Transplanckian axions and the weak gravity conjecture

M. Montero

Instituto de Física Teórica UAM-CSIC

StringPheno
IFT Madrid, June 2015

European Research Council
SPLE Advanced Grant

UNIVERSIDAD AUTONOMA DE MADRID

Instituto de Física Teórica UAM-CSIC

CSIC
Consejo Superior de Investigaciones Científicas

Suddenly a very popular topic!

- *Constraints on Axion Inflation from the Weak Gravity Conjecture*, Tom Rudelius, 1503.00795.
- ***Transplanckian axions !?*, M. M, A. Uranga, I. Valenzuela, 1503.03886.**
- *Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation*, Jon Brown, William Cottrell, Gary Shiu, Pablo Soler 1503.04783.
- *Planckian Axions and the Weak Gravity Conjecture*, Thomas C. Bachlechner, Cody Long, Liam McAllister, 1503.07853.
- *On Axionic Field Ranges, Loopholes and the Weak Gravity Conjecture*, Jon Brown, William Cottrell, Gary Shiu, Pablo Soler, 1504.00659.
- *Large-Field Inflation with Multiple Axions and the Weak Gravity Conjecture*, Daniel Junghans, 1504.03566.

Why transplanckian axions?

- Inflation: Transplanckian field range for sizeable r.
- Need to control Planck-suppressed terms in the potential

$$V(\phi) \sim \left(\frac{\phi}{M_P}\right)^n$$

- Good idea: Use **axions** with shift symmetry $\phi \to \phi + c$ broken to $\phi \to \phi + 2\pi f$ by nonperturbative effects:

$$V(\phi) \sim \cos(\phi/f)$$

- These EFT models might be in the Swampland.

Weak Gravity Conjecture [Arkani-Hamet et al. '06]: For any abelian p-form field, there must be a charged p-dimensional object with tension

$$T \lesssim \frac{g}{\sqrt{G_N}}.$$

- For an axion in 4d, $g = 1/f$ and $T = S$, the object is an instanton. These instantons can generate a potential which prevents transplanckian field excursions.
- Idea: Try to understand WGC-like constraints from studying small black holes/instantons in EFT.

We consider EFT of a single axion coupled to Einstenian gravity

$$\int \left(\frac{-1}{16\pi G} R dV + \frac{f^2}{2} d\phi \wedge *d\phi - V(\phi) dV \right)$$

- The potential $V(\phi)$ does *not* include gravitational instanton contributions.
- We are given an inflationary model and study gravitational instanton effects on top of that.

We consider EFT of a single axion coupled to Einstenian gravity

$$\int \left(\frac{-1}{16\pi G} R dV + \frac{f^2}{2} d\phi \wedge *d\phi - V(\phi) dV \right)$$

- The potential $V(\phi)$ does *not* include gravitational instanton contributions.
- We are given an inflationary model and study gravitational instanton effects on top of that.

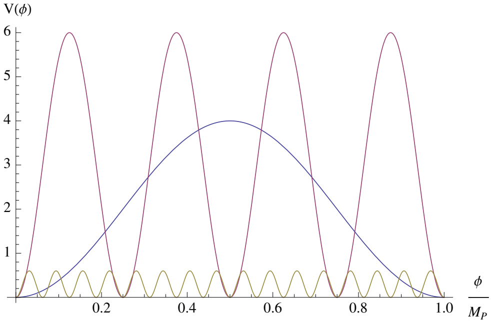
- In the dilute-gas approximation, summing over all instantons with same charge yields a contribution to the path integral of the form

$$S_{inst.} = \int d^4x\sqrt{-g}\mathcal{P}e^{-S_E}\cos(n\phi), \quad S_E = \frac{\sqrt{3\pi}}{16}\frac{M_P n}{f}$$

- $\mathcal{P}$ is a prefactor for which which we **assume** $\mathcal{P} \sim M_P^4$. This is essential. Computation of $\mathcal{P}$ in only a few cases [Gross, Perry& Yaffe '82, Volkov & Wipf '00].

Consequences for a transplanckian axion

The instanton generates a huge potential, cutting down the effective field range to f/n as long as the action is small. $S_E \sim 1$ means $f \sim M_P n$, so the effective range is approximately subplanckian.



Do gravitational instantons pose a threat to multiple-axion inflationary models as well?

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})\mathcal{G}(\partial^\nu \vec{\phi}) - \sum_{i=1}^{N} \Lambda_i^4 (1 - \cos(\phi_i))$$

Here, the axions are periodic $\phi_i \to \phi_i + 2\pi$. This is the *lattice basis*.
With canonical kinetic term

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\zeta}_i)^2 - \sum_{i=1}^{N} \Lambda_i^4 (1 - \cos(R_{ji}\hat{\zeta}_i/f_i)), \quad \hat{\zeta}_i = f_i R_{ij} \phi_j.$$

$\{\hat{\zeta}_i\}$ parametrices the axions in the *kinetic basis*.

Three classes of models

- N-flation [Dimopoulos et al. ' 08]: $\sqrt{N}f$ enhancement
- Kinetic alignment [Bachlechner '15]: $\sqrt{N}f_{\text{max}}$ enhancement
- Lattice alignment [Kim, Nilles, Peloso '05]: parametrically flat direction:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \theta_1)^2 + \frac{1}{2}(\partial_\mu \theta_2)^2 - \Lambda_1^4 \left(1 - \cos\left(\frac{\theta_1}{f} + \frac{\theta_2}{g_1}\right)\right) - \Lambda_2^4 \left(1 - \cos\left(\frac{\theta_1}{f} + \frac{\theta_2}{g_2}\right)\right)$$

As $\epsilon \equiv g_1 - g_2 \to 0$ we get a parametrically flat direction in the potential.

Gravitational instantons for several axions

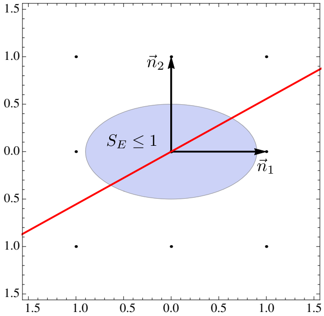
The action of the gravitational instanton becomes

$$S_E = \frac{\sqrt{3\pi}}{16} M_P \sqrt{\vec{n}^T \mathcal{G}^{-1} \vec{n}}$$

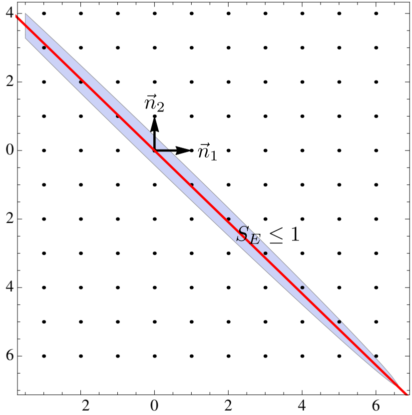
and induces a potential for the system of axions given by

$$V = \mathcal{P}' e^{-S_E} \cos(n_i \phi_i)$$

If $S_E < c$ is smaller than a prescribed value, the instantons will not hinder inflation. These define ellipses in the dual lattice:



So these models evade the effects of gravitational instantons!



- Lots of instantons get vanishing action in the KNP limit.
- The resulting contributions ruin the parametric flatness of the potential.
- Similar to single axion.

String theory is full of axions! A simple choice:

$$\phi \to \int_S C_p, \quad \text{Instanton} \to \text{Euclidean } Dp\text{-brane on } p\text{-cycle.}$$

For $p = 4$, and reduction to $4d$, we have

$$M_P/f \sim \frac{1}{g_s} \quad \Rightarrow \quad S_E \sim \frac{M_P}{f} n \sim \frac{n}{g_s},$$

same parametric dependence as a stack of n euclidean D3's. One can also extend this to the multi-axion setup (in some particular models) to get the right dependence

$$S_{D3} \sim \sqrt{\vec{n}^T \mathcal{G}^{-1} \vec{n}}$$

Summary

- Taking into account gravitational effects is essential for a successful inflationary model.
- For N-flation and kinetic alignment these effects are suppressed.
- For KNP the effects ruin parametric flatness
- Some instantons are absent only if strong form of WGC does not hold.

Outlook:

- Can we embed models with suppressed effects in ST?
- What about monodromy?

Summary

- Taking into account gravitational effects is essential for a successful inflationary model.
- For N-flation and kinetic alignment these effects are suppressed.
- For KNP the effects ruin parametric flatness
- Some instantons are absent only if strong form of WGC does not hold.

Outlook:

- Can we embed models with suppressed effects in ST?
- What about monodromy?

Thank you very much!

When to trust these computations?

$a^{1/4}$ controls size & curvature of the solution. It should be larger than inverse cuttoff and lower than $V(\phi)$ scale

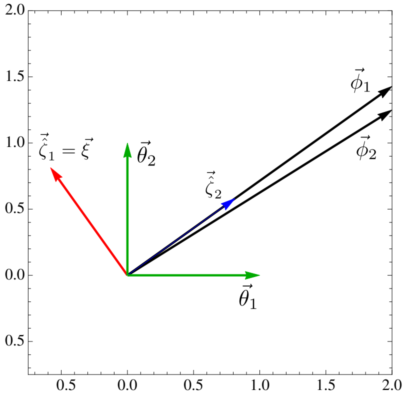
$$\Lambda^{-1} < a^{1/4} < V_0^{-1} \quad \Rightarrow \quad \left(\frac{M_P}{\Lambda}\right)^2 \lesssim S_E < \left(\frac{M_P}{V_0}\right)^2 .$$

- Presumably instantons with $a^{1/4} < \Lambda^{-1}$ still exist (typically, euclidean D-branes).
- Above bounds mean that the terms reducing the field range can only be trusted to do so if Λ is close to M_P.

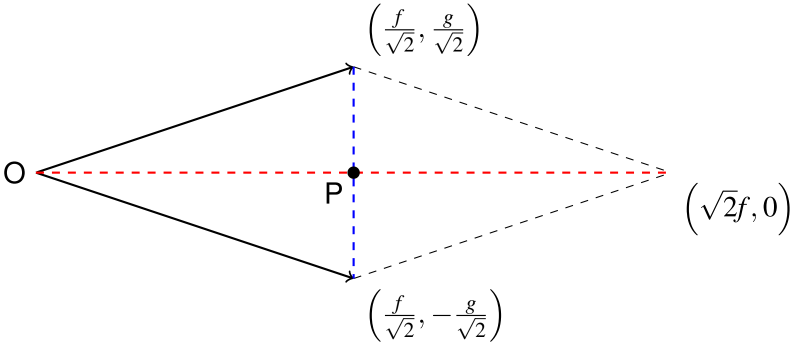
Closer look at KNP

Let us rewrite the KNP model in the lattice basis. The metric is

$$\mathcal{G} = \frac{1}{(g_1 - g_2)^2} \begin{pmatrix} g_1^2(f^2 + g_2^2)^2 & -g_1 g_2(f^2 + g_1 g_2) \\ -g_1 g_2(f^2 + g_1 g_2) & g_2^2(f^2 + g_2^2)^2 \end{pmatrix}$$



In UV: approximate $\mathbb{Z}_2$ symmetry



In other words, when $g \to 0$ we have two sets of gravitational instantons

- Those whose action depends on g and are suppresed
- Those who are not

and the second set obeys periodicities in a more refined lattice.

This violates the *strong* form of the weak gravity conjecture [Brown, Cottrell, Shiu, Soler 1503.04783]. (instanton of lowest charge should have smallest action).