Planckian Axions in String Theory

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Cornell

String Phenomenology 2015, Madrid June 9, 2015

Based on:

Thomas Bachlechner, Mafalda Dias, Jonathan Frazer, L.M.,
“A New Angle on Chaotic Inflation,” 1404.7496.

Thomas Bachlechner, Cody Long, L.M.,
“Planckian Axions in String Theory,” 1412.1093.

Thomas Bachlechner, Cody Long, L.M.,
“Planckian Axions and the Weak Gravity Conjecture,” 1503.07853.

Thomas Bachlechner, Cody Long, L.M., John Stout,
to appear.

Summary

I will characterize the typical diameter of axion field space, in a general theory of N axions, in the absence of monodromy. (Super-Planckian diameter necessary for natural inflation.)

The key phenomenon is kinetic alignment.

The diameter is parametrically larger in N than previous findings (e.g., N-flation): $N^{3/2}$ vs. $N^{1/2}$.

Supporting evidence in an explicit compactification with $h^{1,1}$=51 and diameter 1.1 M_p.

Counterarguments based on Weak Gravity involve strong assumptions about, and beyond, black hole physics.

Motivation

- Observed CMB anisotropies are in superb agreement with predictions of inflation.
- This is evidence that the primordial density perturbations originated from quantum fluctuations of the inflaton.
- No detection of B-modes from primordial gravitational waves. May anticipate upper limits of r<.01, or a detection, in coming years. **Bicep2/Keck/Planck 15** talks by Gratton, Ahmed
- Important to examine possible theoretical limits.
- A prime arena for connecting QG to observations.

see talks by Linde, Kallosh, Silverstein, Blumenhagen, Dudas, Shiu, Hebecker, Fuchs, Landete, Mangat, Plauschinn, Retolaza, Rompineve, Valenzuela, Wieck, Junghans, Kappl, Montero, Otsuka, Ruehle, Soler, Scalisi, Staessens, Witkowski, Atal, Sumitomo, Dutta, Pedro, Welling.

Motivation

- Many authors have argued that super-Planckian field displacements are impossible in quantum gravity.
- Monodromy (repeated traversal of a sub-Planckian axion fundamental domain) gives a plausible counterexample. Is it the only possibility?

Silverstein, Westphal 08; L.M., Silverstein, Westphal 08; Flauger et al. 09; Kaloper, Sorbo 08; Kaloper, Lawrence, Sorbo 11; Marchesano et al. 14; Palti, Weigand 14; Ibanez, Valenzuela 14; Arends et al. 14; L.M. et al. 14; Blumenhagen et al. 14; Hebecker et al. 14

- Question for this talk: what is the diameter of a single fundamental domain in a theory of N axions?
- In EFT and in string theory, without fine-tuning, we will find large diameters.

Plan

I. Introduction and motivation

II. Axion diameters

  a. Alignment is generic at large N

  b. Planckian axions in string theory

  c. Link to the Weak Gravity Conjecture

III. Conclusions

Natural Inflation in string theory?

In natural inflation, the PQ symmetry $\phi \mapsto \phi + const.$ of an axion is invoked to protect the inflaton potential.

Freese, Frieman, Olinto 90

$$\mathcal{L}(\phi) = -\frac{1}{2}(\partial\phi)^2 - \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] + \cdots$$

To address questions of the UV completion of this symmetry, we need to embed natural inflation in string theory.

The super-Planckian axion decay constants required for this model are not possible in presently computable limits of string theory.

Banks, Dine, Fox, Gorbatov 03

$$\frac{f}{M_P} \propto \frac{g_s^\alpha}{\mathcal{V}^\beta} \ll 1$$

$$\alpha, \beta \geq 0$$

Natural Inflation in string theory?

To realize natural inflation in string theory, we will need to relax one or more implicit assumptions.

A. Explore new regimes of strong coupling or small volume where decay constants can be large. **Grimm 14**

B. Incorporate monodromy of an axion: repeatedly traverse the sub-Planckian fundamental period of an axion. **Silverstein, Westphal 08; L.M., Silverstein, Westphal 08; Marchesano et al. 14.**

C. Consider more than one axion. **cf. Assisted Inflation, Liddle et al.**

1. With two or more axions one can fine-tune the decay constants to achieve “lattice alignment”, and a super-Planckian effective period.

   Kim, Nilles, Peloso 04

2. With N >> 1 axions with generic decay constants, the collective displacement can be super-Planckian. Lattice alignment occurs automatically (and more besides).

   Dimopoulos et al. 05; Bachlechner et al. 14; Bachlechner, Long, L.M. 14; Junghans 15

See also: Czerny et al. 14; Tye, Wong 14; Kappl et al. 14; Ben-Dayan et al. 14; Choi et al. 14; Higaki, Takahashi 14

Axion geometry: warmup

$$\mathcal{L} = \frac{1}{2} K_{ij} \partial\theta^i \partial\theta^j - \sum_i \Lambda_i^4 [1 - \cos(\theta_i)]$$

- Naive field space, before periodic identifications, is $\mathbb{R}^N$.
- The identifications $\theta_i \to \theta_i + 2\pi$ define a lattice in this space.
- The fundamental domain is a hypercube of side length 2π.
- The fact that the axions experience periodic identifications defines a preferred coordinate system, the lattice basis, in which the periodicity is manifest, with each axion appearing in a single cosine.
- Define the kinetic basis as the eigenbasis of K_{ij}.
- Lattice basis ≠ kinetic basis in general!

Kinetic alignment

- We can go from

$$\mathcal{L} = \frac{1}{2} K_{ij} \partial\theta^i \partial\theta^j - \sum_i \Lambda_i^4 \left[1 - \cos(\theta_i)\right] \longrightarrow \mathcal{L} = \frac{1}{2} (\partial\phi_i)^2 - \sum_i \Lambda_i^4 \left[1 - \cos(\phi_i/f_i)\right]$$

only if K is diagonal in the lattice basis.

- i.e., if the lattice basis equals the kinetic basis.
- i.e., if the axion fields with distinct identifications have no kinetic mixing.
- This is definitely not generic, in EFT or in string theory.
- But what is the generic relation between the lattice and kinetic basis?
- We will see that for generic mixing, they are aligned in a way that parametrically increases the field space diameter.

Bachlechner, Dias, Frazer, L.M.,14; cf. Shiu, Staessens, Ye 15

Axion geometry

$$\mathcal{L} = \frac{1}{2} K_{ij} \partial\theta^i \partial\theta^j - \sum_{i=1}^{P} \Lambda_i^4 \left[1 - \cos\left(\mathcal{Q}^i{}_j \theta^j\right)\right]$$

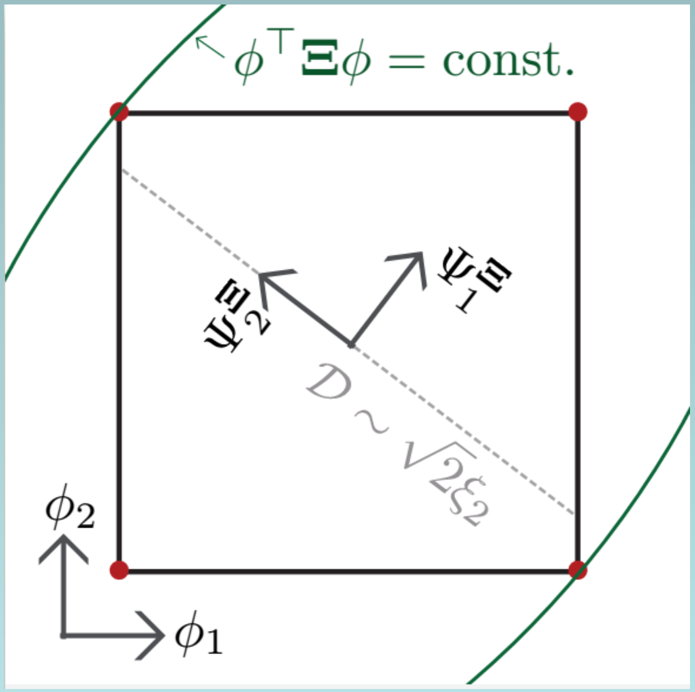
$$\mathcal{Q}^i_j \theta^j \cong \mathcal{Q}^i_j \theta^j + 2\pi$$

When P=N, define: $\boldsymbol{\phi} = \mathbf{Q}\,\boldsymbol{\theta}$ $\quad \boldsymbol{\Xi} = (\mathbf{Q}^{-1})^\top \mathbf{K}\,\mathbf{Q}^{-1}$

$$\mathcal{L} = \frac{1}{2} \partial\boldsymbol{\phi}^\top \boldsymbol{\Xi}\, \partial\boldsymbol{\phi} - \sum_{i=1}^{N} \Lambda_i^4 \left[1 - \cos\left(\phi_i\right)\right]$$

Eigenvector $\boldsymbol{\Psi}_N^{\Xi}$,
largest eigenvalue ξ_N^2

$$\mathcal{D}_{\max} = 2\pi \xi_N \sqrt{N}$$



Eigenvector delocalization

- N basis vectors, 2^N hyperoctants
- If eigenvectors $\vec{\psi}_a$ are uniformly distributed in angle, then at large N, $\psi_a^{(i)} \in \frac{1}{\sqrt{N}}\mathcal{N}(0,1)$
- Constant-distance ellipsoids, $\boldsymbol{\phi}^\top \boldsymbol{\Xi}\, \boldsymbol{\phi} = r^2$, are oriented along diagonals rather than face-normals of the lattice.
- i.e., kinetic basis points along diagonals of lattice
- This eigenvector delocalization is universal in rotationally-invariant random matrix ensembles, but holds more generally. **Tao, Vu 12**
- Eigenvector delocalization implies

$$\boxed{\mathcal{D} \sim \sqrt{N}\xi_N}$$

cf. $\mathcal{D}_{\max} = 2\pi\xi_N\sqrt{N}$

Bachlechner, Dias, Frazer, L.M.,14

Axion geometry

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$$\mathcal{D} \sim \sqrt{N} \xi_N$$

What is the eigenvalue spectrum of $\boldsymbol{\Xi} = (\mathbf{Q}^{-1})^\top \mathbf{K}\,\mathbf{Q}^{-1}$?

Metric eigenvalues

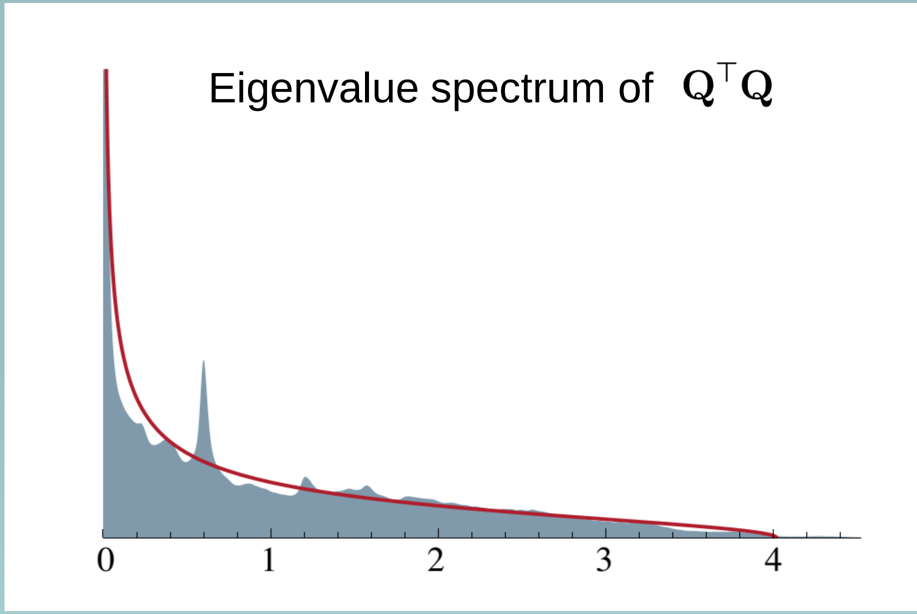
Matrix	Eigenvalues
$\mathbf{K}$	$f_1^2 \leq \ldots \leq f_N^2$
$\mathbf{Q}^\top \mathbf{Q}$	$q_1^2 \leq \ldots \leq q_N^2$
$\boldsymbol{\Xi} = (\mathbf{Q}^{-1})^\top \mathbf{K}\, \mathbf{Q}^{-1}$	$\xi_1^2 \leq \ldots \leq \xi_N^2$

$$\boxed{\xi_N \sim f_N / q_1}$$

$$\mathcal{D} \sim \sqrt{N} \xi_N$$

How small is the smallest eigenvalue of $Q^T Q$?

Charge matrix eigenvalues: Wishart-like spectrum



$$(q_1^2)_{\rm med} = \mathfrak{c}\sigma^2/N$$

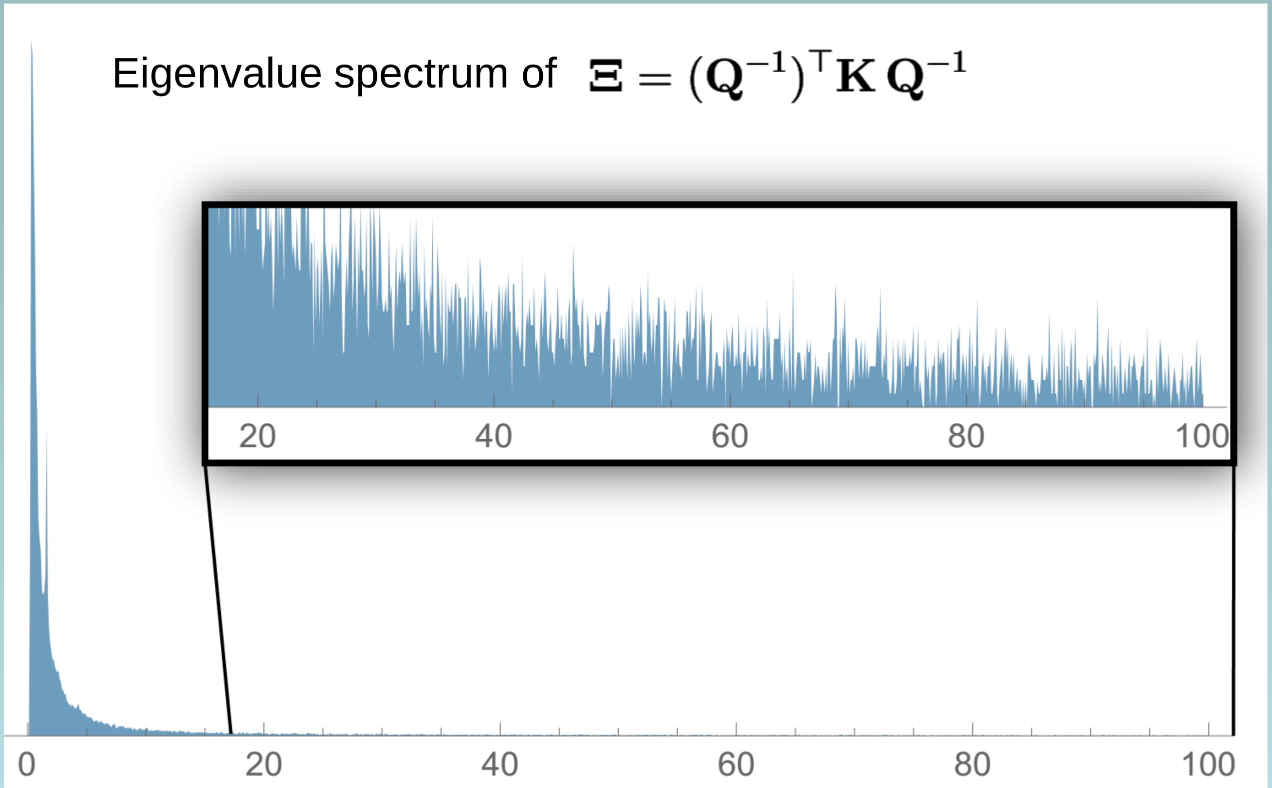
$$\mathfrak{c} = 2 + \log(4) - 2\sqrt{1+\log(4)} \approx 0.30$$

Median size of $(q_1)^{-2}$ is ~N,
but distribution is heavy-tailed: $\rho(\lambda)^{[1/q_1^2]} \propto \lambda^{-3/2}$
Fluctuations to much larger values are common.

cf. Long, L.M., McGuirk 14

Metric eigenvalues: heavy tails

- Metric spectrum has a long tail to large values.
- General consequence of dividing by a quasi-random matrix: here, charge matrix $Q^\top Q$.



Simultaneous kinetic and lattice alignment is generic

$$\mathcal{D} \approx f_N \times \sqrt{N} \times \sqrt{N} \times \frac{1}{\sigma_Q}$$

possible enhancement from sparse charges

eigenvector delocalization
kinetic alignment is generic at large N

eigenvalue repulsion
lattice (KNP) alignment is generic at large N

cf. N-flation, where:

$$\mathcal{D} \approx \sqrt{\sum f_i^2} \ll f_N \sqrt{N}$$

All this is for the typical case.
Fluctuations to much larger diameters are common!

Bachlechner, Dias, Frazer, L.M.,14
Bachlechner, Long, L.M. 14

Recap

In an EFT of N axions:

- Generic diameters are much larger than Pythagorean sum of metric eigenvalues f_i.
  - Kinetic alignment from eigenvector delocalization.
  - Lattice alignment from eigenvalue repulsion.

- In non-generic but not rare cases, diameter can be vastly larger still. Metric spectrum has a heavy tail to large values.

Explicit string theory example

Stabilized compactification:

Orientifold of resolution of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ **Denef et al. 04 (DDFGK)**

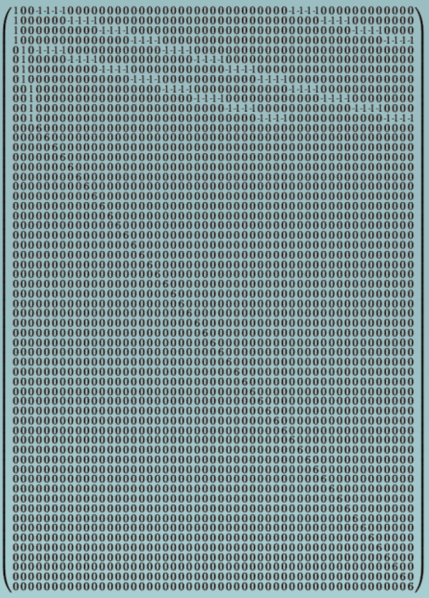
$h_{1,1} = 51$; 48 exceptional divisors, 12 SO(8) stacks on O7-planes

Explicit 3-form flux quanta, explicit moduli stabilization.

$$W = W_0 + \sum_i A_i e^{-q^i{}_j T^j}$$

$$\sigma_Q \approx 0.18$$

$$f_N \approx 0.013\, M_{\rm pl}$$

$\mathbf{q} = \frac{\pi}{3}$ 

Bachlechner, Long, L.M. 14

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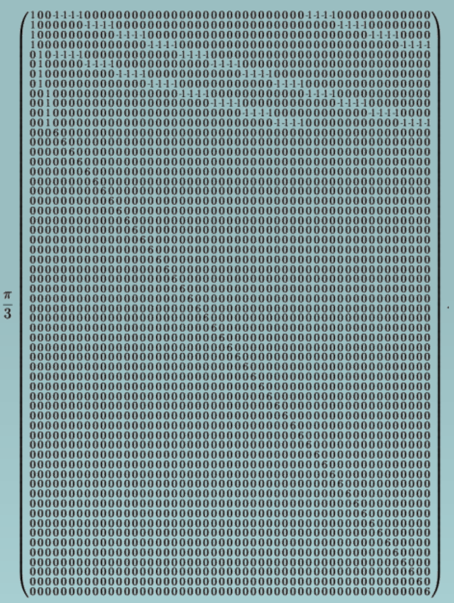

RMT predicts:

$$\mathcal{D} = \sqrt{\frac{51}{2+\log(4)-2\sqrt{1+\log(4)}}}\,\frac{\pi}{2\,\mathrm{erf}^{-1}\left(2^{-1/9}\right)}\,\frac{f_N}{\sigma_Q} \approx 1.21\, M_{\rm pl}$$

Actual diameter:

$$\mathcal{D}_{\rm light} = 1.13\, M_{\rm pl}$$

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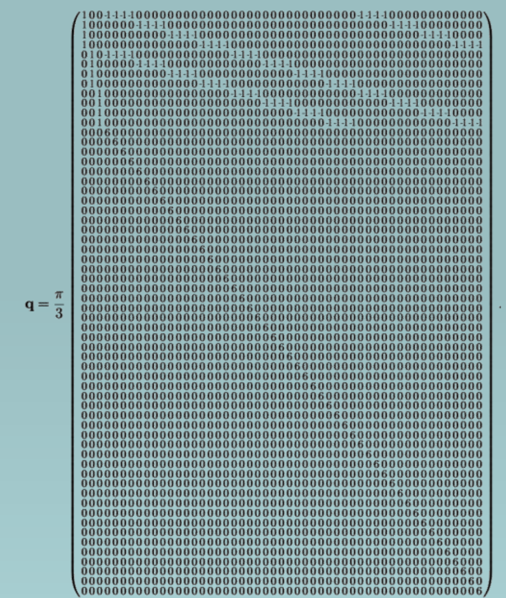


Possible corrections:

Renormalization of G_N? 1% from $\alpha'^3\mathcal{R}^4$; others?
Other α' corrections?
Further instantons, e.g. in K?

Worth examining this model, or a similar vacuum, very closely.

Bachlechner, Long, L.M. 14

The Weak Gravity Conjecture

see talks by Shiu, Hebecker

- Complete evaporation of large black holes destroys global charges.
- Unbroken global symmetries require incomplete decay: ‘remnants’.
- Having infinitely many stable remnants in a finite mass range leads to pathologies **Susskind**
- One could then conjecture that **Arkani-Hamed, Motl, Nicolis, Vafa 06**
  - there should not be infinitely many exactly stable black holes,

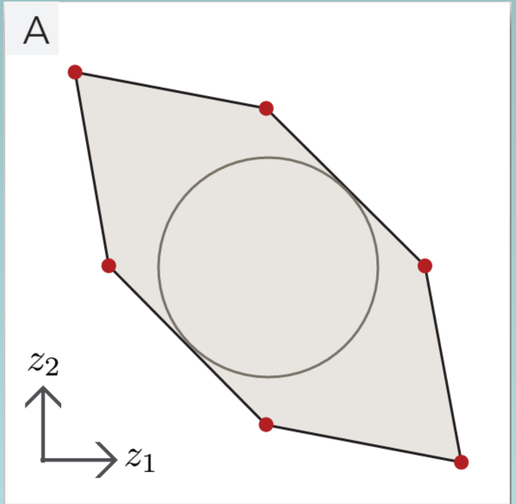
  or even
  - there should not be any.
- Extremal BH decay requires states with Q/M>1.
- The WGC expresses a conjectural necessary condition for QG consistency in terms of the low-energy spectrum.

The Weak Gravity Conjecture

- Conditions for black hole decay in a theory of N U(1)s can be mapped to conditions on a theory of N axions.

  AMNV; Rudelius 14; Montero, Uranga, Valenzuela 15; Brown, Cottrell, Shiu, Soler 15
- The mild form of the WGC, 'there should not be infinitely many exactly stable black holes', maps to Cheung, Remmen
- 'There should exist instantons with charges Q_i and actions S_i, such that the vectors $z_i = Q_i/S_i$ have a convex hull that contains the unit ball'.
- This is a weak constraint on inflation: the instantons that fulfill WGC can be negligible in V.
- One might try to formulate a stronger form of the WGC that does have impact.

  Rudelius 14, Brown et al. 15
- This is subtle!



Convex hull spans unit ball

Bachlechner, Long, L.M. 15; Bachlechner, Long, L.M., Stout 15

Cautions about the WGC

- The mild WGC per se cannot be used to exclude purely low-energy theories: Planckian-mass states can fulfill it.
- The strong form proposed in AMNV does not suffice to remove all stable remnants.
- Knowing whether Planck-mass black holes are problematic for QG is tantamount to knowing the complete spectrum at the Planck scale --- a hard QG problem!
- The assertion ‘EFTs should obey a WGC strong enough to ensure the absence of remnants’ rests on assuming such knowledge.
- Weaker WGCs can be more macroscopically plausible, but are less of a constraint on axion inflation.
- The forms strong enough to exclude large axion diameters are not implied by existing QG or BH arguments.

Towards a counterexample??

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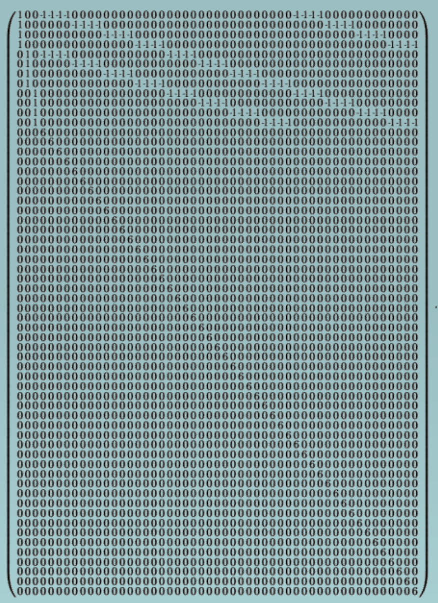
$\mathbf{q} = \frac{\pi}{3}$ 

Strong form of WGC requires: the leading instanton terms have a convex hull that contains the unit ball.

Actual finding: the 60 leading terms **in W** have a convex hull that does not contain the unit ball.

Caution: many issues to check. Other instantons? Corrections to vacuum?

Bachlechner, Long, L.M., Stout 15

WGC Summary

- The mild WGC does not present a stringent constraint.
- Stronger WGCs are less-established (diverse proposals).
- Some claims that WGC prevents large-field axion inflation are formalized statements of expectation, not results following from known black hole physics.
- However, it is possible one could infer a stringent bound by supplementing black hole arguments with something more.
- This is a fascinating interface, where general reasoning about BH in QG could constrain large-field models. But the results are not mature yet. Worth discussing!

Conclusions

- The prospect of near-term observational information about primordial tensors motivates understanding large-field inflation.
- Kinetic alignment is generic in systems of many axions, including in string theory.
- Result: diameters parametrically larger in N than naive Pythagorean estimate.
- Planckian diameter exhibited in an explicit flux compactification, but further checks needed.
- Developing interface with black holes, via WGC.
- Profound connections to quantum gravity promise to illuminate both inflation and string theory.