Yukawa Couplings in Heterotic Calabi-Yau Models



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University of Oxford

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based on:

- 1506.nnnn with Evgeny Buchbinder, Philip Candelas, Stefan Blasneag
- relates to 0904.2186 with Lara Anderson, James Gray, Dan Grayson, Yang-Hui He
- longer-term project, involving Lara Anderson, James Gray, Burt Ovrut

<u>Outline</u>

- Introduction: Heterotic Yukawa couplings
- Yukawa unification?
- Current status of calculation
- Our laboratory: the tetra-quadric CY
- Example 1: up-Yukawa couplings
- Example 2: singlet Yukawa couplings
- Conclusion

Introduction: Heterotic Yukawa couplings

- \bullet Consider heterotic string on CY 3-fold X
- observable bundle $V \to X$ with structure group $H \subset E_8$
- low-energy gauge group $G = \mathcal{C}_{E_8}(H)$
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- matter multiplets from associated bundles $E_i \rightarrow V$, i = 1, 2, 3

Matter multiplets described by harmonic (0,1) forms:

$$\nu_i \in H^1(X, E_i) \qquad \bar{\partial}_{E_i} \nu_i = \bar{\partial}_{E_i}^{\dagger} \nu_i = 0 \qquad i = 1, 2, 3$$

Holomorphic Yukawa couplings:

$$\lambda(\nu_1,\nu_2,\nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3$$

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Holomorphic Yukawa couplings can be computed algebraically!

Normalisation (proportional to):

$$(\nu_i, \mu_i) := \int_X \nu_i \wedge \bar{\star}_{E_i} \mu_i = \frac{1}{2} \int_X J \wedge J \wedge \nu_i \wedge (H\bar{\mu}_i)$$

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Algebraic computation (probably) not possible. Requires methods of differential geometry.

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 $10 \leftrightarrow \nu_1 \in H^1(X, \wedge^2 V)$ $16 \leftrightarrow \nu_{2,3} \in H^1(X, V)$

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up-Yukawa 51010:

5 $\leftrightarrow \nu_1 \in H^1(X, \wedge^2 V^*)$ 10 $\leftrightarrow \nu_{2,3} \in H^1(X, V)$

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down-Yukawa $\bar{5} \, \bar{5} \, 10$:

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SM Yukawa couplings are obtained after taking quotient by discrete symmetry Γ , adding a Wilson line and keeping the Γ -invariant parts.

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upstairs: 6 families

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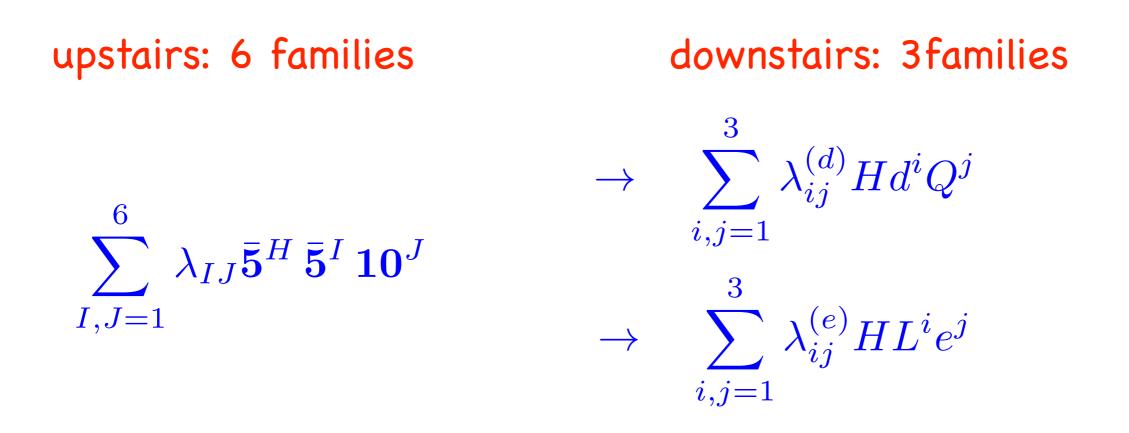
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downstairs: 3 families

$$\sum_{i,j=1}^{3} \lambda_{ij}^{(d)} H d^{i} Q^{j}$$

 $\sum_{i,j=1}^{3} \lambda_{ij}^{(e)} H L^{i} e^{j}$
 $i,j=1$

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Wilson line described by Γ -representations χ_2 , χ_3 satisfying $\chi_2^2 \otimes \chi_3^3 = 1$. For $\Gamma = \mathbb{Z}_2$ we have $\chi_2 = (1)$ and $\chi_3 = (0)$.

$$\chi_{H} = \chi_{2}^{*} = (1) \qquad \begin{array}{l} \chi_{d} = \chi_{3}^{*} = (0) & \chi_{Q} = \chi_{2} \otimes \chi_{3} = (1) \\ \chi_{L} = \chi_{2}^{*} = (1) & \chi_{e} = \chi_{2} \otimes \chi_{2} = (0) \end{array}$$

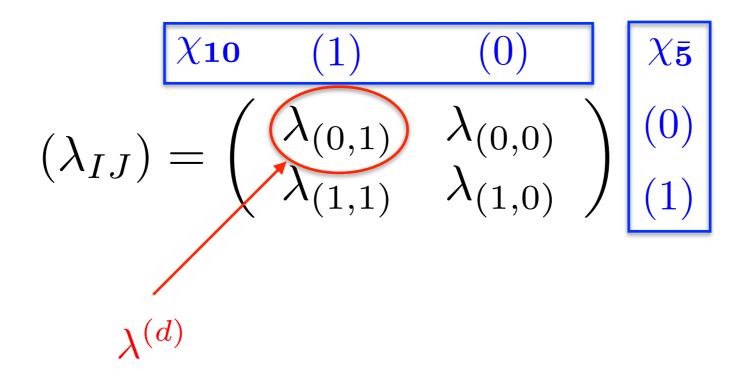
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$$(\lambda_{IJ}) = \begin{pmatrix} \lambda_{(0,1)} & \lambda_{(0,0)} \\ \lambda_{(1,1)} & \lambda_{(1,0)} \end{pmatrix}$$

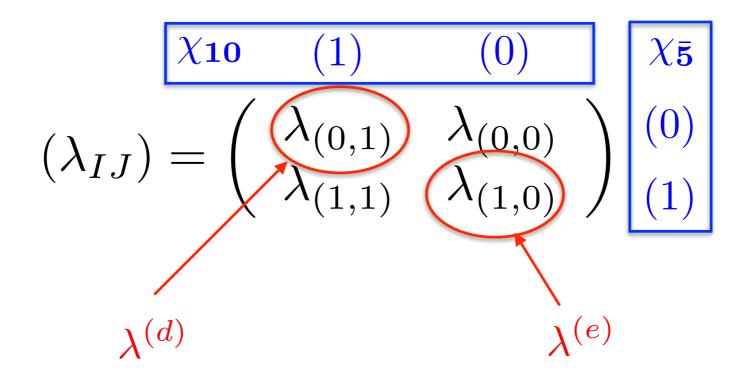
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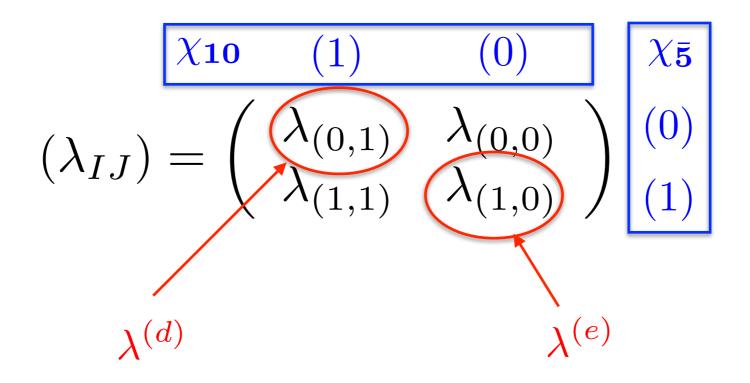
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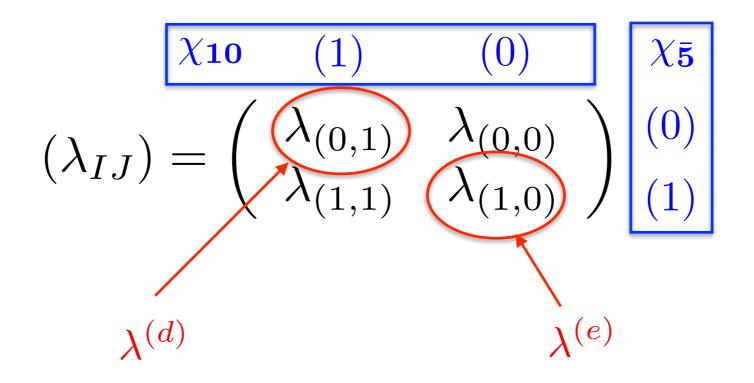


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This holds for any symmetry Γ and all types of Yukawa couplings.

In heterotic GUT models with Wilson line breaking Yukawa unification in the traditional sense (i.e. enforced by the GUT symmetry) never arises. In heterotic GUT models with Wilson line breaking Yukawa unification in the traditional sense (i.e. enforced by the GUT symmetry) never arises.

Yukawa unification may arise from additional symmetries which constraints the upstairs Yukawa couplings λ_{IJ} .

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Similar: Yukawa couplings and normalization determined by complex structure moduli space quantities.

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Phys. Yukawa couplings can be computed for standard embedding

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limitations:

- Sometimes not obvious how to carry out in practice when objects isomorphic to Cech representatives are used.
- Normalisation unkown and cannot be computed in this language.

We would like to....

- understand how to compute hol. Yukawa couplings using differential geometry language.
- clarify how such a differential geometry calculation relates to the algebraic one
- set the scene for a computation of the normalisation which requires differential geometry.

Tetra-quadric: defined as zero locus of multi-degree (2,2,2,2) polynomial in ambient space $\mathcal{A} = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

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Leads to structure groups $S(U(1)^n) \subset SU(n)$ and gauge groups $E_6, SO(10), SU(5)$

Tetra-quadric is simplest CICY which leads to line bundle standard models.

Kozsul sequence: $0 \to N^* \otimes \mathcal{L} \to \mathcal{L} \to U \to 0$ where $N = \mathcal{O}_{\mathcal{A}}(2, 2, 2, 2)$ and $L = \mathcal{L}|_X$

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 $\cdots \to H^{1}(\mathcal{A}, N^{*} \otimes \mathcal{L}) \xrightarrow{p} H^{1}(\mathcal{A}, \mathcal{L}) \xrightarrow{i^{*}} H^{1}(X, L)$ $\xrightarrow{\delta} H^{2}(\mathcal{A}, N^{*} \otimes \mathcal{L}) \xrightarrow{p} H^{2}(\mathcal{A}, \mathcal{L}) \to \cdots$

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harmonic (0,1)-forms: $\alpha = \kappa^{-k} P_{(-k-2)}(\bar{z}) d\bar{z}$ $\kappa = 1 + |z|^2$

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"For negative degree maps, replace variables by derivatives."

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Yukawa couplings vanish due to structure of cohomology.

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Can always be explicitly integrated, or calculated algebraically:

$$\lambda(\nu_1, \nu_2, \nu_3) = 16\pi^3 c\mu(P, Q, R) \qquad \qquad \mu(P, Q, R) = \tilde{P}\tilde{Q}\tilde{R}$$

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 $\mu(P, Q, R) = PQR$

• Case 3: More than one ν_i originates from ambient 2-form

Slightly more complicated but can always be integrated.

Example 1: up-Yukawa couplings

Standard model based on SU(5) GUT with line bundles

$$L_1 = \mathcal{O}_X(-1, 0, 0, 1) , \ L_2 = \mathcal{O}_X(-1, -3, 2, 2) , \ L_3 = \mathcal{O}_X(0, 1, -1, 0)$$
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Relevant line bundles for up-Yukawa coupling:

$$K_{1} = L_{2}^{*} \otimes L_{5}^{*} \qquad 3 \ \mathbf{5}_{2,5}^{H} \qquad \hat{\nu}_{1} = \kappa_{3}^{-2} Q_{(0,2,-2,0)} d\bar{z}_{3}$$

$$K_{2} = L_{5} \qquad 4 \ \mathbf{10}_{2} \qquad \hat{\nu}_{2} = \kappa_{4}^{-2} R_{(1,1,0,-2)} d\bar{z}_{4}$$

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where

$$Q = q_0 + q_1 z_2 + q_2 z_2^2$$

$$R = r_0 + r_1 z_1 + r_2 z_2 + r_3 z_1 z_2$$

$$S = s_0 + s_1 \bar{z}_2 + s_2 \bar{z}_2^2 + s_3 \bar{z}_2^3 + s_4 \bar{z}_1 + s_5 \bar{z}_1 \bar{z}_2 + s_6 \bar{z}_1 \bar{z}_2^2 + s_7 \bar{z}_1 \bar{z}_2^3$$

$$\lambda(Q,R,S) = \frac{1}{\pi} \int_{\mathbb{C}^4} \frac{QRS}{\kappa_1^3 \kappa_2^5 \kappa_3^2 \kappa_4^2} d^4 z \, d^4 \bar{z}$$

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Yukawa couplings, algebraic calculation:

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After taking quotient by $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$ and adding Wilson line:

$$\lambda^{(u)} = \frac{\pi^3}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

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$$K_{1} = L_{2} \otimes L_{4}^{*} = \mathcal{O}_{X}(-2, -4, 3, 3) \rightarrow 12 \mathbf{1}_{2,4} \in \delta^{-1} \operatorname{Ker} \left(H^{2}(\mathcal{O}_{\mathcal{A}}(-4, -6, 1, 1) \xrightarrow{p} H^{2}(\mathcal{O}_{\mathcal{A}}(-2, -4, 3, 3)) \right)$$

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$$\hat{\omega}_1 = \kappa_1^{-4} \kappa_2^{-6} Q_{(-4,-6,1,1)} d\bar{z}_1 \wedge d\bar{z}_2 \quad \text{where} \quad \tilde{p}\tilde{Q} = 0 \hat{\omega}_2 = \kappa_3^{-3} \kappa_4^{-5} R_{(0,0,-3,-5)} d\bar{z}_3 \wedge d\bar{z}_4 \hat{\nu}_3 = \kappa_3^{-2} S_{(0,2,-2,0)} d\bar{z}_3 .$$

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$$\lambda(\nu_1, \nu_2, \nu_3) = -\frac{1}{\pi} \int_{\mathbb{C}^4} \frac{Q\mathcal{R}S}{\kappa_1^4 \kappa_2^6 \kappa_3^4 \kappa_4^5} d^4 z \, d^4 \bar{z}$$

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$$= \frac{\pi^3}{3240} \left(2a_{14}b_1c_1 + 9a_{12}b_0c_2 + 9a_{13}b_0c_2 - 8a_4b_1c_2 - 8a_5b_1c_2 + 3a_{12}b_1c_2 + 3a_{13}b_1c_2 - 36a_7b_0c_3 - 12a_{2}b_1c_3 - 12a_{14}b_0c_4 + 6a_2b_1c_4 + 6a_3b_1c_4 - 6a_6b_1c_4 - 6a_7b_1c_4 + 4a_{14}b_1c_4 - 36a_6b_0c_5 - 12a_3b_1c_5 - 36a_2b_0c_6 - 36a_3b_0c_6 - 12a_6b_1c_6 - 12a_7b_1c_6 \right)$$

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$$= \frac{\pi^3}{3240} (2a_{14}b_1c_1 + 9a_{12}b_0c_2 + 9a_{13}b_0c_2 - 8a_4b_1c_2 - 8a_5b_1c_2 + 3a_{12}b_1c_2 + 3a_{13}b_1c_2 - 36a_7b_0c_3 - 12a_2b_1c_3 - 12a_{14}b_0c_4 + 6a_2b_1c_4 + 6a_3b_1c_4 - 6a_6b_1c_4 - 6a_7b_1c_4 + 4a_{14}b_1c_4 - 36a_6b_0c_5 - 12a_3b_1c_5 - 36a_2b_0c_6 - 36a_3b_0c_6 - 12a_6b_1c_6 - 12a_7b_1c_6)$$

Still need to find kernel $M\mathbf{a} = 0$ where

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 $= \frac{\pi^3}{3240} \left(2a_{14}b_1c_1 + 9a_{12}b_0c_2 + 9a_{13}b_0c_2 - 8a_4b_1c_2 - 8a_5b_1c_2 + 3a_{12}b_1c_2 + 3a_{13}b_1c_2 - 36a_7b_0c_3 - 12a_2b_1c_3 - 12a_{14}b_0c_4 + 6a_2b_1c_4 + 6a_3b_1c_4 - 6a_6b_1c_4 - 6a_7b_1c_4 + 4a_{14}b_1c_4 - 36a_6b_0c_5 - 12a_3b_1c_5 - 36a_2b_0c_6 - 36a_3b_0c_6 - 12a_6b_1c_6 - 12a_7b_1c_6 \right)$

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M =	$/ 24c_6$	0	0	0	$4c_{3}$	$4c_6$	0	0	0	$24c_{5}$	0	0	$3c_4$	0	0
	$24c_5$	0	$6c_{2}$	0	$4c_6$	$4c_3$	0	$6c_2$	0	$24c_{6}$	0	0	$-3c_{4}$	0	0
	$24c_{4}$	$24c_{6}$	0	$6c_2$	$4c_6 - 4c_4$	$4c_3 + 4c_4$	$6c_2$	0	$24c_{5}$	$-24c_{4}$	$12c_{2}$	0	$3c_1$	$3c_4$	$2c_2$
	0	$24c_5$	0	0	$4c_3$	$4c_6$	0	0	$24c_{6}$	0	$12c_{2}$	0	0	$-3c_{4}$	$2c_2$
	$24c_{3}$	0	0	0	$4c_6$	$4c_5$	0	0	0	$24c_{6}$	0	$12c_{2}$	$-3c_{4}$	0	$2c_2$
	$24c_{6}$	$24c_4$	$6c_2$	0	$4c_4 + 4c_5$	$4c_6 - 4c_4$	0	$6c_2$	$-24c_{4}$	$24c_{3}$	0	$12c_{2}$	$3c_4$	$3c_1$	$2c_2$
	0	$24c_{3}$	0	$6c_2$	$4c_5$	$4c_6$	$6c_2$	0	$24c_{6}$	0	0	0	0	$-3c_{4}$	0
	0	$24c_{6}$	0	0	$4c_6$	$4c_5$	0	0	$24c_{3}$	0	0	0	0	$3c_4$	0
	0	0	$12c_{6}$	$12c_{6}$	$8c_2$	$8c_2$	$12c_{3}$	$12c_{5}$	0	0	0	0	0	0	$4c_4$
	0	0	$12c_{5}$	$12c_{3}$	0	0	$12c_{6}$	$12c_{6}$	0	0	0	0	0	0	$-4c_{4}$
	0	0	$12c_{6}$	$12c_{6}$	0	0	$12c_{5}$	$12c_{3}$	0	0	0	0	$6c_2$	$6c_2$	$4c_4$
	0	0	$12c_3 + 12c_4$	$12c_4 + 12c_5$	$8c_2$	$8c_2$	$12c_6 - 12c_4$	$12c_6 - 12c_4$	0	0	0	0	$6c_{2}$	$6c_2$	$4c_1 - 4c_4$ /

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This is generically rank 1, but will be generally rank 2 away from the 5-parameter family. For $c_3 = c_5$ the Higgs remains massless even if $\langle S^i \rangle \neq 0$.

Conclusion

- Calculating Yukawa couplings in string theory is crucial in order to make contact with physics.
- Much remains to be done for Yukawa couplings in heterotic Calabi-Yau models with arbitrary vector bundles.
- We can now compute the holomorphic (perturbative) Yukawa couplings for heterotic line bundle models, both algebraically and in terms of differential geometry.
- First explicit calculation of complex structure dependence: rank of hol. Yukawa couplings can change in complex structure moduli space

Much remains to be done:

- Compute hol. Yukawa couplings for other manifolds.
- Compute hol. Yukawa couplings for non-Abelian bundles.
- Find a way to work out the normalisation.
- Find standard models with realistic Yukawa couplings.

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