Madrid June 2015

F-theory GUTs with Discrete Symmetry Extensions

George Leontaris

Ioannina University

GREECE

Outline of the Talk

- Introductory remarks
- F-theory and Elliptic Fibration
- F-GUTs with discrete symmetries
- Mordell-Weil $U(1)$ and GUTs
- Concluding Remarks

$\mathcal{A}$

Properties of Ordinary $GUTs$

★ interesting features

- ▲ Gauge coupling unification
- ▲ Assembling of SM fermions in a few irreps.
- ▲ Charge Quantisation

★ deficiencies

- ▲ fermion mass hierarchy and mixing not predicted
- ▲ Yukawa Lagrangian poorly constrained
- ▲ Baryon number non-conservation

... Solution requires new insights ... such as:

Discrete and $U(1)$ symmetry extensions

▲ These appear naturally in $\mathcal{F}-\mathcal{THEORY}$ constructions ▲

New Ingredients from F-theory

★ Discrete and $U(1)$ symmetries:

- necessary tools to suppress or eliminate undesired superpotential terms

★ Fluxes :

- ... truncate GUT irreps, eliminate coloured Higgs triplets, induce chirality...

★ "Internal" Geometry :

- ... determines SM arbitrary parameters from a handful of topological properties

$\mathcal{B}$

F-theory and Elliptic Fibration

★ **F-theory** ★

(*Vafa 1996*)

$$\Downarrow$$

Geometrisation of Type II-B superstring

II-B: *closed string spectrum obtained by combining left and right moving open strings with NS and R-boundary conditions:*

$$(NS_+, NS_+),\ (R_-, R_-),\ (NS_+, R_-),\ (R_-, NS_+)$$

Bosonic spectrum:

(NS_+, NS_+): graviton, dilaton and 2-form KB-field:

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \rightarrow B_2$$

(R_-, R_-): scalar, 2- and 4-index fields (p*-form potentials*)

$$C_0, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \rightarrow C_p,\ p = 0, 2, 4$$

Definitions (F*-theory bosonic part*)

1. *String coupling:* $g_s = e^{-\phi}$

2. *Combining the two scalars* $C_0,\ \phi$ *to one modulus:*

$$\tau = C_0 + i\, e^{\phi} \to C_0 + \frac{i}{g_s}$$

IIB - *action (see e.g. Denef, 0803:1194):*

$$S_{IIB} \quad \propto \quad \int d^{10}x\sqrt{-g}\, R - \frac{1}{2}\int \frac{1}{(\mathrm{Im}\tau)^2} d\tau \wedge * d\bar{\tau}$$

$$+ \quad \frac{1}{\mathrm{Im}\tau} G_3 \wedge * \overline{G}_3 + \frac{1}{2}\tilde{F}_5 \wedge * \tilde{F}_5 + C_4 \wedge H_3 \wedge F_3$$

Property:

Invariant under $SL(2,Z)$ S-duality:

$\tau \to \frac{a\tau+b}{c\tau+d}$

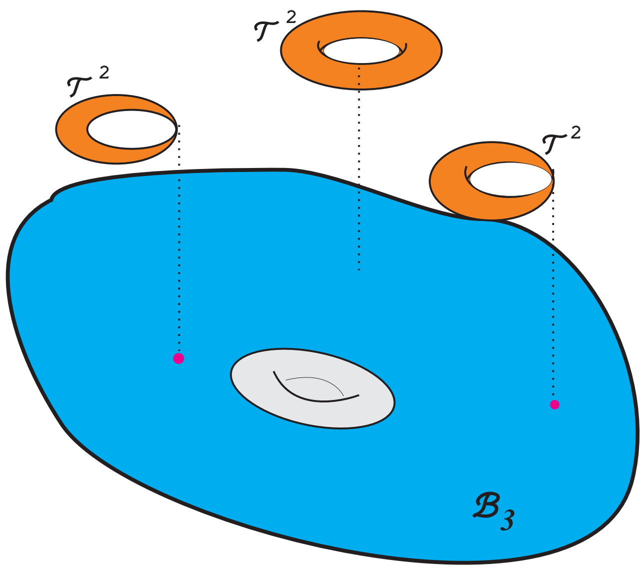
$\mathcal{FIBRATION}$

F-theory $\mathcal{R}^{3,1} \times \mathcal{X}$

$\Rightarrow$ $\mathcal{X}$, elliptically fibered **CY** 4-fold over B_3 $\Leftarrow$

$\Downarrow$

▲ a torus $\tau = C_0 + \imath/g_s$ at each point of B_3 ▲



Elliptic Fibration

described by $\mathcal{W}$eierstraß $\mathcal{E}$quation

$$y^2 \quad = \quad x^3 + f(w)\, xz^4 + g(w)z^6$$

For each point of B_3, the above equation describes a torus

1. x, y, z homogeneous coordinates
2. $f(w),\ g(w) \to 8^{th}$ and 12^{th} degree polynomials.
3. Discriminant

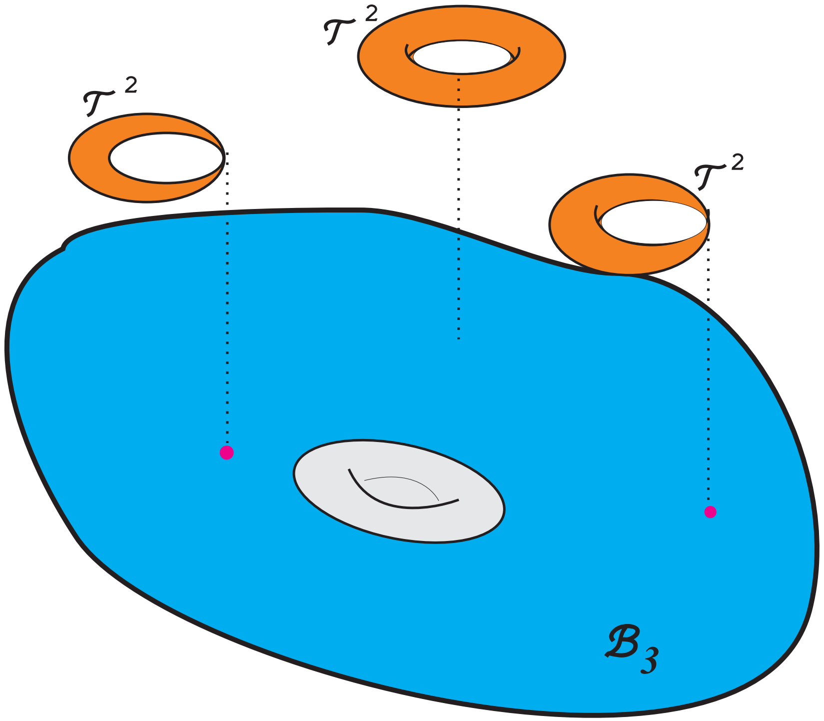
$$\Delta(w) = 4\, f^3 + 27\, g^2$$

Fiber singularities at

$$\Delta(w) = 0 \to 24 \text{ roots } w_i$$

$$\Downarrow$$

Manifold Singularities



CY 4-fold: Red points: pinched torus $\Rightarrow$ 7-branes $\perp B_3$

Kodaira classification:

- Type of Manifold **singularity** is specified by the vanishing order of $f(w),\ g(w)$ and $\Delta(w)$
- **Singularities** are classified in terms of $\mathcal{A}\,\mathcal{D}\,\mathcal{E}$ Lie groups (*Kodaira*).

Interpretation of geometric singularities

$$\Downarrow$$

$$\boxed{CY_4\text{-}\textbf{Singularities} \rightleftarrows \textbf{gauge symmetries}}$$

$$\textbf{Groups} \quad \rightarrow \quad \begin{cases} SU(n) \\ SO(m) \\ \mathcal{E}_n \end{cases}$$

Tate's Algorithm

$$y^2 + \alpha_1 x\, y\, z + \alpha_3 y\, z^3 = x^3 + \alpha_2\, x^2 z^2 + \alpha_4 x\, z^4 + \alpha_6 z^6$$

Table: *Classification of Elliptic Singularities w.r.t. vanishing order of Tate's form coefficients* α_i*:*

Group	α_1	α_2	α_3	α_4	α_6	Δ
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SU(5)$	0	1	2	3	5	5
$SO(10)$	1	1	2	3	5	7
$\mathcal{E}_6$	1	2	3	3	5	8
$\mathcal{E}_7$	1	2	3	3	5	9
$\mathcal{E}_8$	1	2	3	4	5	10

Basic ingredient in F-theory:

$D7$ - brane

GUTs are associated to 7-branes wrapping certain classes of *'internal'* **2-complex dim.** surface $\mathbf{S} \subset B_3$

▲ Gauge symmetry:

$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}$$

▲ $G_{GUT} = SU(5),\ SO(10),\ \ldots$

★ $\mathcal{C}$ *Commutant ...* $\Rightarrow$ *monodromies:*

$$U(1)^n, \text{or discrete symmetry } S_n,\ A_n,\ D_n,\ Z_n$$

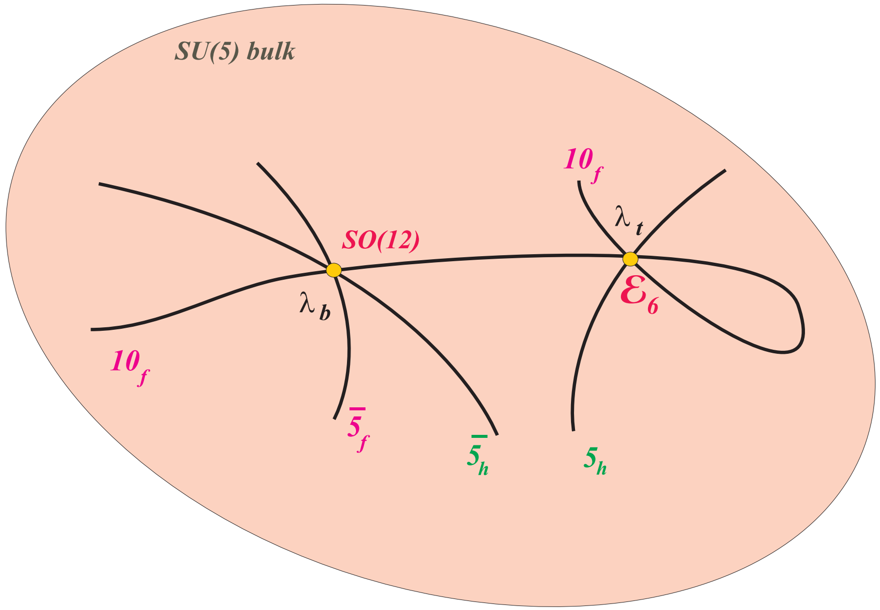
... acting as family or discrete symmetries

Model in this talk: $SU(5) : \mathcal{E}_8 \to SU(5) \times SU(5)_\perp \to \mathcal{C} = SU(5)_\perp$.

Spectral Cover $\mathcal{C}$ described by

$$\mathcal{C} : \sum_k b_k s^{5-k} = 0, \; b_1 = 0, \; \text{roots} \to t_i$$

Matter resides in 10 and $\bar{5}$ along intersections with other 7-branes



$\lambda_{t,b}$-Yukawas at intersections and gauge symmetry enhancements

(Heckman et al 0811.2417; Font et al 0907.4895; GG Ross, GKL, 1009.6000);

(Cecotti et al 0910.0477; Camara et al, 1110,2206; Aparicio et al, 1104.2609,. . .)

$\mathcal{C}$

Non-Abelian Discrete Symmetries

▲ Application: *Spectral Cover splitting*: $\mathcal{C}_5 \to \mathcal{C}_4 \times \mathcal{C}_1$

▲ Motivation: The neutrino sector (*TB-mixing*)

▲ $\mathcal{C}_4 \times \mathcal{C}_1$ implies the splitting of the $\mathcal{C}_5$ polynomial in two factors

$$\sum_k b_k s^{5-k} = \underbrace{(a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4)}_{\mathcal{C}_4} \underbrace{(a_6 + a_7 s)}_{\mathcal{C}_1}$$

Topological properties of a_i are fixed in terms of those of b_k, by equating coefficients of same powers of s

$$b_0 = a_5 a_7, \; b_5 = a_1 a_6, \; etc...$$

Moreover:

▲ $\mathcal{C}_1$: associated to a $\mathcal{U}(1)$

▲ $\mathcal{C}_4$: reduction to

(i) continuous $SU(4)$ subgroup, or

(ii) to Galois group $\in S_4$

(*see Heckman et al, 0906.0581, Marsano et al, 09012.0272, I. Antoniadis and GKL 1308.1581*)

Properties and Residual Spectral Cover Symmetry

- If $\mathcal{H} \in S_4$ the **Galois** group, final symmetry of the model is:

$$SU(5)_{GUT} \times \underbrace{\mathcal{H} \times \mathcal{U}(1)}_{family\ symmetry}$$

- $\mathcal{H} \in S_4$ is linked to specific topological properties of the polynomial coefficients a_i.
- a_i coefficients determine useful properties of the model, such as

i) **Geometric** symmetries $\longrightarrow$ $\mathcal{R}$-parity

ii) **Flux** restrictions on the matter curves

- Fluxes determine useful properties on the matter curves including :

Multiplicities and **Chirality** of matter/Higgs representations

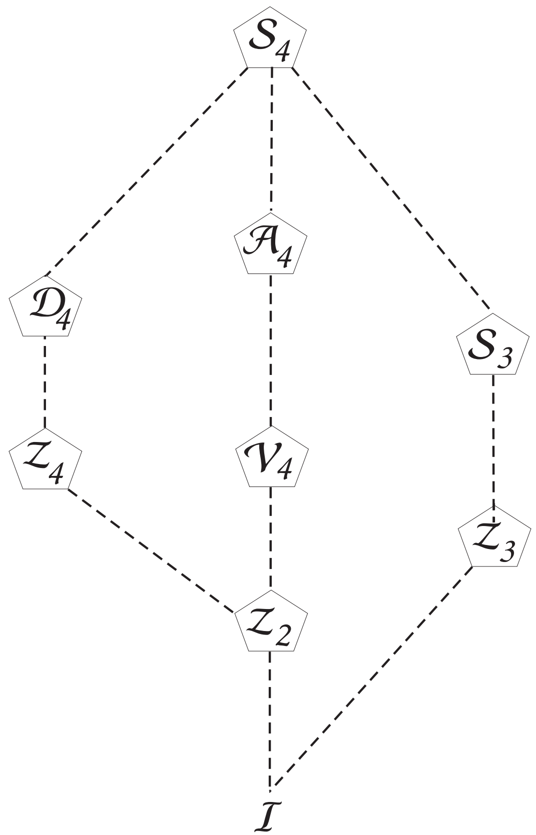


Figure 1: S_4 and the relevant discrete subgroups

The **Galois** group in $\mathcal{C}_4$

Determination of the **Galois** group, requires examination of *(partially) symmetric* functions of roots t_i of the polynomial $\mathcal{C}_4$. For our purposes, it suffices to examine the Discriminant and the Resolvent

$1.)$ **The Discriminant** Δ

$$\Delta = \delta^2 \quad \text{where} \quad \delta = \prod_{i<j} (t_i - t_j)$$

▲ δ *is invariant under* S_4**-even** *permutations* $\Rightarrow \mathcal{A}_4$

Δ symmetric $\rightarrow$ can be expressed in terms of coefficients $a_i \in \mathcal{F}$

$$\Delta(t_i) \; \rightarrow \; \Delta(a_i)$$

If $\Delta = \delta^2$, such that $\delta(a_i) \in \mathcal{F}$, then

$$\mathcal{H} \subseteq \mathcal{A}_4 \text{ or } V_4 \;\; (= Klein\ group)$$

If $\Delta \neq \delta^2$, (i.e. $\delta(a_i) \notin \mathcal{F}$), then

$$\mathcal{H} \subseteq \mathcal{S}_4 \text{ or } \mathcal{D}_4$$

$2.)$ *To study possible reductions of* S_4, A_4 *to their subgroups, we examine the resolvent:*

$$f(x) = (x - x_1)(x - x_2)(x - x_3)$$

$$x_1 = t_1 t_2 + t_3 t_4, \quad x_2 = t_1 t_3 + t_2 t_4, \quad x_3 = t_2 t_3 + t_1 t_4$$

$x_{1,2,3}$ are invariant under the three *Dihedral groups* $D_4 \in S_4$.

Combined results of Δ and $f(x)$:

	$\Delta \neq \delta^2$	$\Delta = \delta^2$
$f(x)$ irreducible	S_4	A_4
$f(x)$ reducible	D_4, Z_4	V_4

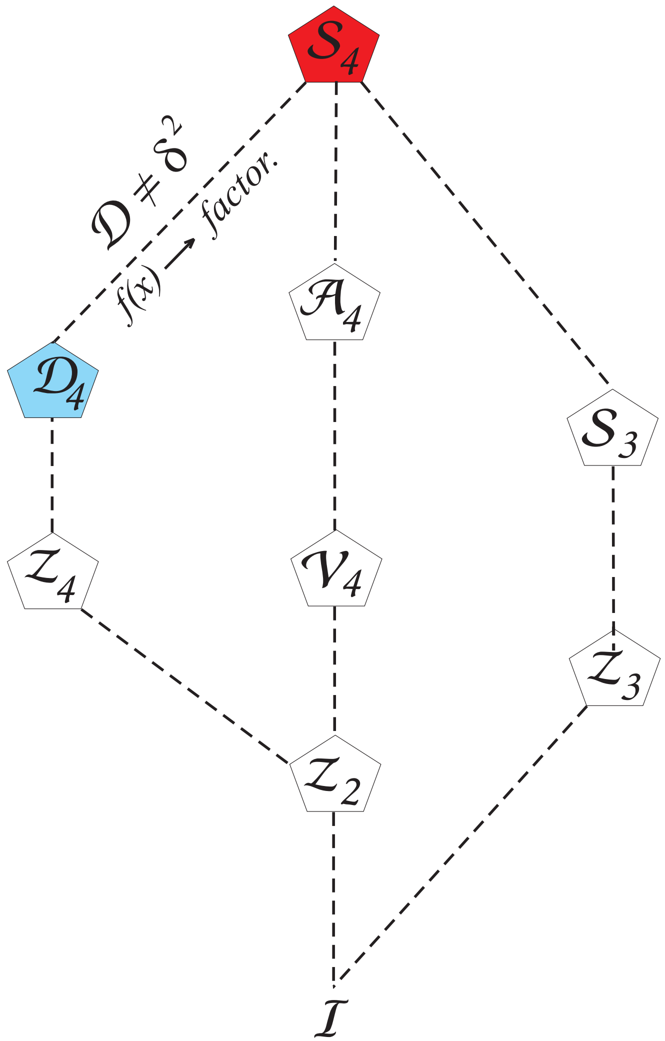


Figure 2: S_4 to D_4

The induced restrictions on the coefficients a_i

1. Tracelessness condition $b_1 = 0$ demands (*Dudas& Palti 1007.1297*)

$$a_4 = a_0 a_6, \qquad a_5 = -a_0 a_7$$

2. For $S_4 \to D_4$, $\Delta \neq \delta^2$ (*arXiv:1308.1581*)

$$\left({a_2}^2 a_5 - {a_4}^2 a_1\right)^2 \neq \left(\frac{16 a_1 a_5 - a_2 a_4}{3}\right)^3$$

3. Reducibility of the function $f(x)$ is achieved if

$$f(0) = 4 a_5 a_3 a_1 - a_1 {a_4}^2 - a_5 {a_2}^2 = 0$$

Matter Parity

Spectral Cover eq. $\sum_k b_k s^{5-k}$, invariant under (*see Hayashi et. al., 0910.2762*)

$$s \to -s, b_k \to (-1)^k e^{i\chi} b_k$$

For C_4 (*see I. Antoniadis, GKL, 1205.6930*)

$$b_k = \sum_{n+m=12-k} a_m a_n \to$$

$$a_n \to e^{i\psi} e^{i(3-n)} a_n$$

Defining Equs of matter curves are expressed in terms of a_n's.

$$\Downarrow$$

... a Geometric Z_2 symmetry assigned to Matter Curves

$SU(5)$	Def. Eqn.	Parity	Content	D_4	t_5
10_1	κ	$-$	$Q_L + u^c_L + e^c_L$	1_{+-}	0
10_2	a_2	$+$	$u^c_L + \bar{e}^c_L$	1_{++}	0
10_3	a_2	$+$	$u^c_L + \bar{e}^c_L$	1_{++}	1
10_4	μ	$-$	$2Q_L + 4e^c_L$	2	0
5_a	a_2	$+$	$2\bar{d}^c_L$	2	0
5_b	a_7	$+$	H_u	1_{++}	0
5_c	κa_7	$-$	$4d^c_L + 3L$	1_{+-}	0
5_d	a_2	$+$	H_d	1_{++}	-1
5_e	a_2	$+$	$\bar{d}^c_L$	1_{+-}	-1
5_f	a_7	$+$	$2d^c_L$	2	-1

Table 1: Full spectrum for $SU(5) \times D_4 \times U(1)_{t_5}$ model.

Low Energy Spectrum	D_4 rep	$U(1)_{t_5}$	Z_2
Q_3, u_3^c, e_3^c	1_{+-}	0	$-$
u_2^c	1_{++}	1	$+$
u_1^c	1_{++}	0	$+$
$Q_{1,2}, e_{1,2}^c$	2	0	$-$
L_i, d_i^c	1_{+-}	0	$-$
ν_3^c	1_{+-}	0	$-$
$\nu_{1,2}^c$	2	0	$-$
H_u	1_{++}	0	$+$
H_d	1_{++}	-1	$+$

Table 2: SM spectrum with $D_4 \times U(1)_{t_5} \times Z_2$ symmetry.

(*Karozas et al 1505.00937*)

$\mathcal{D}_4$

Phenomenology

Neutrino Sector

(*Main Motivation for Non-Abelian Discrete Symmetries*)

$$m_\nu = -m_D M_R^{-1} m_D^T$$

result...

$$m_\nu \propto \begin{pmatrix} 1+(z_1-2y)gz_1 & (1-gyz_1)x_2+(z_1-y)gz_2 & (1-gyz_1)x_3 \\ (1-gyz_1)x_2+(z_1-y)gz_2 & x_2^2-2gyz_2x_2+gz_2^2 & (x_2-gyz_2)x_3 \\ (1-gyz_1)x_3 & (x_2-gyz_2)x_3 & x_3^2 \end{pmatrix}$$

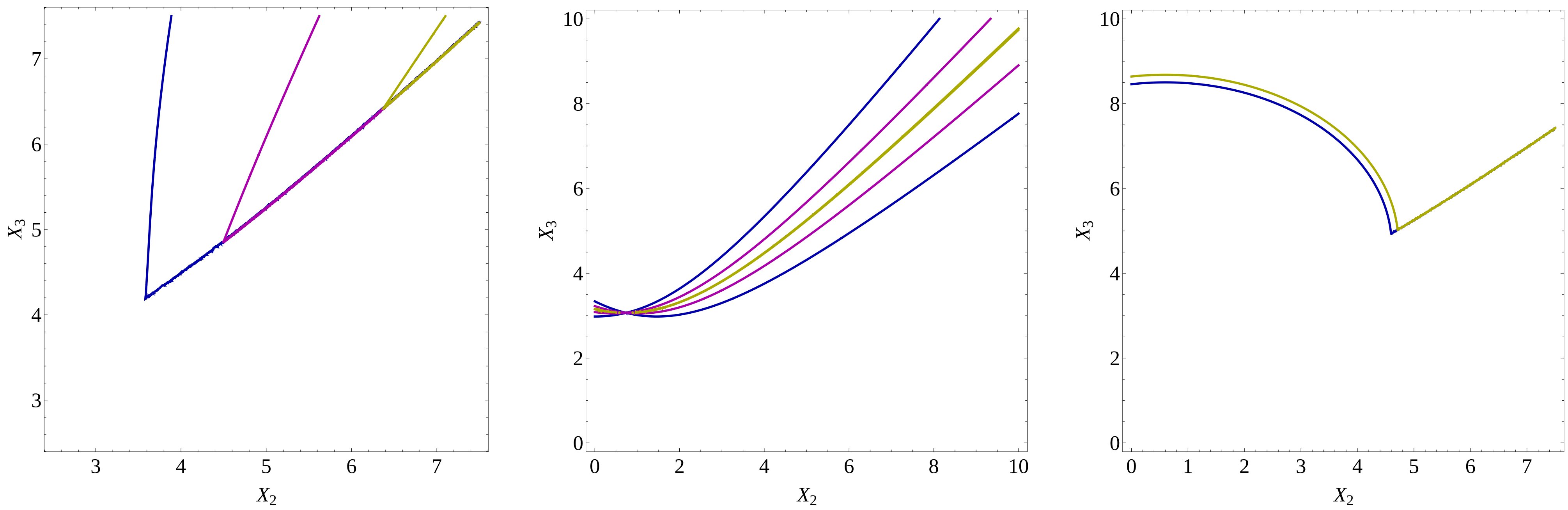


Figure 3: Left: $\sin^2\theta_{12}$ (3σ) (blue-0.270, pink-0.304, yellow-0.344);

Middle: $\sin^2\theta_{23}$ (3σ) (blue-0.382, pink-0.452, yellow-0.5);

Right: $R = \Delta m^2_{23}/\Delta m^2_{12} = 31.34(blue)$ and $R = 34.16$ (yellow).

Baryon Number Violation

eliminated by flux

$$10_2 \rightarrow (\not{Q}, u^c, e^c) \rightarrow (-, u^c, e^c)$$

$\exists$ parity violating term $10_2\bar{5}_c\bar{5}_c \rightarrow \lambda_{dbu} u^c\, d^c\, d^c$ **only!**$\rightarrow$ **Neutron-antineutron oscillations**

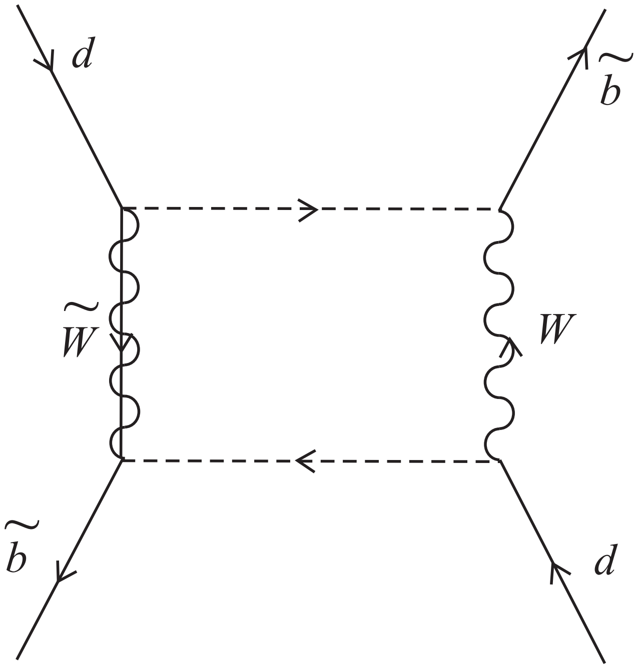


Figure 4: Feynman box graph for $n - \bar{n}$ oscillations (*Goity&Sher PLB 346(1995)69*)

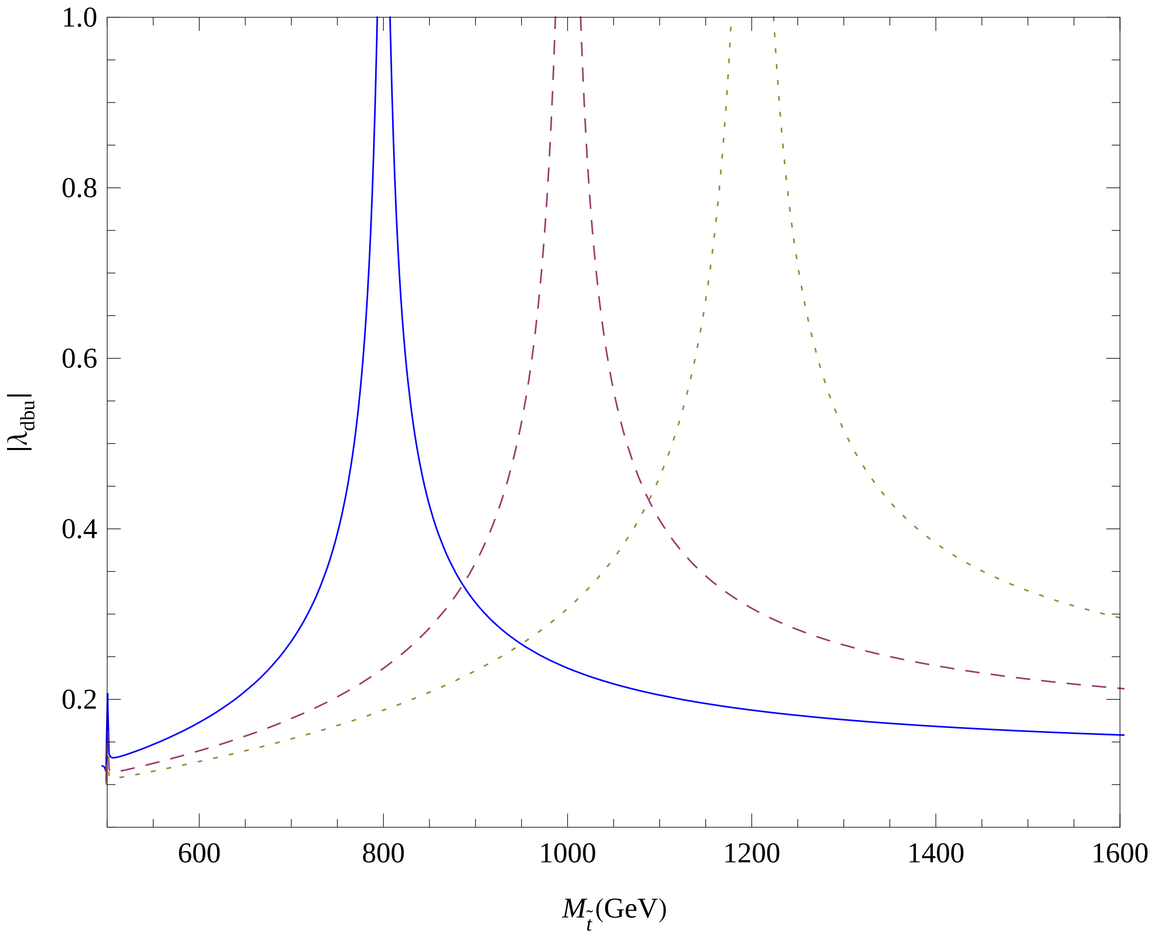


Figure 5: λ_{dbu} bounds for: Blue: $M_{\tilde{u}} = M_{\tilde{c}} = 0.8\,TeV$, Dashed: $M_{\tilde{u}} = M_{\tilde{c}} = 1\,TeV$, Dotted: $M_{\tilde{u}} = M_{\tilde{c}} = 1.2\,TeV$. ($M_{\tilde{b}_L} = M_{\tilde{b}_R} = 500GeV$, $\tau = 10^8 sec.$).

$\mathcal{E}$

Mordell-Weil $U(1)$ *and GUT s*

★ *A new class of Abelian Symmetries associated to Rational Sections of elliptic curves*

Mordell-Weil group ... finitely generated:

$$\underbrace{\mathbb{Z} \oplus \mathbb{Z} \oplus \cdots \oplus \mathbb{Z}}_{r} \oplus \mathcal{G}$$

Abelian group: Rank - r (*unknown*)

Torsion part: $\mathcal{G} \rightarrow$:

$$\mathcal{G} = \begin{cases} \mathbb{Z}_n & n = 1, 2, \ldots, 10, 12 \\ \mathbb{Z}_k \times \mathbb{Z}_2 & k = 2, 4, 6, 8 \end{cases}$$

$\rightarrow$ ... models with new $U(1)$'s and *Discrete Symmetries from Mordell-Weil*

(*Cvetic et al 1210.6094,1307.6425; Mayhofer et al, 1211.6742; Borchmann et al 1307.2902; Krippendorf et al, 1401.7844*)

Simplest (*and perhaps most viable*) Case: *Rank-1 Mordell-Weil*

Sections required: $[u : v : w] = [1 : 1 : 2] \rightarrow$

$\mathbb{P}_{(1,1,2)}$-weighted projective space

... *described by the equation:* (*see Morrison & Park 1208.2695*)

$$w^2 + a_2 v^2 w = u(b_0 u^3 + b_1 u^2 v + b_2 u v^2 + b_3 v^3)$$

Weierstrass model obtained

Birational Map

$$v = \frac{a_2 y}{b_3^2 u^2 - a_2^2 (b_2 u^2 + x)} \tag{1}$$

$$w = \frac{b_3 u y}{b_3^2 u^2 - a_2^2 (b_2 u^2 + x)} - \frac{x}{a_2} \tag{2}$$

$$u = z \tag{3}$$

These lead to the Weierstraß equation in Tate's form

$$
\begin{aligned}
y^2 + 2\frac{b_3}{a_2}xyz \pm b_1 a_2 yz^3 &= x^3 \pm \left(b_2 - \frac{b_3^2}{a_2^2}\right) x^2 z^2 \\
&\quad - b_0 a_2^2 xz^4 - b_0 a_2^2 \left(b_2 - \frac{b_3^2}{a_2^2}\right) z^6
\end{aligned}
$$

but now Tate's coefficients are not all independent !

$$y^2 + 2\frac{b_3}{a_2}xyz \pm b_1 a_2 y z^3 = x^3 \pm \left(b_2 - \frac{b_3^2}{a_2^2}\right) x^2 z^2 - b_0 a_2^2 x z^4 - b_0 a_2^2 \left(b_2 - \frac{b_3^2}{a_2^2}\right) z^6$$

... comparing with standard general Tate's form:

$$y^2 + \alpha_1 xyz + \alpha_3 yz^3 = x^3 + \alpha_2 x^2 z^2 - \alpha_4 x z^4 - \alpha_6 z^6$$

Observation:

$$\boxed{\alpha_6 = \alpha_2 \alpha_4}$$

Implications on the non-abelian structure

Assume local expansion of Tate's coefficients

$$\alpha_k = a_{k,0} + \alpha_{k,1}\xi + \cdots$$

Vanishing orders for $SU(2n)$:

$$\begin{aligned} \alpha_2 &= a_{2,1}\,\xi + \cdots \\ \alpha_4 &= \alpha_{4,n}\xi^n + \cdots \\ \alpha_6 &= \alpha_{6,2n}\xi^{2n} + \cdots \end{aligned}$$

$$\alpha_6 = \alpha_2\alpha_4 \to \alpha_{2,1}\alpha_{4,n}\xi^{n+1} = \alpha_{6,2n}\xi^{2n} \;\Rightarrow\; n = 1$$

...from $SU(n)$ series, compatible are Only:

$$SU(2), \text{ and } SU(3)$$

... extending the analysis to exceptional groups...

Viable non-Abelian GUTs with $U(1)_{MW}$

and the vanishing order of the coefficients $a_2 \sim a_{2,m}\xi^m, b_k \sim b_{k,n}\xi^n$

Group	a_2	b_0	b_1	b_2	b_3
$\mathcal{E}_6$	1	1	1	2	2
	0	3	1	2	1
$\mathcal{E}_7$	1	1	2	2	2
	0	3	3	2	1

This simple property ... perhaps suggestive for a model

$$\mathcal{E}_6 \times U(1)_{\mathcal{MW}}$$

Remarks

Spectral Cover:

- Analysis of model with gauge symmetry

$$SU(5) \times \mathcal{D}_4 \times U(1)$$

- Non-abelian discrete symmetries naturally incorporated
- $n - \bar{n}$ oscillations, suppressed proton decay

Mordell-Weil:

- *... gauge symmetries with one abelian Mordell-Weil:*

$$\mathcal{E}_6 \times U(1)_{MW}, \ \mathcal{E}_7 \times U(1)_{MW}$$

- ... extra $U(1)_{MW}$ might have interesting implications to Model building ...
- Torsion group: possible explanation of discrete symmetries...


STRING PHENO 2016
15th conference in the
String Phenomenology Conference series
Ioannina,Greece, June 20-24
http://stringpheno2016.physics.uoi.gr
e-mail: stringpheno2016@conf.uoi.gr