String Phenomenology 2015, IFT, Madrid
10th of June, 2015

New F-theory compactifications with U(1)'s and discrete gauge groups

- particle physics applications -

Denis Klevers

arXiv:1408.4808: D.K., D. Mayorga Peña, P. Oehlmann, H. Piragua, J. Reuter
arXiv:1503.02968: M. Cvetič, D.K., D. Mayorga Peña, P. Oehlmann, J. Reuter
arXiv:1502.06953: R. Donagi, M. Cvetič, D.K., H. Piragua, M. Poretschkin
arXiv:1506.nnnnn: M. Cvetič, D.K., H. Piragua, W. Taylor

Motivation

Global F-theory: advances and goals

1) Local model building:

- Demonstration that F-theory can yield GUT models with promising particle physics & cosmology: features not accessible in perturbative IIB (E_6 to E_8, 10x10x5)
  [Donagi,Wijnholt; Beasley,Heckman,Vafa;… many works]

2) Global models:

- *Past focus:* embed local models into compact CY-fourfolds.
  [Blumenhagen,Grimm,Jurke,Weigand;Marsano,Saulina,SchäferNameki;… many works]
- *Recent approaches* (yearly F-theory workshops for complete list):
  - construct new F-theory vacua with new features: *U(1)*'s, discrete gauge groups…
  - develop new tools: interplay physics/math, duality techniques,…
  - model independence: study whole families of vacua & their generic properties
  - analyze transitions between vacua: Higgs effect ⟷ geometric transitions,…

Goal: Explicit construction of all F-theory models & understanding their physics.

Goals of this talk

I. Study phenomenologically interesting class of CY-manifolds = fibrations of 2D toric hypersurfaces:

- have intrinsic gauge groups, matter contents & Yukawas
- are connected in network of Higgsings.
- admit global 4D three-family Standard, Pati-Salam & Trinification Models.
- include $\mathbb{Z}_n$ discrete gauge groups.

II. Analyze $\mathbb{Z}_n$ discrete gauge groups in M-/F-theory duality: focus on $\mathbb{Z}_3$

III. Construct moduli space of rank two non-Abelian gauge theories in F-theory

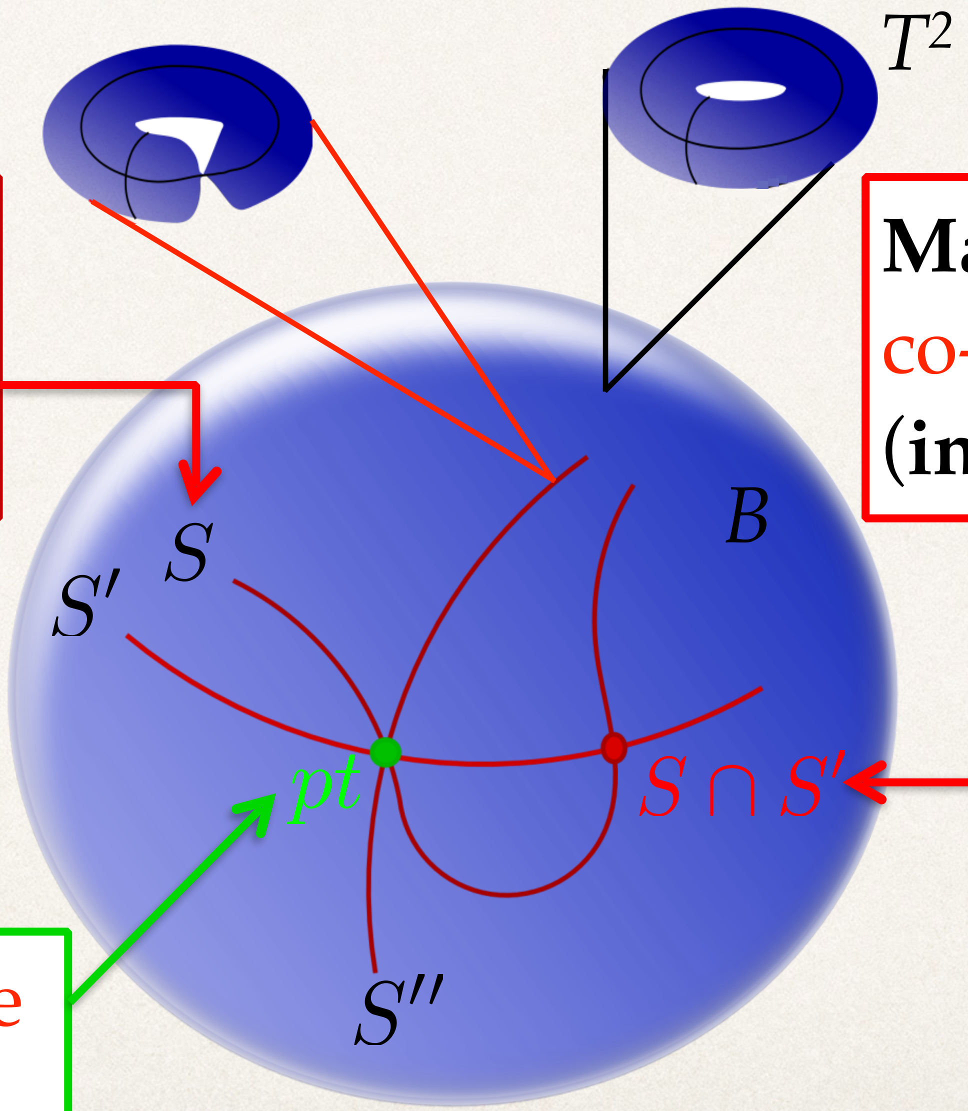
- Propose new non-toric model with $U(1)^2$ gauge group needed to describe this moduli space.

➡ Present first realization of $SU(3)$ gauge theories with symmetric matter representations in global F-theory.

Review: constructing F-theory vacua

What is F-theory vacuum?

Singular torus-fibered Calabi-Yau X over base B ⟷ globally well-defined setup of intersecting (p,q)7-branes



Gauge theory in **8D:**
co-dim. one singularity
(**7-branes**)

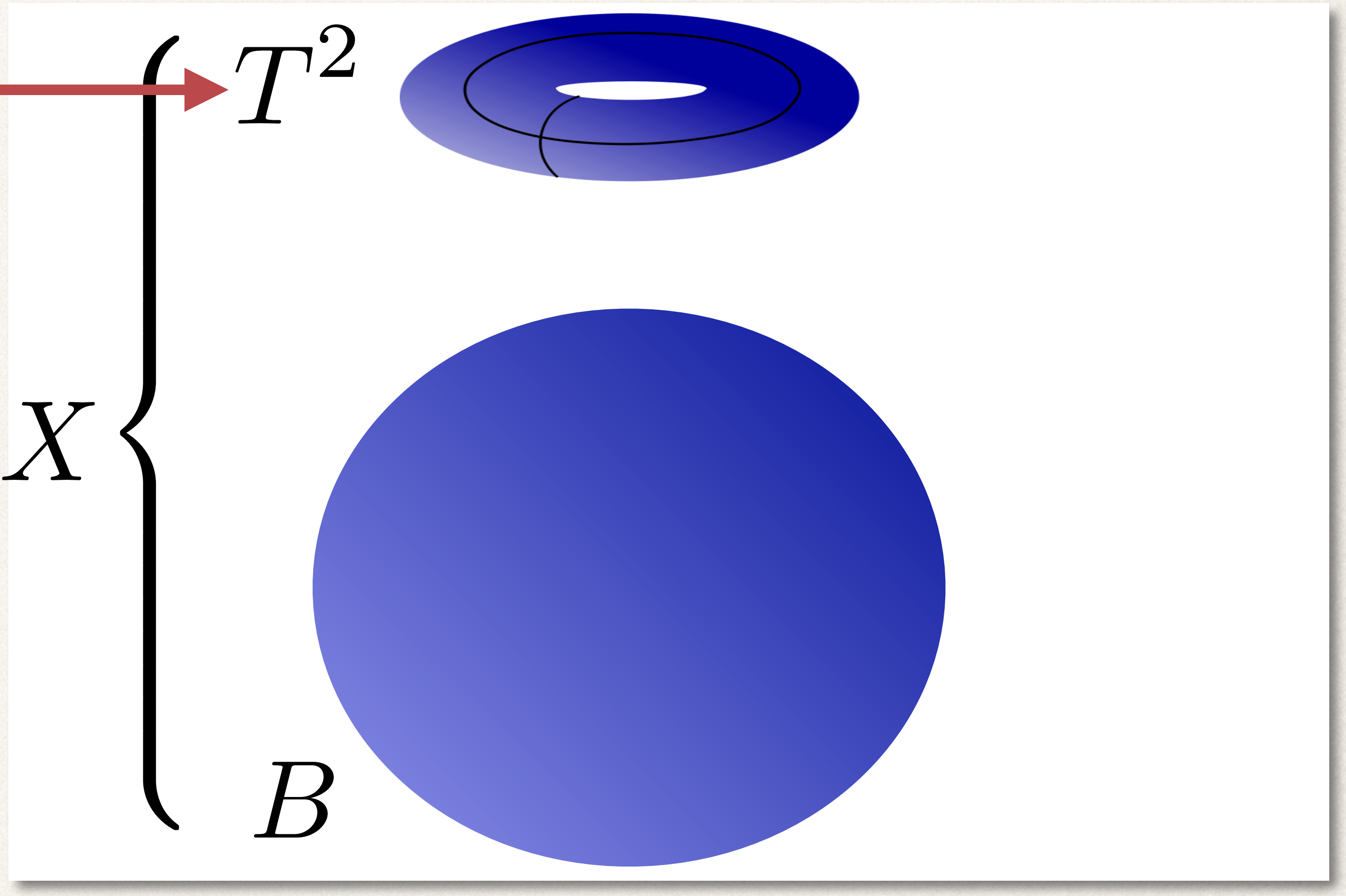
Matter in **6D :**
co-dim. two sing.
(**intersec. 7-branes**)

[Katz,Vafa]

4D Yukawa: co-dim three
$pt = S \cap S' \cap S''$

Building blocks of torus-fibered Calabi-Yau X

1. Fiber torus of X



Today: keep analysis base-independent (*B arbitrary*), focus on torus fiber T^2 of X

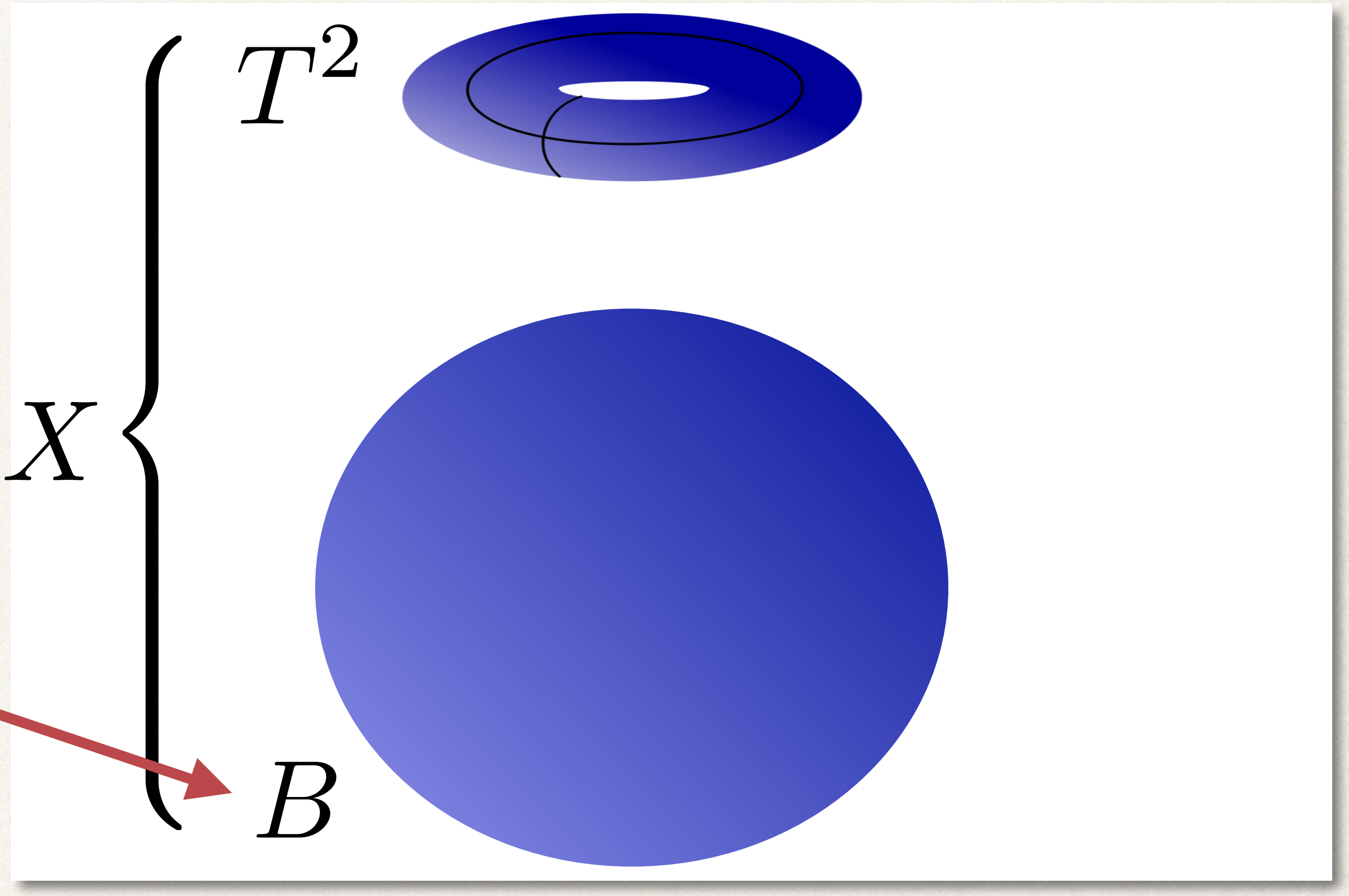
Classification of *B*: [Morrison,Taylor;Martini,Taylor;Johnson,Taylor;Taylor,Wang]

→ talk by Taylor

Building blocks of torus-fibered Calabi-Yau X

1. Fiber torus of X

2. Base B of X



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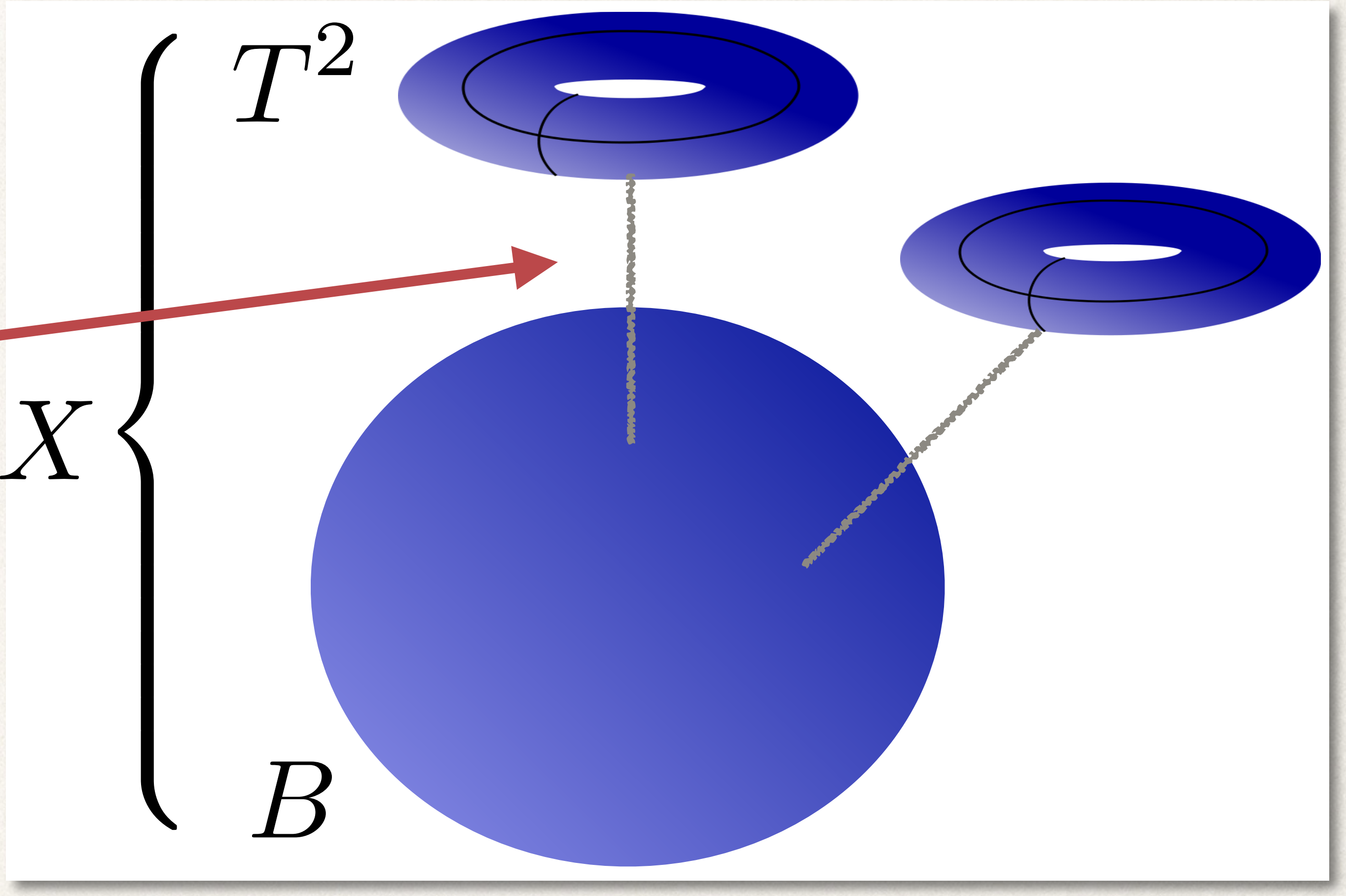
Building blocks of torus-fibered Calabi-Yau X

1. Fiber torus of X

3. Fibration data of X

2. Base B of X



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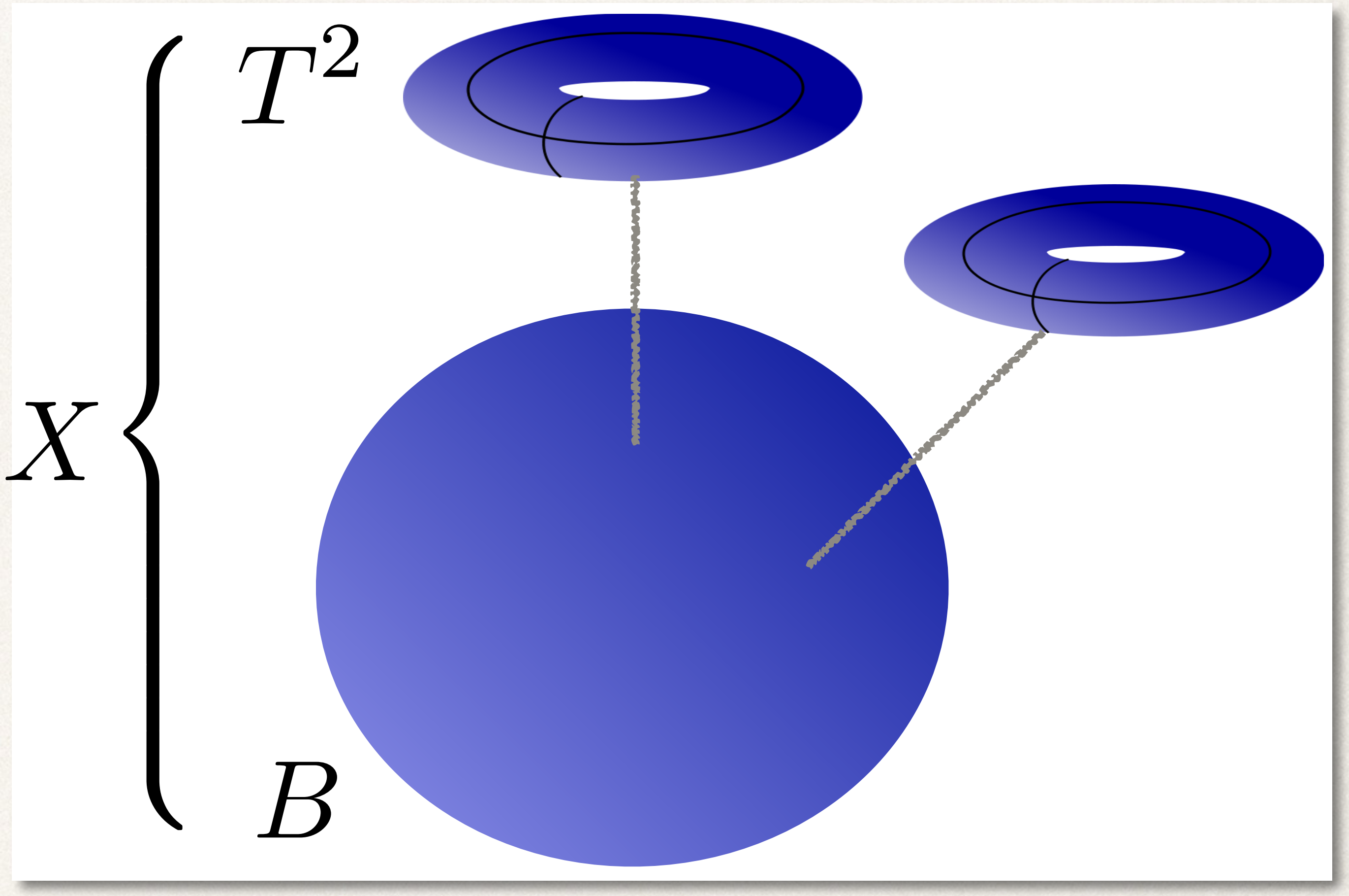

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→ **talk by Taylor**

I. F-theory on toric hypersurface fibrations

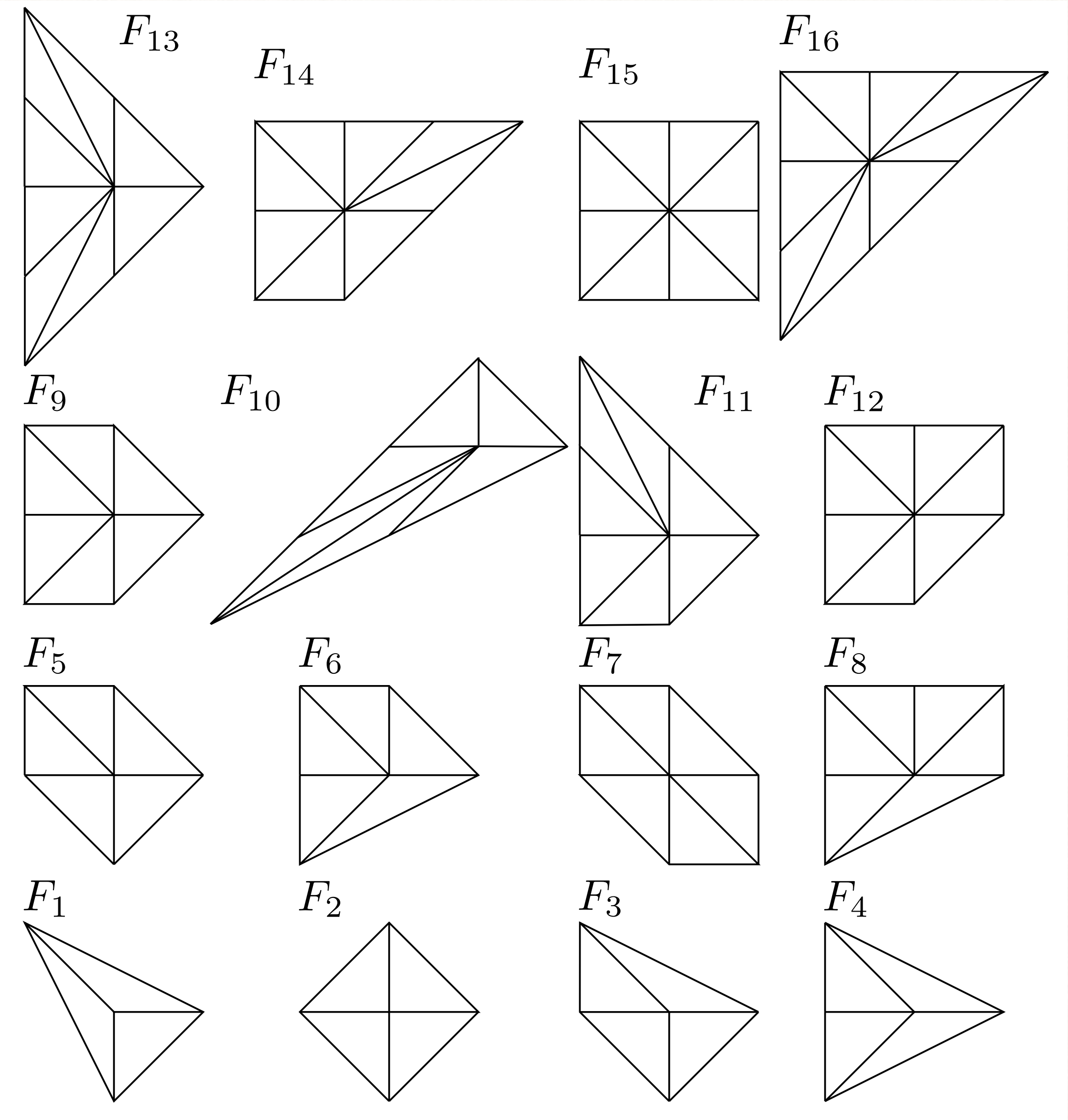
Elliptic CY-manifolds with other toric fibers for F-theory:
[Aldazabal,Font,Ibanez,Uranga;Klemm,Mayr,Vafa;Candelas,Font;Klemm,Lian,Roan,Yau;…….]
A lot of recent activity:
[Grimm,Weigand;Grimm,Hayashi;Morrison,Park;Braun,Grimm,Keitel;Borchmann,Mayrhofer,Palti,Weigand; Cvetič,Klevers,Piragua;Grimm,Kapfer,Keitel;Cvetič,Grassi,Klevers,Piragua;Küntzler,SchäferNameki; CaboBizet,Klemm,Lopes;Braun,Morrison;Morrison,Taylor;Mayrhofer,Morrison,Till,Weigand;Anderson,Haupt,Lukas; Anderson,GarciaEtxebarria,Grimm,Keitel;Mayrhofer,Palti,Till,Weigand;GarciaEtxebarria,Grimm,Keitel;Lawrie,Sacco; Lawrie,SchäferNameki,Wong…]

➡ [DK,Mayorga-Peña,Piragua,Oehlmann,Reuter]

1) Construction of toric hypersurface fibrations

→ **more details in talk by Paul Oehlmann in parallel session at 17:30**

Toric varieties from reflexive polytopes



- Toric variety $\mathbb{P}_{F_i}$ associated to 16 reflexive polytopes F_i in 2D.
- Each $\mathbb{P}_{F_i}$ has corresponding genus-one curve $\mathcal{C}_{F_i}$={ $p_{F_i} = 0$}

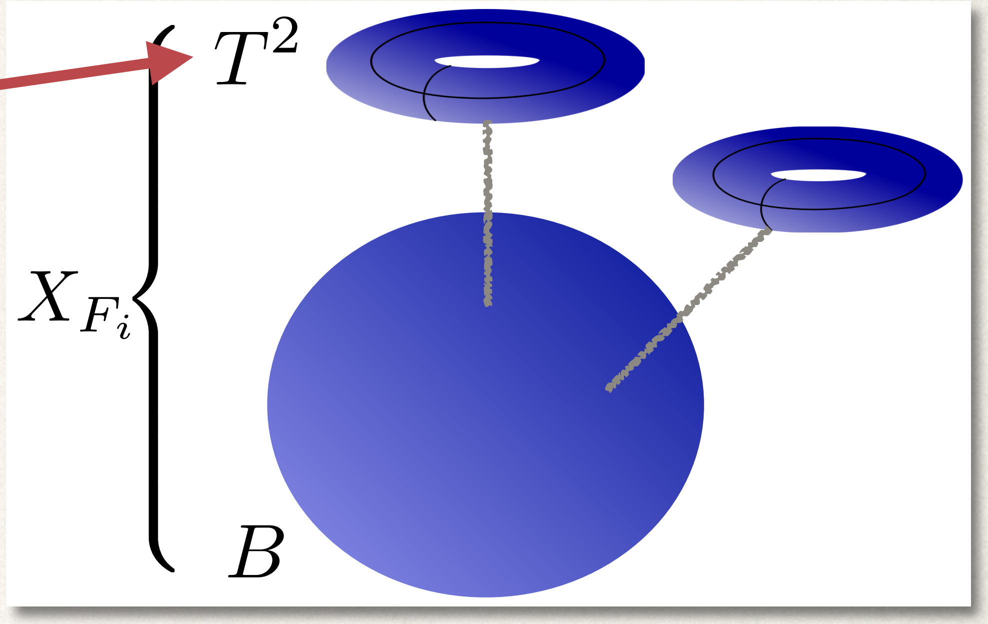
  = anti-canonical divisor in $\mathbb{P}_{F_i}$.

Construction of toric hypersurface fibration X_{F_i}

Conceptually:

- Take fiber torus $T^2 = \mathcal{C}_{F_i}$
- Choose arbitrary base B
- Fibration data: choice of line bundles on B for two local coordinates of $\mathcal{C}_{F_i}$ (denoted $\mathcal{S}_7$, $\mathcal{S}_9$)

➡ discrete families of CY-manifolds $X_{F_i}(\mathcal{S}_7, \mathcal{S}_9)$.



➡ Derive the effective theory of F-theory for all these X_{F_i}.

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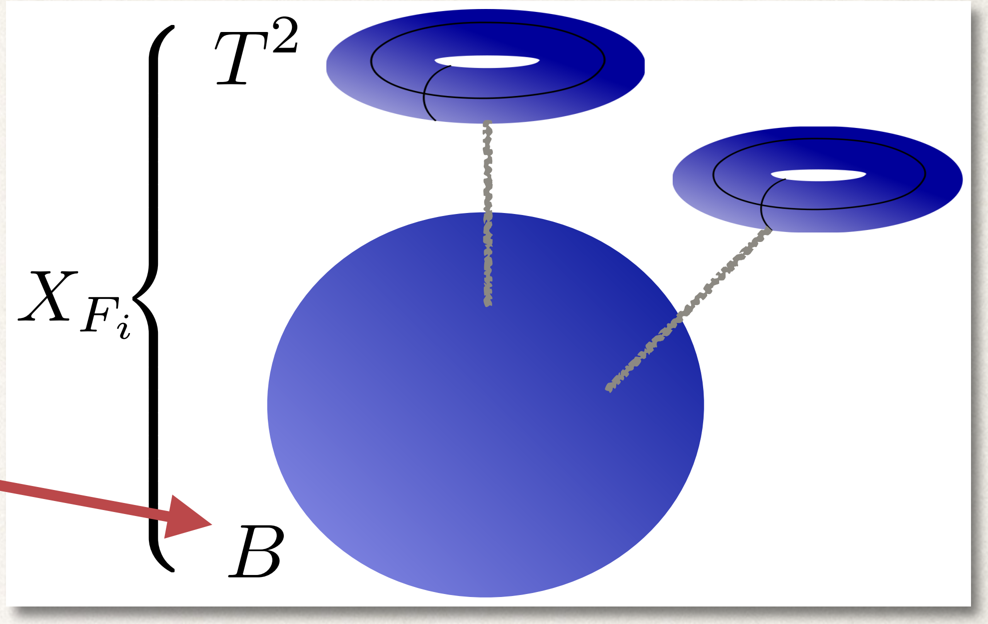

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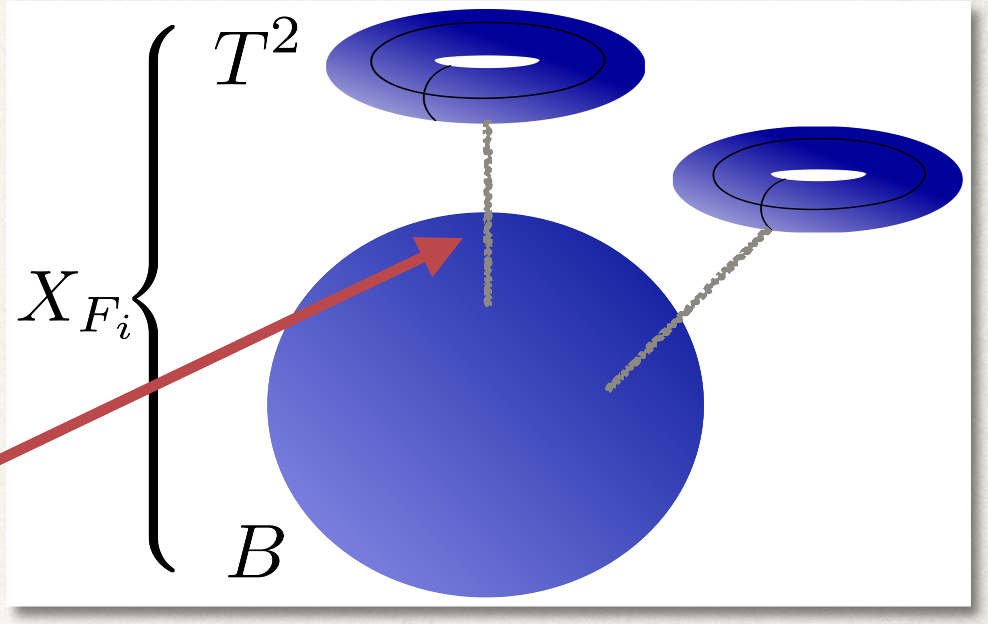

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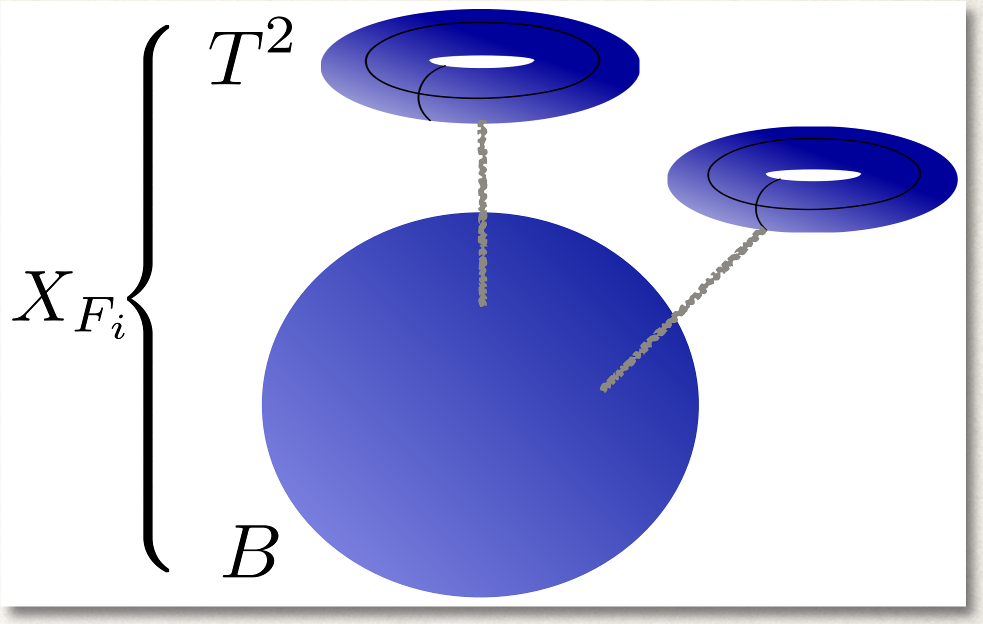
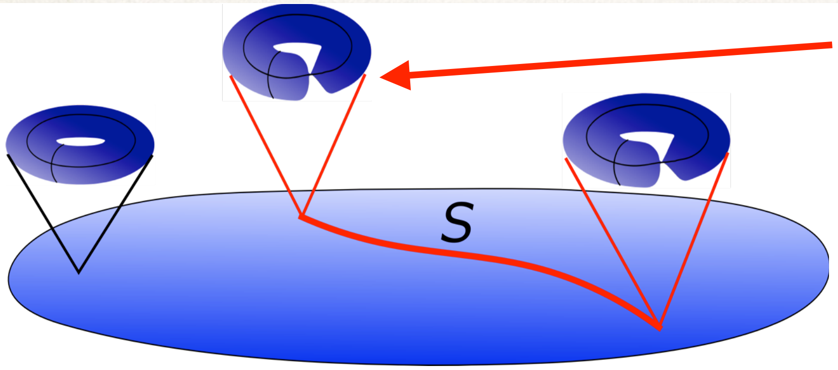


➡ Derive the effective theory of F-theory for all these X_{F_i}.

2) The low-energy effective theory

Non-Abelian Gauge Group

Gauge theory at singularities of elliptic fibration of X:
[Vafa;Morrison,Vafa;Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa]



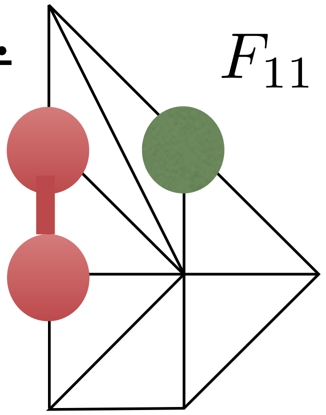
Singularity

- classified by Lie algebra G [Kodaira;Tate]
- resolve: Cartan matrix of G by $\mathbb{P}^1$s over S
- M2's on shrinkable $\mathbb{P}^1$s: G becomes gauge group in eff. theory

X_{F_i} have codim. 1 singularities and intrinsic gauge group G_{F_i}

- can read off G_{F_i} from toric polytope
  - Points inside edges = nodes in Dynkin diagram

e.g.



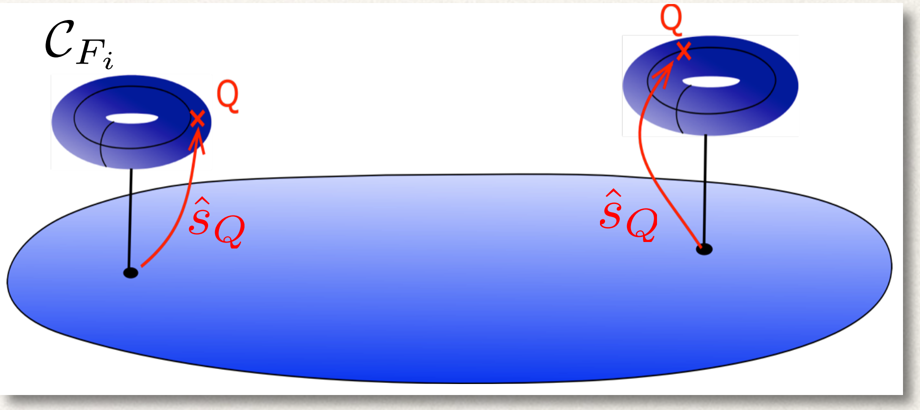
SU(3)xSU(2) $\subset G_{F_{11}}$

Abelian Gauge Group

U(1)-symmetries ⟷ Mordell-Weil group of rational sections of elliptic fibrations X_{F_i}. [Morrison,Vafa]

- rational section is map $\hat{s}_Q\,:\;B\rightarrow X_{F_i}$ induced by rational point Q on $\mathcal{C}_{F_i}$.

  → **talk by Schäfer-Nameki**

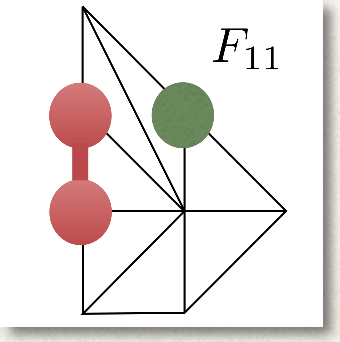


Number of U(1)'s / rational sections from toric polytope:

➡ number of U(1)'s = #(vertices of F_i)-3 (some sections non-toric)

Toric MW-group: [Braun,Grimm,Keitel]

<u>Example:</u>



➡

$4-3=1$ U(1):

$G_{F_{11}}$ = **SU(3)**x**SU(2)**xU(1)

Effective theories of the 16 toric hypersurface fibrations

[D.K.,Mayorga Peña,Oehlmann,Piragua,Reuter]

Intrinsic gauge group G_{F_i} of all 16 toric hypersurface fibrations X_{F_i}

G_{F_1}	$\mathbb{Z}_3$	G_{F_7}	$\mathrm{U}(1)^3$		
G_{F_2}	$\mathrm{U}(1)\times\mathbb{Z}_2$	G_{F_8}	$\mathrm{SU}(2)^2\times\mathrm{U}(1)$	$G_{F_{13}}$	$(\mathrm{SU}(4)\times\mathrm{SU}(2)^2)/\mathbb{Z}_2$
G_{F_3}	$\mathrm{U}(1)$	G_{F_9}	$\mathrm{SU}(2)\times\mathrm{U}(1)^2$	$G_{F_{14}}$	$\mathrm{SU}(3)\times\mathrm{SU}(2)^2\times\mathrm{U}(1)$
G_{F_4}	$(\mathrm{SU}(2)\times\mathbb{Z}_4)/\mathbb{Z}_2$	$G_{F_{10}}$	$\mathrm{SU}(3)\times\mathrm{SU}(2)$	$G_{F_{15}}$	$\mathrm{SU}(2)^4/\mathbb{Z}_2\times\mathrm{U}(1)$
G_{F_5}	$\mathrm{U}(1)^2$	$G_{F_{11}}$	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	$G_{F_{16}}$	$\mathrm{SU}(3)^3/\mathbb{Z}_3$
G_{F_6}	$\mathrm{SU}(2)\times\mathrm{U}(1)$	$G_{F_{12}}$	$\mathrm{SU}(2)^2\times\mathrm{U}(1)^2$		

Non-simply connected groups: [Aspinwall,Morrison;Mayrhofer,Morrison,Till,Weigand]

- up to three U(1)'s, non-simply connected & discrete gauge groups $\mathbb{Z}_2$,$\mathbb{Z}_3$,$\mathbb{Z}_4$

  $\mathbb{Z}_2$ discrete group: [Morrison,Taylor;Anderson,García-Etxebarria,Grimm,Keitel;GarcíaEtxebarria,Grimm,Keitel; Mayrhofer,Palti,Till,Weigand]

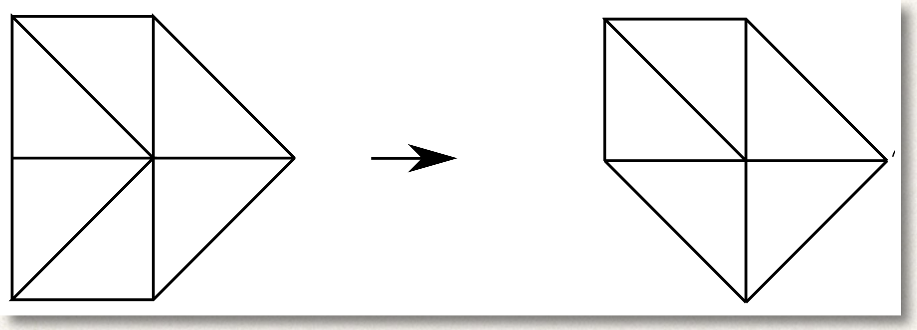
- for any B: 6D matter (= 4D non-chiral) spectrum & 4D Yukawas derived
  - used techniques from computational algebraic geometry
- all theories anomaly-free. ✔

3) A Higgs network

Higgs transitions between toric hypersurface fibrations

All toric hypersurface fibrations X_{F_i} are connected: change of torus fibers $\mathcal{C}_{F_i}$

- Described in toric polytope as cutting corners (= extremal transition in X_{F_i})



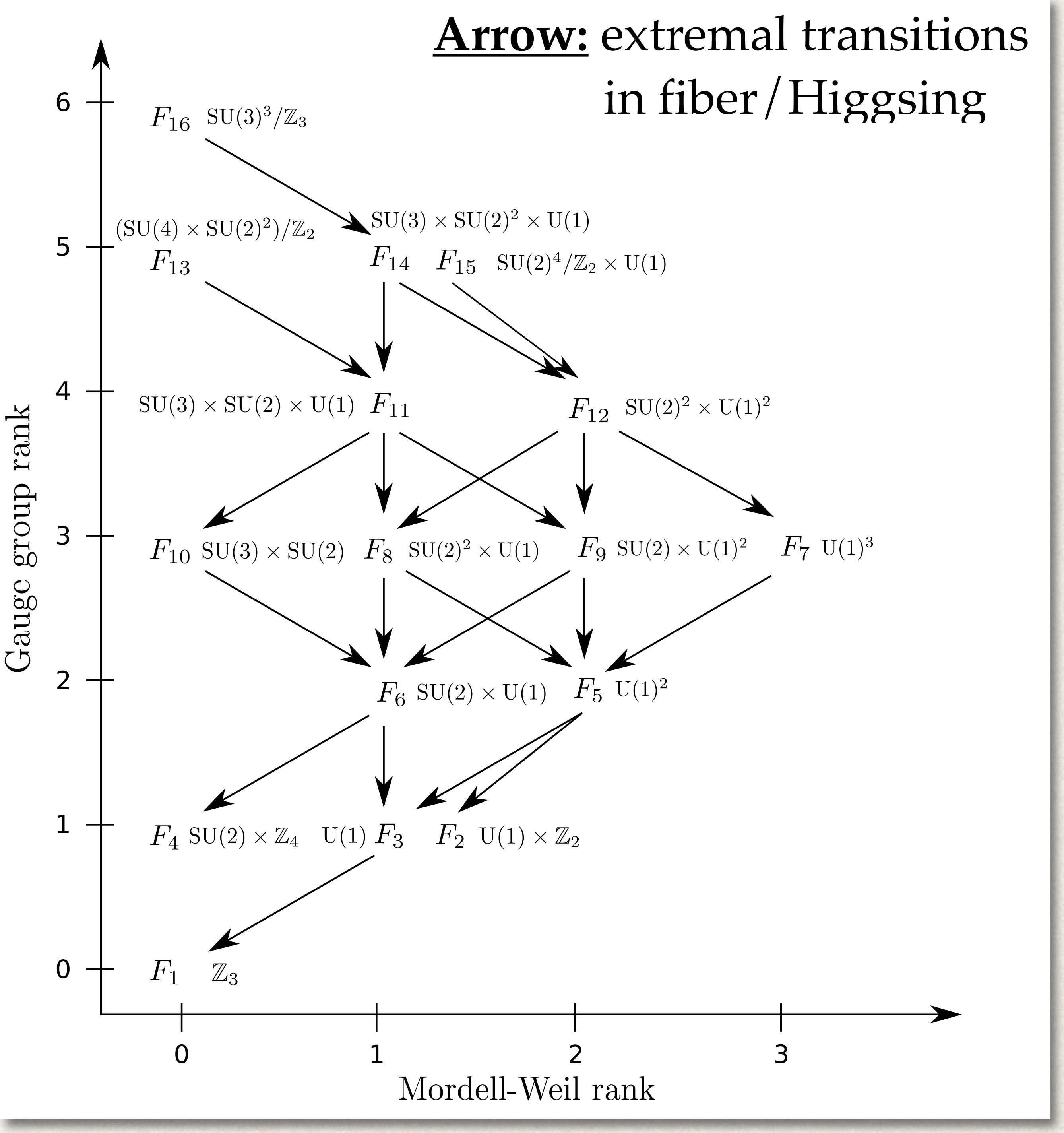
Corresponds to Higgsing in effective field theory

- ➡ worked out full network of all such Higgsings,
- ➡ generates subbranch of moduli space of field theory: “toric Higgs branch”.

Toric Higgs branch

[DK,Mayorga-Pena,Oehlmann,Reuter,Piragua]

- matched full 6D spectra (charged & uncharged).
- all theories in one moduli space of maximal models F_{13}, F_{15}, F_{16}.
- all models with discrete gauge groups arise from Higgsing gauged U(1)’s:
  - ➡ consistent with quantum gravity constraint that every global symmetry has to be gauged ✔


Arrow: extremal transitions in fiber/Higgsing

4) Three family models in toric unification

[Cvetič, DK, Mayorga-Pena, Oehlmann, Reuter]

→ **more details in talk by Damian Mayorga in parallel session at 17:45**

Phenomenologically interesting examples

[DK,Mayorga-Pena,Oehlmann,Reuter,Piragua]

Natural unification structure in toric Higgs branch:

Trinification

Pati-Salam

$$\begin{tikzcd}
F_{16}\ \mathrm{SU}(3)^3/\mathbb{Z}_3 \arrow[dr] & \\
(\mathrm{SU}(4)\times\mathrm{SU}(2)^2)/\mathbb{Z}_2\ F_{13} \arrow[dr] & F_{14} \arrow[d] \\
& \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\ F_{11}
\end{tikzcd}$$

Standard Model

1. Standard-Model-like theory: $X_{F_{11}}$

Representation	$(\mathbf{3},\mathbf{2})_{1/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{-1}$
Multiplicity	$\mathcal{S}_9([K_B^{-1}]+\mathcal{S}_7-\mathcal{S}_9)$	$\mathcal{S}_9(2[K_B^{-1}]-\mathcal{S}_7)$	$\mathcal{S}_9(5[K_B^{-1}]-\mathcal{S}_7-\mathcal{S}_9)$	$([K_B^{-1}]+\mathcal{S}_7-\mathcal{S}_9)$ $\times(6[K_B^{-1}]-2\mathcal{S}_7-\mathcal{S}_9)$	$(2[K_B^{-1}]-\mathcal{S}_7)$ $\times(3[K_B^{-1}]-\mathcal{S}_7-\mathcal{S}_9)$

- All gauge invariant 4D Yukawas realized.

SM via tops of dP$_2$: [Lin,Weigand]
using NHC: [Grassi,Halverson,Shaneson,Taylor]

2. Pati-Salam-like theory: $X_{F_{13}}$
3. Trinification-like theory: $X_{F_{16}}$

$\Big\}$ spectrum & Yukawas

Construction of three family models in 4D

[Cvetič,DK,Mayorga-Pena,Oehlmann,Reuter]

Model Building Strategy:

1. Construct G_4-flux by computing $H^{(2,2)}_V(X_{F_i})$ for Standard, Pati-Salam and Trinification Models following **[Cvetič,Grassi,DK,Piragua]** .

   see also:[Marsano,Schäfer-Nameki;Grimm, Hayashi;Cvetič,Grimm,DK;Cvetič,Grassi,DK,Piragua]

2. Compute chiralities

$$\chi(\mathbf{R}) = -\frac{1}{4}\int_{\mathcal{C}_{\mathbf{R}}} G_4$$

   [Donagi,Wijnholt;Hayashi,Tatar,Toda,Watari, Yamazaki;Braun,Collinucci,Valandro;Marsano, Schäfer-Nameki]

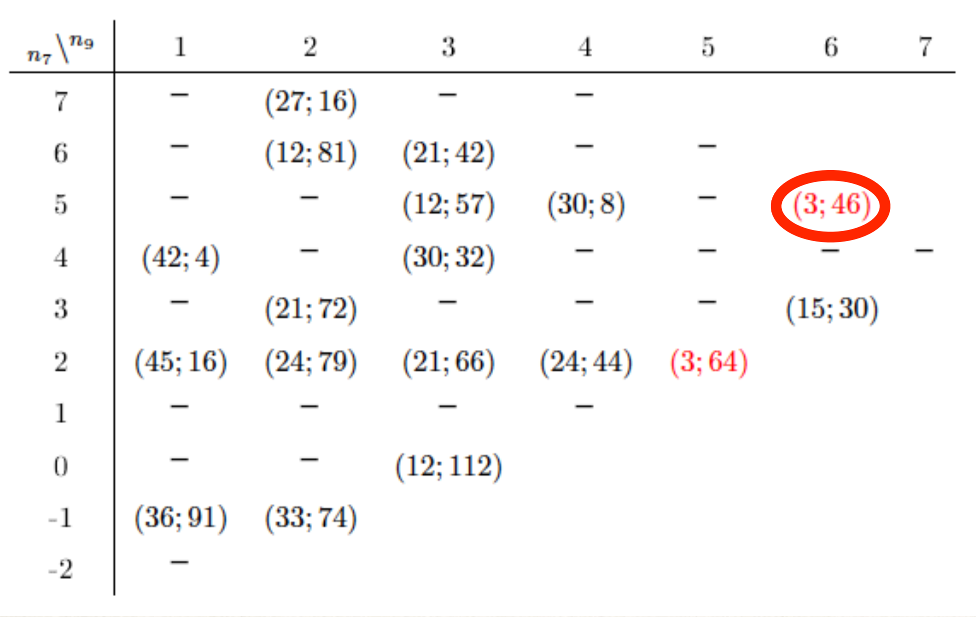
3. Determine minimal number of families so that n_{D3} is integral & positive and G_4-flux quantized ⟺ 3D CS-terms are integral (in dual M-theory).

➡ Explicit results for concrete fourfolds with base $B = \mathbb{P}^3$.

Construction of three family models in 4D

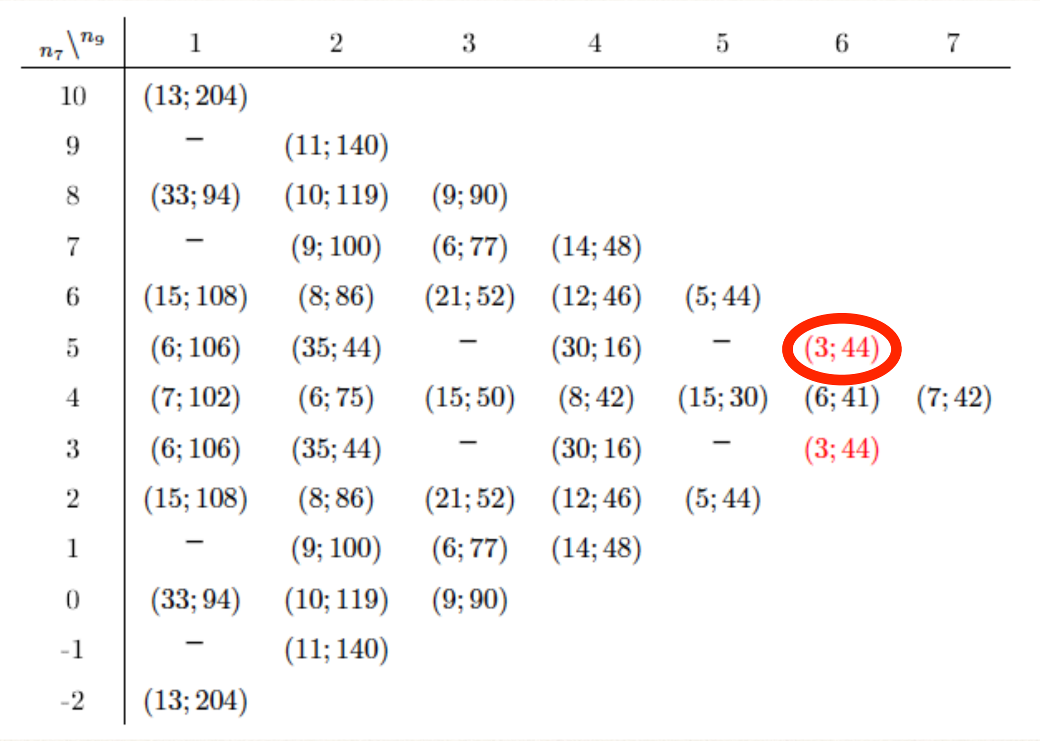
[Cvetič,DK,Mayorga-Pena,Oehlmann,Reuter]

Standard Model: (#(families),n_{D3})



$n_7\backslash^{n_9}$	1	2	3	4	5	6	7
7	−	(27; 16)	−	−			
6	−	(12; 81)	(21; 42)	−	−		
5	−	−	(12; 57)	(30; 8)	−	(3; 46)	
4	(42; 4)	−	(30; 32)	−	−	−	−
3	−	(21; 72)	−	−	−	(15; 30)	
2	(45; 16)	(24; 79)	(21; 66)	(24; 44)	(3; 64)		
1	−	−	−	−			
0	−	−	(12; 112)				
-1	(36; 91)	(33; 74)					
-2	−						

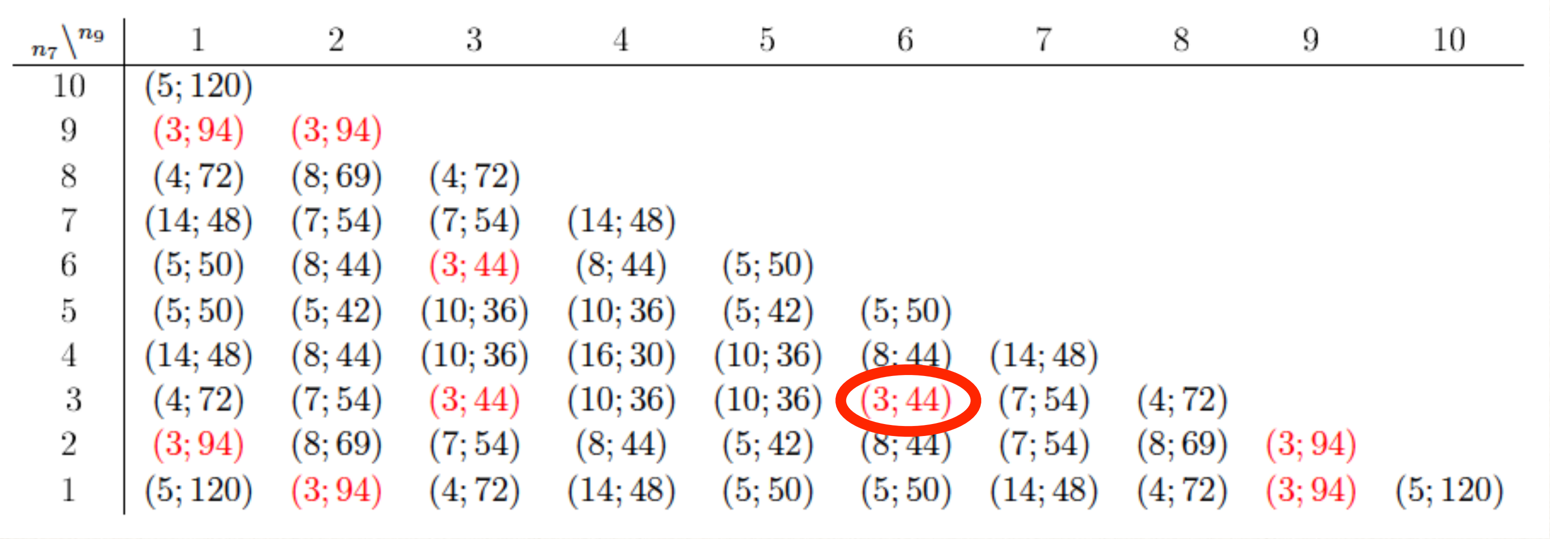
Pati-Salam: (#(families),n_{D3})



$n_7\backslash^{n_9}$	1	2	3	4	5	6	7
10	(13; 204)						
9	−	(11; 140)					
8	(33; 94)	(10; 119)	(9; 90)				
7	−	(9; 100)	(6; 77)	(14; 48)			
6	(15; 108)	(8; 86)	(21; 52)	(12; 46)	(5; 44)		
5	(6; 106)	(35; 44)	−	(30; 16)	−	(3; 44)	
4	(7; 102)	(6; 75)	(15; 50)	(8; 42)	(15; 30)	(6; 41)	(7; 42)
3	(6; 106)	(35; 44)	−	(30; 16)	−	(3; 44)	
2	(15; 108)	(8; 86)	(21; 52)	(12; 46)	(5; 44)		
1	−	(9; 100)	(6; 77)	(14; 48)			
0	(33; 94)	(10; 119)	(9; 90)				
-1	−	(11; 140)					
-2	(13; 204)						

Parameters labelling models $(\mathcal{S}_7,\mathcal{S}_9) \rightarrow (n_7,n_9)$

Trinification: (#(families),n_{D3})



$n_7\backslash^{n_9}$	1	2	3	4	5	6	7	8	9	10
10	(5; 120)									
9	(3; 94)	(3; 94)								
8	(4; 72)	(8; 69)	(4; 72)							
7	(14; 48)	(7; 54)	(7; 54)	(14; 48)						
6	(5; 50)	(8; 44)	(3; 44)	(8; 44)	(5; 50)					
5	(5; 50)	(5; 42)	(10; 36)	(10; 36)	(5; 42)	(5; 50)				
4	(14; 48)	(8; 44)	(10; 36)	(16; 30)	(10; 36)	(8; 44)	(14; 48)			
3	(4; 72)	(7; 54)	(3; 44)	(10; 36)	(10; 36)	(3; 44)	(7; 54)	(4; 72)		
2	(3; 94)	(8; 69)	(7; 54)	(8; 44)	(5; 42)	(8; 44)	(7; 54)	(8; 69)	(3; 94)	
1	(5; 120)	(3; 94)	(4; 72)	(14; 48)	(5; 50)	(5; 50)	(14; 48)	(4; 72)	(3; 94)	(5; 120)

- All models admit three families
- Unification with three families possible.

II. $\mathbb{Z}_n$ discrete gauge groups beyond *n*=2

[DK,Mayorga-Pena,Oehlmann,Reuter,Piragua; Cvetič, Donagi, DK, Piragua, Poretschkin]

$\longrightarrow$ **talk by Leontaris for discrete symmetries in F-theory GUTs**

F-theory phenomenology:[Karozas,King,Leontaris,Meadowcroft;Leontaris]

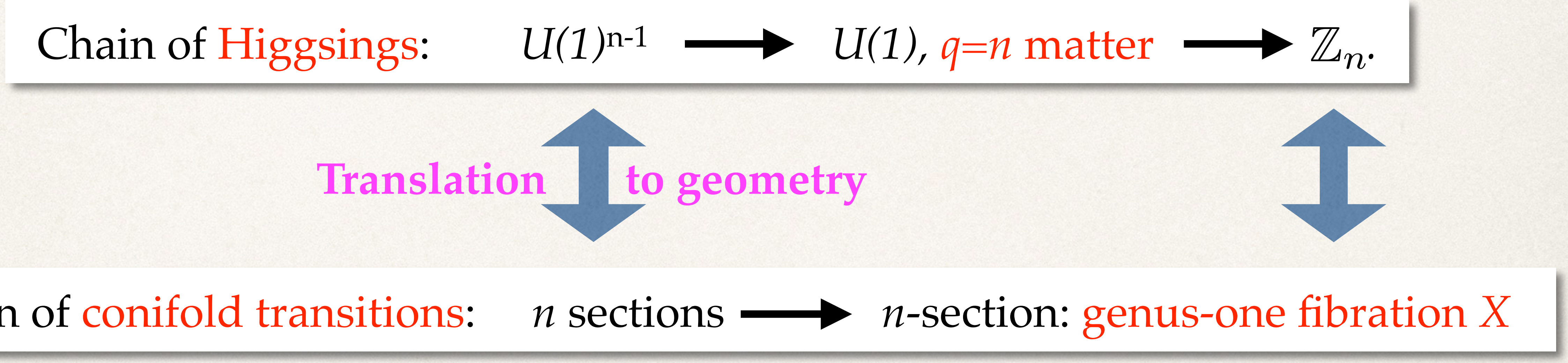
$\mathbb{Z}_2$ completely understood: (1) in M-theory (2) torsion homology of *J(X)* $\longrightarrow$ **see talk by Palti**

**[Braun,Morrison; Morrison,Taylor;Anderson,García-Etxebarria,Grimm,Keitel;
García-Etxebarria,Grimm,Keitel;Mayrhofer,Palti,Till,Weigand]**

The geometry of Abelian discrete symmetries

Question: What is geometrical object associated to discrete gauge groups in F-theory?

- know in field theory: $\mathbb{Z}_n$ from Higgsing a theory with $U(1)$ by $q=n$ matter.



Chain of Higgsings: $U(1)^{\text{n-1}} \longrightarrow U(1)$, $q=n$ matter $\longrightarrow \mathbb{Z}_n$.

Translation to geometry

Chain of conifold transitions: n sections $\longrightarrow$ n-section: genus-one fibration X

Proposal: Tate-Shafarevich group of genus-one fibration X → $\text{Ш}(J(X)) \supset \mathbb{Z}_n$

[Witten;deBoer,Dijkgraaf,Hori,Keurentjes,Morgan,Morrison,Sethi]

- $\text{Ш}(J(X))$ only visible after circle compactification: labels different M-theory vacua

Here: Check for $\mathbb{Z}_3$ → need recent progress on understanding of U(1) models.

Global models with $\mathbb{Z}_3$ discrete gauge groups

Field theory: Higgs dP_2-model with $U(1)^2$ ⟶ $U(1)$ with $q=3$ matter
[Borchmann,Mayrhofer,Palti,Weigand; Cvetič,Klevers,Piragua]

Geometry: CY with $q=3$ identified as dP_1-fibration X_{F3} [D.K.,Mayorga,Oehlmann,Piragua,Reuter]

$$\begin{tikzcd}
\text{F-theory: } U(1)_{6d},\ \Phi_{(3)} \arrow[rr, "\langle\Phi_{(3)}\rangle\neq0"'] \arrow[ddd, "S^1\text{ with flux } \xi=\int_{S^1}A_{6d}"'] & & \mathbb{Z}_3 \arrow[d, "S^1\text{ with discrete flux } \xi=\int_{S^1}A_{mass.}"] & \\
& & U(1)_{KK}\times\mathbb{Z}_3 & \\
& & & \text{Ш}(J(X)) \\
\text{M-theory: } U(1)\times U(1)_{KK} \arrow[uurr, "\xi=0:\langle\Phi_{(3,0)}\rangle\neq0"] \arrow[rr, "\xi=1/3:\langle\Phi_{(3,-1)}\rangle\neq0"'] \arrow[drr, "\xi=2/3:\langle\Phi_{(3,-2)}\rangle\neq0"'] & & U'(1)_{KK} & \\
\text{KK-tower } \Phi_{(3,k)},\ m=|3q+k| & & U''(1)_{KK} &
\end{tikzcd}$$

➡ Identification of different elements in $\text{Ш}(J(X))$ by finding Higgs-curves.

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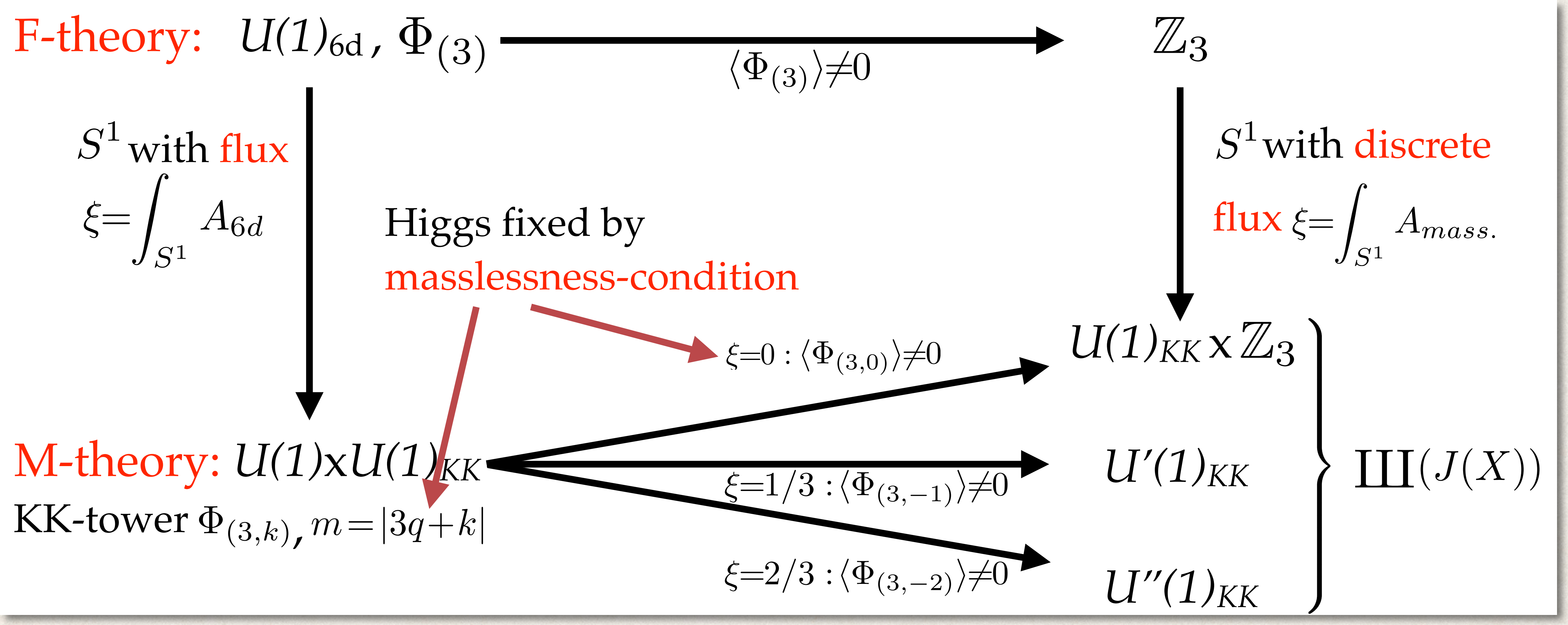

F-theory: $U(1)_{6d}$, $\Phi_{(3)}$ $\xrightarrow{\langle\Phi_{(3)}\rangle\neq 0}$ $\mathbb{Z}_3$

S^1 with flux $\xi=\int_{S^1} A_{6d}$

S^1 with discrete flux $\xi=\int_{S^1} A_{mass.}$

Higgs fixed by masslessness-condition

M-theory: $U(1)\text{x}U(1)_{KK}$

KK-tower $\Phi_{(3,k)}$, $m=|3q+k|$

$\xi=0 : \langle\Phi_{(3,0)}\rangle\neq 0$ → $U(1)_{KK}\,\text{x}\,\mathbb{Z}_3$

$\xi=1/3 : \langle\Phi_{(3,-1)}\rangle\neq 0$ → $U'(1)_{KK}$

$\xi=2/3 : \langle\Phi_{(3,-2)}\rangle\neq 0$ → $U''(1)_{KK}$

$Ш(J(X))$

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➡ Identification of different elements in $\text{Ш}(J(X))$ by finding Higgs-curves.

Global models with $\mathbb{Z}_3$ discrete gauge groups

Higgs field = shrinkable curves at codimension two in X_{F3}

- $\xi{=}1/3, \langle\Phi_{(3,-1)}\rangle{\neq}0$ and $\xi{=}2/3, \langle\Phi_{(3,-2)}\rangle{\neq}0$: shrinking Higgs curves yields general cubic-fibration X_{F_1}

$$\boxed{G_{F_1} = \mathbb{Z}_3}$$

[D.K.,Mayorga Peña,Oehlmann,Piragua,Reuter]

- Third vacuum $\xi{=}0, \langle\Phi_{(3,0)}\rangle{\neq}0$ more involved [Cvetic,Donagi,D.K.,Piragua,Poretschkin]
  - candidate curve identified: class $[c_{(3,-1)}+ T^2]$ has right charges (3,0)
  - indirect evidence from Gromov-Witten invariant $\boxed{N_{c_{(3,-1)}+T^2}{=}1}$
  - holom. curve in $[c_{(3,-1)}+T^2]$ explicitly visible in complete intersection resolution.

III. The moduli space of rank two gauge groups in F-theory

[Cvetič, DK, Piragua, Taylor]

The general question

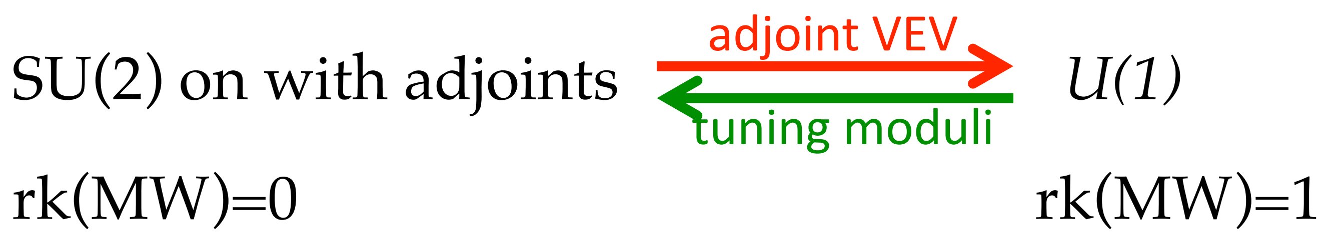
Question: What are the geometries describing adjoint Higgsings $G \rightarrow U(1)^{\mathrm{rk}(G)}$?

➡ Elliptic fibrations with higher rank MW-group crucial for F-theory moduli space.

Rank one case understood:

6D F-theory with SU(2) Higgses to model with $U(1)$ described by quartic in $\mathbb{P}^2(1,1,2)$.

[Morrison,Taylor]



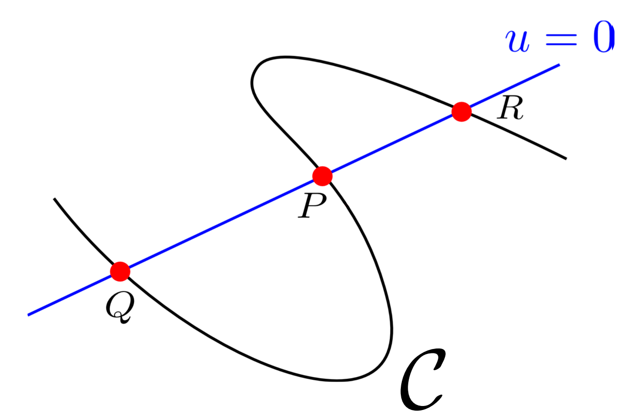
Here: Work out answer for rank two gauge groups $SU(2)\mathrm{x}SU(2)$, $SU(3)$

- Higgsings to $U(1)^2$ not seen in toric network
  - ➡ need more general, non-toric $U(1)^2$ model.

1) A more general model with $U(1)^2$

Construction of non-toric model with U(1)^2

Any elliptic fibration X with MW-rank two is cubic. [Deligne;Borchmann,Mayrhofer,Palti,Weigand; Cvetič,DK,Piragua]



$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$$f_2 = s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2$$

- Keep three rational points in elliptic fiber $\mathcal{C}$ at general positions

$$P = [0 : -b_1 : a_1] \quad Q = [0 : -b_2 : a_2] \qquad R = [0 : -b_3 : a_3]$$

➡ Coefficients a_i, b_i non-trivial polynomials: not considered before.

- Construct elliptically fibered CY-manifold X from $\mathcal{C}$ and arbitrary B.

[Cvetič, DK, Piragua, Taylor]

2) The low-energy effective theory

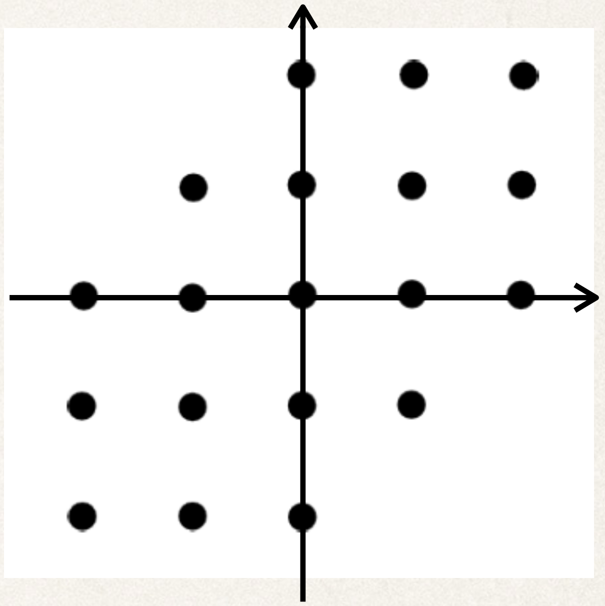
Full 6D spectrum over general base B

[Cvetič, DK, Piragua, Taylor]

Charges	Multiplicity
(2,0)	$x_{(2,0)} = [a_2] \cdot [b_2]$
(0,2)	$x_{(0,2)} = [a_3] \cdot [b_3]$
(-2,-2)	$x_{(-2,-2)} = [a_1] \cdot [b_1]$
(-1,1)	$x_{(-1,1)} = (2[b_2] + [s_3]) \cdot ([a_3] + [b_2]) - 2x_{(2,0)}$
(-2,-1)	$x_{(-2,-1)} = (2[b_3] + [s_3]) \cdot ([a_1] + [b_3]) - 2x_{(0,2)}$
(-1,-2)	$x_{(-2,-1)} = (2[b_1] + [s_3]) \cdot ([a_1] + [b_2]) - 2x_{(-1,1)}$
(1,1)	$x_{(1,1)} = ([a_1^4 a_2 b_3 s_8^2]) \cdot ([a_1^4 a_2^2 s_8^3]) - 2x_{(2,0)} - 8x_{(-2,-1)} - 4x_{(-1,-2)} - 20x_{(-2,-2)}$
(1,0)	$x_{(1,0)} = 4[b_1^3 b_2^3 s_3^3] \cdot ([a_1 b_2] - [K_B]) - 16x_{(2,0)} - 16x_{(-2,-1)} - x_{(-1,-2)} - 16x_{(-2,-2)} - x_{(-1,1)} - x_{(1,1)}$
(0,1)	$x_{(0,1)} = 4[b_1^3 b_3^3 s_3^3] \cdot ([a_1 b_3] - [K_B]) - x_{(-2,-1)} - 16x_{(0,2)} - 16x_{(-1,-2)} - 16x_{(-2,-2)} - x_{(-1,1)} - x_{(1,1)}$

➡ model specified by four divisor classes $[a_1]$, $[a_2]$, $[a_3]$, $[s_8]$ on B

U(1)xU(1) charge lattice



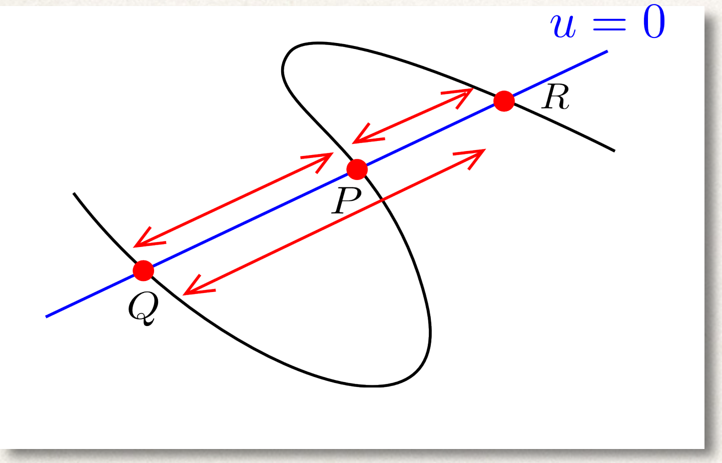
- rich particle spectrum with new types of representations
- checked cancellation of all 6D anomalies. ✔

3) The moduli space

Geometry of Higgs transitions in F-theory

<u>General strategy to reduce Mordell-Weil rank of X</u>

- tune moduli of X to place rational points on top of each other



$$u f_2(u, v, w) + \prod_{i=1}^{3} (a_i v + b_i w) = 0$$

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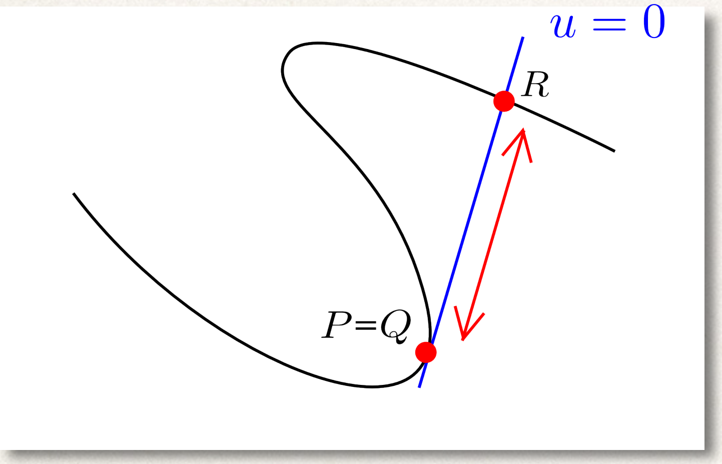


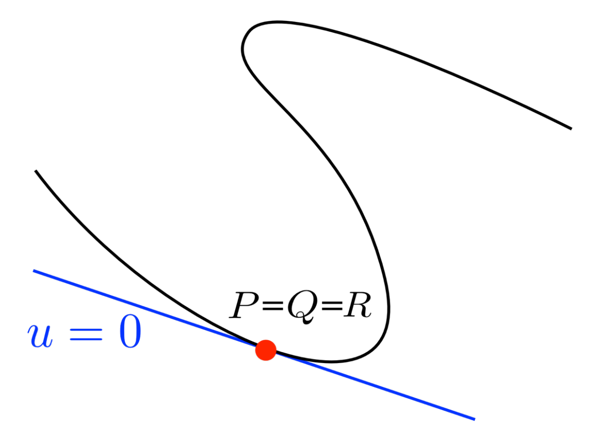
$$uf_2(u,v,w) + \lambda_1(a_1 v + b_1 w)^2(a_3 v + b_3 w) = 0$$

- rk(MW)=2 → 1: $\overline{PQ} \longrightarrow 0$

Geometry of Higgs transitions in F-theory

General strategy to reduce Mordell-Weil rank of *X*

- tune moduli of *X* to place rational points on top of each other



$$uf_2(u,v,w) + \lambda_1\lambda_2(a_1v + b_1w)^3 = 0$$

- rk(MW)=2 → 1: $\overline{PQ} \longrightarrow 0$ not possible in
- rk(MW)=1 → 0: $\overline{PR} \longrightarrow 0$ toric models

No *U(1)'s*, tuning induces non-Abelian group *G* in *X*

G=SU(2)xSU(2)xSU(3)

Higgsing back to Abelian theory by bifundamentals:

$$U(1)^2 \xleftarrow{\mathbf{(2,1,2)}} SU(2)xU(1)xSU(2) \xleftarrow{\mathbf{(1,2,3)}} SU(2)xSU(2)xSU(3)$$

Special cases: smaller *G* and Higgsing by adjoints.

Special cases: SU(2)^2 and SU(3)

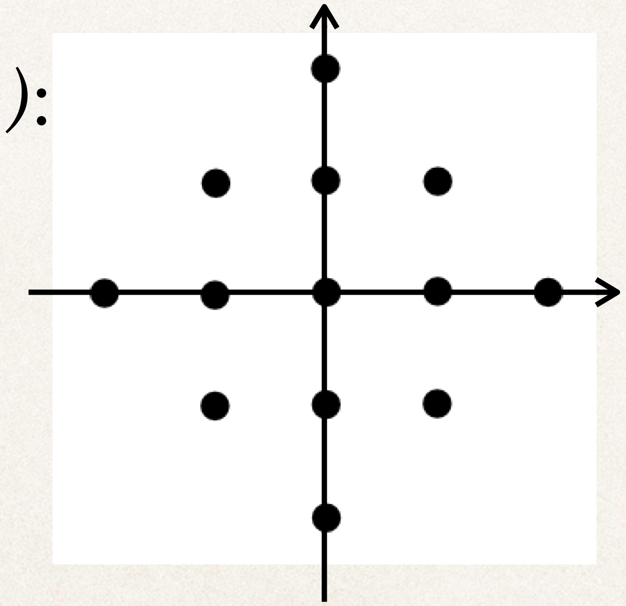
Different strata in moduli space of *X* yield ``unHiggsed'' models with

1. *G*=*SU(2)*x*SU(2)* and adjoints
2. *G*=*SU(3)* and adjoints

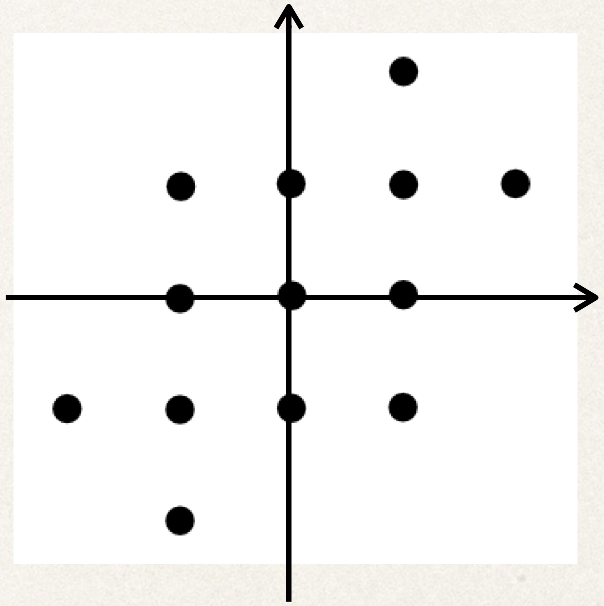
➡ Reproduce exactly their most general Weierstrass models. ✔

➡ Full spectra match. ✔

1. *SU(2)*x*SU(2)*:



2. *SU(3)*:



Can add matter with charges ±(2,2) to *SU(3)*-spectrum:

➡ Get first *SU(3)*-model with symmetric representation in global F-theory.

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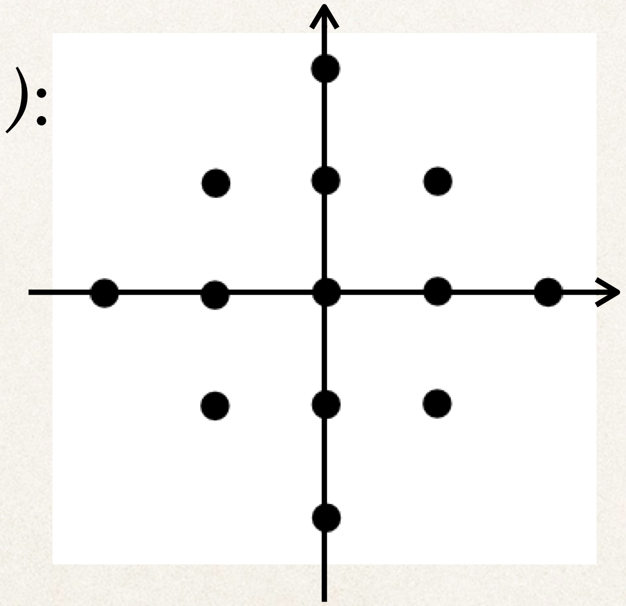
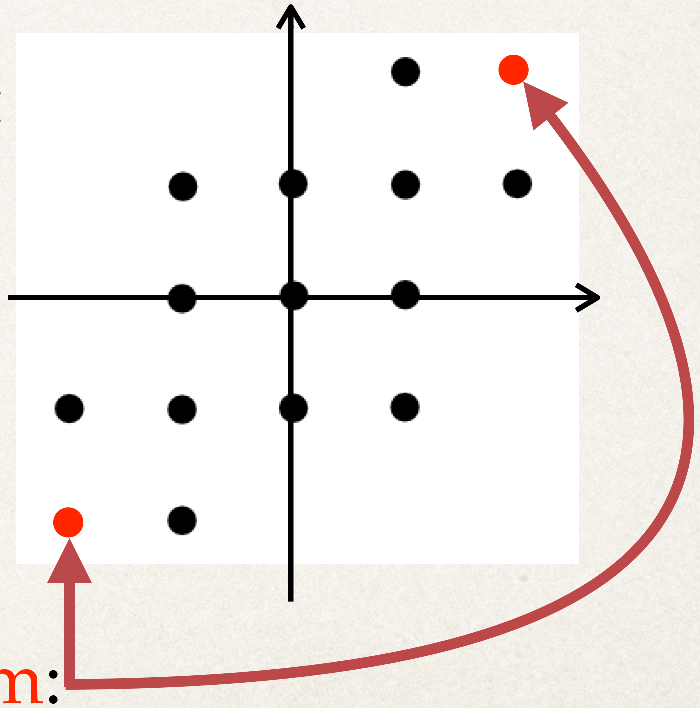


2\. *SU(3)*:



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4) Global F-theory with symmetrics of SU(3)

Global models of SU(3) with symmetric matter representations

[Cvetič, DK, Piragua, Taylor]

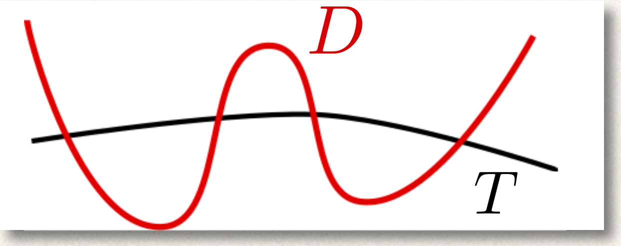
Tuned Abelian model has I_3-singularity over divisor with ordinary double point singularity at $a_1=b_1=0$:

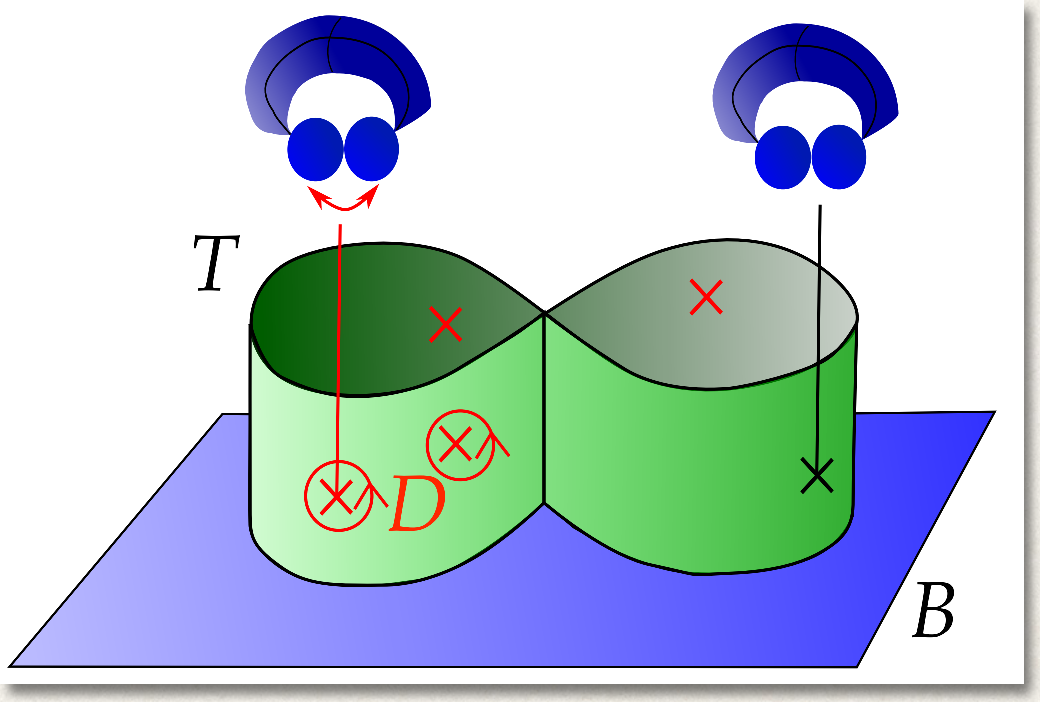
$$T = \{a_1^2 s_8 - a_1 b_1 s_6 + b_1^2 s_3 = 0\}$$

- Weierstrass model looks like I_3^{ns}: monodromy cover is irreducible

$$\psi^2 + (s_6^2 - 4 s_3 s_8) = 0$$
$$D := \{s_6^2 - 4 s_3 s_8 = 0\}$$

Generic situation: Z_2-monodromy exchanges nodes in fiber around $D \cap T \rightarrow I_3^{ns}$ yielding $G=SU(2)$





- Interplay between structure of T and fiberation: double point of T not deformable [Morrison, Taylor]

➡ first global example with symmetric + antisymmetric matter at $a_1=b_1=0$

Global models of SU(3) with symmetric matter representations

[Cvetič, DK, Piragua, Taylor]

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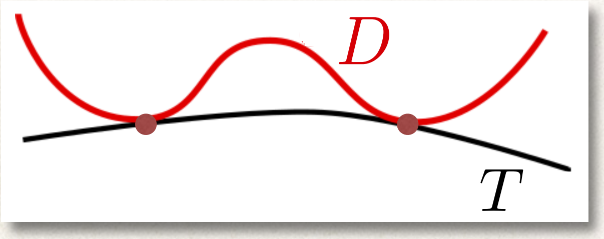
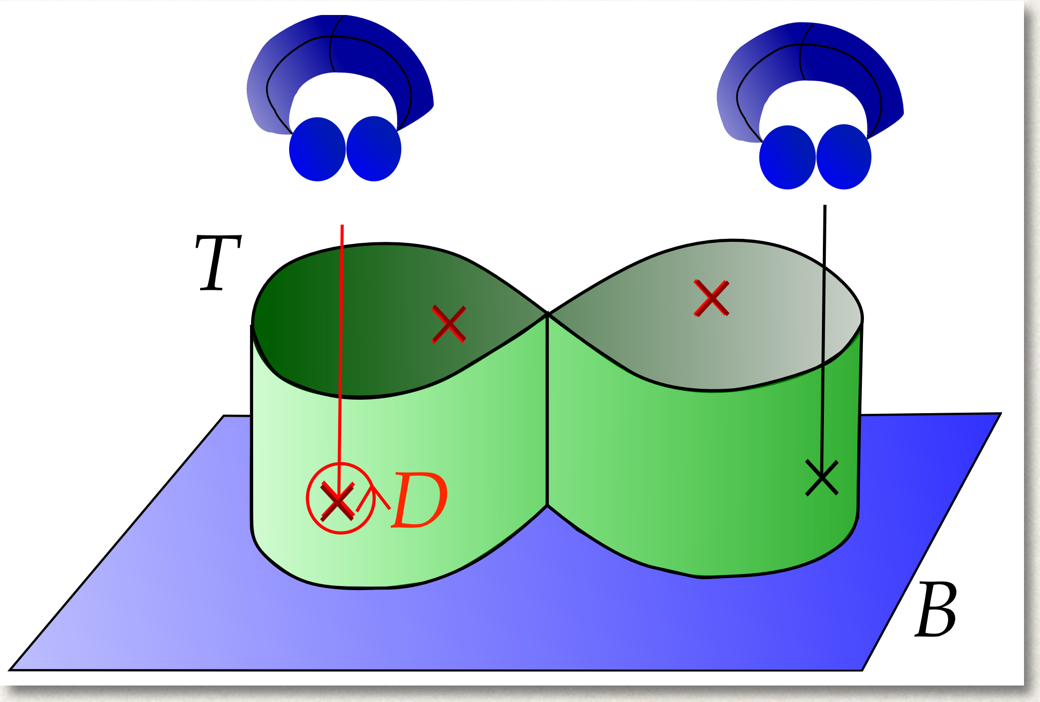
Here: D = discriminant locus of T

➡ intersection points in $D \cap T$ pair up: no monodromy



➡ I_3 is split: I_3^s yielding $SU(3)$



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III. Conclusions & Outlook

Summary

1. Analyzed all genus-one fibrations with fiber $\mathcal{C}_{F_i}$ in toric varieties associated to 16 2D reflexive polytopes F_i.
    - Full effective theories in 6D (= non-chiral 4D) determined: discrete gauge groups,…
    - Network of Higgsings relating all effective theories studied.
    - Construction of explicit three family SM, PS & Trinif. models.

2. Analysis of F-/M-theory vacua with Z_3 discrete gauge group: all Higgs-curves found.

3. Construction of general, non-toric CY-elliptic fibration with $U(1)^2$
    - Determination of full effective theory in 6D (= non-chiral 4D).
    - All models with rank two non-Abelian gauge groups embedded.
    - First explicit construction of $SU(3)$ gauge theory with two-times symmetric representation in global F-theory.

Outlook

- Explore further phenomenology of all toric hypersurface fibrations: add G_4-flux, compute chiralities… work with them!
- Study non-toric models for phenomenology: new features expected.

Thank you