mathbb{R}^+ \times S^1_- \times S_- \cong M_-, \\ & (\theta_-, t, \theta_+, x) & \longmapsto & (\theta_+, T+1-t, \theta_-, r(x)) \end{array}$$

- Kovalev's Theorem: (rough: see paper for more)

Given such $M_\pm$ and r, F, can glue to get TCS seven-manifold X. X has a natural G_2 form related to G_2 forms of $M_\pm$ and it has a torsion free def within its cohomology class. With that metric, X is G_2 mfld.

- People (e.g. CHNP) have gotten *really* good at constructing ($M_\pm$, r, F) to the tune of ***50 million examples.*** Their progress: V = Z\S, Z weak Fano.

TCS Topology [CHNP]

- Let's be specific about the topology since it matters for physics.
- Typically get $V_\pm$ from other alg. threefolds $Z_\pm$ as $V_\pm = Z_\pm \setminus S_\pm$
- There are natural restriction maps with kernel $K_\pm$ and image $N_\pm$. $\rho_\pm : \quad H^2(V_\pm, \mathbb{Z}) \longrightarrow H^2(S_\pm, \mathbb{Z})$
- Then for the second and third cohomology we have:

$$H^2(X, \mathbb{Z}) = (N_+ \cap N_-) \oplus K_+ \oplus K_-$$

$$H^3(X, \mathbb{Z}) \supset H^3(Z_+, \mathbb{Z}) \oplus H^3(Z_-, \mathbb{Z}) \oplus K_+ \oplus K-$$

- Note: some two-forms and three-forms "come together."
- Thm: if C is rig hol. curve in V then def $C \times S^1$ is rig associative in X.

Broad Assessment of TCS G_2 Landscape

[J.H., MORRISON]

- If M on TCS G2 X is vacuum of broken NAGT, *nearly all Higgs branches*.

  (semi-Fano building blocks of CHNP large numbers have K=0, together with simplest gluing gives $b_2(X) = 0$).

  MATH ARTIFACT OF GLUING DIFFICULTY.

- Fluxes: never *have* to turn them on in G_2. [BEASLEY, WITTEN]

- Instantons: due to CHNP rigid associated theorem, for the first time we have M2-instantons with no deformation modes. *Must be concerned about Wilson line modulini!*

- *Some* G_2 transitions can be understood in terms of top trans in $V_{\pm}$.

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- Get three U(1)'s since dim(K)=3.
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  (specifically: one gen. of pencil is non-generic quartic $x_0x_1x_2x_3=0$ in P^3)
- Particle charges: intersections in X via intersections in 6-mfld.

	Q_1	Q_2	Q_3
Ψ^k_{12}	-2	-1	-1
Ψ^k_{13}	-1	-2	-1
Ψ^k_{14}	-1	-1	-2
Ψ^k_{23}	1	-1	0
Ψ^k_{24}	1	0	-1
Ψ^k_{34}	0	1	-1

4 (vector-pairs of chiral multiplets) of each type.

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HARVEY, MOORE

- One V has 24 rigid holomorphic curves, giving 24 assoc.

$$W \supset 4(A_1 e^{-\Phi_1} + A_2 e^{-\Phi_2} + A_3 e^{-\Phi_3} + A_4 e^{\Phi_1 - \Phi_2} + A_5 e^{\Phi_1 - \Phi_3} + A_6 e^{\Phi_2 - \Phi_3})$$

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- Deform. Since S^3 is sLag in V, it's associative in X. (use other CHNP thm)
- But we know what we're Higgsing! Field theory predicts def top.

GAUGE ENHANCEMENT

how do you get it, globally!?

Families of metrics in non-compact examples are great,
but we don't have that luxury in compact examples
and want a language / approach appropriate for compact examples.

Related: in the last example we ran into an important problem
but didn't really discuss it. Let's do that now.

The Problem

Why does gauge enhancement work for CY (F-th)? What is G2 obstruction?

- CY singularities are well understood. (defining equations, CAG, etc)
- More specifically: know how certain singularities relate to families of two-cycles going to zero volume via variation in Kahler moduli.

$$vol(C) = \int_C J \mapsto 0$$

- Dimensionality of family M determines spacetime quantum numbers.

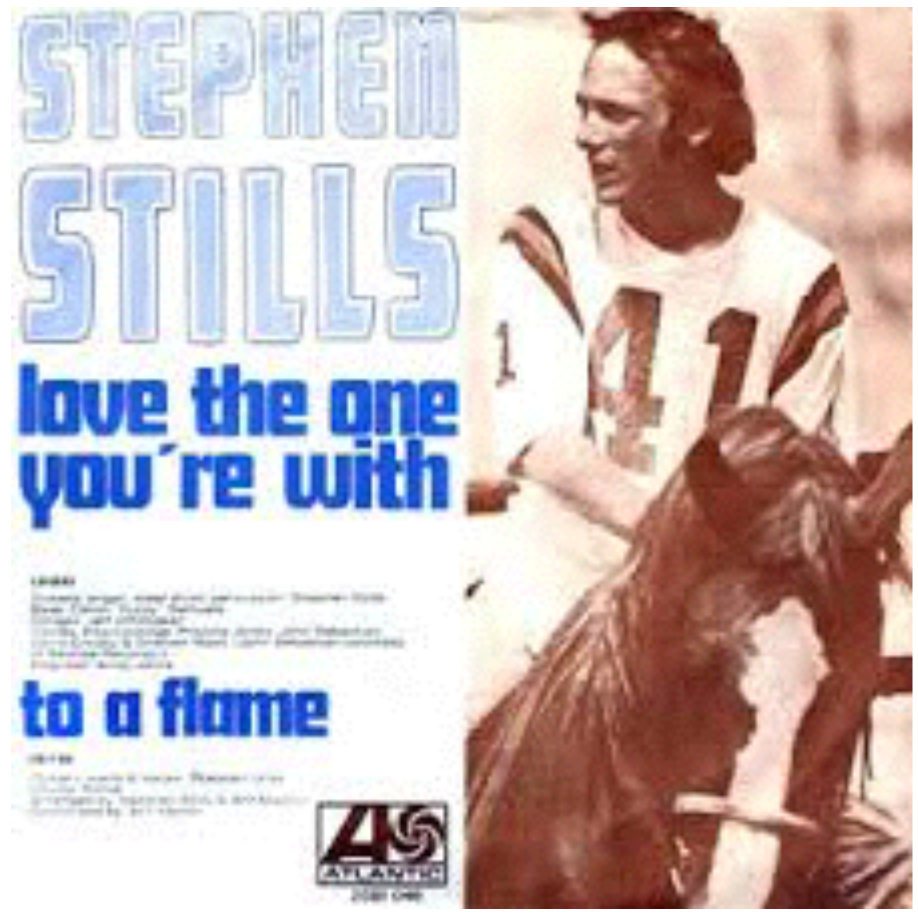
[Witten] [Aspinwall, Katz, Morrison]

- So varying J and studying dim(M), we can identify limits that give massless charged matter and / or massless charged W-bosons.
- Problem: G_2 has no calibrated two-cycles! What to do!?

A Proposal

[J.H., MORRISON]

WHEN ROCK AND ROLL FITS YOUR TALK, YOU'VE GOT TO USE IT



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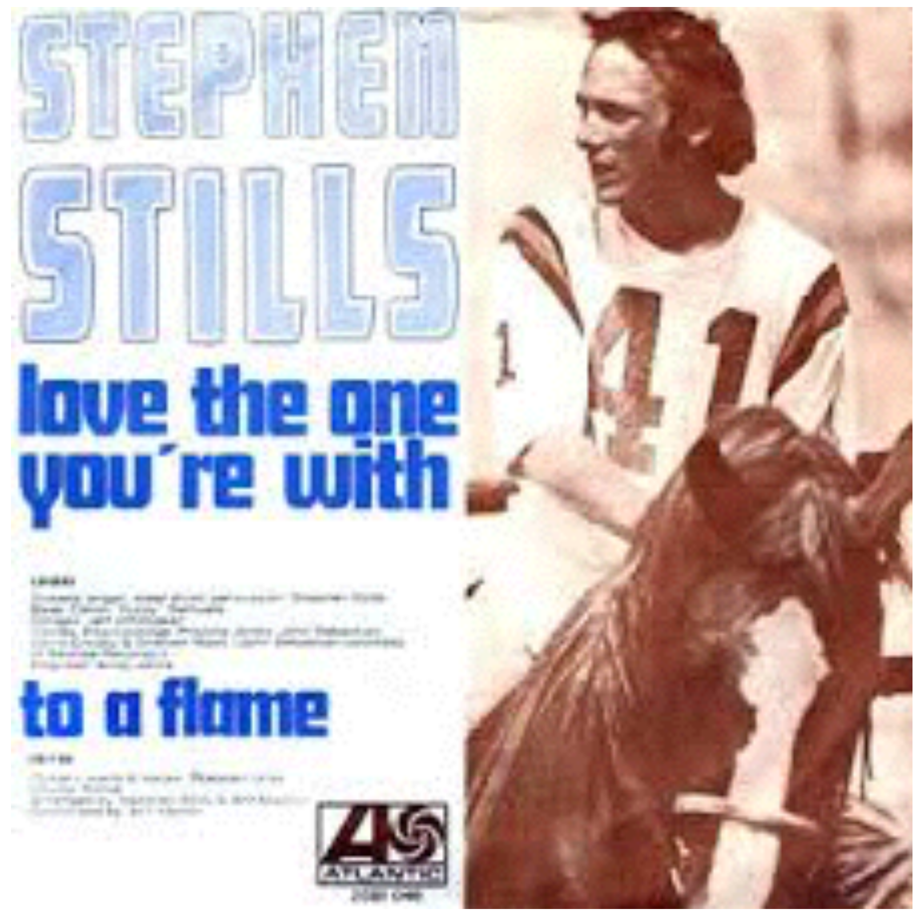

And if you can't be with the one you love, honey,

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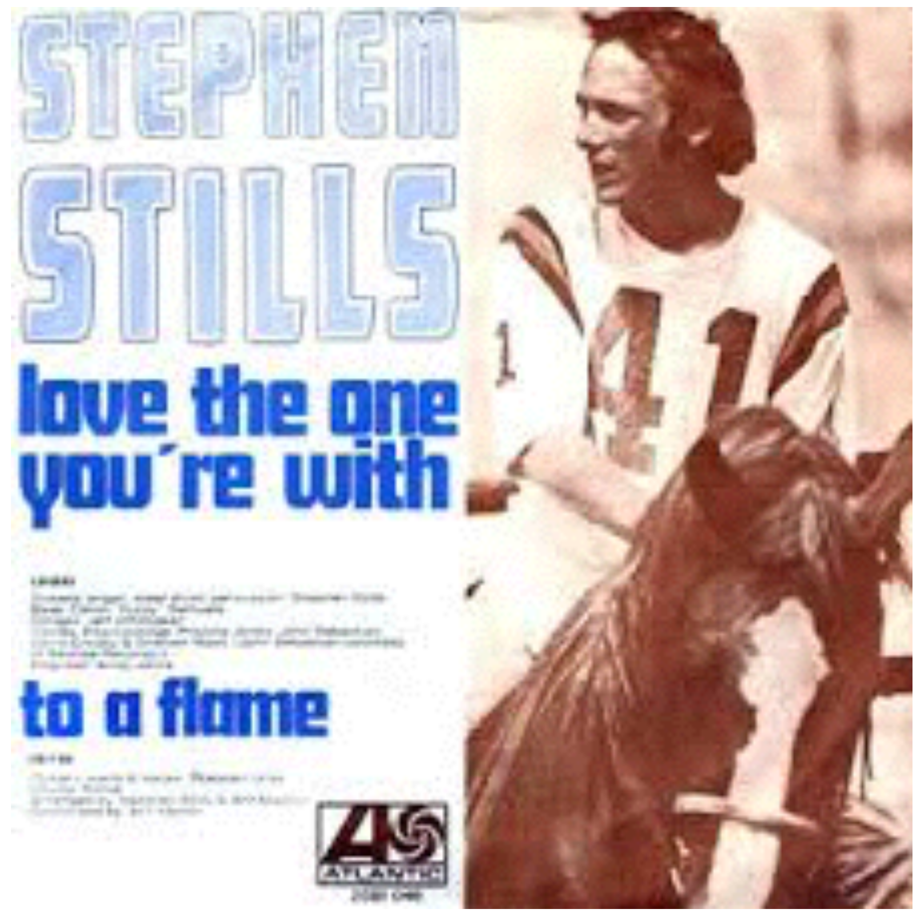

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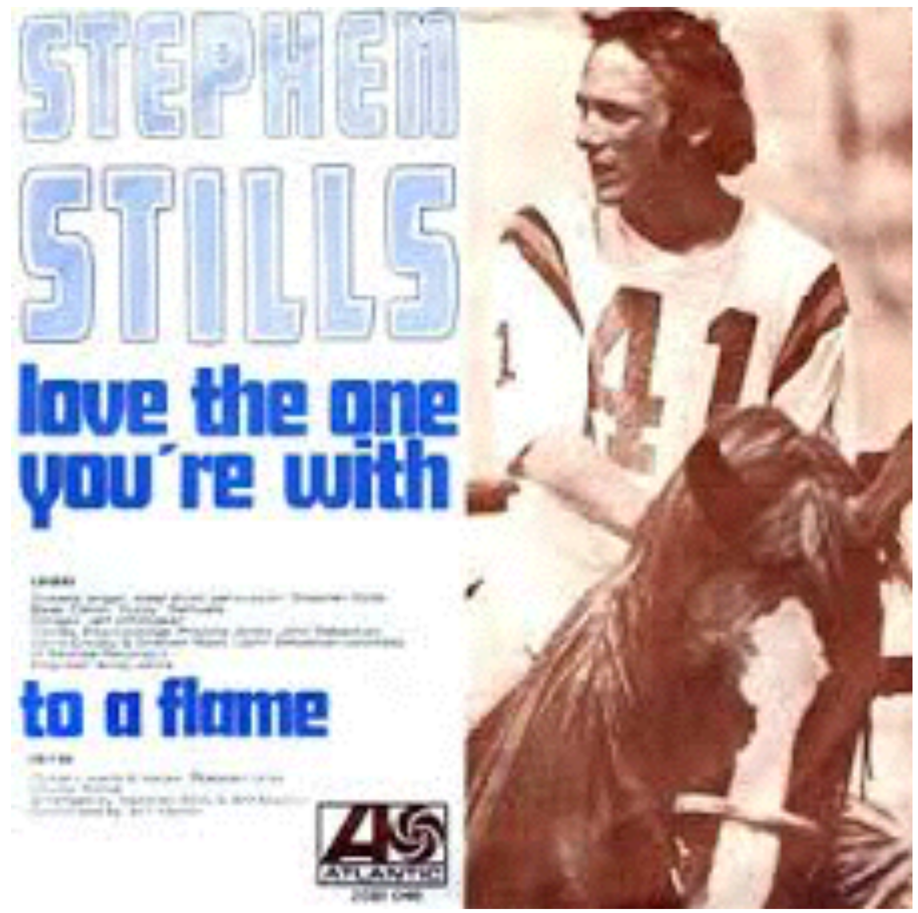

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HOLOMORPHIC CURVES

ASSOC AND COASSOC SUBMANIFOLDS

A Proposal

[J.H., Morrison]

Use the calibrations you have! 3- and 4-mflds.

- Recall: associative 3-mflds and coassociative 4-mflds are calibrated, so we can control their volumes as a function of moduli.
- Idea: define cones of assoc. and coassoc. analogous to Kahler cone. Get singularities by collapsing associatives or coassociatives.
- Math trick) assoc or coassoc collapse via collapsing two-cycle in them.
  particle masses to zero.
- Physics option) Use other signatures of symmetry breaking. (e.g. defects)

$$M_W = g\,v \qquad M_m = v\,/\,g \qquad T_{ANO} = 2\pi\, v^2$$

- Some defects arise from calibrated cycles. (e.g. strings, inst, dom walls)
  Some not from not-calibrated cycles (e.g. monopoles, but they're still useful).

WHAT DO WE WANT?

Ideally, a natural 3 or 4-cycle associated with symmetry

Physics of a Lemma of Joyce

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- Consider any non-trivial class $[\sigma] \in H^2(X,\mathbb{R})$ then:

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- For ***any*** U(1) in M on X have a non-triv 3-cycle $[D_\Sigma]$ = - PD[σ cup σ] !

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- For ***any*** U(1) in M on X have a non-triv 3-cycle $[D_\Sigma]$ = - PD[σ cup σ] !
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- Upshot: finite g, and if D_Σ exists, a place to wrap gauge instantons!

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- Recall: even when broken to H, G-instantons play a role.
  e.g. the so-called "'t Hooft term" $\Delta S = 2\pi^2 |v|^2 \rho^2$
- Furthermore: though for v non-zero ρ not a modulus the
  zero size instanton still solves EOM. (e.g. use "constrained instantons" of Affleck)

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- Instanton behavior follows from $vol(D_\Sigma) \sim \frac{1}{g^2}$

Example: Coulomb Branches

Landing naturally on a fibration-like structure

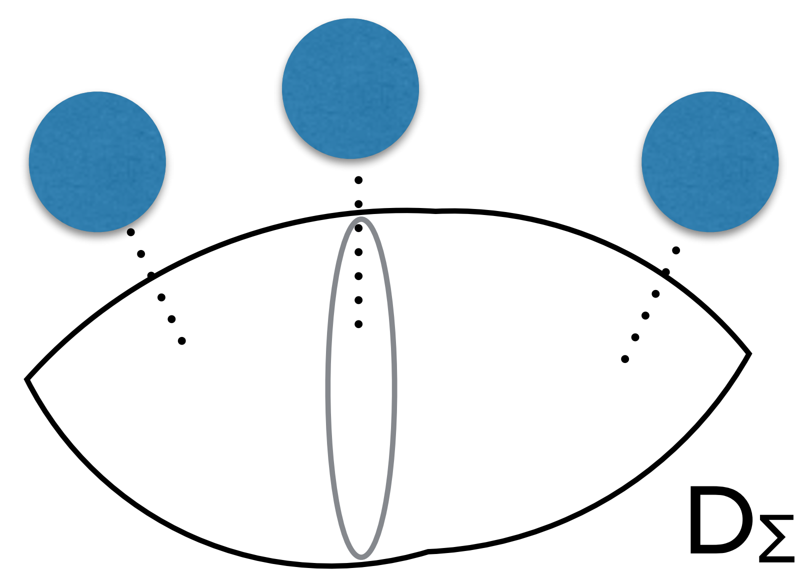
- Geometrically, we have three volumes, of $\Sigma \quad \tilde{\Sigma} \quad D_\Sigma$
  and two physical parameters, g and v (f moduli)
  Overconstrained system?
- More specifically: $M_W \propto g_{YM}|v| \propto vol(\Sigma)$ and $M_M \propto \frac{|v|}{g_{YM}} \propto vol(\tilde{\Sigma})$
- The gauge coupling is computed by the volume of D_Σ so

$$M_M \propto \frac{|v|}{g_{YM}} \propto \frac{M_W}{g_{YM}^2} \propto vol(\tilde{\Sigma}) \propto vol(\Sigma)vol(D_\Sigma)$$

- Volume relation suggests $\tilde{\Sigma}$ fibered over D_Σ by curves of class Σ

Example: Coulomb Branches

LANDING NATURALLY ON A FIBRATION-LIKE STRUCTURE



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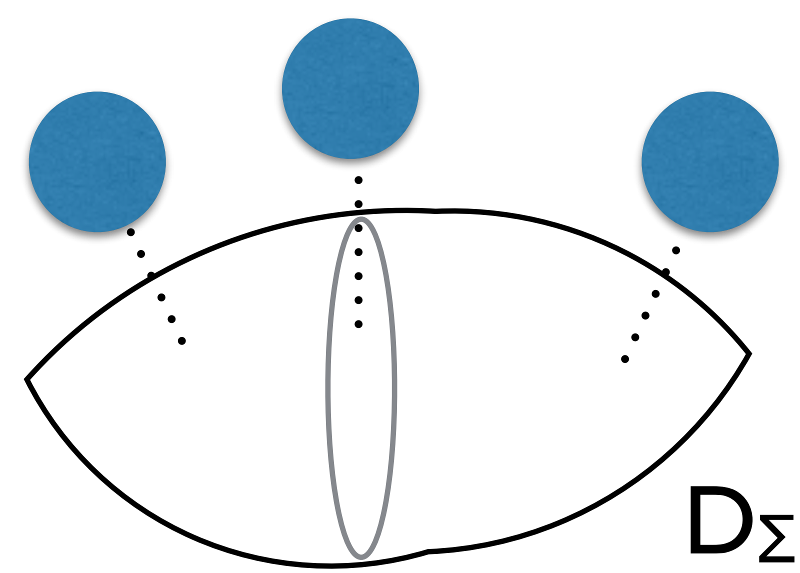

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No adjoint chiral in the story yet. D_Σ can have topology.
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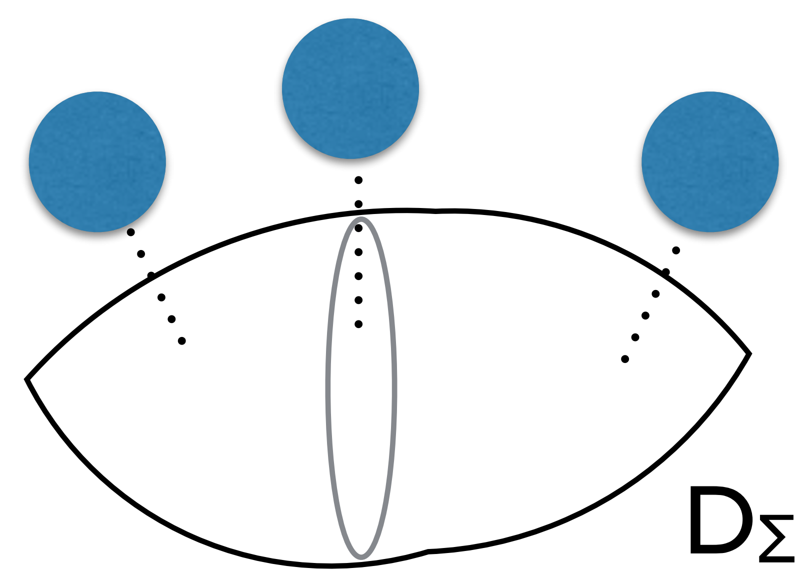

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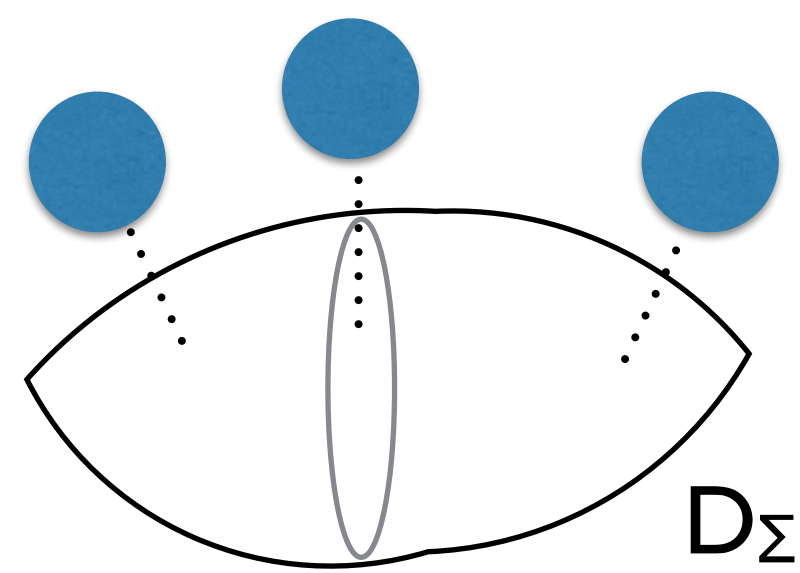

- Joyce example: SU(2) to U(1) gives S^2 fib over T^3. 3 adj chiral mult.

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  Math trick (two-spheres collapse)
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- Coulomb branch: rather general physics arguments and a simple G_2 fact lead to standard fibration picture and three-cycles to collapse.

Thanks **so much** to the organizers for a truly great conference!