DIPARTIMENTO
DI FISICA
E ASTRONOMIA
Galileo Galilei

Warping the Kähler potential of flux compactifications

Luca Martucci

University of Padova

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0902.4031

hep-th/0703129 with P. Koerber

StringPheno2015

Flux compactifications and warping

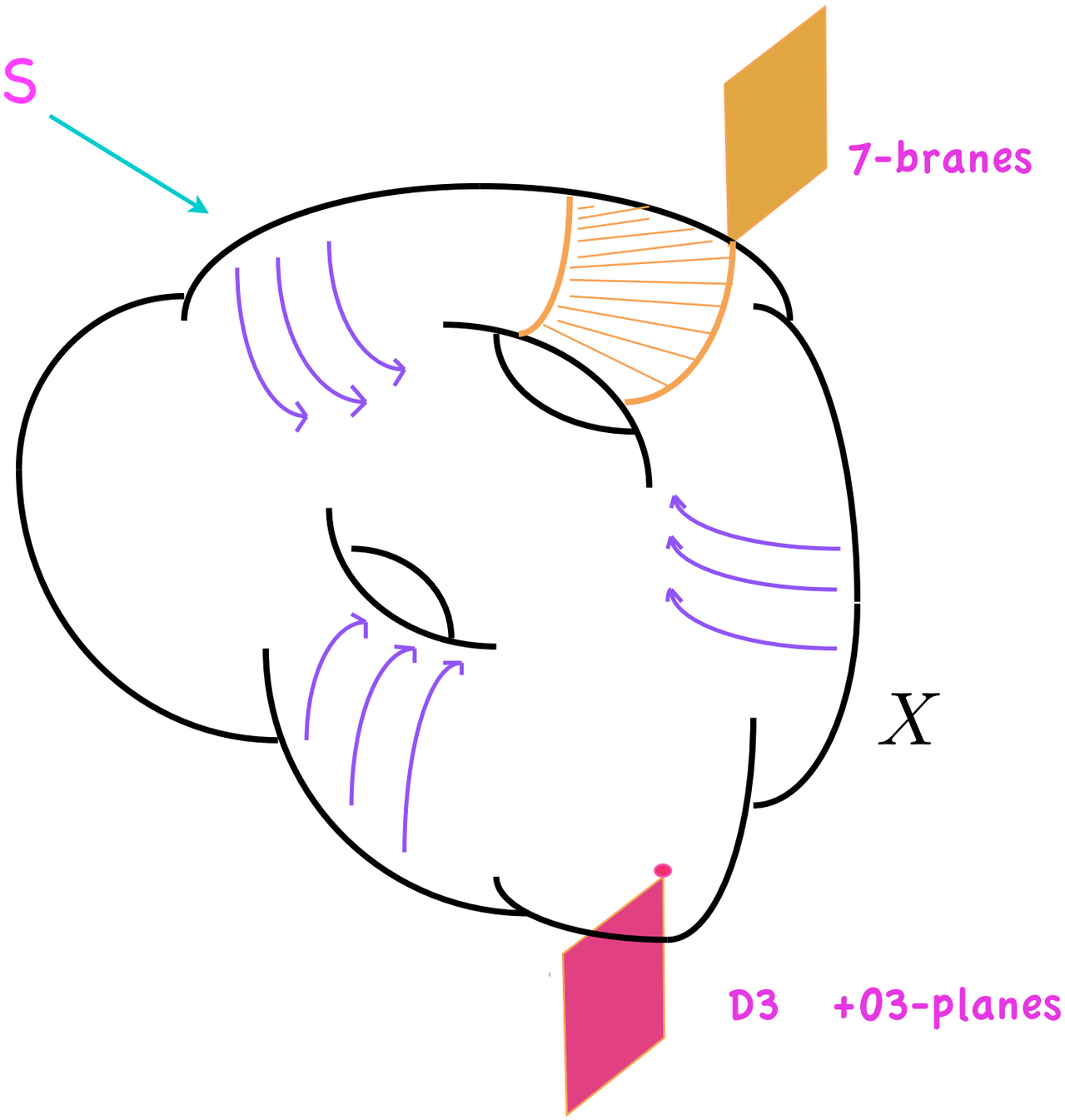
FLUXES

$$W_{\text{flux}} = \int_X \Omega \wedge G_3$$

[Gukov-Vafa-Witten]

* moduli stabilization
* SUSY breaking
* warping!

$$\mathrm{d}s_{10}^2 = e^{2A(y)}\mathrm{d}s_4^2 + e^{-2A(y)}\mathrm{d}s_X^2$$



[Becker-Becker,Grana-Polchinski, Gubser, Giddings-Kachru-Polchinski]

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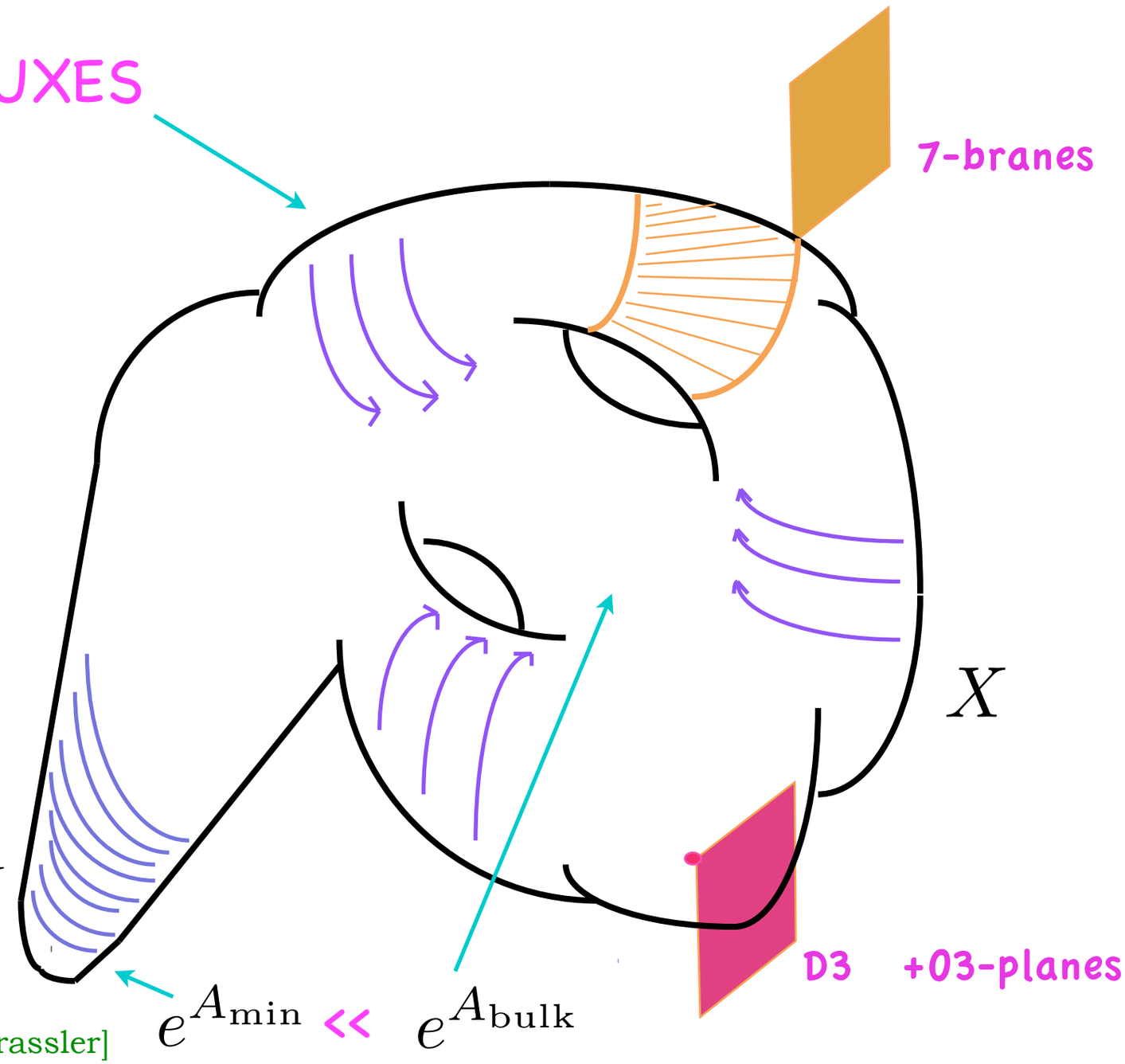

[Klebanov-Strassler]

[Becker-Becker,Grana-Polchinski, Gubser, Giddings-Kachru-Polchinski]

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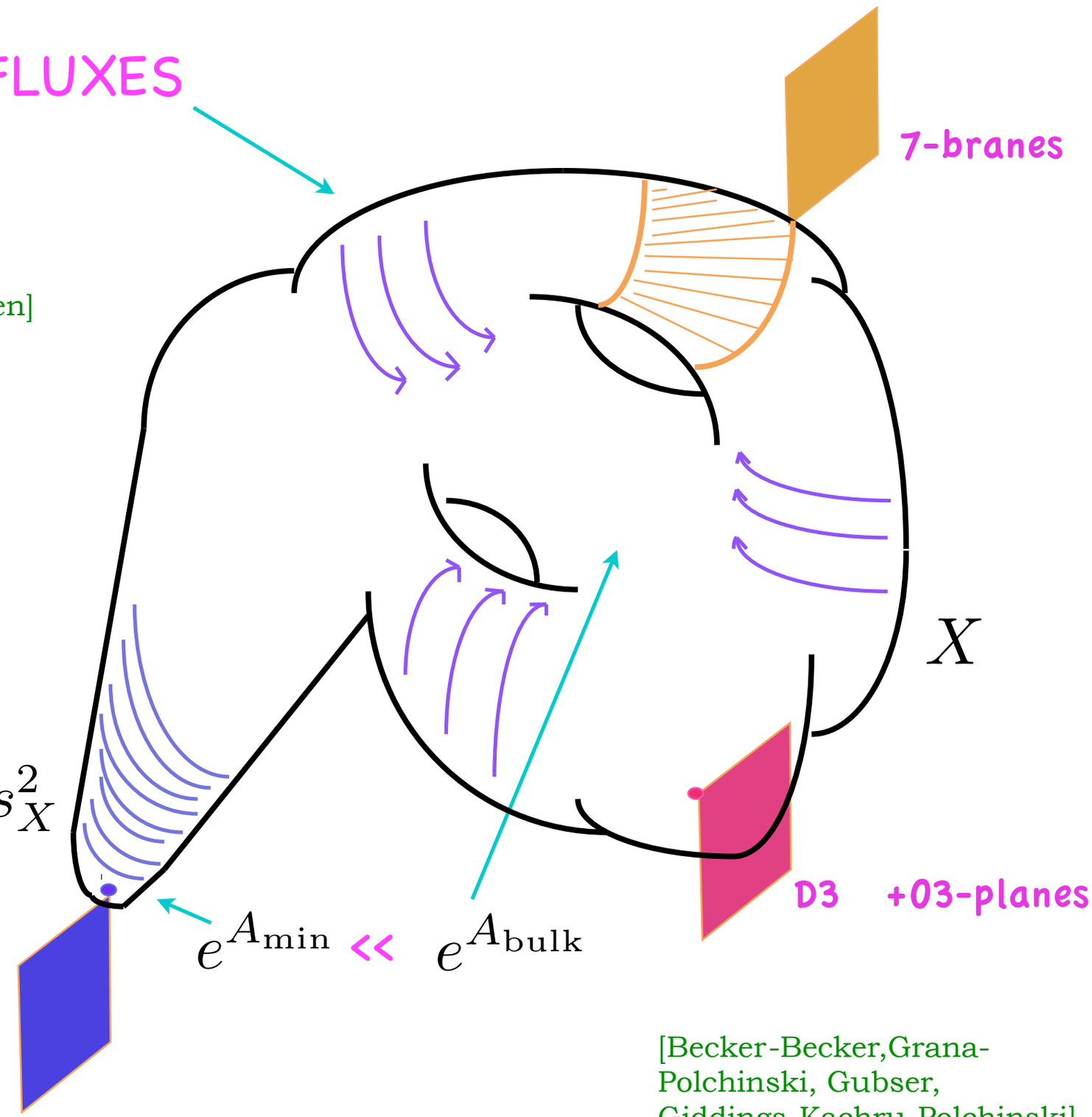

"de Sitter uplift"

[KKL(MM)T,...]

(not further considered in this talk)

[Becker-Becker,Grana-Polchinski, Gubser, Giddings-Kachru-Polchinski]

4D warped effective theory?

- The non-trivial warp factor complicates the standard KK reduction

  [DeWolfe-Giddings, Giddings-Maharana, Frey-Maharana, Burgess-Cåmara-deAlwis-Giddings-Maharana-Quevedo-Suruliz, Marchesano-McGuirk-Shiu, ...]

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- Usual solution: Ignore the warp factor!

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- Usual solution: Ignore the warp factor!

  ↓

  Physical quantities from pseudo-topological data (underlying N=2 structure) → nice effective action

  [..., Grimm-Louis, Graña-Grimm-Jockers-Louis, Jockers-Louis, Denef, Grimm, ...]

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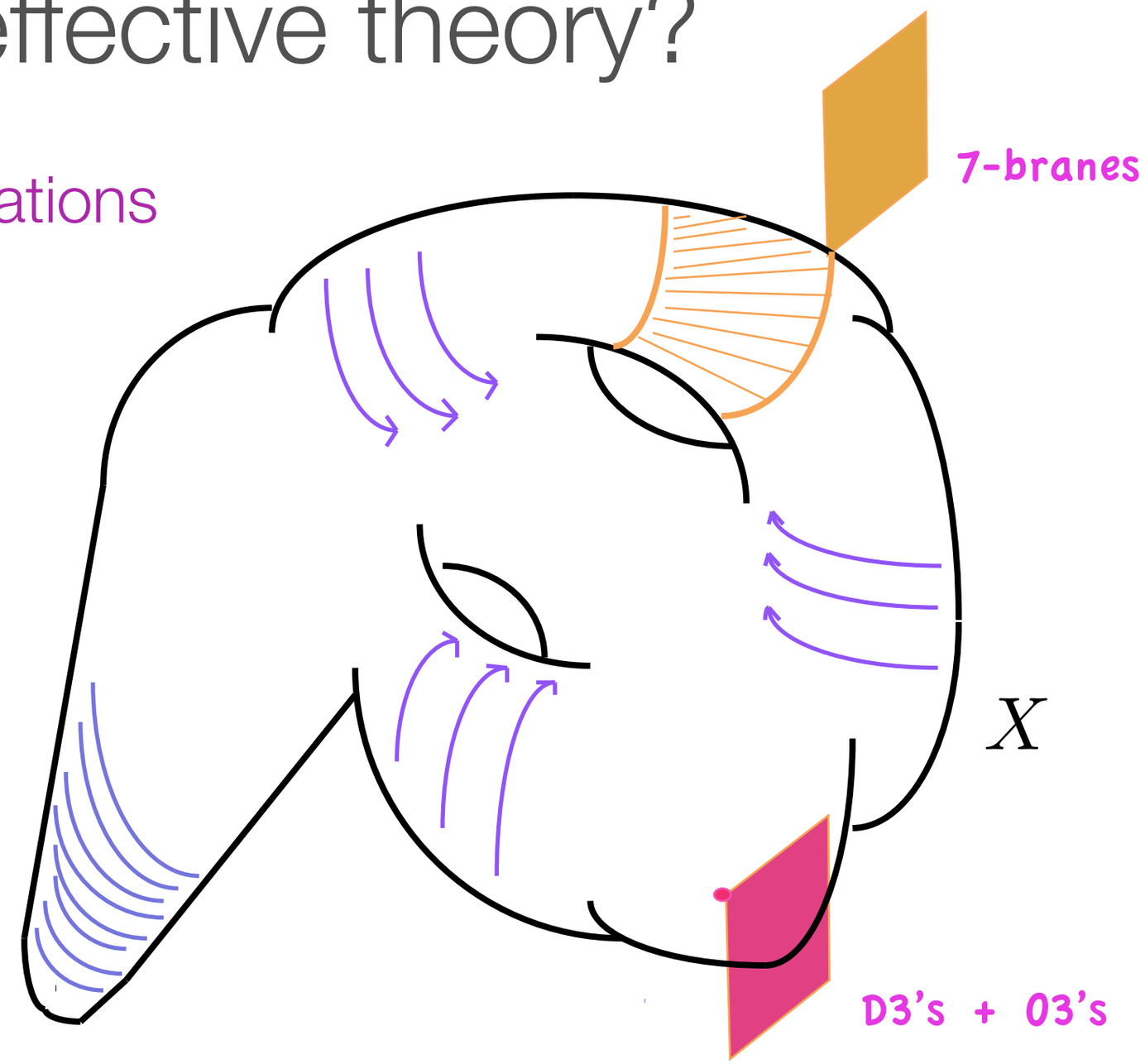
… however in warped compactifications

* X is still complex and Kähler!
* preserved N=1 supersymmetry (+ possible no-scale SB)
* superpotential insensitive to warping



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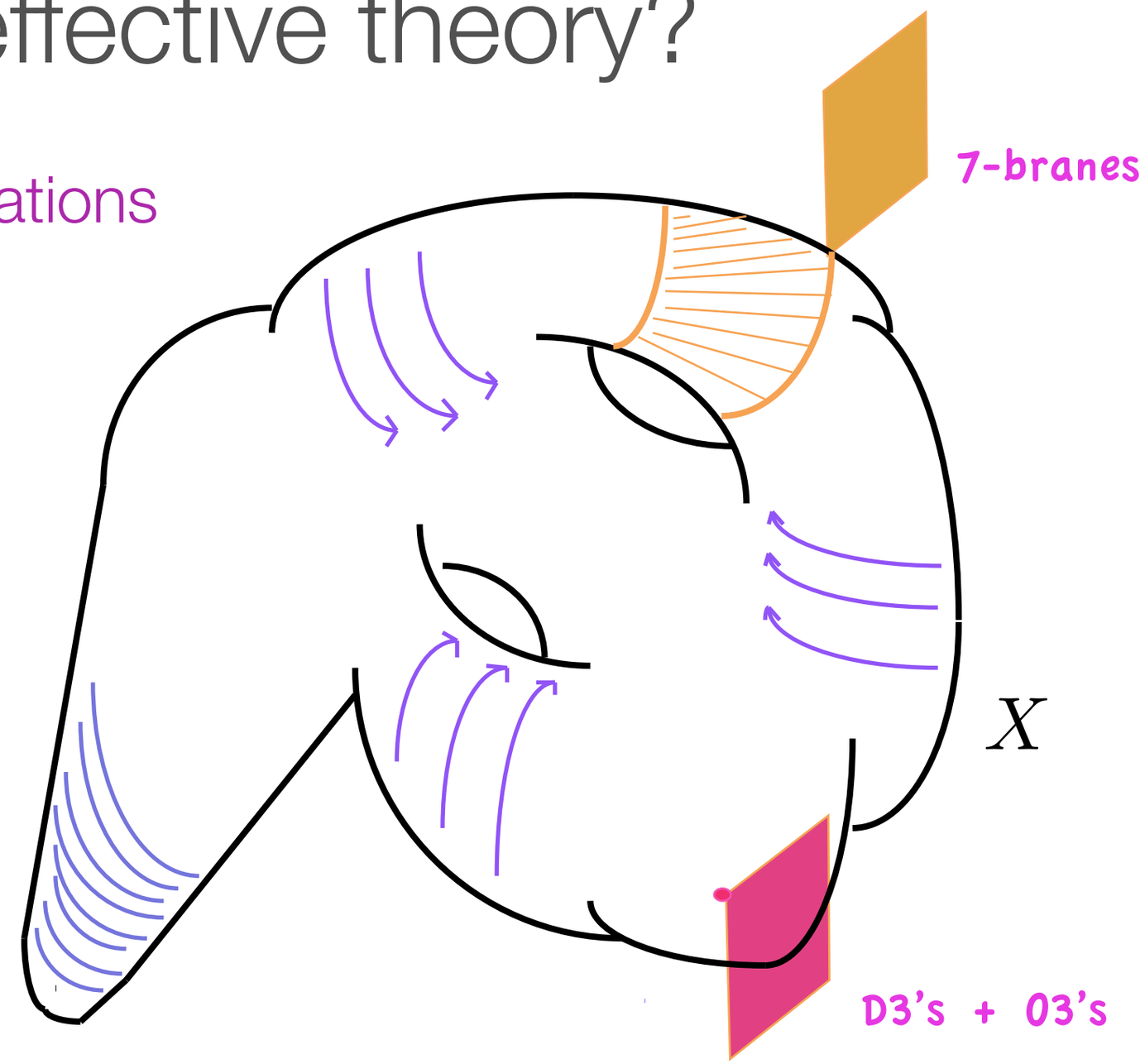

Question:

Natural and 'simple' incorporation of the warping in the Kähler potential ?

The warped Kähler potential

Step 0: reassembling Kähler moduli

(Complex structure and axio-dilaton moduli assumed frozen)

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D3-charge density

- $$\Delta e^{-4A} = *Q_6 \quad \text{with} \quad Q_6 = F_3 \wedge H_3 + \sum_{I \in \mathrm{D3's}} \delta_I^6 - \frac{1}{4} \sum_{O \in \mathrm{O3's}} \delta_O^6 + \ldots$$

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$$\Rightarrow \quad e^{-4A(y)} = a + e^{-4A_0(y)}$$

[Giddings-Maharana]

UNIVERSAL MODULUS a

$$\int_X G(y,y') Q_6(y')$$

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Kähler form: $J = v_a \omega^a$

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$\Rightarrow$ Naturally split $h^{1,1}$ Kähler moduli: a , v_a

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$$\int_{M_4\times X} R_{10} = \int_{M_4} \mathrm{vol_w}(X) R_4 + \ldots \qquad \text{with} \qquad \mathrm{vol_w}(X) = \int_X e^{-4A}\mathrm{dvol}_X$$

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- Superconformal argument fixes: cf. also [Haack-Louis]

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cf. [DeWolfe-Giddings]

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cf. [DeWolfe-Giddings]

- Inserting $e^{-4A(y)} = a + e^{-4A_0(y)}$ and using $\int_X e^{-4A_0}\mathrm{dvol}_X = 0$:

$\Rightarrow$

$$\boxed{K = -3\log a}$$

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$$\Rightarrow \quad \boxed{K = -3\log a}$$

simple but implicit!

Step 2: chiral coordinates

- Sector we focus on:
  - D3 positions: $Z_I^i \qquad I = 1, \ldots, n_{\mathrm{D3}}$ good chiral fields
  - Kähler moduli: a , v_a + axionic partners $\longrightarrow$ $h^{1,1}$ chiral fields ρ^a

    $a = 1, \ldots, h^{1,1}$

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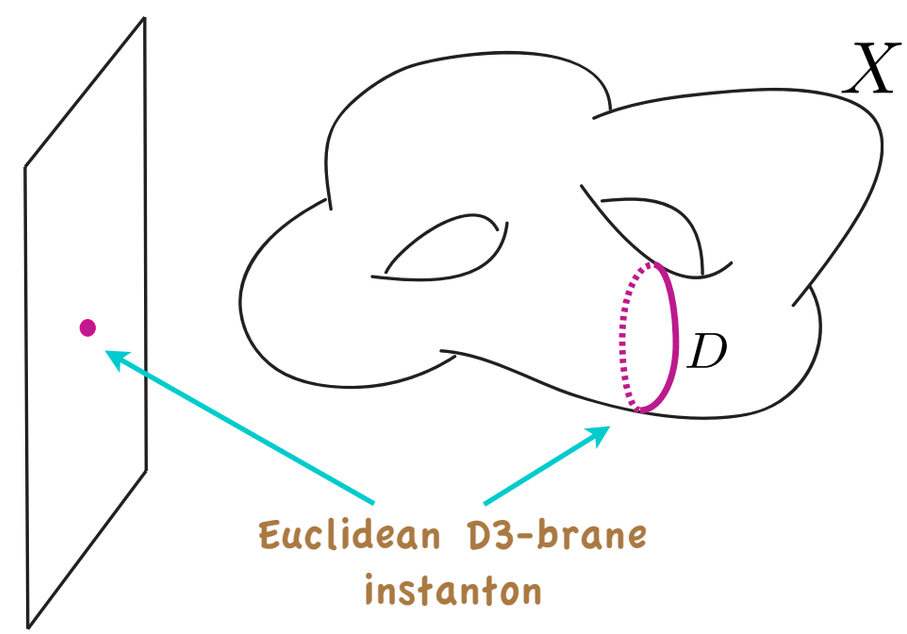
HOW??

- Probing the geometry by SUSY D3 instantons

F-terms $\sim \quad e^{-S_{\mathrm{D3}}}$

$$S_{\mathrm{D3}} = \frac{1}{2}\int_D e^{-4A} J \wedge J + \quad \ldots$$

must depend holomorphically on chiral fields $\rho^a\,,\ Z_I^i$



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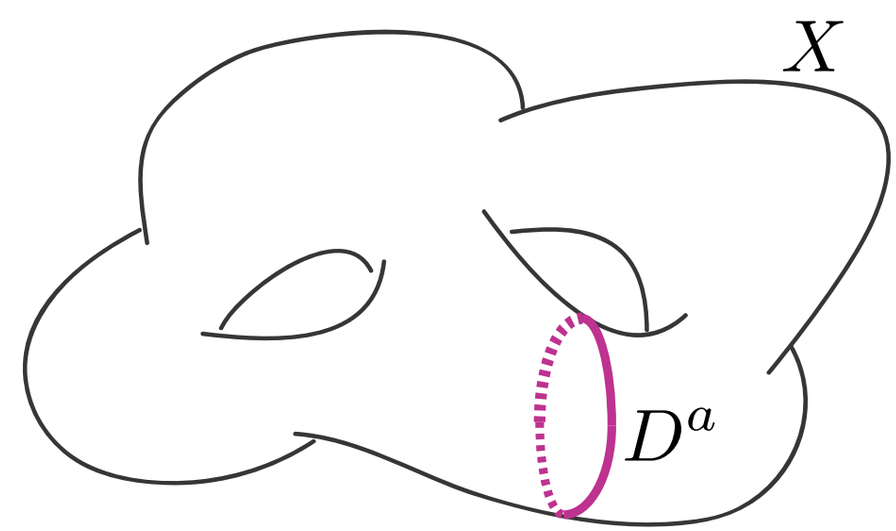
HOW??

- Natural choice: ρ^a such that

$$\mathrm{Re}\,\rho^a = \frac{1}{2}\int_{D^a} e^{-4A} J \wedge J$$

$$[D^a] = \omega^a$$

harmonic $H^2(M, \mathbb{Z})$ basis



cf. [Giddings-Maharana]

Step 2: chiral coordinates

- By solving the warping:

$$\boxed{\mathrm{Re}\,\rho^a = \frac{1}{2} a\,\mathcal{I}^{abc} v_a v_b + h^a(v) + \frac{1}{2}\sum_{I\in \mathrm{D3's}} \kappa^a(Z_I, \bar{Z}_I; v)}$$

fluxes + other D3-charge sources

D3-branes contribution

* $\mathcal{I}^{abc} = D^a \cdot D^b \cdot D^c$
* $\kappa^a(z, \bar{z}; v)$ `potentials': $\mathrm{i}\partial\bar{\partial}\kappa^a = \omega^a$
* $h^a(v) \equiv \int_X (\kappa^a - \mathrm{Re}\log\zeta^a) Q_6^{\mathrm{bg}}$

section of $\mathcal{O}(D^a)$

background D3-charge

$$F_3 \wedge H_3 - \frac{1}{4}\sum_{O\in \mathrm{O3's}} \delta_O^6 + \ldots$$

Summarising

$$K = -3\log a$$

$$\mathrm{Re}\,\rho^a = \frac{1}{2}a\,\mathcal{I}^{abc}v_a v_b + h^a(v) + \frac{1}{2}\sum_{I\in \mathrm{D3's}} \kappa^a(Z_I, \bar{Z}_I; v)$$

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Final step:

find $a(\mathrm{Re}\,\rho, Z, \bar{Z}) \longrightarrow K(\rho, \bar{\rho}, Z, \bar{Z}) \equiv -3\log a(\mathrm{Re}\,\rho, Z, \bar{Z})$

not possible in general! (as in the unwarped approximation)

Some implications: Old and new results

Particular limits

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- Just universal modulus ($h^{1,1} = 1$)

$$K = -3 \log \left(\rho + \bar{\rho} \right)$$

cf. [Frey-Torroba-Underwood-Douglas]

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cf. [DeWolfe-Giddings, Giddings-Maharana, Baumann et al, Chen-Nakayama-Shiu]

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- Constant warping, no fluxes and weakly fluctuating D3-branes

$$K = -2\log\left(\mathcal{I}^{abc}v_a v_b v_c\right)$$

v_a unconstrained

$$\mathrm{Re}\,\rho^a = \frac{1}{2}\mathcal{I}^{abc}v_a v_b - \frac{\mathrm{i}}{2}\sum_{I\in\mathrm{D3's}} \omega^a_{i\bar{\jmath}}\Phi^i_I\bar{\Phi}^{\bar{\jmath}}_I$$

cf. [Graña-Grimm-Jockers-Louis]

Large moduli limit

- Assuming $\mathrm{Re}\,\rho^a \gg 1$ one can approximate

$$K \simeq -2\log V + \frac{1}{V}\left[h(v) + \frac{1}{2}\sum_{I\in \mathrm{D3's}} k(Z_I, \bar{Z}_I; v)\right] + \ldots$$

* $V = \frac{1}{6}\mathcal{I}^{abc} v_a v_b v_c$ $\qquad$ v_a unconstrained

* $\mathrm{Re}\,\rho^a = \frac{1}{2}\mathcal{I}^{abc} v_a v_b$ $\qquad$ unwarped chiral coordinates

* $h(v) \equiv \int_X [k(v) - v_a \mathrm{Re}\,\zeta^a] Q_6^{\mathrm{bg}}$

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$\sim \mathcal{O}(V^{-\frac{2}{3}})$

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Kinetic terms

- Even though the explicit $K(\rho, \bar{\rho}, Z, \bar{Z})$ is not known in general, one can compute the explicit form of the kinetic terms:

$$\mathcal{L}_{\rm kin} = -\mathcal{G}^{\rm w}_{ab} \nabla_\mu \rho^a \nabla^\mu \bar{\rho}^b - \frac{1}{2a} \sum_I g_{i\bar{\jmath}}(Z_I, \bar{Z}_I) \partial_\mu Z^i_I \partial^\mu \bar{Z}^{\bar{\jmath}}_I$$

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D3-branes kinetic terms

$\longrightarrow$ matches what obtained by probe approximation

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* $\mathcal{G}^{\rm w}_{ab} \equiv \frac{1}{8a^2} v_a v_b - \frac{1}{4a} (M^{-1}_{\rm w})_{ab}$

* $M^{ab}_{\rm w} \equiv \int_X e^{-4A} J \wedge \omega^a \wedge \omega^b$

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warping of Kähler structure moduli space!

(absent if X is geometrically formal)

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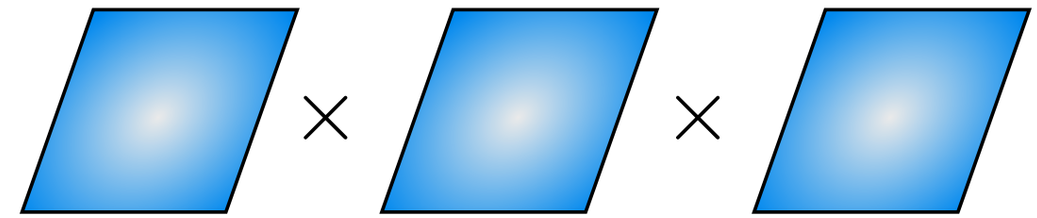
Furthermore:

$K(\rho, \bar{\rho}, Z, \bar{Z})$ is no-scale:

$$K^{A\bar{B}} K_A K_{\bar{B}} = 3$$

A simple class of solvable models

- O3-compactification on $X =$ 

$$z^i = x^i + \lambda y^i$$

$$\tau = \lambda = e^{\frac{2\pi \mathrm{i}}{3}}$$

[Kachru-Schulz-Trivedi]

$$G_3 \sim \mathrm{d}z^1 \wedge \mathrm{d}z^2 \wedge \mathrm{d}\bar{z}^3 + \mathrm{d}z^2 \wedge \mathrm{d}z^3 \wedge \mathrm{d}\bar{z}^1 + \mathrm{d}z^3 \wedge \mathrm{d}z^1 \wedge \mathrm{d}\bar{z}^1$$

- 6 ρ^a moduli + D3's:

$$\kappa^1 = \frac{z^1 \bar{z}^1}{\mathrm{Im}\,\lambda} \qquad \kappa^2 = \frac{z^2 \bar{z}^2}{\mathrm{Im}\,\lambda} \qquad \kappa^3 = \frac{z^3 \bar{z}^3}{\mathrm{Im}\,\lambda}$$

$$\kappa^4 = \frac{\mathrm{Re}\,(z^2 \bar{z}^3)}{\mathrm{Im}\,\lambda} \qquad \kappa^5 = \frac{\mathrm{Re}\,(z^3 \bar{z}^1)}{\mathrm{Im}\,\lambda} \qquad \kappa^6 = \frac{\mathrm{Re}\,(z^1 \bar{z}^2)}{\mathrm{Im}\,\lambda}$$

- $$K = -\log\left[T^1 T^2 T^3 + 2T^4 T^5 T^6 - T^1 \left(T^4\right)^2 - T^2 \left(T^5\right)^2 - T^3 \left(T^6\right)^2\right]$$

with $T^a \equiv \mathrm{Re}\,\rho^a - \frac{1}{2} \sum_I \kappa^a (Z_I, \bar{Z}_I)$

no warping of ρ^a moduli space

Summary

- Very simple (implicit) Kähler potential $K = -3\log a$
  + flux and D3 dependent chiral coordinates for Kähler structure moduli

- D3's automatically incorporated

- The Kähler structure moduli space gets warped

- At large moduli, warping induced corrections to K of order $\mathcal{O}(V^{-\frac{2}{3}})$

- B_2 - C_2 and D7 axions can be easily incorporated (at weak coupling)

Future directions

- Develop efficient computational methods

- Incorporation of complex structure and 7-brane moduli

  Complex structure enters the definition of the ρ^a moduli:

  → mixing of complex and Kähler structures

$$K_{\text{tot}} \neq K_{\text{cs}}(U,\bar{U}) + K_{\text{ks}}(\rho,\bar{\rho},Z_I,\bar{Z}_I)$$

[Graña-Grimm-Jockers-Louis]

- Incorporation of gauge sector and charged matter cf. [Marchesano-McGuirk-Shiu] [Grimm-Klevers-Poretschkin]

- Combined warping and higher order effects? cf. [Grimm-Pugh-Weissenbacher]

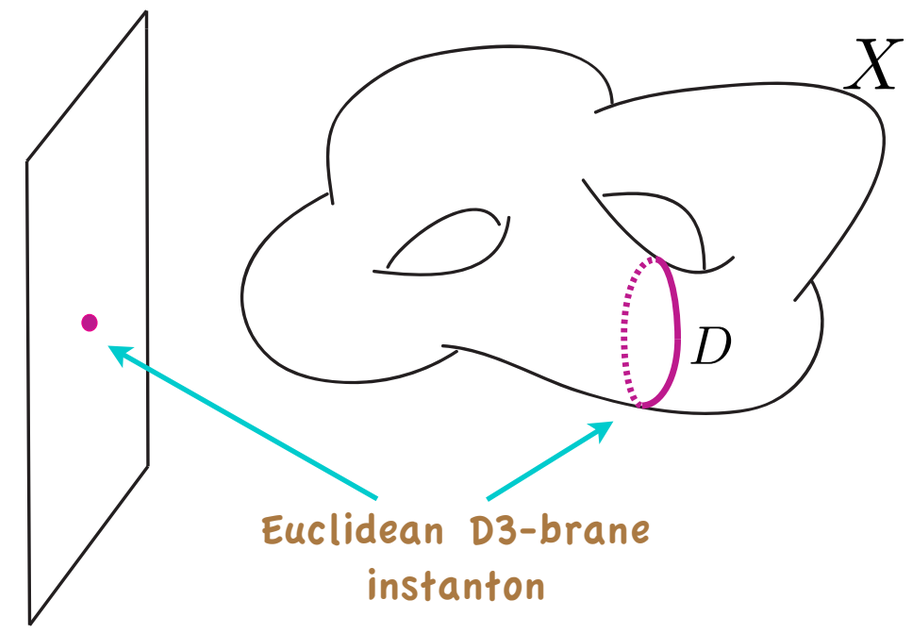
- Phenomenological implications?

A by-product

- Supersymmetric D3-brane instanton wrapping D :

$$W_{\rm np} \sim \exp\Big[-\frac{1}{2}\int_D e^{-4A} J\wedge J + \ldots\Big]$$

$$\sim \prod_{I\in \text{D3's}} \zeta_D(Z_I)\exp(-\rho)$$



[Ganor]

[Baumann-Dymarsky-Klebanov-Maldacena-McAllister]