Charting Class $\boldsymbol{S}_\mathrm{k}$ Territory

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Based on:
[arXiv:1504.05988] with Sebastián Franco and Angel Uranga

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1. Introduction

- Class ***S*** theories :

Compactifying M5-branes (or the 6d (2,0) theory) on a Riemann surface with punctures yields a 4d $\mathcal{N}$ = 2 or $\mathcal{N}$ = 1 superconformal field theory.

Gaiotto 09

- However, class ***S*** theories do not include many interesting 4d $\mathcal{N}$ = 1 theories, like 4d $\mathcal{N}$ = 1 chiral theories.

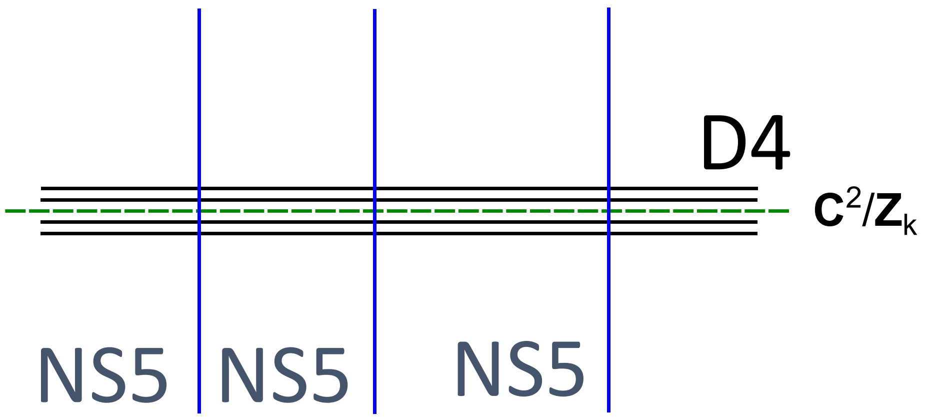
- Recently, the construction was extended to so-called class S_k theories, which in fact include certain 4d $\mathcal{N}$ = 1 chiral theories.

Gaiotto, Razamat 15

- Class $\boldsymbol{S}_k$ theories :

Compacfiying M5-branes at an orbifold singularity $\mathbf{C}^2/\mathbf{Z}_k$ (or the corresponding 6d (1,0) theory) on a Riemann surface with punctures.

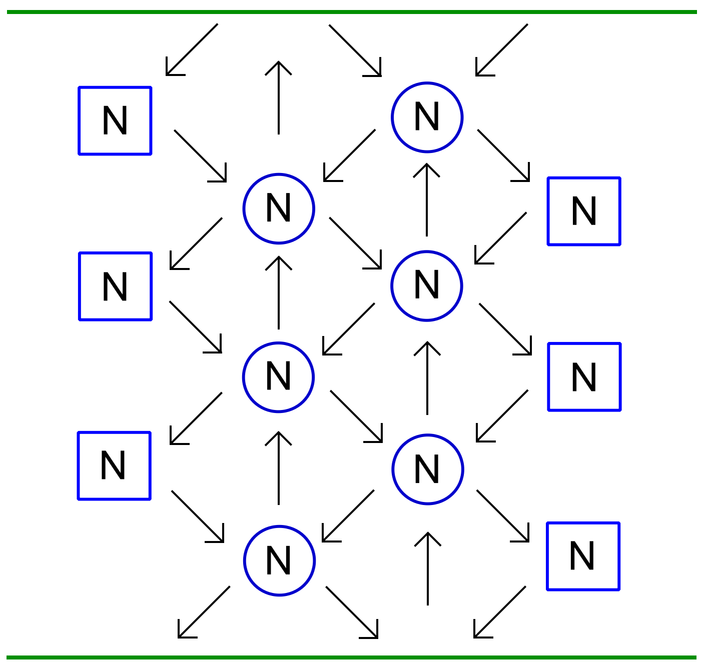
Ex: type IIA picture for a sphere with two maximal punctures and three minimal punctures:



Lykken, Poppitz, Trivedi 97
Uranga 98

- It gives a 4d $\boldsymbol{N} = 1$ quiver theory on a cylinder:

Ex. k=3



N: the number of D4-branes
(or M5-branes)

circle → SU(N) gauge group
box → SU(N) global symmetry
arrow → bi-fund. chiral multiplet

- It has the intrinsic flavor symmetry $U(1)_\beta^{k-1}$ x $U(1)_\gamma^{k-1}$ x $U(1)_t$ which originates from 6d $SU(k)^2$ flavor symmetry.

- It was shown that this class of theories also possess the basic features necessary for the Riemann surface picture.

Gaiotto, Razamat 15

Checks :

1. The number of the complex structure moduli = the number of the exactly marginal deformations.

2. Exchanging punctures corresponds to dualities.

3. etc…

- Their primary focus was special class $\boldsymbol{S}_k$ theories whose type IIA description is given by D4-branes at the singularity with parallel NS5-branes.

- In this talk, we would like to extend the analysis to more general quiver theories by introducing rotated NS5-branes which we denote by NS5'-branes.

Franco, H.H., Uranga 15
Hanany, Maruyoshi 15

- We find that the theories also admit the Riemann surface picture. In particular, we will focus on the check of dualities in this talk.

2. Including NS5'-branes

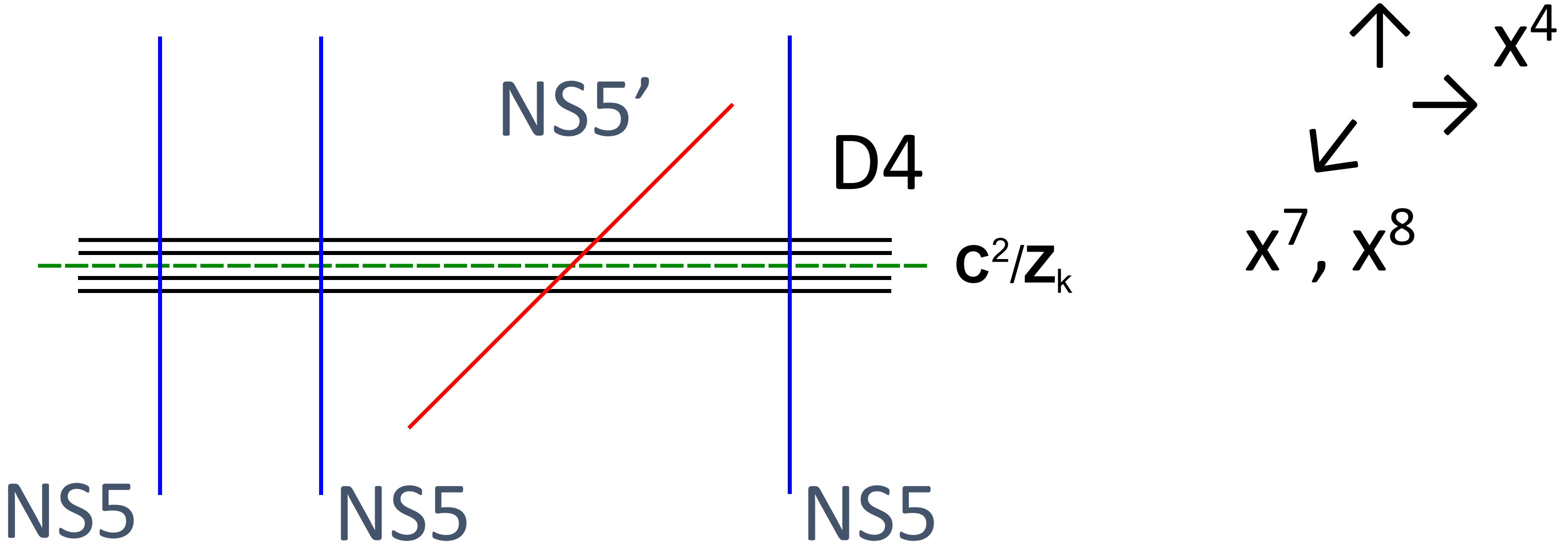
and the Riemann surface picture

- Type IIA brane setup

C^2/Z_k

	0	1	2	3	4	5	6	7	8	9
D4-brane	×	×	×	×	×					
NS5-brane	×	×	×	×		×	×			
NS5'-brane	×	×	×	×				×	×	

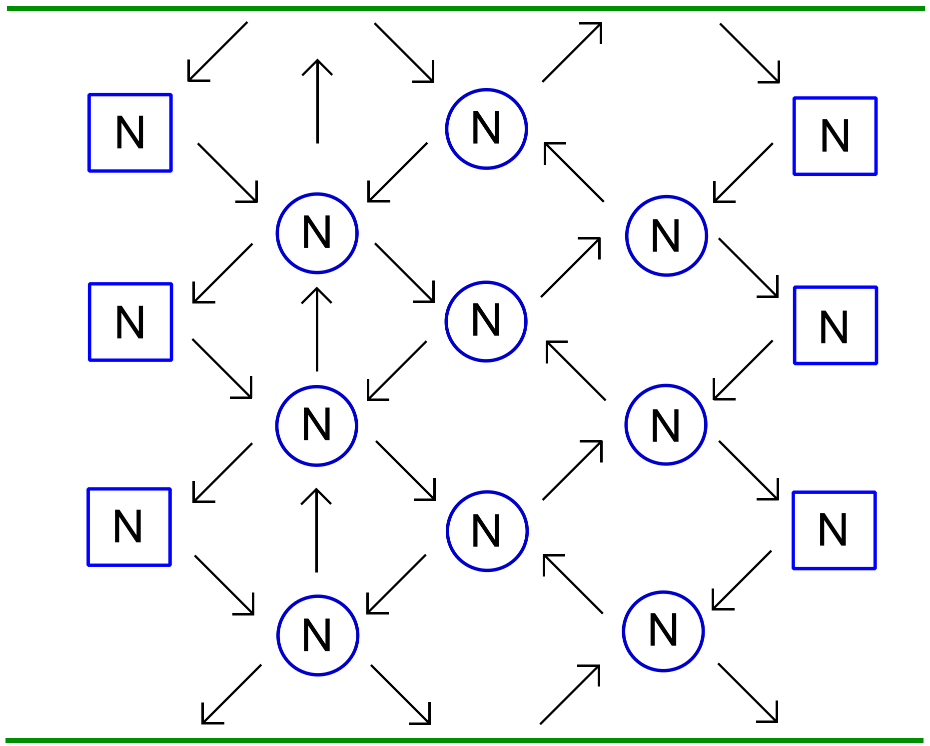
Example:



- The brane setup gives a 4d quiver theory on a cylinder and it is in a general class of bipartite field theory (BFT).

Ex. k=3

Franco 12
Xie, Yamazaki 12

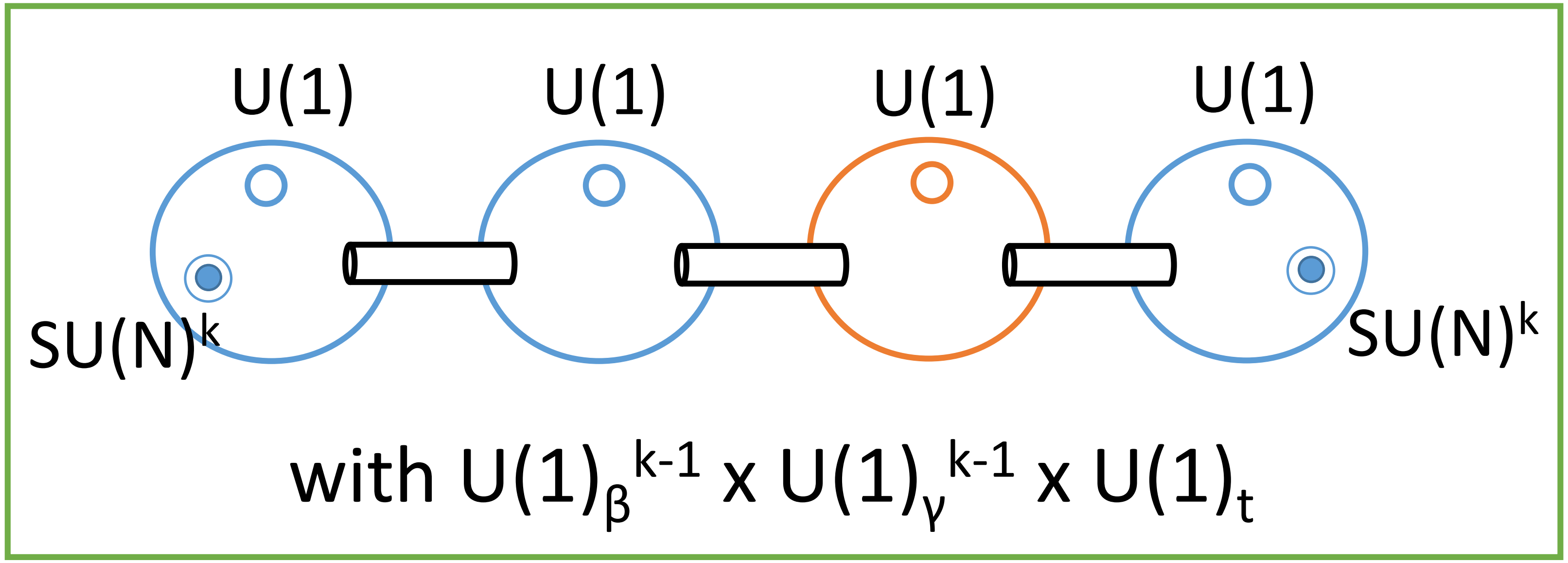


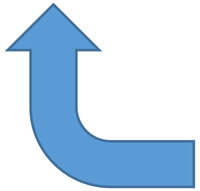
Differences:

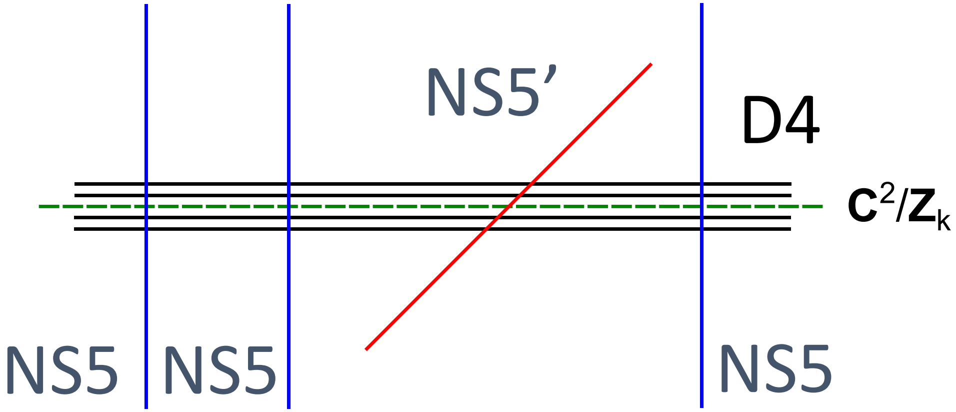
1. Quartic superpotentials.

2. Non-trivial anomalous dimensions at the IR fixed point.

- The expected Riemann surface picture







3. Dualities and 4d superconformal index

- Check: Dualities as exchanging punctures

One useful quantity to check dualities is the 4d ***N*** = 1 superconformal index :

Romelsberger 05
Kinney, Maldacena, Minwalla, Raju 05

$$\mathcal{I}(p,q;\mathbf{u}) = \mathrm{Tr}_{\mathcal{H}_{S^3}}(-1)^F e^{-\beta\{Q,Q^\dagger\}} p^{j_1+j_2-\frac{R}{2}} q^{j_1-j_2-\frac{R}{2}} \prod_{a\in G_F} u_a^{f_a}$$

F : Fermion numbers,

j_1, j_2 : the charges under the SU(2) x SU(2) isometry of S^3

R : $U(1)_R$ charge

f_a : the charges of other flavor symmetries G_F

- It counts gauge invariant BPS operators.

- The SCI only depends on the fugacities of flavor symmetries. In particular, it does not depend on any continuous parameters nor the RG scale. Therefore, we can compute the SCI at UV where the theory is free.

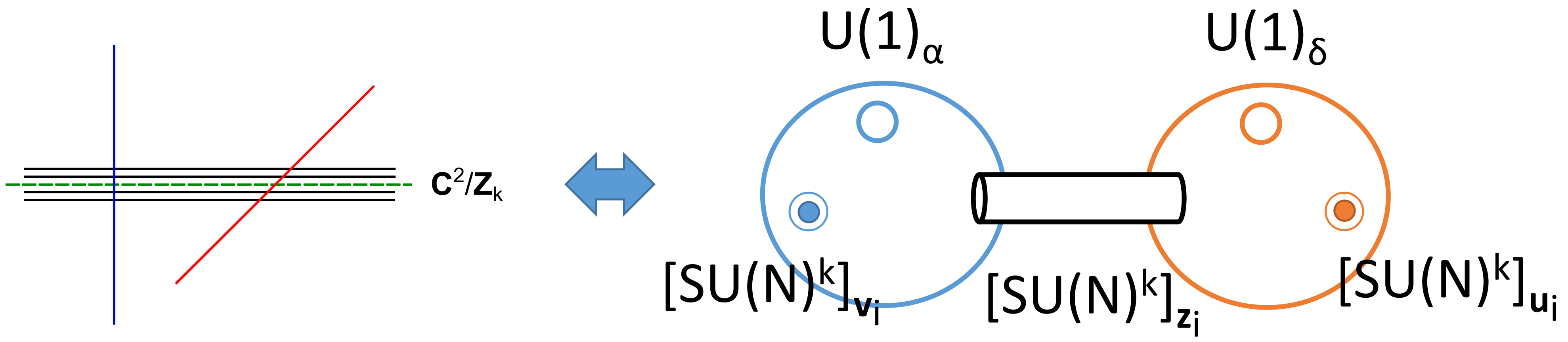
Romelsberger 05, 07, Festuccia, Seiberg 11

- Gauging a flavor symmetry corresponds to introducing a contribution of vector multiplets, and the projection onto only gauge invariant operators.

- The SCI should be the same under dualities. It has been used to check various dualities.

Ex. Romelsberger 07, Dolan, Osborn 08
Gadde, Pomoni, Rastelli, Razamat 09

- Example : the basic core theory



- The index is given schematically by

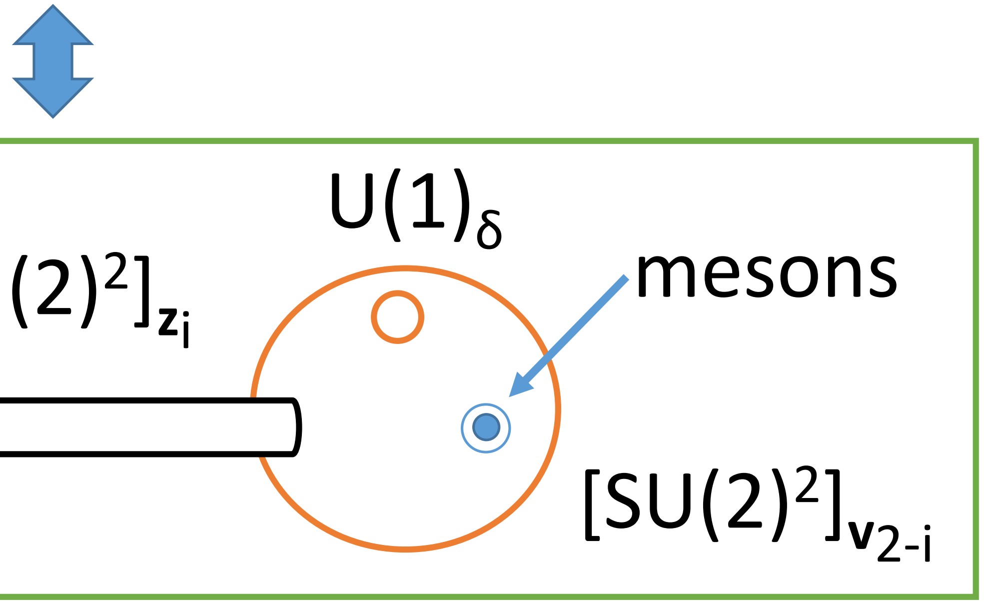
$$\mathcal{I}^{core}_{\mathbf{v}_i\alpha\delta\mathbf{u}_i}(\beta_i,\gamma_i,t)=\int[d\mathbf{z}]_{SU(N)^k}\,\mathcal{I}^{ft}\left(\mathbf{v}_i^{-1},\alpha,\mathbf{z}_{k-i+1}^{-1};\gamma_i,\beta_{i+1},t^{-1}\right)\,\mathcal{I}^{ft}\left(\mathbf{z}_i,\delta,\mathbf{u}_i;\beta_i,\gamma_i,t\right)$$

- We will mainly focus on N=2 and k=2 for simplicity.

1. Exchanging maximal punctures

By applying a mathematical theorem, (in [Rains 10])

$$\mathcal{I}^{core}_{\mathbf{v}_i \alpha \delta \mathbf{u}_i}(\beta, \gamma, t) = \mathcal{I}^{core}_{\mathbf{u}^{-1}_{2-i} \alpha^{-1} \delta^{-1} \mathbf{v}^{-1}_{2-i}}(\gamma^{-1}, \beta^{-1}, t)\, \mathcal{I}_{\text{mesons}}$$



This is essentially the Seiberg duality!

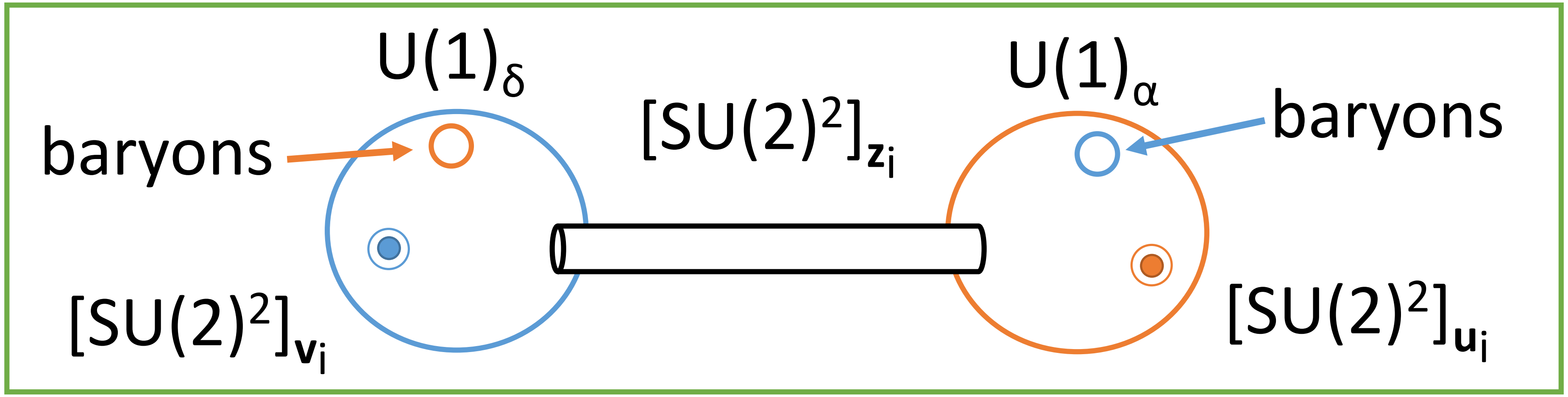
Franco, H.H., Uranga 15

2. Exchanging minimal punctures

By applying a mathematical theorem, (in [Spiridonov 03])

$$\mathcal{I}^{core}_{\mathbf{v}_i \alpha \delta \mathbf{u}_i}(\beta, \gamma, t) = \mathcal{I}^{core}_{\mathbf{v}_i \delta \alpha \mathbf{u}_i}(\beta, \gamma, t)\, \mathcal{I}_{\text{baryons}}$$

$$\Updownarrow$$



This also corresponds to another duality!

Franco, H.H., Uranga 15

4. Conclusion

- We investigate class $\boldsymbol{S}_{\mathrm{k}}$ theories by including NS5'-branes.
- The theories have novel features like quartic superpotentials and non-trivial anomalous dimensions, and still possess the Riemann surface picture.
- We have checked that exchanging punctures corresponds to dualities of the 4d theory. In particular, the duality relations suggest that we need to enlarge the kind of punctures and allow the decoration of punctures by mesons or baryons.