Tuned Large Field Inflation Models in the Flux Landscape

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Motivation: Limits of fine-tuning?

- Idea: Landscape of flux vacua can in principle accommodate models with highly fine-tuned parameters; see e.g. applications to the cosmological constant [Bousso, Polchinski, 2000].
- The numbers of supersymmetric flux vacua in type IIB and F-theory have already been estimated, e.g. in [Denef, Douglas, 2004, Denef, 2008,...].
  Results often briefly summarised: 10^{500} (type IIB) flux vacua.
- Suppose we want to realise a model with 100 fine-tuned parameters in a landscape of 10^{500} vacua. Naive expectation: fine-tuning of at most $1 : 10^5$ per parameter.
- F-term axion monodromy inflation models with complex structure moduli seem to go along with potentially very many fine-tuned parameters. What is the price to pay?
  [A. Hebecker, PM, F. Rompineve, L.T. Witkowski, 2014]

Steps towards an inflation model with F-term axion monodromy, see [A. Hebecker, PM, F. Rompineve, L.T. Witkowski, 2014]:

1. Choose a CY 3- or CY 4-fold.
2. Compute the Kähler potential K, the Kähler metric $K_{i\bar{\jmath}}$ from the period vectors.
3. Identify one complex structure modulus u in the large complex structure limit, s.t. it occurs shift-symmetric in K. Inflaton $y \equiv \mathrm{Re}(u)$.
4. By flux choice, ensure that the superpotential looks like
$$W = W_0 + a(z)u.$$
5. F-term scalar potential contains terms
$$V \sim |K_u a|^2 y^2 + |\partial_z a + K_z a|^2 y^2 + \ldots$$
6. Thus, have to tune small $|a(z)|$ and all the $|\partial_z a(z)|$, i.e. potentially many terms! No-Go theorem for CY 3-folds!

Counting susy F-Theory flux vacua [Denef, Douglas, 2004, Denef, 2008]

- G_4-flux numbers N^I can't be chosen arbitrarily. Let X be a CY 4-fold, we have to satisfy the D3-tadpole condition
$$\frac{1}{2}\int_X G_4 \wedge G_4 = \frac{\chi(X)}{24} - N_{\mathrm{D3}} \leq \frac{\chi(X)}{24} \equiv L_\star$$
if $N_{\mathrm{D3}} \geq 0$. (Anti-D3-branes decay in flux-background, so $N_{\mathrm{D3}} \ll 0$ is no option.)
- In the case of SUSY we have $G_4 = \star G_4$, hence:
$$0 \leq \frac{1}{2}\int_X G_4 \wedge G_4 \leq L_\star$$
- By $[G_4] = N^I \Sigma_I$, with 4-cycles Σ_I, we can rewrite the inequality as:
$$0 \leq \frac{1}{2} N^I Q_{IJ} N^J \leq L_\star,$$
with positive definite matrix Q. Hence: finitely many choices of flux numbers!

Counting susy F-Theory flux vacua
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- Thus, we need to determine how many sets of flux vectors satisfy
$$0 \leq L \equiv \frac{1}{2} N^I Q_{IJ} N^J \leq L_\star .$$
- Let b be the dimension of the flux space. Approximate the number of solutions by the volume of a b-dimensional ball with radius $\sqrt{2L_\star}$.
Analogy: b-dimensional ball described by $x_1^2 + \dots + x_b^2 \leq R^2$.
- Actual computation rather technical. Result:
$$\mathcal{N}_{\text{vac}}(L \leq L_\star) = \frac{(2\pi L_\star)^{b/2}}{\left(\frac{b}{2}\right)! \sqrt{\det Q_{IJ}}} \times \underbrace{\int_{\mathcal{M}} e(\nabla)}_{\substack{\text{integral of Euler density} \\ \text{over compl. str. moduli space } \mathcal{M}}}$$
- $\mathbb{CP}^3$-fibration: $b = 23320$, $L_\star = 972 \Rightarrow \mathcal{N}_{\text{vac}} \sim 10^{1700}$.
- Caution: Formula unterestimates true $\mathcal{N}_{\text{vac}}$ for large b.

Constraining the flux landscape by fine-tuning conditions
[A. Hebecker, PM, F. Rompineve, L.T. Witkowski, 2014]

- Starting point of more correct computation of $\mathcal{N}_{\text{vac}}(L \leq L_\star)$:

$$\mathcal{N}_{\text{vac}}(L \leq L_\star) = \sum_{\text{susy vacua}} \theta(L_\star - L),$$

  rewrite θ-fct. as contour integral over auxiliary parameter, see [Denef, Douglas, 2004].

- Include J_t *real* tuning constraints $|a_i| < \epsilon$ of *complex* parameters a_i as follows:

$$\mathcal{N}_{\text{vac}}(L \leq L_\star, \{|a_i| < \epsilon\}) = \sum_{\text{susy vacua}} \theta(L_\star - L) \prod_{i=1}^{J_t/2} \theta(\epsilon - |a_i|)$$

- F-term AMI: Make J_f flux choices. $J_t/2 - 1$ counts number of z that enter $a(z)$.

Constraining the flux landscape by fine-tuning conditions

[A. Hebecker, PM, F. Rompineve, L.T. Witkowski, 2014]

- Each flux choice and each (real) tuning condition lower the dimension of flux space by one, respectively.
- Each $\theta(\epsilon - |a_i|)$ contributes a factor $\pi\epsilon^2$.
- Result:

$$\mathcal{N}_{\rm vac}(L \le L_\star, |a_i| < \epsilon) = \frac{(2\pi L_\star)^{(b-J_f-J_t)/2}}{\left(\frac{b-J_f-J_t}{2}\right)!\sqrt{\det Q_{IJ}}} \times (\pi\epsilon^2)^{J_t/2} \times \int_{\mathcal{M}} \dots$$

- Consequences: $\epsilon \lesssim 0.04$ for ϕ^2-inflation. Take $\epsilon = 0.04$: Consider again $\mathbb{CP}^3$-fibration and allow only 300 of the 3878 compl. str. moduli to enter a. Hence: $J_t = 600$. Assume $J_f = 0$.
  Result:

$$\sim 10^{300}$$

  out of originally 10^{1700} vacua remain!

Summary and Prospect

- F-Term Axion Monodromy inflation with breaking of the shift-symmetry via $W \supset a(z)u$ can only be achieved by accepting a **high amount of fine-tuning**.

- Even if only very view complex structure moduli backreact with the inflaton, the **flux landscape will be strongly reduced**.

- It would be interesting to construct concrete of F-term axion monodromy inflation models on concrete geometries and work out the required flux choices and tuning conditions in detail.

- Quantifying how many vacua are appropriate for various stringy inflation models can tell us how generic they are in the string landscape.

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Thank you!