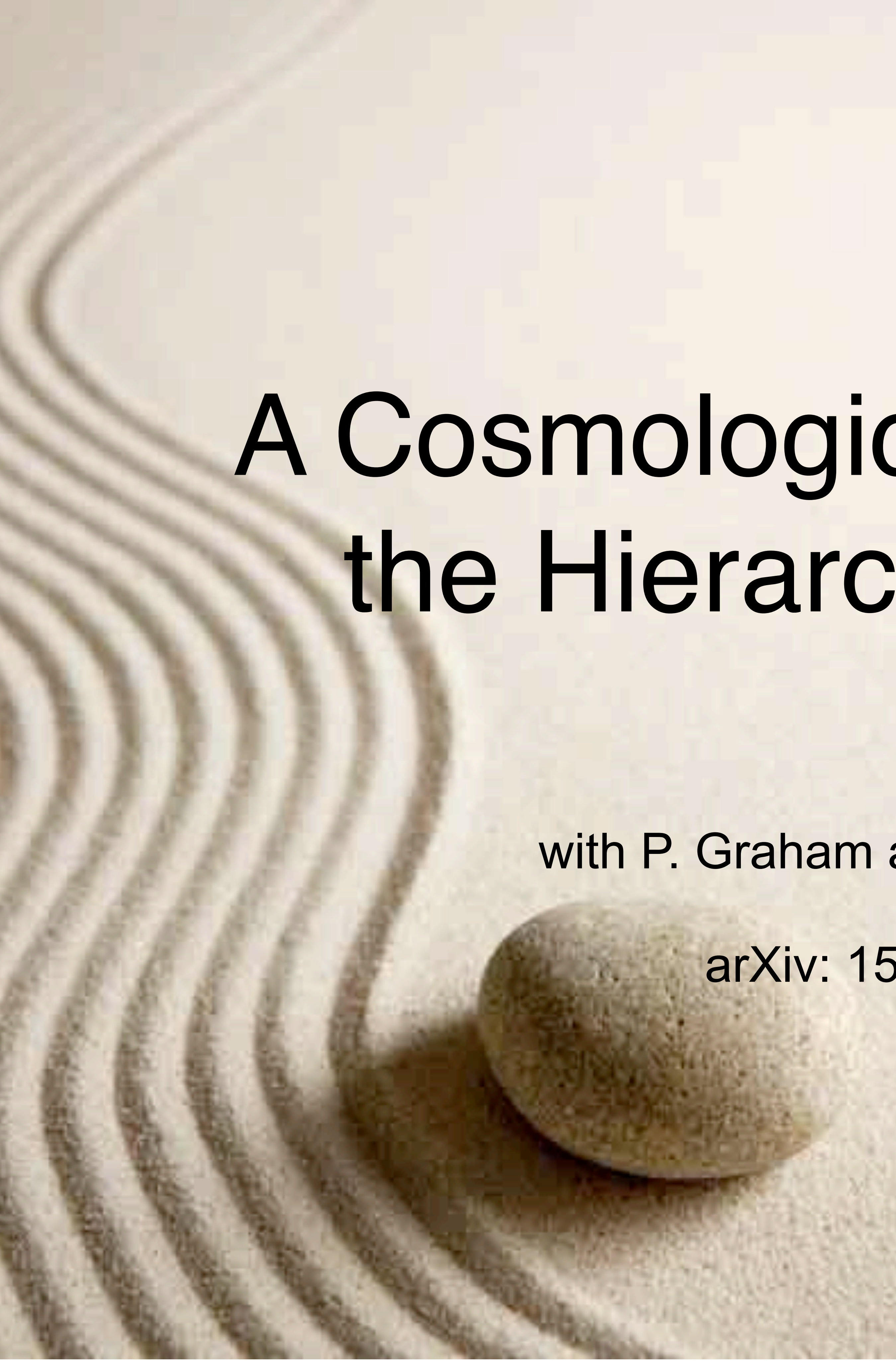


A Cosmological Solution to the Hierarchy Problem

with P. Graham and S. Rajendran

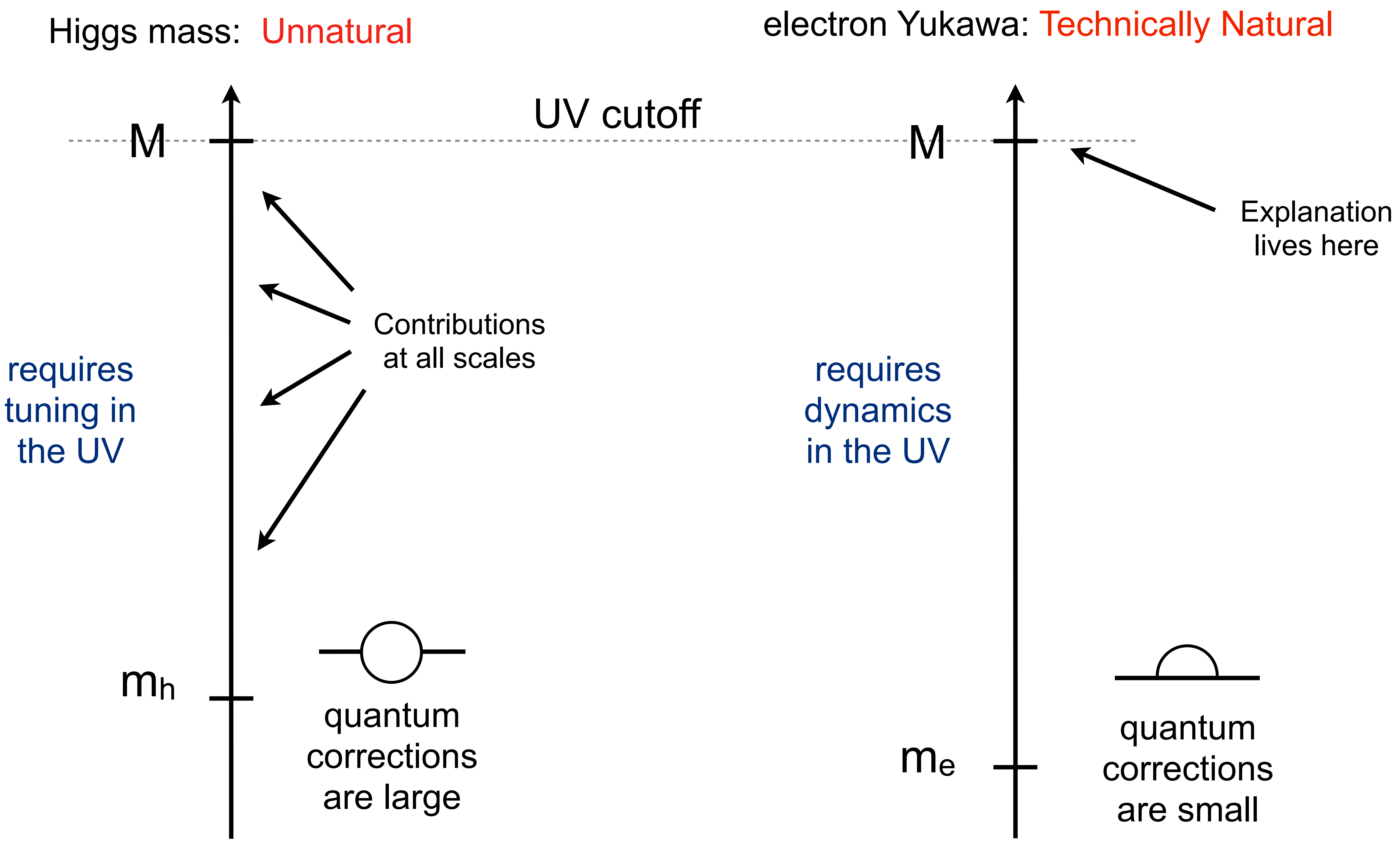
arXiv: 1504.07551

the Relaxion

The Hierarchy Problem

The Higgs mass in the standard model is sensitive to the ultraviolet.

Unnatural vs. Technically Natural in the SM



The Hierarchy Problem

The Higgs mass in the standard model is sensitive to the ultraviolet.

Two approaches to explain:

- New symmetry or new dynamics realized at the electroweak scale. (SUSY, composite Higgs, EOFT)
- An anthropic explanation for fine tuning of ultraviolet parameters. (Multiverse)

We Propose:
A Dynamical Solution

- Higgs mass-squared promoted to a field.
- The field evolves in time in the early universe.
- The mass-squared relaxes to a small negative value.
- The electroweak symmetry breaking stops the time-dependence.
- The small electroweak scale is fixed until today.

Caveats

The solution:

- is only technically natural.
- requires large field excursions (larger than the scale that cuts off loops).
- requires a very long period of inflation.
- can only push the cutoff up to 10^8 GeV.

Simplest Model

Standard Model plus QCD axion

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 \qquad \cdots + \frac{\phi}{32\pi^2 f} G^{\mu\nu}\tilde{G}_{\mu\nu}$$

M is the cutoff.

The axion here is non-compact.

Simplest Model

Standard Model plus QCD axion

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Simplest Model

Standard Model plus QCD axion

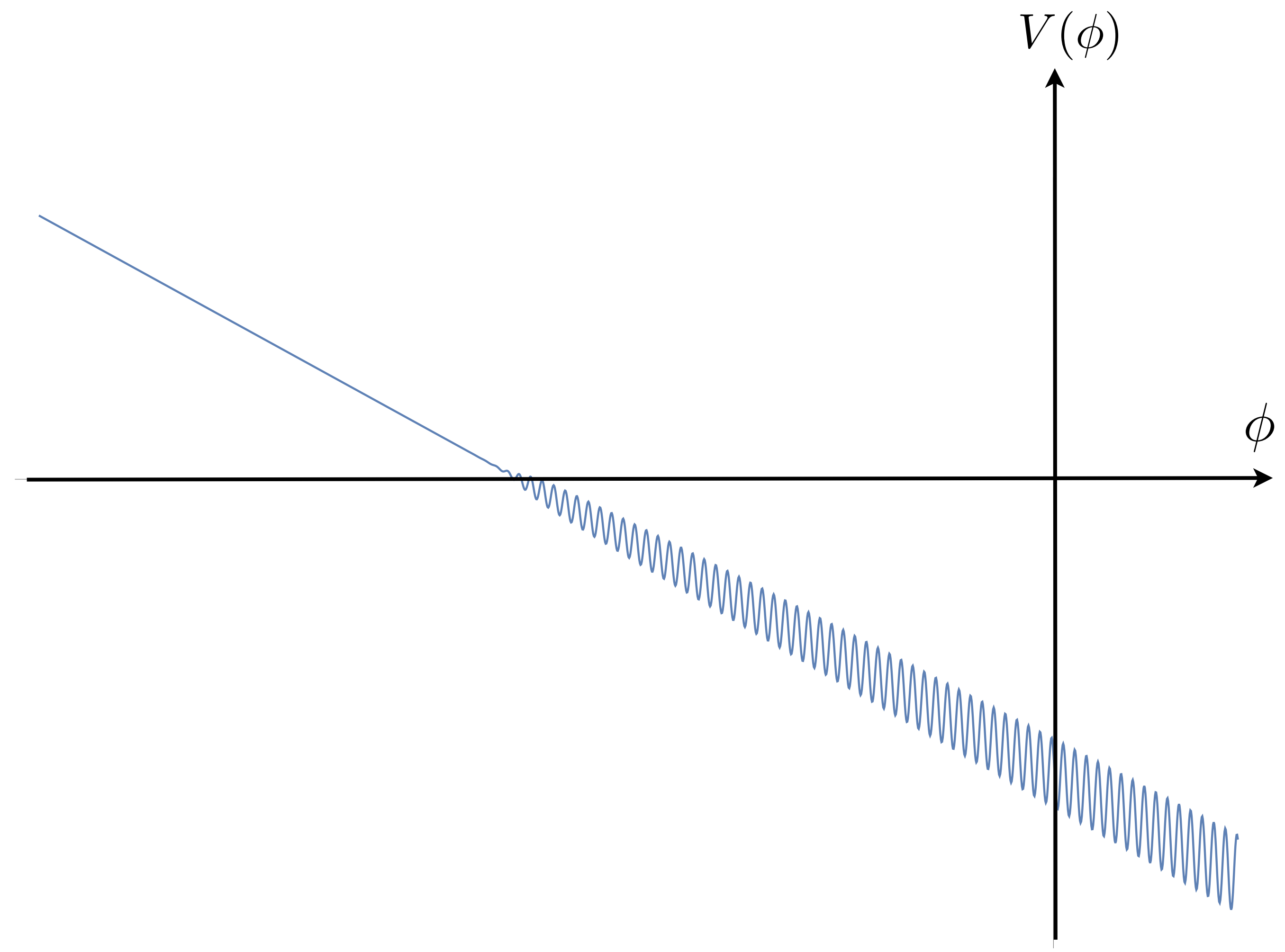
$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + gM^2\phi + g^2\phi^2 + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

Conservative effective field theory regime: $\phi \lesssim \dfrac{M^2}{g}$

(Assuming expansion of $V(g\phi)$ in powers of $\left(\frac{g\phi}{M^2}\right)$)

Chronology

- Take initial ϕ value such that $m_h^2 > 0$.
- During inflation, ϕ slow-rolls, scanning physical Higgs mass.
- ϕ hits value where ~ m_h^2 crosses zero.
- Barriers grow until rolling has stopped.



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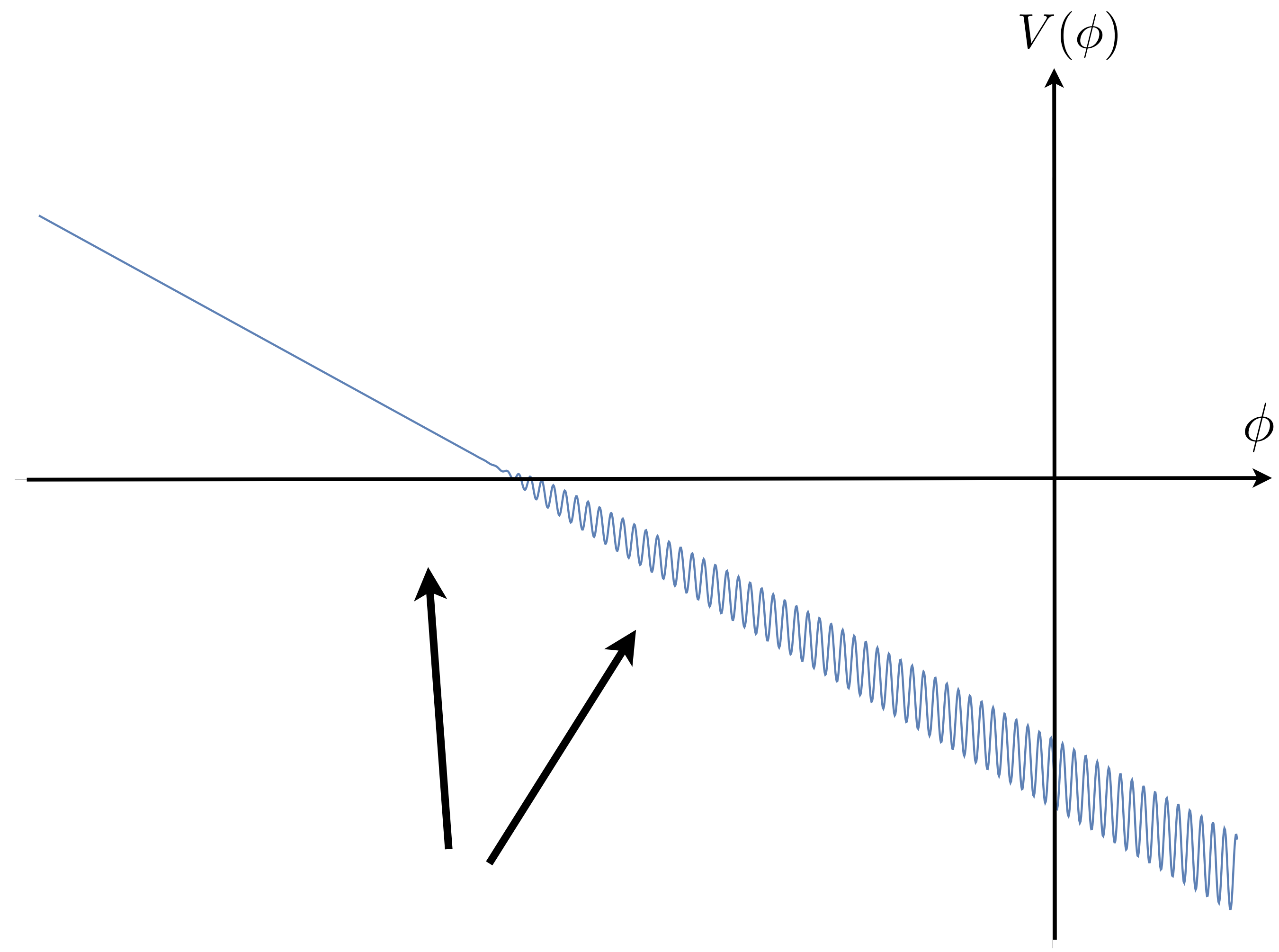

Key: Barriers grow because they depend on the Higgs vev.

Higgs vev and the Periodic Potential

Barrier height (axion potential) can be approximated in the chiral Lagrangian (2 flavors):

$$V_{\text{axion}}\left(\frac{\phi}{f}\right) \sim \Lambda^4 \cos\frac{\phi}{f}$$

Around the normal EW scale: $\Lambda^4 \sim f_\pi^2 m_\pi^2 \left(\frac{\min(m_u, m_d)}{m_u + m_d}\right)$

$$m_\pi^2 \propto (y_u + y_d)\langle h \rangle$$

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Barrier height grows with the Higgs vev.

Parameter Requirements

φ stops rolling and Higgs vev stops growing when slope turns around:

$$\partial_\phi (gM^2\phi + \Lambda^4 \cos(\phi/f)) \sim 0$$

or

$$\boxed{gM^2 f \sim \Lambda^4}$$

$$\Lambda^4 \sim 100 \text{ MeV}$$

fixed parameters

changes with Higgs vev

$$gM^2 f \sim f_\pi^2 \mu (y_u + y_d)\langle h \rangle$$

Parameter Requirements

1) Vacuum energy density during inflation $> M^4$

$$\boxed{H_{\rm infl} > \frac{M^2}{M_{\rm pl}}}$$

2) Barriers can form in Hubble volume:

$$\boxed{H_{\rm infl} < \Lambda}$$

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Plugging in for g, and using 1) and 2):

$$\boxed{M^2 < \Lambda M_{\text{pl}}}$$

Bound on cutoff…

$$M < 3 \times 10^8 \ \mathrm{GeV}$$

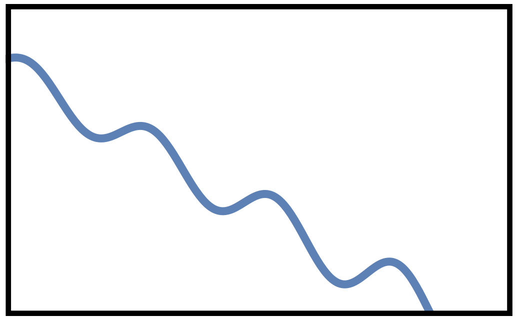
Usual strong CP solutions don't work.

Bound on cutoff...

$$M < 3 \times 10^8 \text{ GeV}$$

However,...

$$\boxed{gM^2 f \sim \Lambda^4}$$



$$\theta_{\rm QCD} \simeq \pi/2$$

Prediction: $d_n \simeq few \times 10^{-16} e\,\text{cm}$

Usual strong CP solutions don't work.

Solve Strong CP

Dynamical one -- Drop the slope:

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + \kappa\sigma^2\phi + gM^2\phi + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

inflaton - drops at end of inflation

$$gM^2 \simeq \theta \times \kappa\sigma^2 \longrightarrow$$

$$gM^2 f \sim \theta\,\Lambda^4$$

$$H_{\rm infl} > \theta^{-\frac{1}{2}}\,\frac{M^2}{M_{\rm pl}}$$

$$H_{\rm infl} < \Lambda$$

Bound on cutoff!

$$M^2 < \theta^{\frac{1}{2}} \Lambda M_{\mathrm{pl}}$$

or

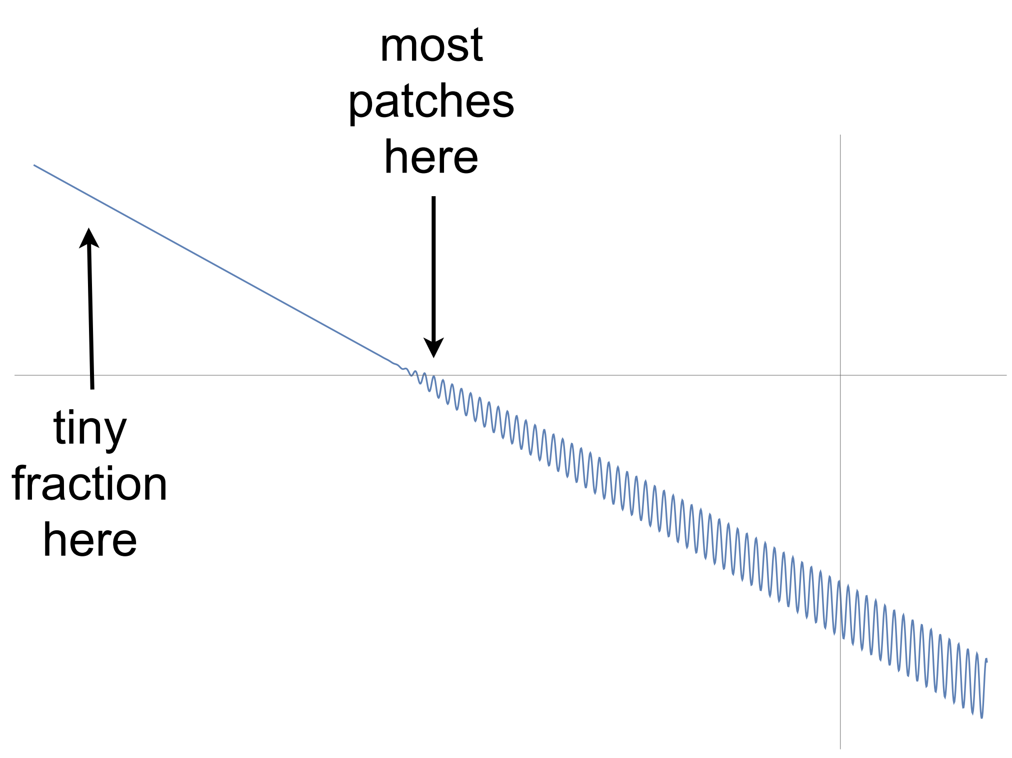
$$M < 1000\ \mathrm{TeV} \left(\frac{\theta}{10^{-10}} \right)^{\frac{1}{4}}$$

Quantum vs. Classical evolution

Additional constraint can come from requiring classical evolution to dominate.

$$\frac{\dot{\phi}}{H_{\rm infl}} > H_{\rm infl}$$

otherwise:



Quantum vs. Classical evolution

Additional constraint can come from requiring classical evolution to dominate. $\longrightarrow$ $\frac{\dot{\phi}}{H_{\rm infl}} > H_{\rm infl}$

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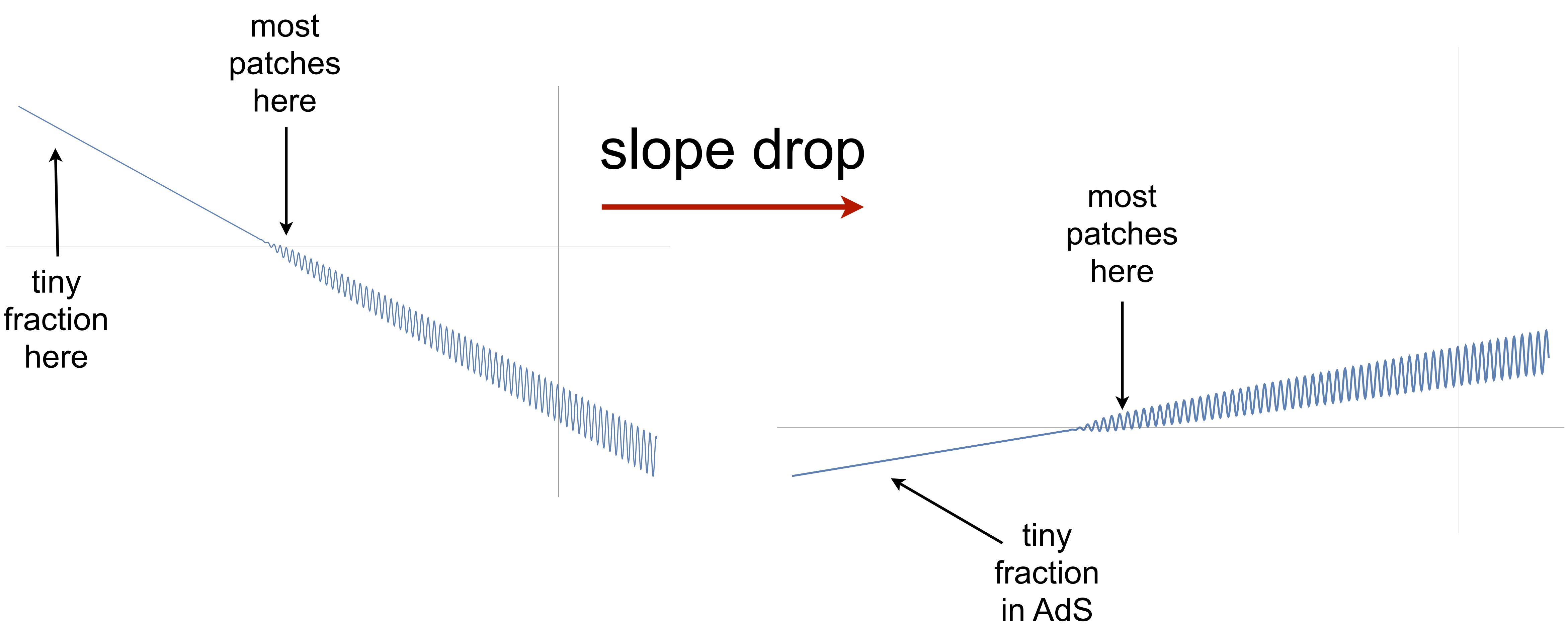


Solve Strong CP (2)

(Model 2)

Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y h L N^c + \tilde{y} h^\dagger L^c N$$

	$SU(3)$
L, N	$\square$
L^c, N^c	$\overline{\square}$

assume:

$$m_L \gg f_{\pi'} \gg m_N$$

NDA: $\Lambda^4 \simeq 4\pi f_{\pi'}^3 m_{N_1}$

Model 2

Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

Higher induced: $\delta m_{N_1} \simeq \frac{y\tilde{y}\langle h\rangle^2}{m_L}$ "Bare": $m_N \gtrsim \frac{y\tilde{y}}{16\pi^2} m_L \log\frac{M}{m_L}$

Require: $$m_L < \frac{4\pi\langle h\rangle}{\sqrt{\log M/m_L}}$$

Bounds: $$m_L \gtrsim 250\ \mathrm{GeV}$$

Parameter Requirements

$$\boxed{H_{\rm infl} > \frac{M^2}{M_{\rm pl}}}$$

$$\boxed{H_{\rm infl}^3 < gM^2} \quad \frac{\dot{\phi}}{H_{\rm infl}} > H_{\rm infl}$$

$$gM^2 f \sim \Lambda^4$$

$$M < 3 \times 10^8\ \text{GeV} \left(\frac{f_{\pi'}}{30\ \text{GeV}}\right)^{\frac{3}{7}} \left(\frac{y\tilde{y}}{10^{-2}}\right)^{\frac{1}{7}} \left(\frac{250\ \text{GeV}}{m_L}\right)^{\frac{1}{7}} \left(\frac{M}{f}\right)^{\frac{1}{7}}$$

Parameter Requirements

$$\boxed{H_{\text{infl}} > \frac{M^2}{M_{\text{pl}}}}$$

$$\boxed{H_{\text{infl}}^3 < gM^2} \quad \frac{\dot{\phi}}{H_{\text{infl}}} > H_{\text{infl}}$$

Plugging in for g, ($gM^2 f \sim \Lambda^4$):

$$\boxed{M^6 < \frac{\Lambda^4 M_{\text{pl}}^3}{f}}$$

$$M < 3 \times 10^8 \text{ GeV} \left(\frac{f_{\pi'}}{30 \text{ GeV}}\right)^{\frac{3}{7}} \left(\frac{y\tilde{y}}{10^{-2}}\right)^{\frac{1}{7}} \left(\frac{250 \text{ GeV}}{m_L}\right)^{\frac{1}{7}} \left(\frac{M}{f}\right)^{\frac{1}{7}}$$

Inflation

To achieved the relaxed value,
inflation has to last long enough:

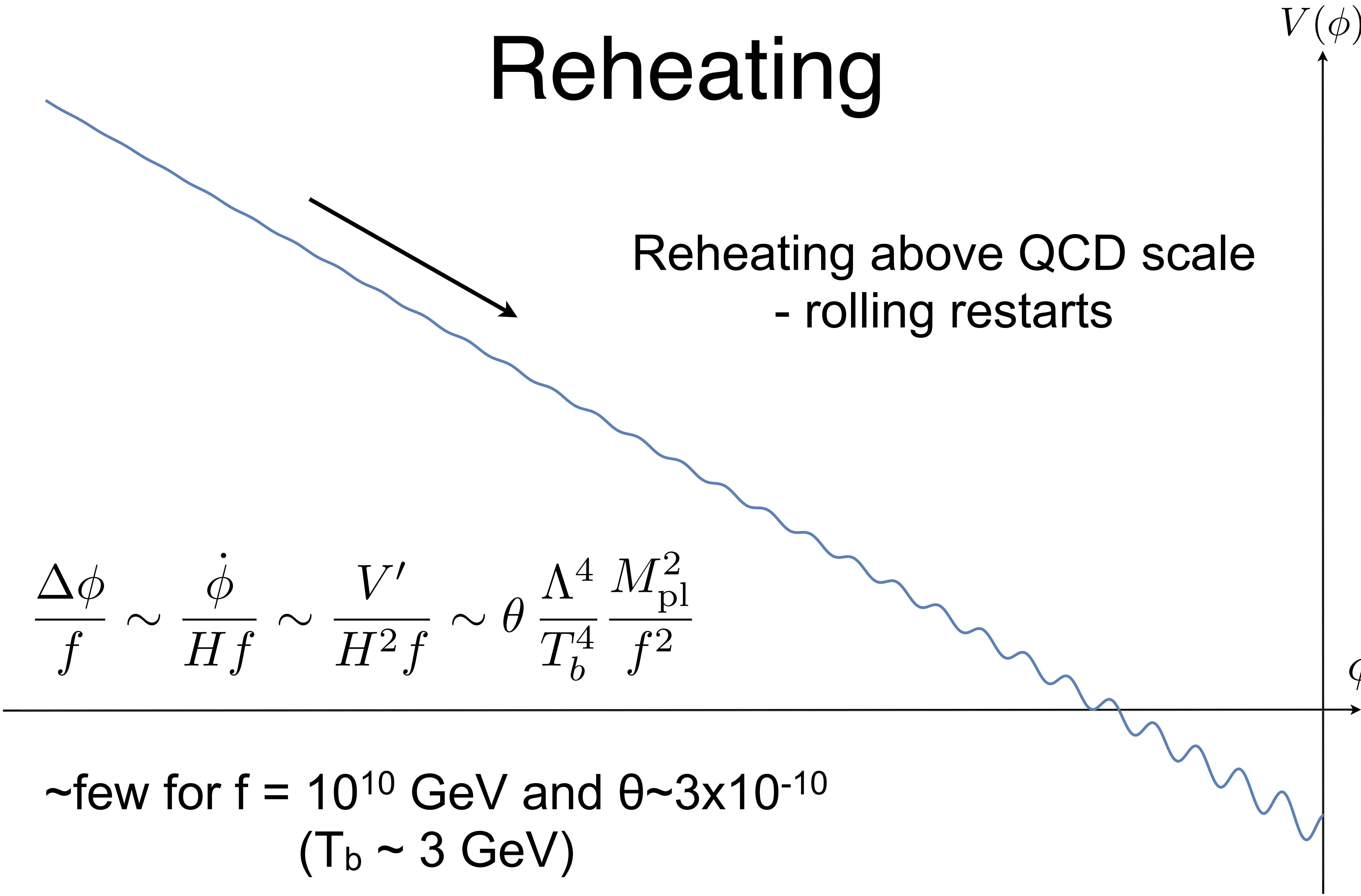
$$\Delta\phi \sim \frac{\dot{\phi}}{H_{\rm infl}} N \sim \frac{\partial_\phi V}{H_{\rm infl}^2} N \sim \frac{g M^2}{H_{\rm infl}^2} N$$

We require:

$$\Delta\phi \gtrsim \left(\frac{M^2}{g}\right)$$

$$N \gtrsim \frac{H_{\rm infl}^2}{g^2} \sim 10^{48}, 10^{37}$$ (Model 1,2 saturated)

Reheating



Reheating above QCD scale
- rolling restarts

$$\frac{\Delta\phi}{f} \sim \frac{\dot{\phi}}{Hf} \sim \frac{V'}{H^2 f} \sim \theta\,\frac{\Lambda^4}{T_b^4}\,\frac{M_{\rm pl}^2}{f^2}$$

~few for f = 10^{10} GeV and $\theta \sim 3\mathrm{x}10^{-10}$
(T_b ~ 3 GeV)

(Rel)axion DM?

~few for $f = 10^{10}$ GeV and $\theta \sim 3 \times 10^{-10}$

$$\theta_0 \sim \left(\frac{10^{10}\ \text{GeV}}{f}\right)^2 \left(\frac{\theta_{QCD}}{3 \times 10^{-10}}\right)$$

for $f < 10^{10}$ GeV, axion rolls over barriers initially, extra kinetic energy can add to DM abundance.

To Do

- Better Inflation models - can the relaxion be the inflaton?

$$N \sim \left(\frac{M}{\Lambda}\right)^8 \left(\frac{f}{M_{\rm pl}}\right)^2$$

- Better models - can the field range be reduced?

$$\Delta\phi \sim \left(\frac{M}{\Lambda}\right)^4 f$$

- Phenomenology - New non-collider experiments?
- UV completion - axion monodromy?
- Cosmological Constant - new solution?

Thank you!

Extra: Inflation

Single field: $V(\Phi) = m^2\Phi^2$

$$N = \int H dt \sim \int \frac{H^2}{\partial_\Phi V} d\Phi \sim \frac{\Phi_i^2}{M_{\rm pl}^2}$$

Classical rolling:

$$\frac{\dot{\Phi}}{H_{\rm infl}} < H_{\rm infl} \longrightarrow \frac{m\Phi_i^2}{M_{\rm pl}^3} < 1 \longrightarrow V(\Phi_i) < \frac{M_{\rm pl}^4}{N}$$

$$\longrightarrow N < \left(\frac{M_{\rm pl}}{M}\right)^4 (\times\theta)$$

$$N \gtrsim \frac{H_{\rm infl}^2}{g^2} \longrightarrow M < 10^5, 10^{8.75}\ \mathrm{GeV}$$

Reheating requires additional dynamics (e.g., hybrid)

T-dependence of barriers