Cosmological attractors
and initial conditions for inflation

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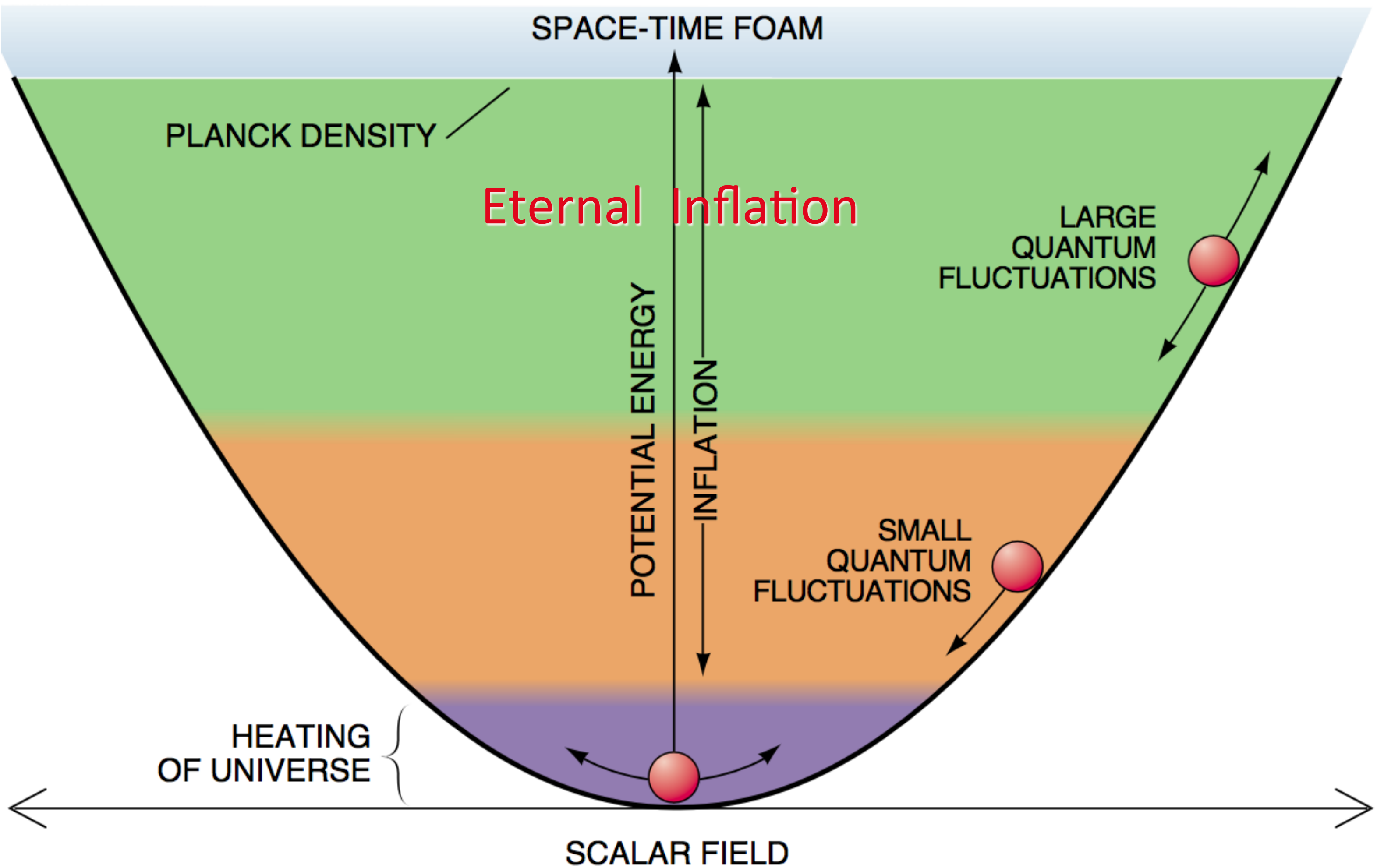
with Kallosh, Carrasco, Ferrara, Roest, Galante, Scalisi

Our goal is to find inflationary models which are flexible enough to fit the data, which can be implemented in string theory or supergravity, and which may tell us something interesting and instructive.

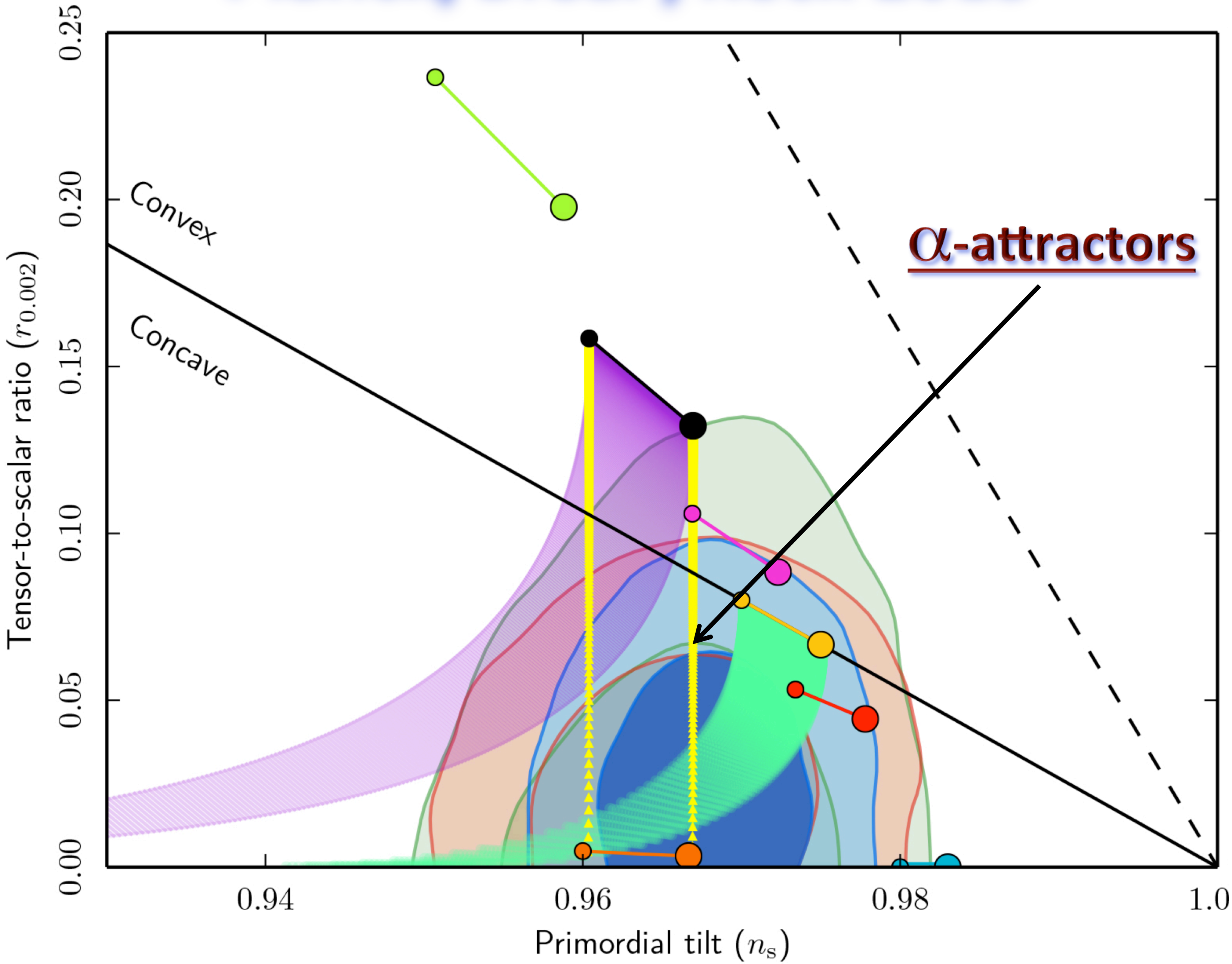
The simplest chaotic inflation model

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Planck data suggest that this simple model should be modified. The two vertical yellow lines in the next slide will show the consequences of a minor modification of this simple chaotic inflation model versus the results of Planck 2015.

Planck/BICEP/Keck 2015



What is the meaning of α-attractors?

Start with the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

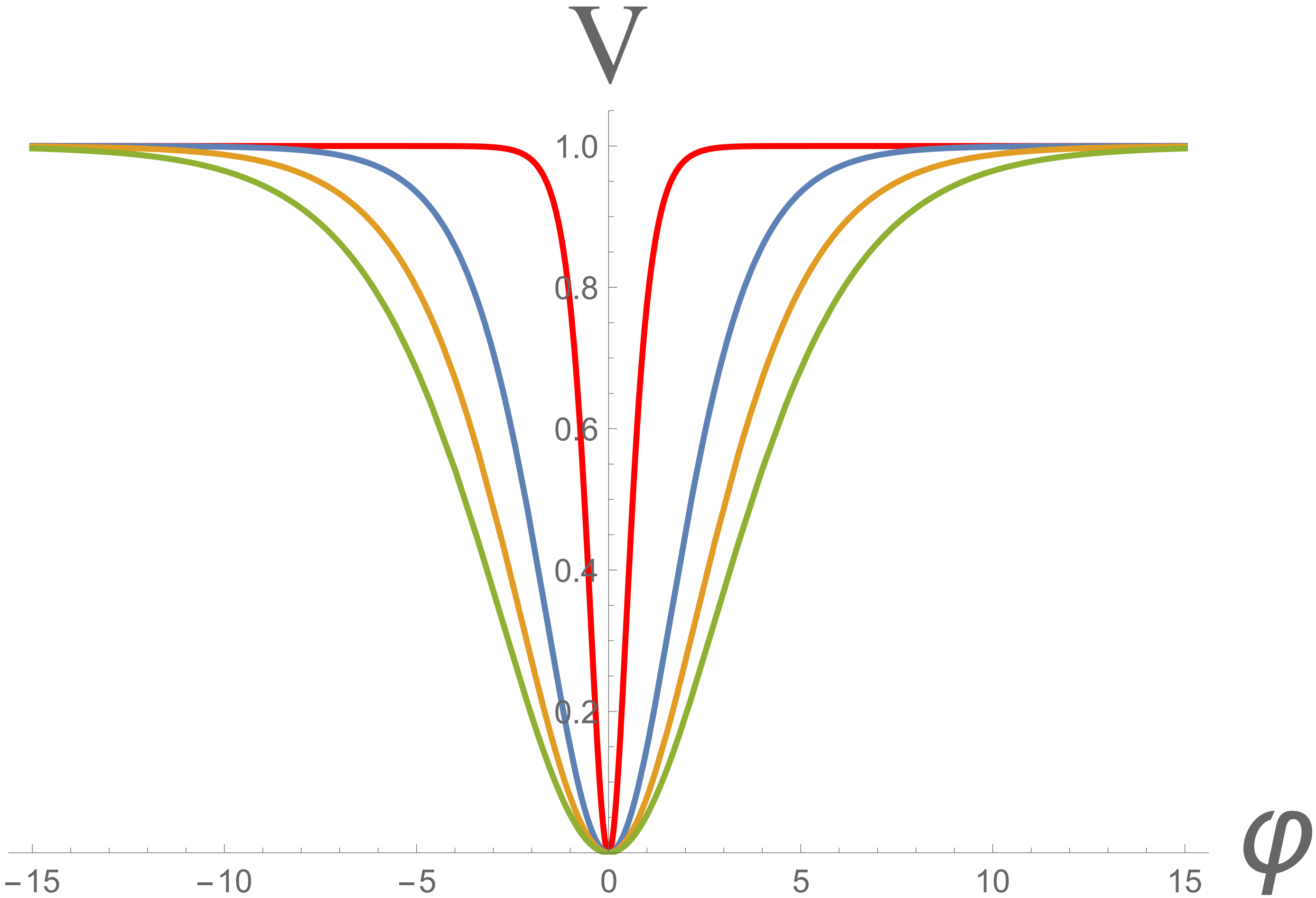
The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

T-models

$$V = f(\tanh^2 \frac{\varphi}{\sqrt{6\alpha}})$$

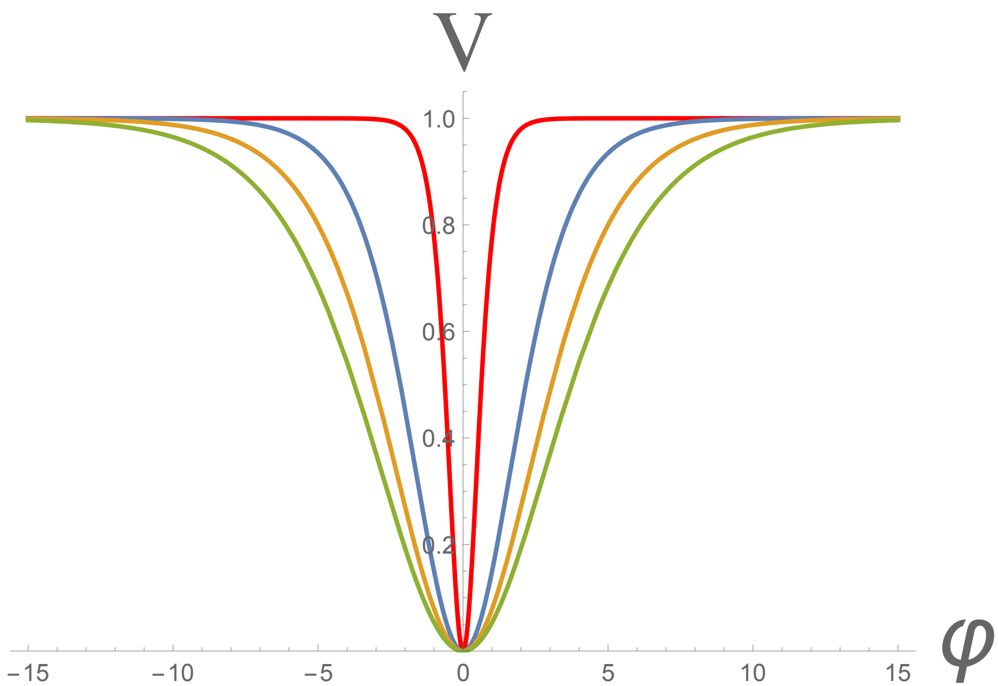
$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha \frac{12}{N^2}$$



Similar models have been discussed for the first time by Goncharov and A.L. Phys. Lett. 139B (1984) 28. It was the first paper on chaotic inflation in supergravity, but then it was nearly forgotten. It corresponds to $\alpha = 1/9$

$$n_s = 1 - \frac{2}{N} \approx 0.967, \qquad r \sim 4 \times 10^{-4}$$

Red line – GL model 1984

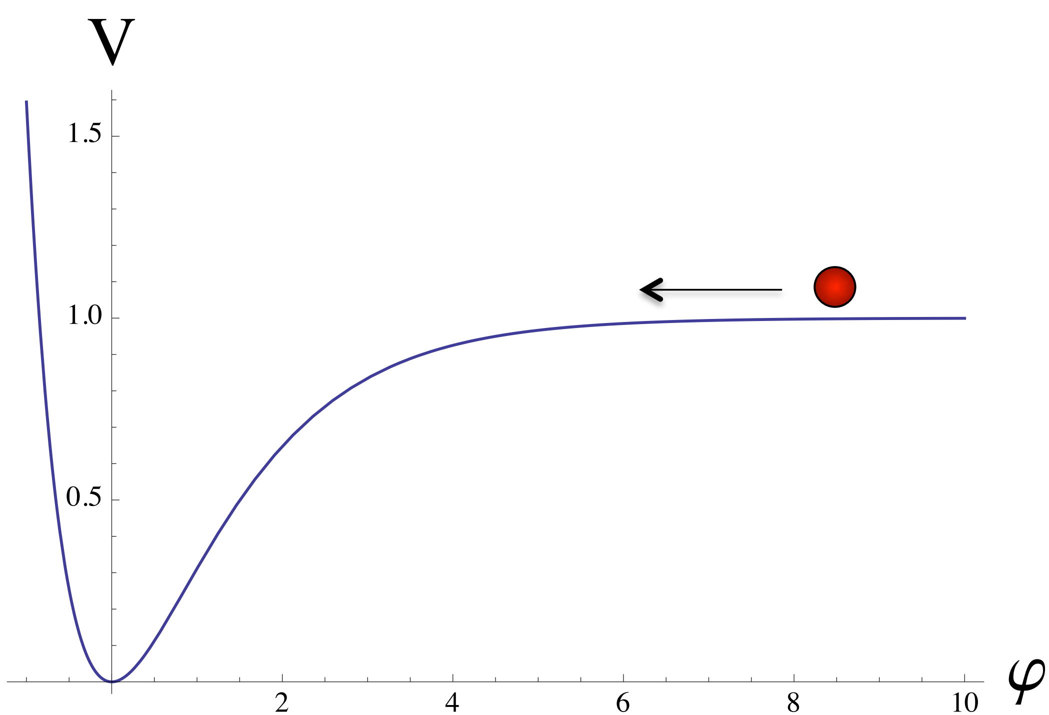


Starobinsky model

$$L = \sqrt{-g}\left(\frac{1}{2}R + \frac{R^2}{12M^2}\right)$$

$$\tilde{g}_{\mu\nu} = (1 + \phi/3M^2)g_{\mu\nu} \qquad \varphi = \sqrt{\tfrac{3}{2}}\ln\left(1 + \tfrac{\phi}{3M^2}\right)$$

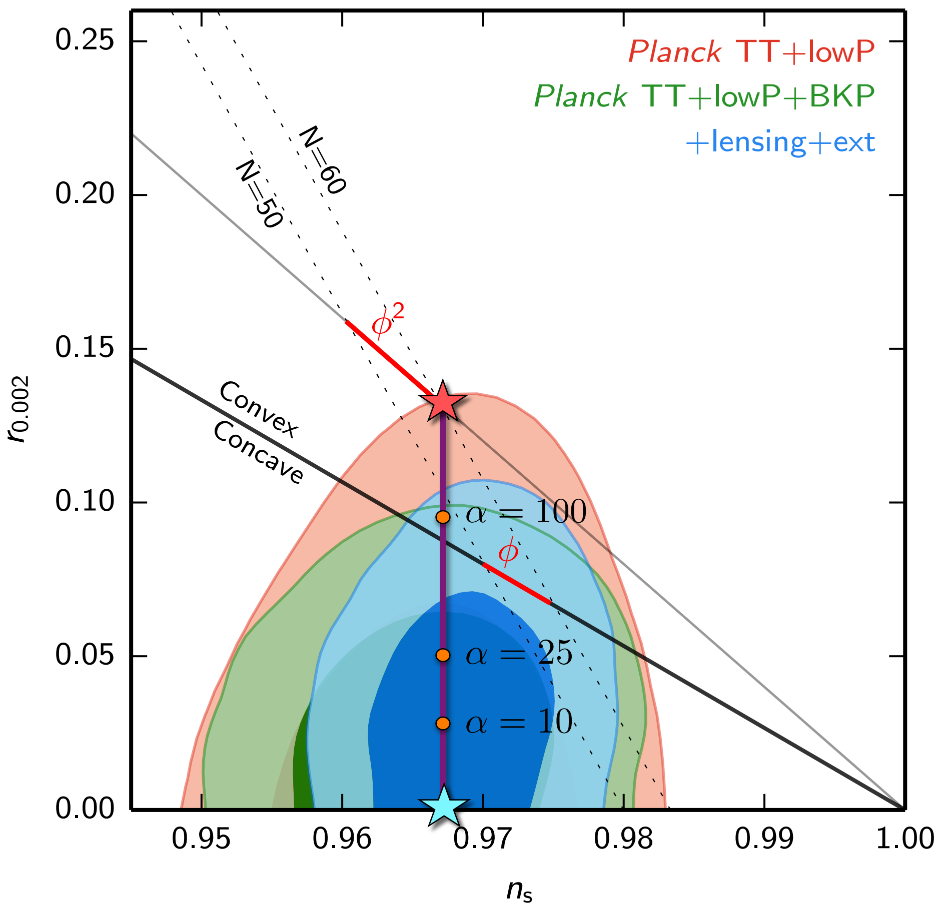
$$L = \sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\,\varphi}\right)^2\right]$$

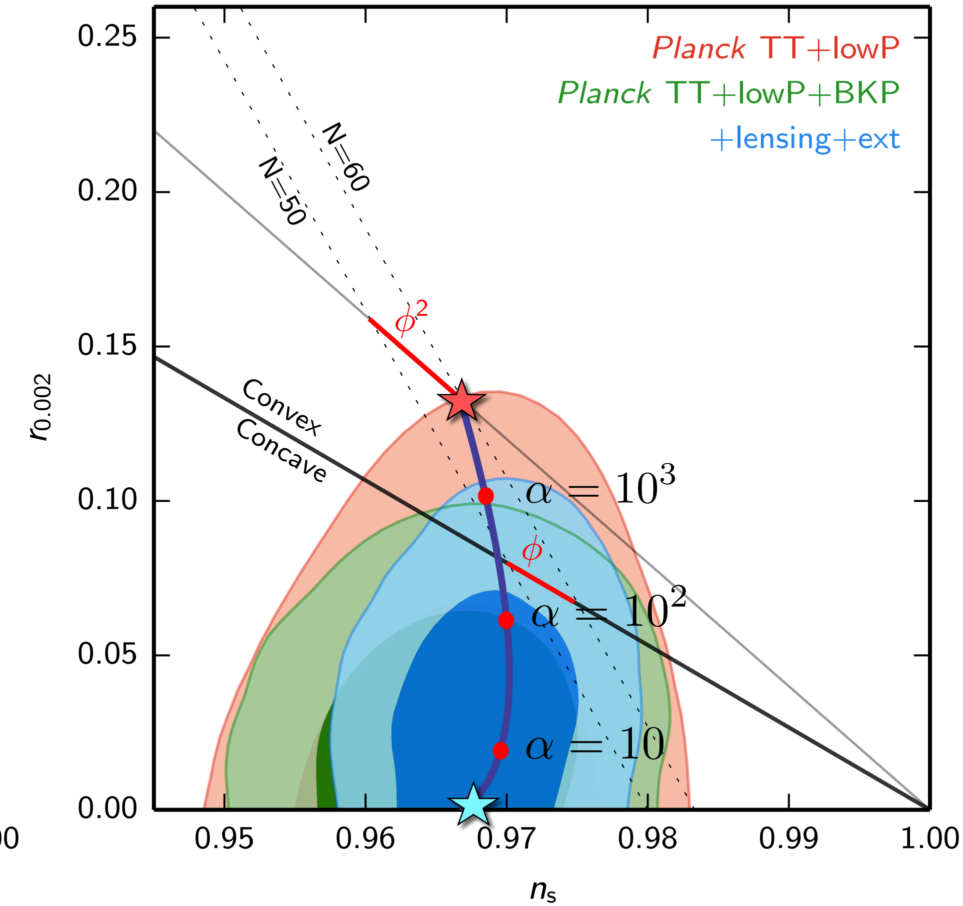


Whitt 1984

Identified with the Starobinsky model only in 1988:

Barrow 1988
Maeda 1988
Coule, Mijic 1988





$$\frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2$$

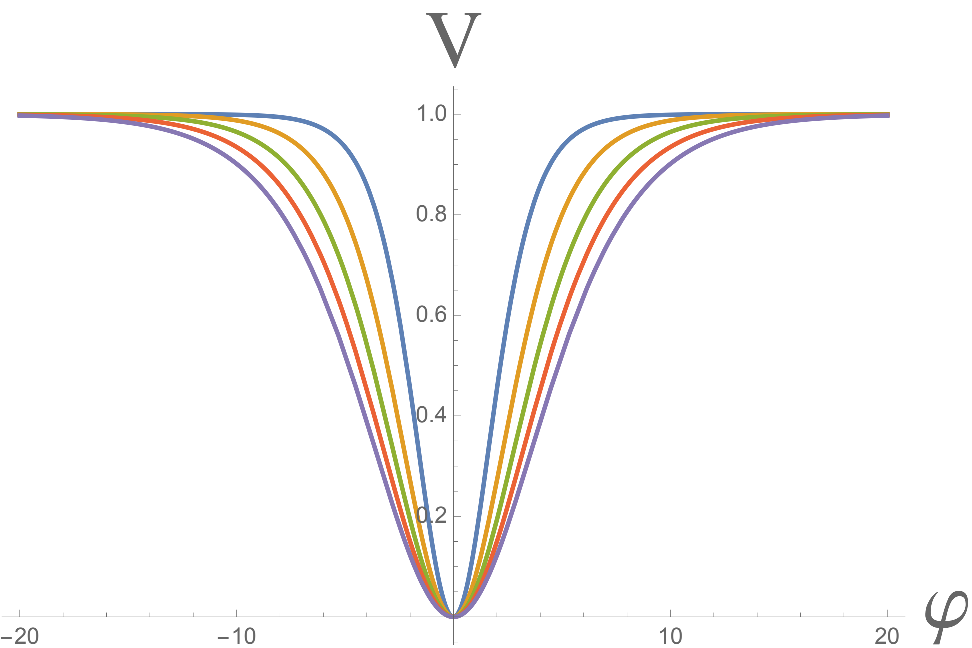
$$\frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\frac{\phi^2}{(1+\frac{\phi}{\sqrt{6\alpha}})^2}$$

Simplest T-models

Simplest E-models

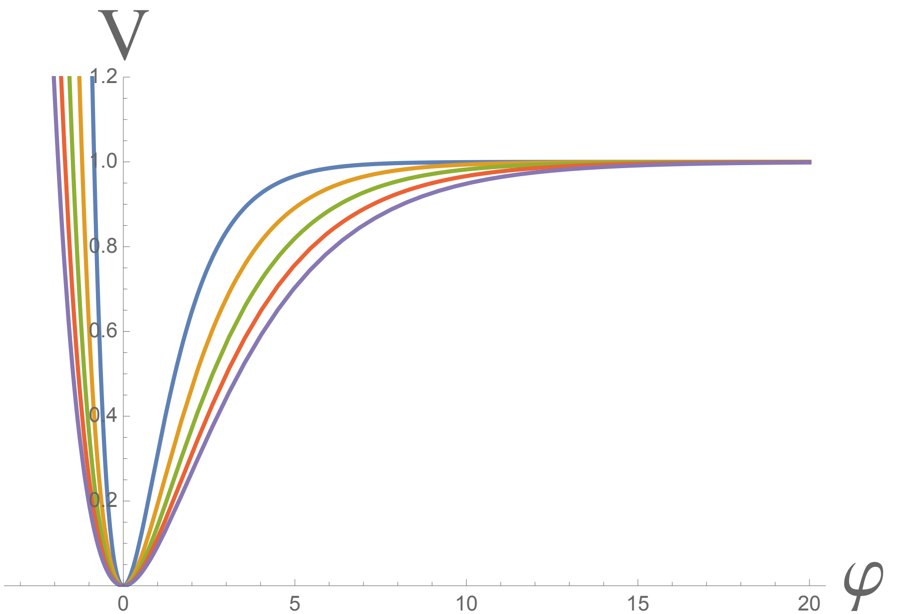
Ferrara, Kallosh, AL, Porrati, 2013, Kallosh, AL, Roest 2013

Simplest T-models



$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\Big(\tanh\frac{\varphi}{\sqrt{6\alpha}}\Big)^2$$

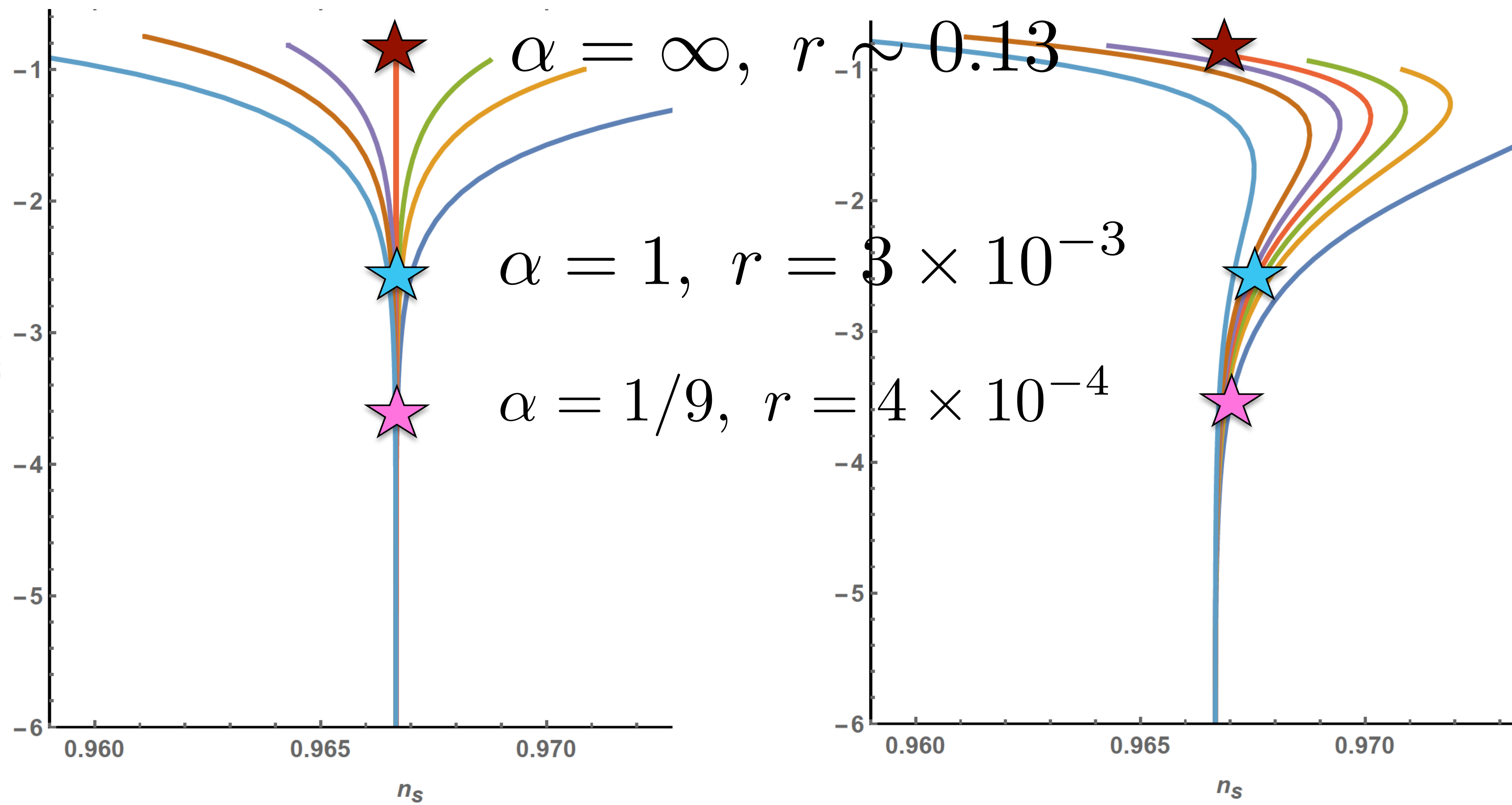
Simplest E-models



$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\Big(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

α-attractors

T-models **E-models**



$$\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^{2n}$$

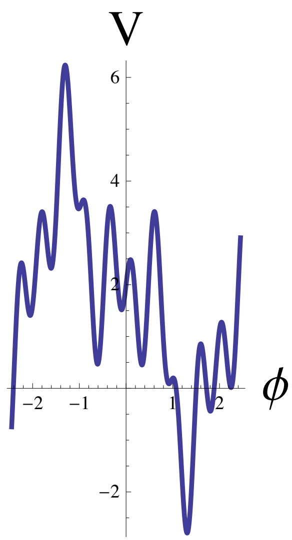
$$\left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^{2n}$$

Stretching and flattening of the potential is similar to stretching of inhomogeneities during inflation

Kallosh, AL 2013

Potential in the **original variables** with kinetic term

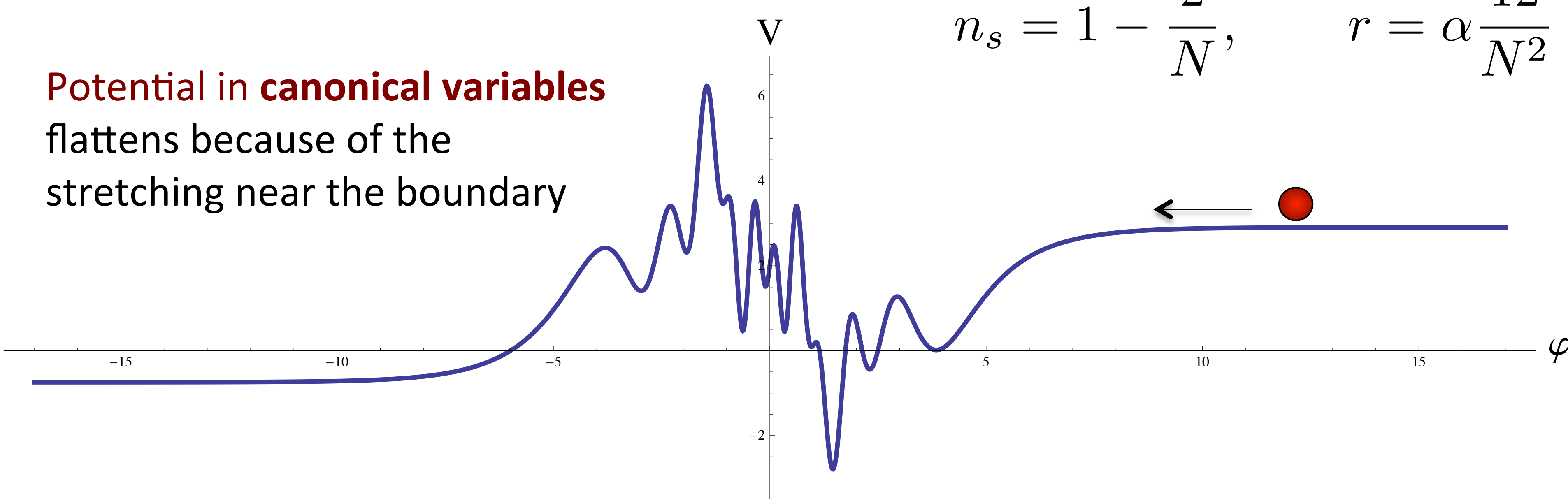
$$\frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2}$$



All of these models predict

$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha\frac{12}{N^2}$$

Potential in **canonical variables** flattens because of the stretching near the boundary



The essence of α-attractors

Galante, Kallosh, AL, Roest 1412.3797

$$\frac{1}{2}R - \frac{3}{4}\alpha\left(\frac{\partial t}{t}\right)^2 - V(t)$$

Suppose inflation takes place near the pole at t = 0, and

$V(0) > 0 \qquad V'(0) < 0$, and V has a minimum nearby

Then in canonical variables

$$\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_0(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + \ldots)$$

and, in the leading approximation in 1/N, almost independently on V(t)

$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha\frac{12}{N^2}$$

The essence of α-attractors

THE BASIC RULE:

For a broad class of cosmological attractors, the spectral index n_s depends mostly on the order of the pole in the kinetic term, while the tensor-to-scalar ratio **r** depends on the residue. Choice of the potential, as long as it is non-singular near the pole, almost does not matter. Geometry of the moduli space, not the potential, determines much of the answer.

THE REMAINING PROBLEM:

Can we get a pole in the kinetic term from something more fundamental than a theory of a single scalar field, for example in supergravity?

Is there any hidden symmetry, interesting physics and math here, beyond describing poles in kinetic terms?

A hint: Properties of kinetic terms in supergravity are controlled by Kahler geometry.

α–attractors in supergravity

General setting:

$$K_{\mathbb{D}} = -\frac{3\alpha}{2}\log\left[\frac{(1-Z\bar{Z})^2}{(1-Z^2)(1-\bar{Z}^2)}\right] + S\bar{S}$$

$$W_{\mathbb{D}} = A(Z) + S\,B(Z)\ \ .$$

Simplest model:

$$W\ =\sqrt{\alpha}\mu\,S\,Z$$

$$V\sim\alpha\,\mu^2\tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

In this simple model SUSY is unbroken and V = 0 in the minimum (no cosmological constant). Can be modified to account for SUSY breaking and cosmological constant (the talk by Kallosh).

Initial conditions for inflation

In the simplest chaotic inflation model $m^2\phi^2$, inflation begins at the Planck density under a trivial condition: the potential energy should be greater than the kinetic and gradient energy in a smallest possible domain of a Planckian size.

However, in a broad class of cosmological attractor models, inflation can begin only when the energy density drops from its Planck value by 10 orders of magnitude. Is it a problem?

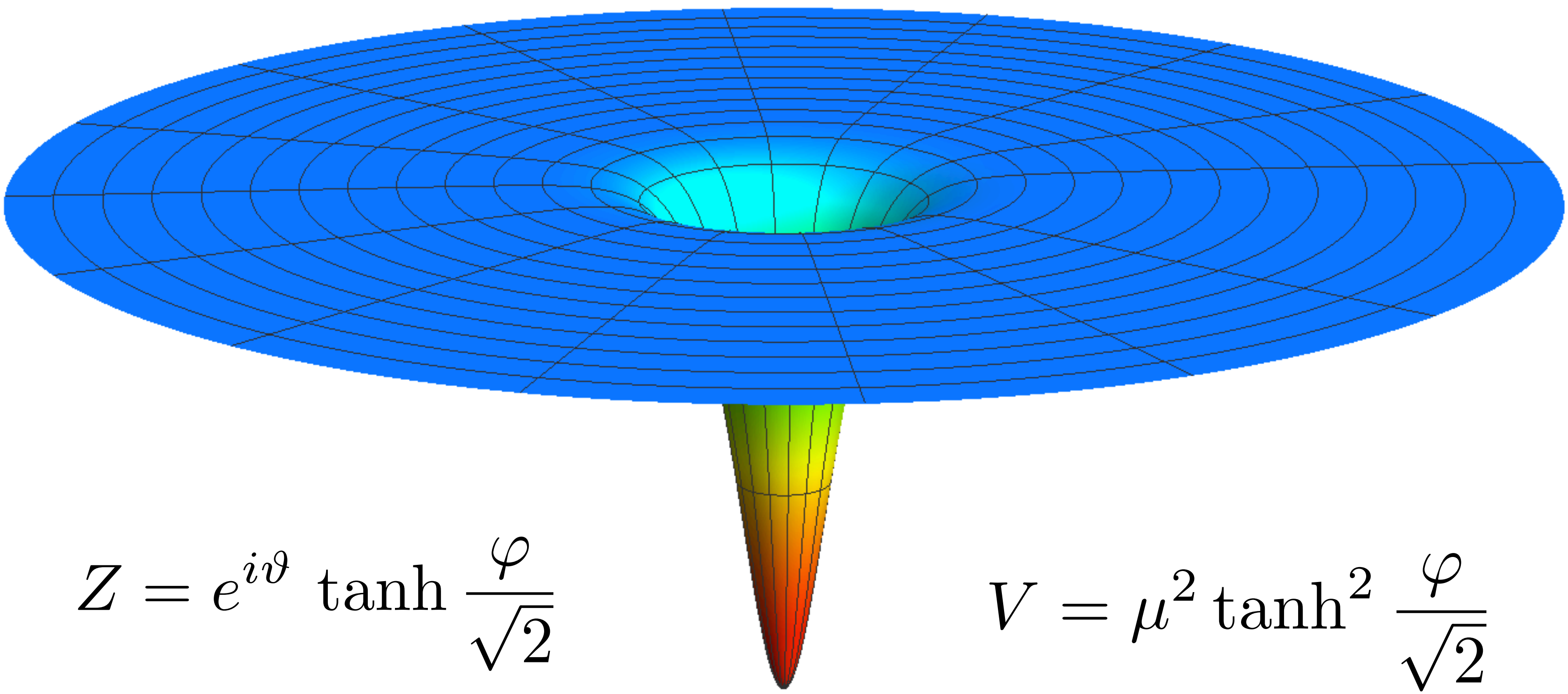
Carrasco, Kallosh, AL 1506.0936

Moduli space and Escher's Angels and Devils

This is the simplest quadratic inflationary potential, with angels and devils concentrated near the boundary of the moduli space

The same potential in terms of the canonical inflaton field for α = 1/3

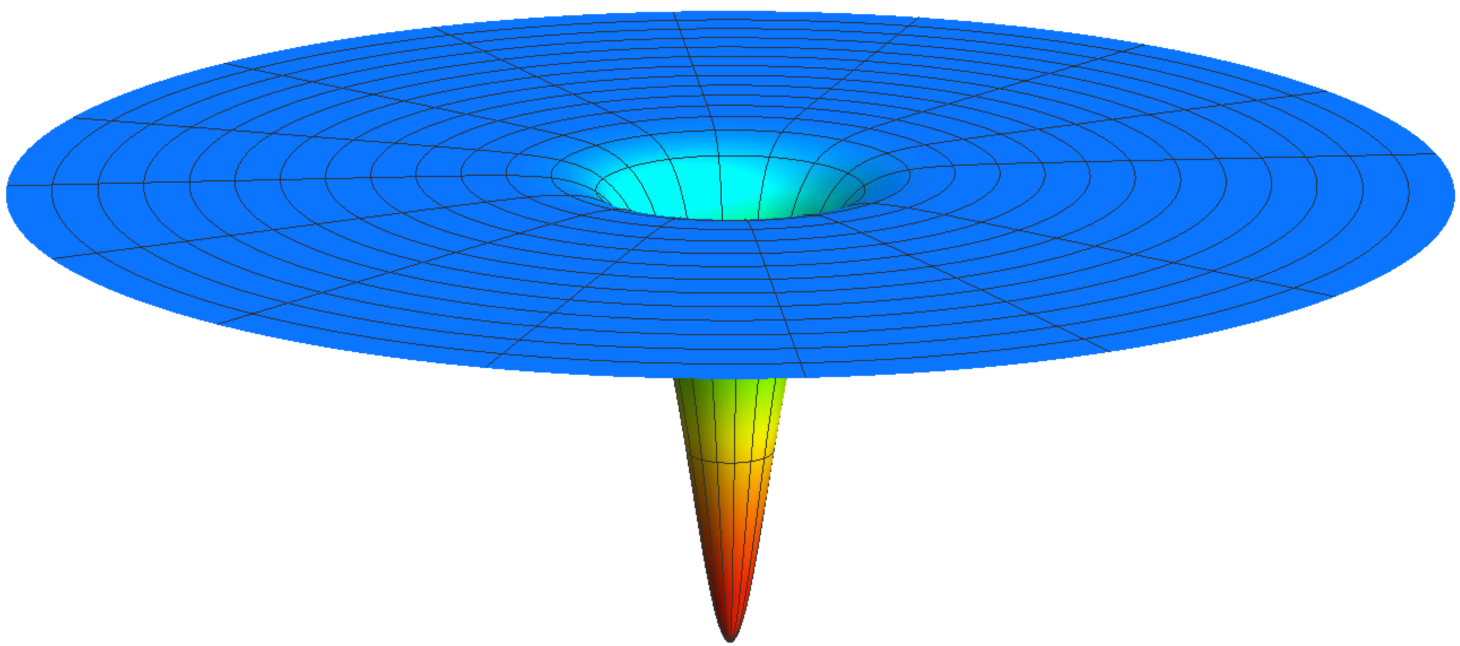
$$K = -\log\left[\frac{1 - Z\bar{Z} - S\bar{S}}{\sqrt{(1-Z^2)(1-\bar{Z}^2)}}\right] \qquad W = \mu\, S\, Z$$



$$Z = e^{i\vartheta}\,\tanh\frac{\varphi}{\sqrt{2}} \qquad V = \mu^2\tanh^2\frac{\varphi}{\sqrt{2}}$$

Potential defines infinite dS space, everywhere except a small vicinity of the minimum

The universe is born at the Planck density, 10 orders of magnitude above the dS disk. It may be very inhomogeneous, but if it expands, density of matter decreases. In 10^{-28} seconds it becomes dominated by dS energy density. After that, the field slowly rolls to the minimum. This solves the problem of initial conditions for inflation

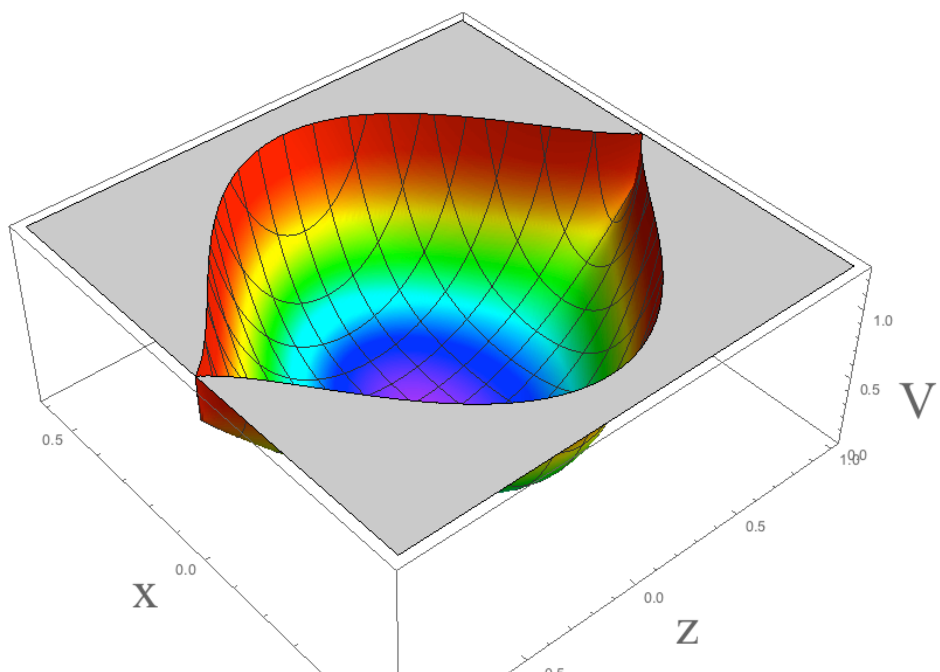


α–attractors with dS valley

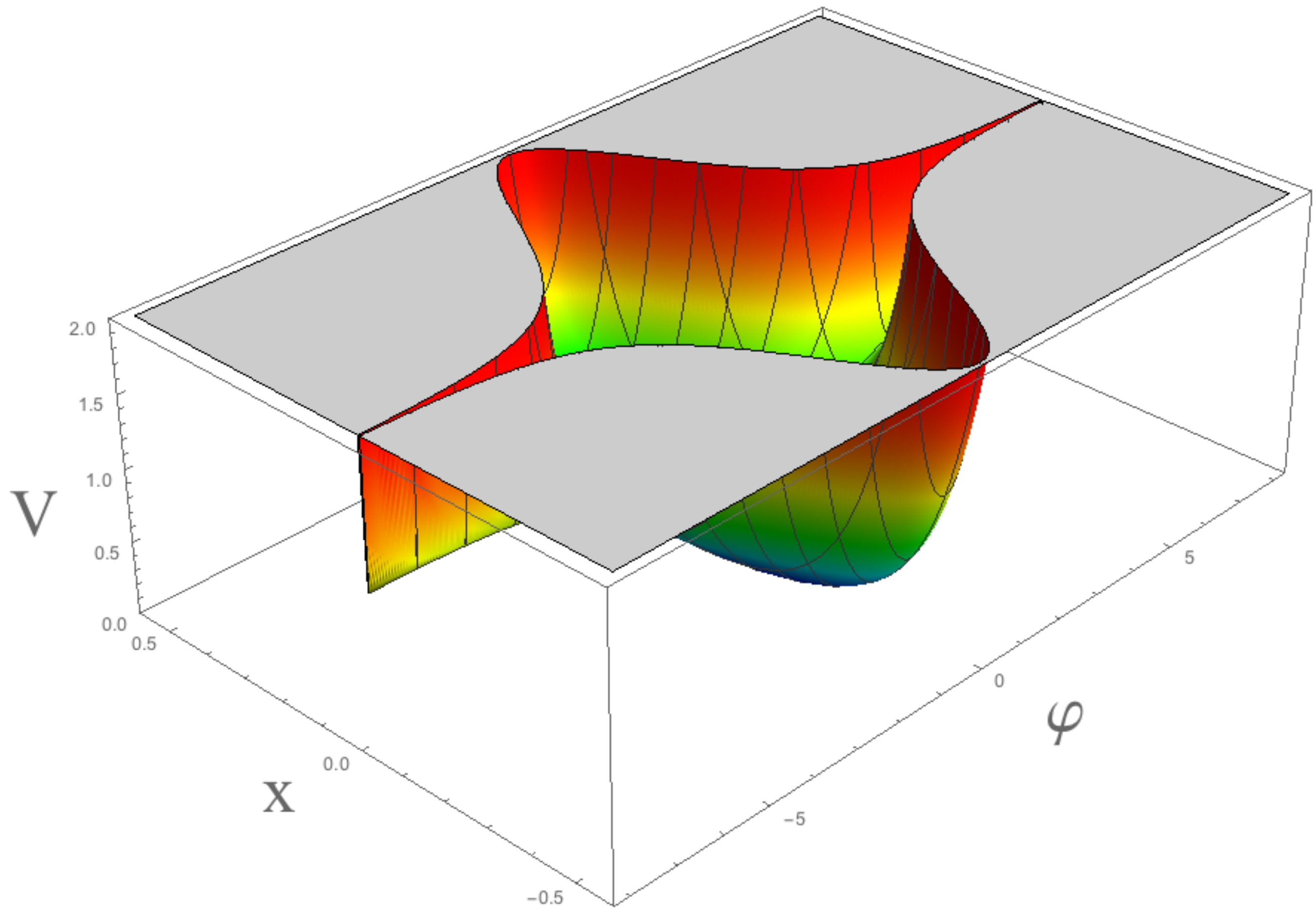
$$K_{\mathbb{D}} = -\frac{3\alpha}{2} \log\left[\frac{(1-Z\bar{Z})^2}{(1-Z^2)(1-\bar{Z}^2)}\right] + S\bar{S} \qquad W = \mu\, S\, Z$$

$$V = \alpha\mu^2\ (z^2+x^2)\left[\frac{z^4+2z^2(x^2-1)+(x^2+1)^2}{(1-z^2-x^2)^2}\right]^{\frac{3\alpha}{2}}$$

In terms of the original variables Z = z+ ix, the potential looks like a little boat. Where is the place for inflation to begin?



Things start looking better if one goes from z = Re Z to a canonically normalized inflaton field φ: the potential has an infinite dS valley.

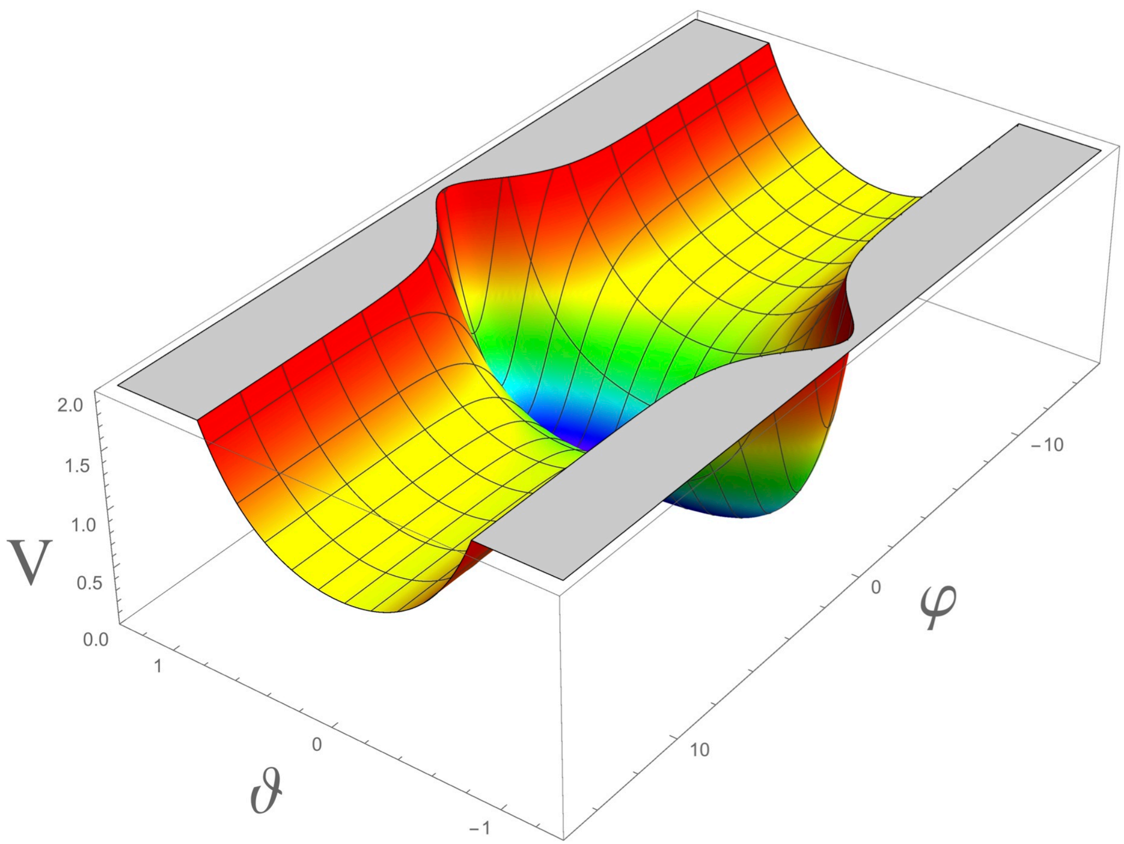


But these coordinates are a bit misleading: We see an infinitely long gorge with rapidly growing curvature, but the calculation of the mass squared of the field x shows that it is large and constant…

More appropriate coordinates:

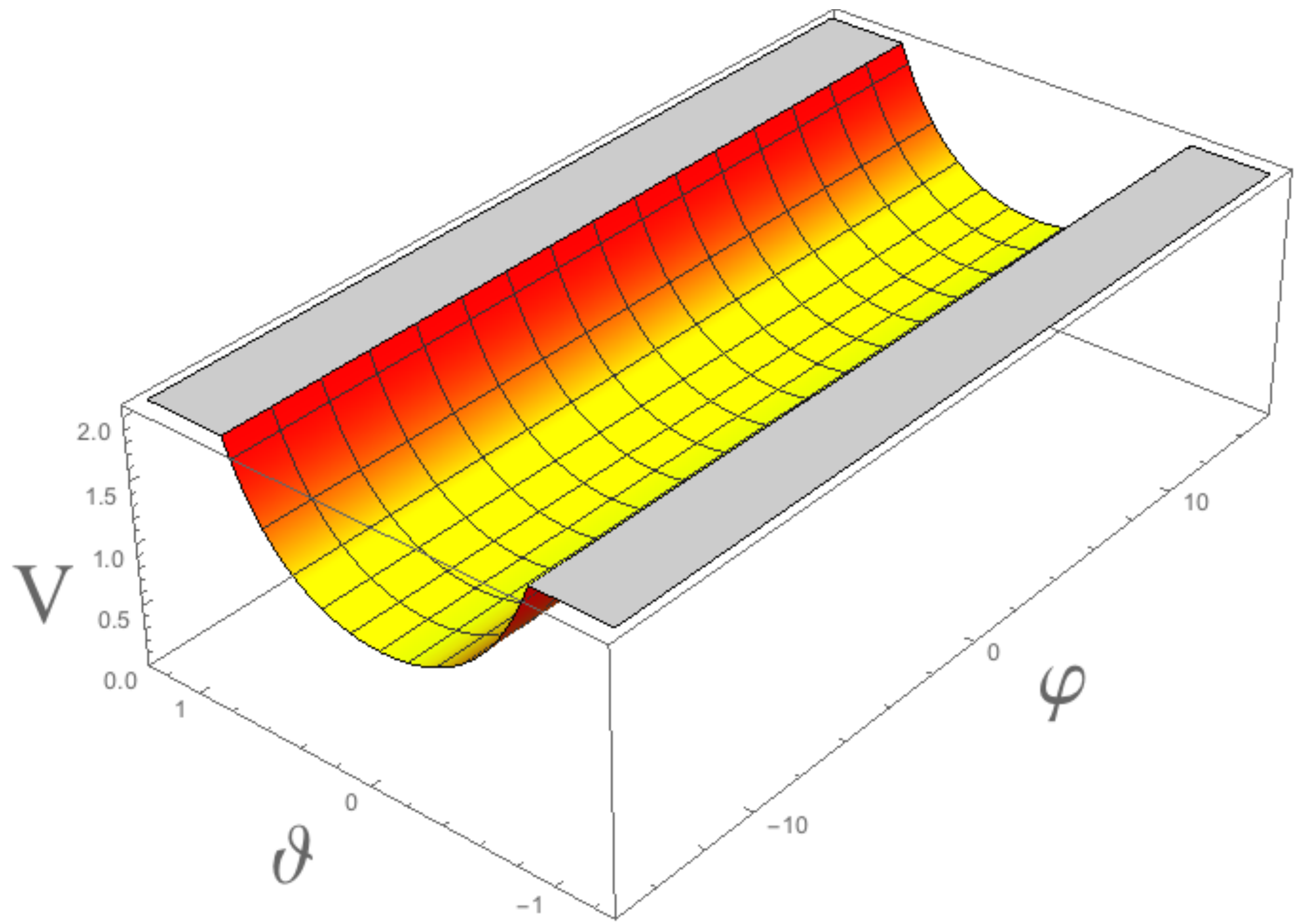
$$ds^2 = \frac{d\varphi^2 + d\vartheta^2}{2\cos^2\sqrt{\frac{2}{3\alpha}}\vartheta}$$

$$V = \alpha\mu^2 \left|\tanh\frac{\varphi + i\vartheta}{\sqrt{6\alpha}}\right|^2 \cdot \left(\cos\sqrt{\frac{2}{3\alpha}}\vartheta\right)^{-3\alpha}$$

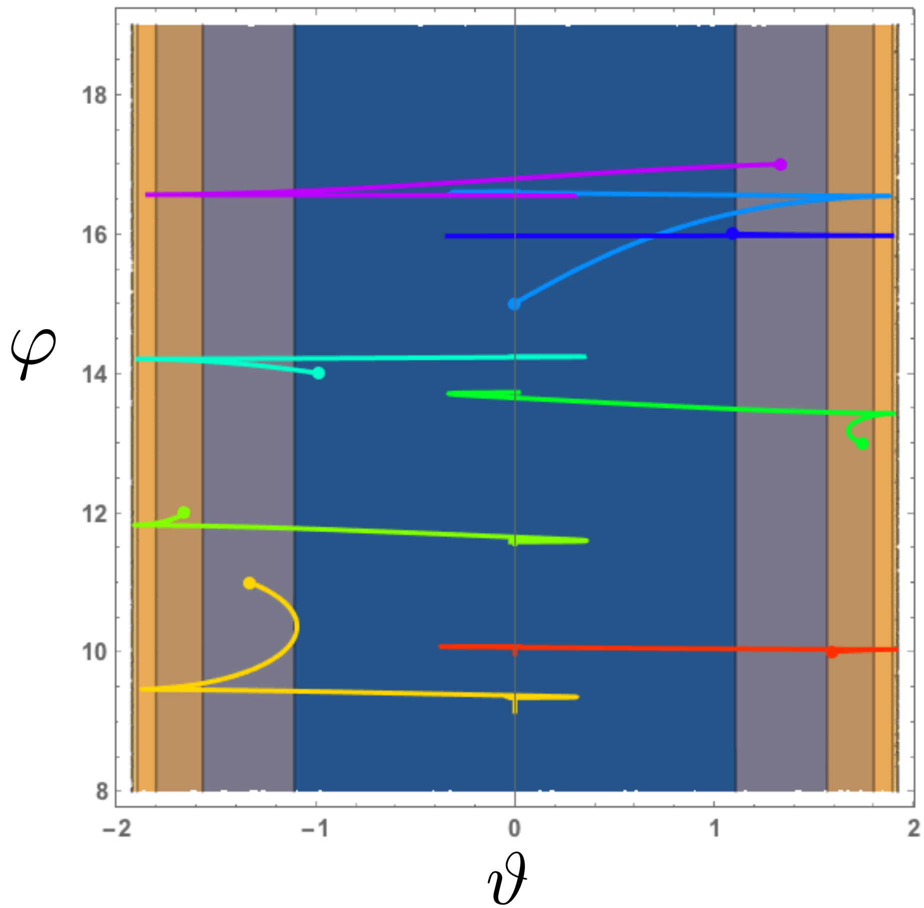


At large values of the inflaton field everything becomes <u>shift-symmetric</u>

$$V = \alpha\mu^2 \left(\cos\sqrt{\frac{2}{3\alpha}}\vartheta\right)^{-3\alpha}$$



Independently of its initial velocity, **the inflaton field starting at the Planck energy does not move by more than 10 until it stops and the slow-roll inflation begins**. The problem of initial conditions in a homogeneous universe is solved due to the existence of an infinite shift-symmetric dS valley



These considerations apply not only to cosmological attractors in supergravity, but to any inflationary model with a sufficiently long and flat potential.

What can go wrong with this argument?

The universe as a whole may collapse within 10^{-28} second

Three options:

1) Universe is small and closed. If id does not inflate at birth, it instantly dies, so in a sense, it is like a virtual particle, not even born.

2) Universe is infinite. Then there always will be non-collapsed parts, which leads to inflation.

3) Universe is open or flat, but COMPACT, like a torus. It may easily become inflationary.

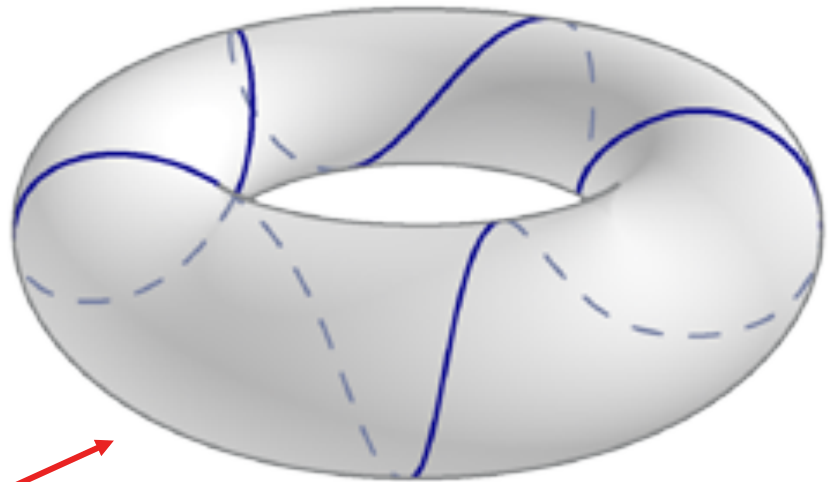
Take a box (a part of a flat universe) and glue its opposite sides to each other. What we obtain is a **torus**, which is a topologically nontrivial flat universe.

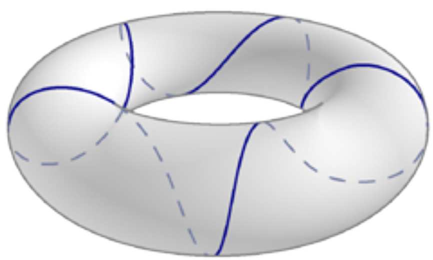
No need to tunnel: A compact open inflationary universe may be arbitrarily small

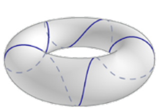
Chaotic mixing

Cornish, Starkman, Spergel 1996;   A.L. 2004

The size of a torus (our universe) with relativistic matter grows as $t^{1/2}$, whereas the mean free path of a relativistic particle grows much faster, as t

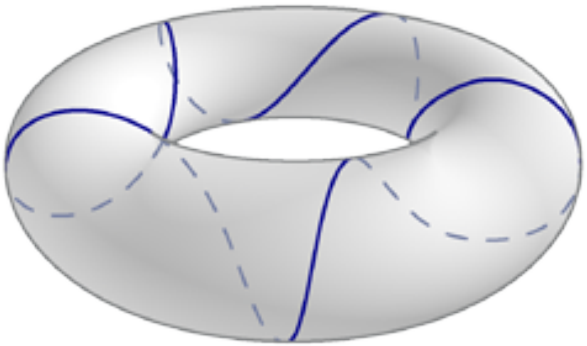






Therefore until the beginning of inflation the universe remains smaller that the size of the horizon ~ t

If the universe initially had a Planck size, then within the cosmological time $t >> 1$ each particle runs around the torus **many times** and appear in all parts of the universe with equal probability, which makes the universe homogeneous and keeps it homogeneous until the beginning of inflation



Thus chaotic mixing keeps the universe uniform until the onset of inflation, even if it can occur only at $V << 1$. This is yet another solution of the problem of initial conditions.

Can we do it even simpler, starting inflation at Planck density?

Yes, for a << 1/3. In this case the potential in the direction perpendicular to the inflaton field θ becomes flat. Inflation may begin at the Planck density when the field θ was falling towards the dS valley. After that, inflation in the ϕ direction begins.

From inflation to dark energy and SUSY breaking

Ultimately, we want these models to describe not only inflation, but also dark energy and SUSY breaking.

There is some urgency in learning about the interplay of SUSY and cosmology: LHC restarted in March 2015, the first collisions observed in May. Will supersymmetry be discovered? It will affect cosmological models.

α–attractors with SUSY breaking and a cosmological constant

$$W = \Big(S + \frac{1-Z^2}{b}\Big)(\sqrt{3}\alpha\, m^2\, Z^2 + M)$$

S – nilpotent superfield (no scalar component)

m - inflaton mass scale

M - SUSY breaking mass scale

For $b = \sqrt{3}$ one has $\Lambda = 0$. Changing b gives any desirable value of the cosmological constant.

Kallosh, AL 1502.07733, Carrasco, Kallosh, AL 1506.01708

More about it – in the talk by Renata Kallosh