Three family models of particle physics from global F-theory compactifications

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Plan

- Introduction and motivation
- Tools and Strategies for Model Building
- An MSSM-like model
- Pati-Salam
- Trinification
- Conclusions and Prospects

Introduction and motivation

- F-theory constitutes a rich background to realize many features of particle physics models: gauge symmetries, chiral matter, Yukawas...

See talks from today and tomorrow's session

- Most model building efforts have contemplated an underlying SU(5) GUT...

Antoniadis, Beasley, Braun, Colinucci,Dolan, Dudas, Grimm, Hayashi, Heckman, Keitel, Leontaris, Marchesano, Marsano, Mayrhofer, Palti, Schäfer Nameki, Vafa,Valandro, Watari, Weigand,...

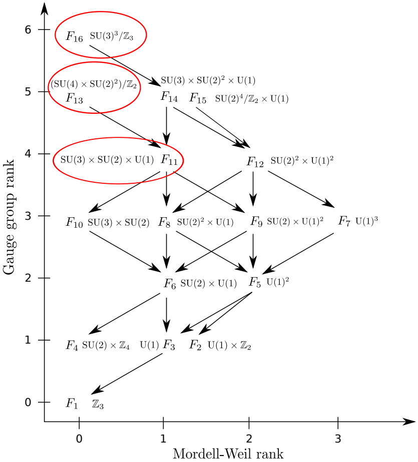
...there are (in principle) other alternatives

Lin, Weigand'14

- Here we consider some of such alternatives which stemmed from our study of toric hypersurface fibers...the bare MSSM, Pati-Salam, trinification

Klevers, DM, Oehlmann, Piragua, Reuter'14

Introduction and motivation



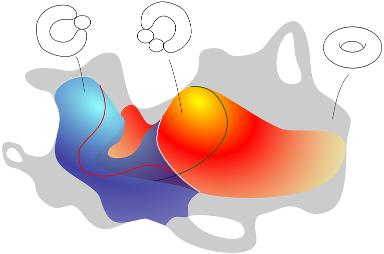
See P. Oehlmann's talk

Out of the 16 types of toric fibrations, three lead to the gauge group and (plausibly) the representations needed for the MSSM (F_{11}), Pati-Salam (F_{13}) and trinification (F_{16}).

- So far only 6D theories discussed. Here we discuss the simplest type of geometry leading to 4D models with the desired features.
- We need to consider G_4-flux in order to generate the right chiralities.

Tools and strategies for model building

F-theory Geometry:



$\Rightarrow$ This leaves us with the sections $S_0,\ldots,S_m$, the gauge divisors $D^I_{i_I},\ldots$

- Cut the fiber $\mathcal{C}_{F_i}$ as a restricted cubic in any of the toric ambient spaces F_{11}, F_{13} or F_{16}.
  $\Rightarrow$ Map the cubic to the Weierstraß equation using Nigell's algorithm.
- The CY elliptic fibration results from promoting the coefficients of the cubic as sections of the (simplest) base $B = \mathbb{P}^3$

$$\begin{array}{ccc} \mathcal{C}_{F_i} & \longrightarrow & X_{F_i}(\mathcal{S}_7, \mathcal{S}_9) \\ & & \downarrow \\ & & \mathbb{P}^3 \end{array} \qquad (1)$$

Tools and strategies for model building

G_4**-flux:**

- To construct the (chirality inducing) G_4 flux we have to work out $H_V^{(2,2)}(X_{F_i})$.
- G_4 is subjected to a quantization condition and must allow for a positive (integer) tadpole cancelling number of D3 branes

Witten'96, Sethi, Vafa, Witten'96, Gukov, Vafa, Witten'99

$$G_4 + \frac{c_2(X)}{2} \in H^4(X, \mathbb{Z}) \,, \quad \frac{\chi(X)}{24} - \frac{1}{2}\int_X G_4 \wedge G_4 = n_{\mathrm{D3}} \in \mathbb{N} \,.$$

- There are also constraints on certain CS terms $\Theta_{AB} = \int_X G_4 \wedge D_A \wedge D_B$

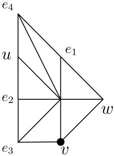
Marsano, Schäfer-Nameki'11, Grimm, Hayashi'12, Cvetič, Grimm, Klevers'12

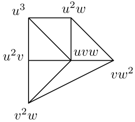
...we do not have an integral basis to check flux quantization. Instead we demand $\Theta_{AB} \in \mathbb{Z}/2$.

Intrilligator, Jockers, Mayr, Morrison, Plesser'12

- The chiralities: $\chi(\mathbf{R}) = n(\mathbf{R}) - n(\bar{\mathbf{R}}) = \int_{\mathcal{C}^w_{\mathbf{R}}} G_4$.

An MSSM-like model





- **sections:** $S_0 \;:\; v = 0$, $S_1 \;:\; e_4 = 0$
- **Non Abelian factors:**
  - SU(3) at $\{s_9 = 0\}$
  - SU(2) at $\{s_3 = 0\}$

The fiber is cut by the cubic

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 \\ + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

as $\{p_{F_{11}} = 0\}$. In the fibration, the div. classes of the s_i are written in terms of two divisors $\mathcal{S}_7$, $\mathcal{S}_9$.

For the specific case of base $\mathbb{P}^3$ one can write these in terms of the hyperplane class H_B.

$$\mathcal{S}_7 = n_7 H_B \;, \quad \mathcal{S}_9 = n_9 H_B \;, \quad [K_B^{-1}] = 4 H_B \,.$$

...with (n_7, n_9) being constrained by *effectiveness*.

An MSSM-like model

- **at codimension two:**

Representation	Locus
$(\mathbf{3},\mathbf{2})_{1/6}$	$V(I_{(1)}) := \{s_3 = s_9 = 0\}$
$(\mathbf{1},\mathbf{2})_{-1/2}$	$V(I_{(2)}) := \{s_3 = s_2 s_5^2 + s_1(s_1 s_9 - s_5 s_6) = 0\}$
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$V(I_{(3)}) := \{s_5 = s_9 = 0\}$
$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$V(I_{(4)}) := \{s_9 = s_3 s_5^2 + s_6(s_1 s_6 - s_2 s_5) = 0\}$
$(\mathbf{1},\mathbf{1})_{1}$	$V(I_{(5)}) := \{s_1 = s_5 = 0\}$

- **at codimension three:**

Yukawa	Locus
$(\mathbf{3},\mathbf{2})_{1/6} \cdot (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \cdot \overline{(\mathbf{1},\mathbf{2})_{-1/2}}$	$s_3 = s_5 = s_9 = 0$
$(\mathbf{3},\mathbf{2})_{1/6} \cdot (\bar{\mathbf{3}},\mathbf{1})_{1/3} \cdot (\mathbf{1},\mathbf{2})_{-1/2}$	$s_3 = s_9 = 0 = s_1 s_6 - s_2 s_5$
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3} \cdot \overline{(\bar{\mathbf{3}},\mathbf{1})_{1/3}} \cdot (\mathbf{1},\mathbf{1})_{1}$	$s_1 = s_5 = s_9 = 0$
$(\mathbf{3},\mathbf{2})_{1/6} \cdot (\mathbf{3},\mathbf{2})_{1/6} \cdot \overline{(\bar{\mathbf{3}},\mathbf{1})_{1/3}}$	$s_3 = s_9 = s_6 = 0$
$(\mathbf{1},\mathbf{2})_{-1/2} \cdot (\mathbf{1},\mathbf{2})_{-1/2} \cdot (\mathbf{1},\mathbf{1})_{1}$	$s_1 = s_5 = s_3 = 0$
$(\bar{\mathbf{3}},\mathbf{1})_{1/3} \cdot (\bar{\mathbf{3}},\mathbf{1})_{1/3} \cdot (\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$s_5 = s_6 = s_9$

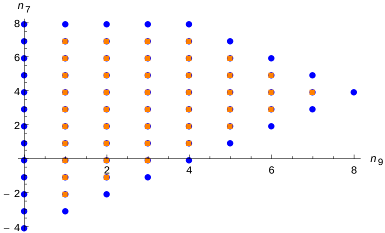
An MSSM-like model

The flux is of the form

$$G_4 = a_6 H_B \cdot \sigma(\hat{s}_1) - a_7 \left[\tilde{S}_0^2 + (20 n_7 - n_7^2 + 8 n_9 - n_7 n_9 - 92) H_B^2 \right] ,$$

Flux solutions could break the hypercharge generator. Only flux solutions preserving a family structure allowed.

$$a_6 = -\frac{6 a_7 (n_7 - 8)(4 + n_7 - n_9)}{3 n_7 + n_9 - 36} = \frac{6(\sharp\text{Families})}{n_9 (4 + n_7 - n_9)} .$$



For each (n_7, n_9) one has to check for $\sharp$Families which give half-integral CS terms and allows for tadpole cancelation.

An MSSM-like model

The smallest ♯Families for a positive n_{D3}:

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	—	(27; 16)	—	—			
6	—	(12; 81)	(21; 42)	—	—		
5	—	—	(12; 57)	(30; 8)	—	**(3; 46)**	
4	(42; 4)	—	(30; 32)	—	—	—	—
3	—	(21; 72)	—	—	—	(15; 30)	
2	(45; 16)	(24; 79)	(21; 66)	(24; 44)	**(3; 64)**		
1	—	—	—	—			
0	—	—	(12; 112)				
-1	(36; 91)	(33; 74)					
-2	—						

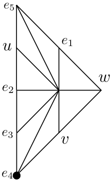
- Two strata for which one gets a three family solution as a posibility.
- Not much to say about the vector-like sector from which the Higgses emerge.

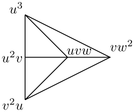
Q_i	$\bar{u}_i$	$\bar{d}_i$	L_i	$\bar{e}_i$	H_u, H_d
$(\mathbf{3},\mathbf{2})_{1/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{1}$	$(\mathbf{1},\mathbf{2})_{\pm 1/2}$

the following couplings will be induced

$$\mathcal{W} \supset Y^u_{i,j} Q_i \bar{u}_j H_u + Y^d_{i,j} Q_i \bar{d}_j H_d + Y^L_{i,j} \bar{e}_i \bar{L}_j H_d$$
$$\lambda^{(0)}_{i,j,k} Q_i \bar{d}_j L_k + \lambda^{(1)}_{i,j,k} \bar{e}_i L_j L_k + \lambda^{(2)}_{i,j,k} \bar{u}_i \bar{d}_j \bar{d}_k + \mu H_u H_d$$

Pati-Salam





- **Gauge group:** $SU(4) \times SU(2)^2/\mathbb{Z}_2$
- **Flux:** one parameter $\Rightarrow$ complete families!

$$p_{F_{13}} = s_1 e_1^2 e_2^2 e_3 e_5^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 e_5^2 u^2 v + s_3 e_2^2 e_3^3 e_4^4 u v^2 + s_6 e_1 e_2 e_3 e_4 e_5 uvw + s_9 e_1 v w^2 \,,$$

At codimension two:

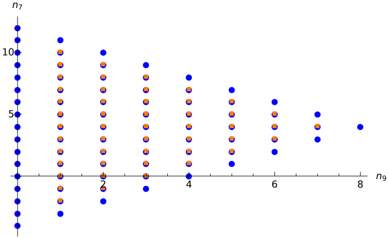
Representation	Locus
$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$V(I_{(1)}) := \{s_1 = s_3 = 0\}$
$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$V(I_{(2)}) := \{s_1 = s_9 = 0\}$
$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$V(I_{(3)}) := \{s_3 = s_9 = 0\}$
$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$V(I_{(4)}) := \{s_6 = s_9 = 0\}$

At codimension three:

Yukawa	Locus
$(\mathbf{1}, \mathbf{2}, \mathbf{2}) \cdot \overline{(\mathbf{4}, \mathbf{2}, \mathbf{1})} \cdot (\mathbf{4}, \mathbf{1}, \mathbf{2})$	$s_1 = s_3 = s_9 = 0$
$\overline{(\mathbf{6}, \mathbf{1}, \mathbf{1})} \cdot (\mathbf{4}, \mathbf{2}, \mathbf{1}) \cdot (\mathbf{4}, \mathbf{2}, \mathbf{1})$	$s_1 = s_6 = s_9 = 0$
$\overline{(\mathbf{6}, \mathbf{1}, \mathbf{1})} \cdot (\mathbf{4}, \mathbf{1}, \mathbf{2}) \cdot (\mathbf{4}, \mathbf{1}, \mathbf{2})$	$s_3 = s_6 = s_9 = 0$

Pati-Salam

the allowed region for (n_7, n_9)



- One parameter flux of solutions.
- Three families possible.
- No control on the real representations.

$n_7 \backslash n_9$	1	2	3	4	5	6	7
10	(13; 204)						
9	—	(11; 140)					
8	(33; 94)	(10; 119)	(9; 90)				
7	—	(9; 100)	(6; 77)	(14; 48)			
6	(15; 108)	(8; 86)	(21; 52)	(12; 46)	(5; 44)		
5	(6; 106)	(35; 44)	—	(30; 16)	—	(3; 44)	
4	(7; 102)	(6; 75)	(15; 50)	(8; 42)	(15; 30)	(6; 41)	(7; 42)
3	(6; 106)	(35; 44)	—	(30; 16)	—	(3; 44)	
2	(15; 108)	(8; 86)	(21; 52)	(12; 46)	(5; 44)		
1	—	(9; 100)	(6; 77)	(14; 48)			
0	(33; 94)	(10; 119)	(9; 90)				
-1	—	(11; 140)					
-2	(13; 204)						

Pati-Salam

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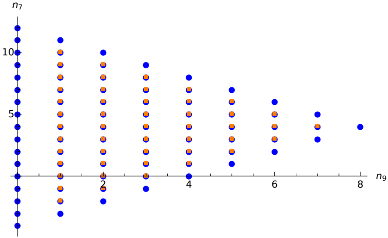

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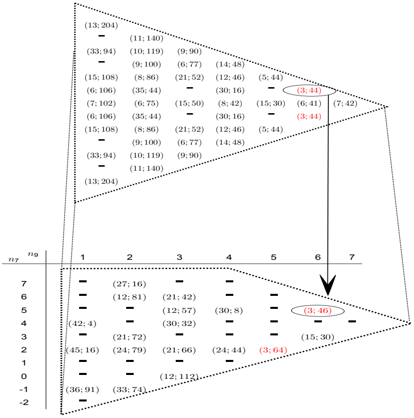

n_7 \ n_9	1	2	3	4	5	6	7
7	–	(27; 16)	–	–			
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1	–	–	–	–			
0	–	–	(12; 112)				
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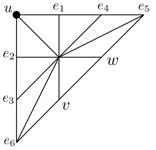
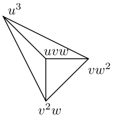
-2	–						

Trinification





- Gauge group:

$$\mathrm{SU}(3)^3/\mathbb{Z}_3$$

$n_7 \backslash n_9$	1	2	3	4	5	6	7	8	9	10
10	(5; 120)									
9	(3; 94)	(3; 94)								
8	(4; 72)	(8; 69)	(4; 72)							
7	(14; 48)	(7; 54)	(7; 54)	(14; 48)						
6	(5; 50)	(8; 44)	(3; 44)	(8; 44)	(5; 50)					
5	(5; 50)	(5; 42)	(10; 36)	(10; 36)	(5; 42)	(5; 50)				
4	(14; 48)	(8; 44)	(10; 36)	(16; 30)	(10; 36)	(8; 44)	(14; 48)			
3	(4; 72)	(7; 54)	(3; 44)	(10; 36)	(10; 36)	(3; 44)	(7; 54)	(4; 72)		
2	(3; 94)	(8; 69)	(7; 54)	(8; 44)	(5; 42)	(8; 44)	(7; 54)	(8; 69)	(3; 94)	
1	(5; 120)	(3; 94)	(4; 72)	(14; 48)	(5; 50)	(5; 50)	(14; 48)	(4; 72)	(3; 94)	(5; 120)

- Matter: $(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$, $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$, $(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})$
- Over one stratum, two consecutive Higgsings could lead to the MSSM.

Conclusions and prospects

- We constructed globally defined models of particle physics that have
  - three chiral families
  - the exact MSSM, Pati-Salam and Trinification gauge group and matter content
  - the anomalies and n_{D3} tadpoles canceled
- At certain strata the models allow for a Higgs transition among them.
- No distinction between Leptons and H_d, no control over the μ term in the models.
- Vector-like sector and real reps. not visible.

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Gracias!