Generalized CICY three-folds

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gCICY 3-folds

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. . . . . Smoothness

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String Phenomenology and Calabi-Yau Manifolds

- String Phenomenology
  - Connect string theory to phenomenology
  - Desirably obtain the SM in its N=1 SUSY version

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- N=1 Compactifications
  - Het on CY3 with a gauge background
  - Type II branes on CY3 [IIA, IIB, I]
  - M on G2 (IIA with D6)
  - F on elliptic CY4 (IIB with D7/D3)

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- **The Quintic 3-fold**
  - Zero locus in $\mathbb{P}^4$ of a quintic polynomial, $X = \{[\mathbf{x}] \,|\, p^{(5)}(\mathbf{x}) = 0\} \subset \mathbb{P}^4_{\mathbf{x}}$

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$(h^{1,1}, h^{2,1}) = (1, 101)$
$\chi = -200$

$c_2(X) = 10J^2$

$\int_X J^3 = 5$

Positive half-line,
for Kaehler/Mori cones

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  - Common zero locus of homogeneous polynomials in $\prod_{r=1}^{m} \mathbb{P}^{n_r}$
  - 7890 such CY3's; extensively used for string models

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$$\longleftrightarrow \quad X \sim \left[\begin{array}{c||cc} \mathbb{P}^2 & 2 & 1 \\ \mathbb{P}^3 & 2 & 2 \end{array}\right]$$

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CICY 3-folds

- **CICY3 Configuration Matrix**

$$X \sim \left[\begin{array}{c\|cccc} \mathbb{P}^{n_1} & a_1^1 & a_2^1 & \cdots & a_K^1 \\ \mathbb{P}^{n_2} & a_1^2 & a_2^2 & \cdots & a_K^2 \\ \vdots & \vdots & \ddots & \vdots & \\ \mathbb{P}^{n_m} & a_1^m & a_2^m & \cdots & a_K^m \end{array}\right]$$

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, with $a^r_\alpha \geq 0$

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$$\begin{array}{c} \sum_{r=1}^{m} n_r \\ \parallel \\ 3+K \end{array}$$

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- **Why positive-semidefinite configurations?**
  - Polynomial sections (in the ambient homogeneous coordinates)

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- **Analyzing CICYs**
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- **Why positive-semidefinite configurations?**
  - Polynomial sections (in the ambient homogeneous coordinates)
  - However, only needs to be able to "sequentially" construct sections!

gCICY 3-folds $\cdots$ Illustration

Generalities

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, with $a_\alpha^r \geq 0$ & $\begin{array}{c} \sum_{r=1}^{m} n_r \\ \| \\ 3 + K + L \end{array}$ & $\begin{array}{c} \sum_{\alpha=1}^{K} a_\alpha^r \\ + \\ \sum_{\mu=1}^{L} b_\mu^r \\ \| \\ 1 + n_r \end{array}$

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- **gCICY3 Configuration Matrix**

$$X \sim \left[\begin{array}{c||cccc|ccc} & p_1 & & & & & & \\ \mathbb{P}^{n_1} & a_1^1 & a_2^1 & \cdots & a_K^1 & b_1^1 & \cdots & b_L^1 \\ \mathbb{P}^{n_2} & a_1^2 & a_2^2 & \cdots & a_K^2 & b_1^2 & \cdots & b_L^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ \mathbb{P}^{n_m} & a_1^m & a_2^m & \cdots & a_K^m & b_1^m & \cdots & b_L^m \\ & \mathcal{M}_1 & & & & & & \end{array}\right]$$

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$\mathcal{M}_1 \supset \mathcal{M}_2$

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$$X \sim \left[\begin{array}{c||cccc|ccc} & p_1 & p_2 & \cdots & p_K & q_1 & & \\ \mathbb{P}^{n_1} & a^1_1 & a^1_2 & \cdots & a^1_K & b^1_1 & \cdots & b^1_L \\ \mathbb{P}^{n_2} & a^2_1 & a^2_2 & \cdots & a^2_K & b^2_1 & \cdots & b^2_L \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ \mathbb{P}^{n_m} & a^m_1 & a^m_2 & \cdots & a^m_K & b^m_1 & \cdots & b^m_L \end{array}\right]$$

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$$\mathcal{M}_1 \supset \mathcal{M}_2 \supset \cdots \supset \mathcal{M}_K \supset \mathcal{M}_{K+1}$$

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 & p_1 & p_2 & \cdots & p_K & q_1 & \cdots & q_L \\
\mathbb{P}^{n_1} & a_1^1 & a_2^1 & \cdots & a_K^1 & b_1^1 & \cdots & b_L^1 \\
\mathbb{P}^{n_2} & a_1^2 & a_2^2 & \cdots & a_K^2 & b_1^2 & \cdots & b_L^2 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\
\mathbb{P}^{n_m} & a_1^m & a_2^m & \cdots & a_K^m & b_1^m & \cdots & b_L^m
\end{array}\right] \text{, with } a_\alpha^r \geq 0 \quad \& \quad \text{3-fold} \quad \& \quad \text{CY}
$$

$$
\mathcal{M}_1 \supset \mathcal{M}_2 \supset \cdots \supset \mathcal{M}_K \supset \mathcal{M}_{K+1} \supset \quad \supset \mathcal{M}_{K+L} \equiv X
$$

- Motivations for going beyond CICYs?

gCICY 3-folds $\cdots$ Illustration

Generalities

- **gCICY3 Configuration Matrix**

$$X \sim \left[\begin{array}{c||cccc|ccc} & p_1 & p_2 & \cdots & p_K & q_1 & \cdots & q_L \\ \mathbb{P}^{n_1} & a_1^1 & a_2^1 & \cdots & a_K^1 & b_1^1 & \cdots & b_L^1 \\ \mathbb{P}^{n_2} & a_1^2 & a_2^2 & \cdots & a_K^2 & b_1^2 & \cdots & b_L^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ \mathbb{P}^{n_m} & a_1^m & a_2^m & \cdots & a_K^m & b_1^m & \cdots & b_L^m \end{array}\right]$$

, with $a^r_\alpha \geq 0$ & 3-fold & CY

$$\mathcal{M}_1 \supset \mathcal{M}_2 \supset \cdots \supset \mathcal{M}_K \supset \mathcal{M}_{K+1} \supset \quad \supset \mathcal{M}_{K+L} \equiv X$$

- **Motivations for going beyond CICYs?**
  - A new class of CY 3-folds

gCICY 3-folds $\cdots$ Illustration

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  - A new class of CY 3-folds
  - Chance to achieve a positive Euler number **cf. Toric construction [Batyrev], [Kreuzer, Skarke]**

gCICY 3-folds ···· Illustration

Generalities

- **gCICY3 Configuration Matrix**

$$X \sim \left[\begin{array}{c||cccc|ccc} & p_1 & p_2 & \cdots & p_K & q_1 & \cdots & q_L \\ \mathbb{P}^{n_1} & a_1^1 & a_2^1 & \cdots & a_K^1 & b_1^1 & \cdots & b_L^1 \\ \mathbb{P}^{n_2} & a_1^2 & a_2^2 & \cdots & a_K^2 & b_1^2 & \cdots & b_L^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ \mathbb{P}^{n_m} & a_1^m & a_2^m & \cdots & a_K^m & b_1^m & \cdots & b_L^m \end{array}\right]$$

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gCICY 3-folds ···· Illustration

Generalities

- **gCICY3 Configuration Matrix**

$$X \sim \begin{array}{c} \begin{array}{cccccccc} \phantom{\mathbb{P}^{n_m}} & p_1 & p_2 & \cdots & p_K & q_1 & \cdots & q_L \end{array} \\ \left[\begin{array}{c||cccc|ccc} \mathbb{P}^{n_1} & a_1^1 & a_2^1 & \cdots & a_K^1 & b_1^1 & \cdots & b_L^1 \\ \mathbb{P}^{n_2} & a_1^2 & a_2^2 & \cdots & a_K^2 & b_1^2 & \cdots & b_L^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ \mathbb{P}^{n_m} & a_1^m & a_2^m & \cdots & a_K^m & b_1^m & \cdots & b_L^m \end{array}\right] \end{array}$$

, with $a_\alpha^r \geq 0$ & 3-fold & CY

$$\mathcal{M}_1 \supset \mathcal{M}_2 \supset \cdots \supset \mathcal{M}_K \supset \mathcal{M}_{K+1} \supset \quad \supset \mathcal{M}_{K+L} \equiv X$$

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  - A new class of CY 3-folds
  - Chance to achieve a positive Euler number **cf. Toric construction [Batyrev], [Kreuzer, Skarke]**
  - Elliptic-fibration **cf. [Gray' talk]**, but in many cases with singularities **cf. [Taylor's talk]**

gCICY 3-folds .... Illustration

Generalities

- gCICY3 Configuration Matrix

$$X \sim \left[\begin{array}{c||cc} \mathbb{P}^1 & 3 & -1 \\ \mathbb{P}^4 & 2 & 3 \end{array}\right]$$

- Motivations for going beyond CICYs?
  - A new class of CY 3-folds
  - Chance to achieve a positive Euler number **cf. Toric construction [Batyrev], [Kreuzer, Skarke]**
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gCICY 3-folds $\cdots$ Illustration

Generalities

- gCICY3 Configuration Matrix

$$X \sim \left[\, \underbrace{\boxed{\begin{array}{c||c} \mathbb{P}^1 & \overset{p}{3} \\ \mathbb{P}^4 & 2 \end{array}}}_{\mathcal{M}} \; \begin{array}{c} -1 \\ 3 \end{array} \right]$$

- Motivations for going beyond CICYs?
  - A new class of CY 3-folds
  - Chance to achieve a positive Euler number **cf. Toric construction [Batyrev], [Kreuzer, Skarke]**
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gCICY 3-folds $\cdots$ Illustration

Generalities

- gCICY3 Configuration Matrix

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1 & 3 & -1 \\ \mathbb{P}^4 & 2 & 3 \end{array}\right]$$

$$\mathcal{M} \supset X$$

- Motivations for going beyond CICYs?
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gCICY 3-folds ···· Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||c c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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gCICY 3-folds $\cdots$ Illustration

Analysis: Section Construction

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$$\mathcal{M} \supset X$$

$$p \in \Gamma(\mathbb{P}^1 \times \mathbb{P}^4, \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(3,2))$$

$$q \in \Gamma(\mathcal{M}, \mathcal{O}_{\mathcal{M}}(-1,3))$$

gCICY 3-folds ···· Illustration

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$p \in \Gamma(\mathbb{P}^1 \times \mathbb{P}^4, \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(3,2))$ ~ dim=60

$q \in \Gamma(\mathcal{M}, \mathcal{O}_{\mathcal{M}}(-1,3))$

gCICY 3-folds $\cdots$ Illustration

Analysis: Section Construction

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gCICY 3-folds ···· Illustration

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- **Sequential Section Construction**

gCICY 3-folds $\cdots$ Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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- Sequential Section Construction

gCICY 3-folds …. Illustration

Analysis: Section Construction

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- **Sequential Section Construction**
  - Divisor of zeros and of poles

gCICY 3-folds .... Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||cc} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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- **Sequential Section Construction**
  - Divisor of zeros and of poles **cf. [Gao's talk]**

gCICY 3-folds ···· Illustration

Analysis: Section Construction

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- **Sequential Section Construction**
  - Divisor of zeros and of poles cf. [Gao's talk]

$$\boxed{\begin{matrix}-1\\3\end{matrix}} = \boxed{\begin{matrix}0\\3\end{matrix}} - \boxed{\begin{matrix}1\\0\end{matrix}}$$

gCICY 3-folds ···· Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||cc} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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$$q \searrow \frac{N^{(0,3)}(\mathbf{x},\mathbf{y})}{D^{(1,0)}(\mathbf{x},\mathbf{y})}$$

- **Sequential Section Construction**
  - Divisor of zeros and of poles cf. [Gao's talk]

$$\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

gCICY 3-folds .... Illustration

Analysis: Section Construction

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$$q \longrightarrow \frac{N^{(0,3)}(\mathbf{x},\mathbf{y})}{D^{(1,0)}(\mathbf{x},\mathbf{y})}$$

- **Sequential Section Construction**
  - Divisor of zeros and of poles
  - $D = 0$ in $\mathcal{M}$ should imply $N = 0$

gCICY 3-folds ···· Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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  - Divisor of zeros and of poles
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Key $p(\mathbf{x},\mathbf{y}) \equiv 0$

gCICY 3-folds ···· Illustration

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- **Sequential Section Construction**
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Key $p(\mathbf{x},\mathbf{y}) \equiv 0 \rightarrow \boxed{x_0^3\,\Pi_1^{(2)}(\mathbf{y}) + x_0^2 x_1\,\Pi_2^{(2)}(\mathbf{y}) + x_0 x_1^2\,\Pi_3^{(2)}(\mathbf{y}) + x_1^3\,\Pi_4^{(2)}(\mathbf{y}) \equiv 0}$

gCICY 3-folds $\cdots$ Illustration

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  - E.g., with $D^{(1,0)}(\mathbf{x},\mathbf{y}) = x_0$

gCICY 3-folds $\cdots$ Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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$$q \searrow \frac{N^{(0,3)}(\mathbf{x},\mathbf{y})}{D^{(1,0)}(\mathbf{x},\mathbf{y})}$$

- **Sequential Section Construction**
  - Divisor of zeros and of poles
  - $D=0$ in $\mathcal{M}$ should imply $N=0$

Key $p(\mathbf{x},\mathbf{y}) \equiv 0 \rightarrow \boxed{x_0^3\,\Pi_1^{(2)}(\mathbf{y}) + x_0^2 x_1\,\Pi_2^{(2)}(\mathbf{y}) + x_0 x_1^2\,\Pi_3^{(2)}(\mathbf{y}) + x_1^3\,\Pi_4^{(2)}(\mathbf{y}) \equiv 0}$

  - E.g., with $D^{(1,0)}(\mathbf{x},\mathbf{y}) = x_0$, $D=0$ in $\mathcal{M}$ implies $\Pi_4^{(2)}(\mathbf{y}) = 0$

gCICY 3-folds .... Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c|c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

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$$q \rightarrow \frac{N^{(0,3)}(\mathbf{x},\mathbf{y})}{D^{(1,0)}(\mathbf{x},\mathbf{y})}$$

- **Sequential Section Construction**
  - Divisor of zeros and of poles
  - $D = 0$ in $\mathcal{M}$ should imply $N = 0$

Key $p(\mathbf{x},\mathbf{y}) \equiv 0 \rightarrow \boxed{x_0^3\,\Pi_1^{(2)}(\mathbf{y}) + x_0^2 x_1\,\Pi_2^{(2)}(\mathbf{y}) + x_0 x_1^2\,\Pi_3^{(2)}(\mathbf{y}) + x_1^3\,\Pi_4^{(2)}(\mathbf{y}) \equiv 0}$

  - E.g., with $D^{(1,0)}(\mathbf{x},\mathbf{y}) = x_0$, $D = 0$ in $\mathcal{M}$ implies $\Pi_4^{(2)}(\mathbf{y}) = 0$ and hence, $\frac{\Pi_4^{(2)}(\mathbf{y})}{x_0}\, l^{(1)}(\mathbf{y})$ are holomorphic sections of $O_{\mathcal{M}}(-1,3)$

gCICY 3-folds ···· Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

$$\mathcal{M} \supset X$$

$p \in \Gamma(\mathbb{P}^1 \times \mathbb{P}^4, \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(3,2))$ ~ dim=60

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- **Sequential Section Construction**
  - Divisor of zeros and of poles
  - $D = 0$ in $\mathcal{M}$ should imply $N = 0$

Key $p(\mathbf{x},\mathbf{y}) \equiv 0 \rightarrow \boxed{x_0^3\,\Pi_1^{(2)}(\mathbf{y}) + x_0^2 x_1\,\Pi_2^{(2)}(\mathbf{y}) + x_0 x_1^2\,\Pi_3^{(2)}(\mathbf{y}) + x_1^3\,\Pi_4^{(2)}(\mathbf{y}) \equiv 0}$

- $D^{(1,0)}(\mathbf{x},\mathbf{y}) = x_0$ gives 5 sections of $O_{\mathcal{M}}(-1,3)$:

$$\frac{\Pi_4^{(2)}(\mathbf{y})}{x_0}\,y_i \text{ for } i = 0, ..., 4$$

gCICY 3-folds ···· Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c|c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

$$\mathcal{M} \supset X$$

$p \in \Gamma(\mathbb{P}^1 \times \mathbb{P}^4, \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(3,2))$ ~ dim=60

$q \in \Gamma(\mathcal{M}, \mathcal{O}_{\mathcal{M}}(-1,3))$ ~ dim=15

$$q \rightarrow \frac{N^{(0,3)}(\mathbf{x},\mathbf{y})}{D^{(1,0)}(\mathbf{x},\mathbf{y})}$$

- **Sequential Section Construction**
  - Divisor of zeros and of poles
  - $D = 0$ in $\mathcal{M}$ should imply $N = 0$

Key $p(\mathbf{x},\mathbf{y}) \equiv 0 \rightarrow \boxed{x_0^3\,\Pi_1^{(2)}(\mathbf{y}) + x_0^2 x_1\,\Pi_2^{(2)}(\mathbf{y}) + x_0 x_1^2\,\Pi_3^{(2)}(\mathbf{y}) + x_1^3\,\Pi_4^{(2)}(\mathbf{y}) \equiv 0}$

- $D^{(1,0)}(\mathbf{x},\mathbf{y}) = x_0$, $x_0 - x_1$, and $x_0 + x_1$ each gives 5 sections of $O_{\mathcal{M}}(-1,3)$:

$$\frac{\Pi_4^{(2)}(\mathbf{y})}{x_0}\, y_i, \quad \frac{\sum_{\alpha=1}^{4} \Pi_\alpha^{(2)}(\mathbf{y})}{x_0 - x_1}\, y_i, \text{ and } \frac{\sum_{\alpha=1}^{4} (-1)^\alpha\, \Pi_\alpha^{(2)}(\mathbf{y})}{x_0 + x_1}\, y_i \quad \text{for } i = 0, \ldots, 4$$

gCICY 3-folds $\cdots$ Illustration

Analysis: Section Construction

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

$\mathcal{M} \supset X$

$p \in \Gamma(\mathbb{P}^1 \times \mathbb{P}^4, \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(3,2))$ ~ dim=60

$q \in \Gamma(\mathcal{M}, \mathcal{O}_{\mathcal{M}}(-1,3))$ ~ dim=15

$$\searrow \frac{N^{(0,3)}(\mathbf{x},\mathbf{y})}{D^{(1,0)}(\mathbf{x},\mathbf{y})}$$

- **Sequential Section Construction**
  - Divisor of zeros and of poles
  - $D = 0$ in $\mathcal{M}$ should imply $N = 0$

Key $p(\mathbf{x},\mathbf{y}) \equiv 0 \rightarrow \boxed{x_0^3\,\Pi_1^{(2)}(\mathbf{y}) + x_0^2 x_1\,\Pi_2^{(2)}(\mathbf{y}) + x_0 x_1^2\,\Pi_3^{(2)}(\mathbf{y}) + x_1^3\,\Pi_4^{(2)}(\mathbf{y}) \equiv 0}$

- $D^{(1,0)}(\mathbf{x},\mathbf{y}) = x_0$, $x_0 - x_1$, and $x_0 + x_1$ each gives 5 sections of $O_{\mathcal{M}}(-1,3)$:

$$\frac{\Pi_4^{(2)}(\mathbf{y})}{x_0}\, y_i,\ \frac{\sum_{\alpha=1}^4 \Pi_\alpha^{(2)}(\mathbf{y})}{x_0 - x_1}\, y_i \text{, and } \frac{\sum_{\alpha=1}^4 (-1)^\alpha\, \Pi_\alpha^{(2)}(\mathbf{y})}{x_0 + x_1}\, y_i \text{ for } i = 0, \ldots, 4$$

Can check these 15 are linearly independent & no more independent sections arise!

gCICY 3-folds ···· Illustration

Analysis: Smoothness

$$X \sim \left[\begin{array}{c||c|c} & p & q \\ \mathbb{P}^1_{\mathbf{x}} & 3 & -1 \\ \mathbb{P}^4_{\mathbf{y}} & 2 & 3 \end{array}\right]$$

$\mathcal{M} \supset X$

$p \in \Gamma(\mathbb{P}^1 \times \mathbb{P}^4, \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(3,2))$ ~ dim=60

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gCICY 3-folds $\cdots$ Illustration

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- **Remark**
  - Smoothness is hard to achieve since section space often factorizes as a whole (i.e., a generic section factorizes in many cases)

gCICY 3-folds $\cdots$ Illustration

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  - Smooth and reduced

gCICY 3-folds ···· Illustration

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- **The Techniques for CICYs Still Apply**
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gCICY 3-folds …. Illustration

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- **Simplifications**
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- **The Techniques for CICYs Still Apply**
  - Hodge numbers $(h^{1,1}, h^{2,1}) = (2, 46);\ \chi = -88$
  - Chern class $\mathbf{c}_2 = 24\,\nu^{\mathbf{x}} + 46\,\nu^{\mathbf{y}}$, where $\int_X J_s \wedge \nu^t = \delta^t_s$ and $H^{1,1}(X) = \text{Span}\,\langle J_{\mathbf{x}}, J_{\mathbf{y}} \rangle$
  - Intersection numbers $d_{\mathbf{xyy}} = 6;\ d_{\mathbf{yyy}} = 7,$ where $d_{stu} = \int_X J_s J_t J_u$

Summary

- **CICYs have been generalized to gCICYs** by allowing for negative entries in the configuration matrix.
- In this new class, there are **new smooth CY3's** that had not been found/understood; **their topology has been analyzed.**

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Outlook

- **A systematic scan** -- *more CY3 geometries* and/or *gCICY3 classification*
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THANK YOU