

# Natural inflation with and without modulations in string theory

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(Waseda University)**

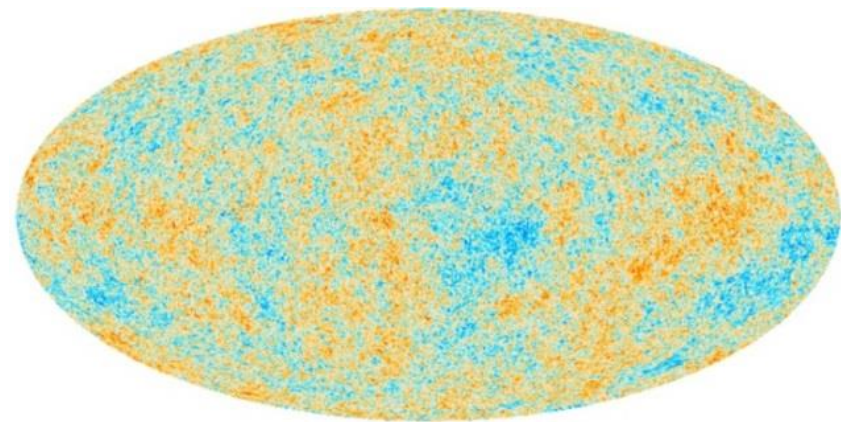
**with**

**Hiroyuki Abe (Waseda Univ.)  
Tatsuo Kobayashi (Hokkaido Univ.)**

**based on JHEP 1504 (2015) 160**

# Inflation

- i) Solving the fine tuning problem  
(Horizon problem and flatness problem)
- ii) Producing the origin of the density perturbations



$$\frac{\Delta T}{T} \sim 10^{-5}$$

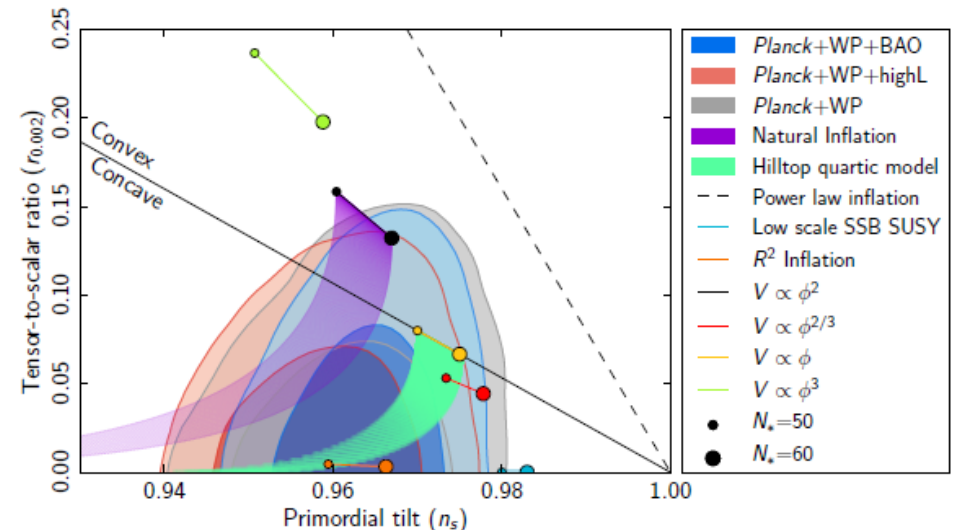
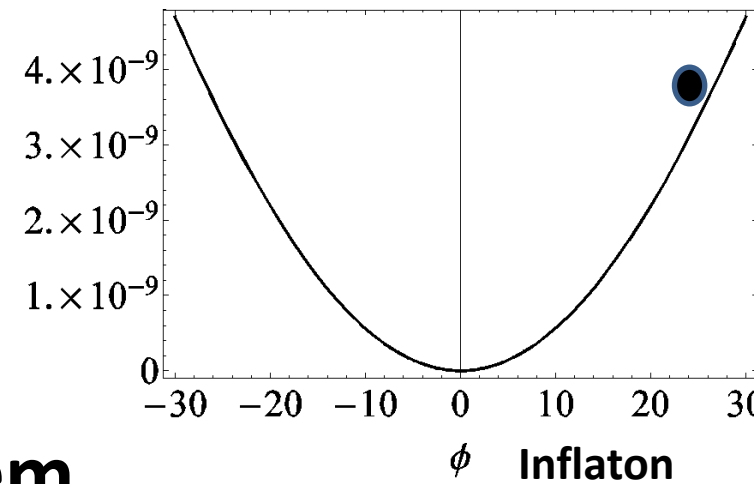
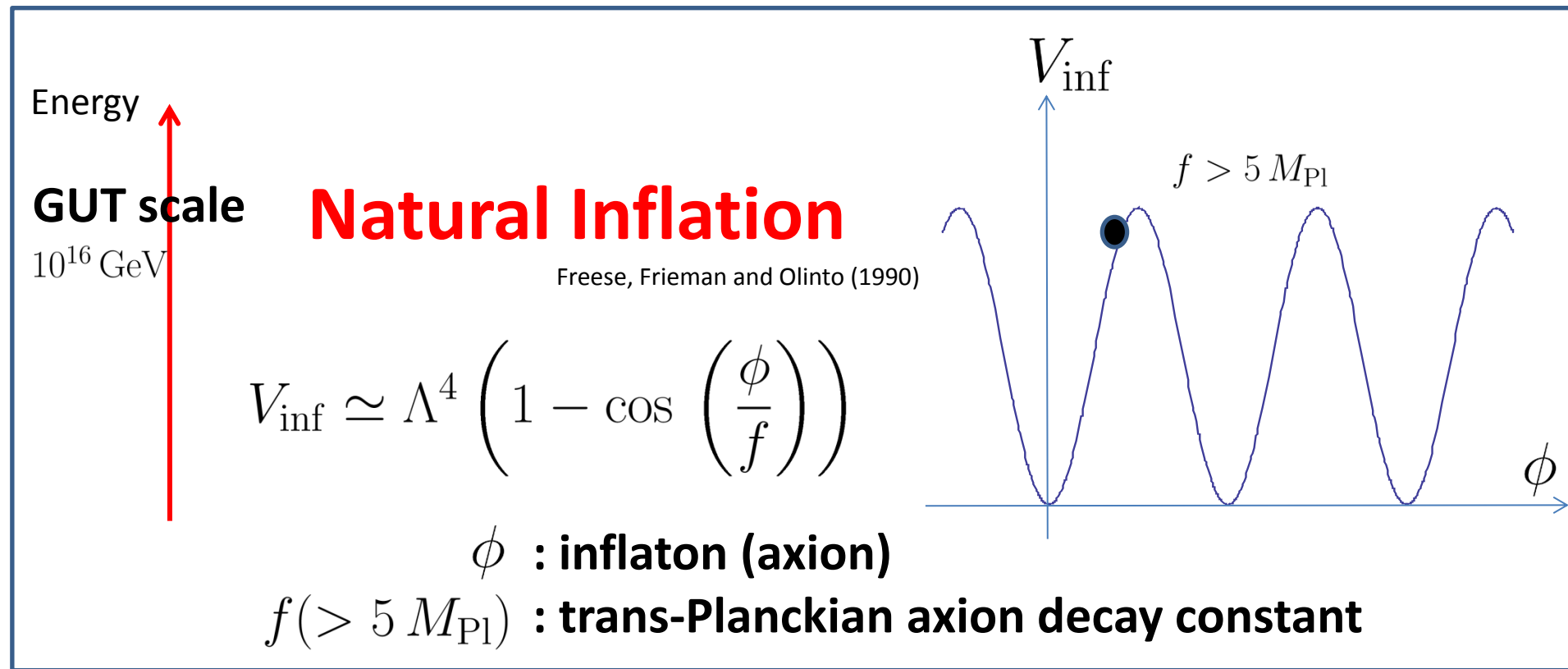


Fig. 1. Marginalized joint 68% and 95% CL regions for  $n_s$  and  $r_{0.002}$  from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

- A lot of axions in string theory
  - Perturbative flat potential (shift symmetry)
- ➡ A good candidate for the inflaton



From the dimensional reduction, the fundamental axion decay constants are  $f \ll M_{\text{Pl}}$

K. Choi and J. E. Kim (1985),  
T. Banks, et al. (2003),...

**Axion decay constant is severely constrained,**

$$f \ll M_{\text{Pl}}$$

**In order to enhance the axion decay constant,**

**Multiple axions**

**○ Alignment mechanism, N-flation, . . .**

*J. E. Kim, H. P. Nilles and M. Peloso ( '04), S. Dimopoulos, S. Kachru,  
J. McGreevy, J. G. Wacker ( '05)*

**Single axion**

**○ Threshold correction**

*H. Abe, T. Kobayashi, and H. O. ( '14)*

**We propose the **single-field natural inflation**  
by (closed) string axion.**

# Outline

i) Introduction

**ii) Trans-Planckian axion decay constant**

**iii) Natural inflation with modulations  
(type IIB string)**

**iv) Conclusion**

# Type IIB string on toroidal orientifold and orbifold with D3/D7-branes

$$T^6/(Z_2 \times Z_2)$$

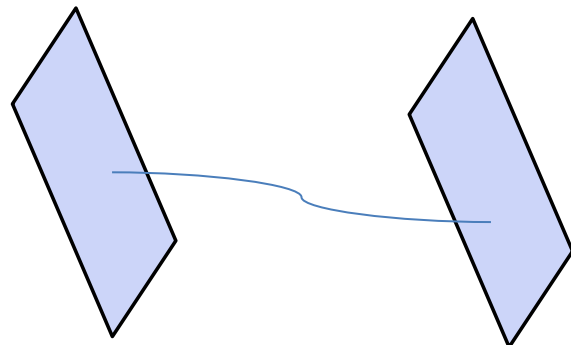
On D7-branes, the gauge coupling receives  
the moduli-dependent threshold corrections

$$\frac{1}{g^2} = \frac{T}{4\pi} + \frac{\Delta(U)}{16\pi^2}$$

$U$  :Complex structure moduli  
 $T$  :Kähler moduli

*D. Lüst and S. Stieberger, ( '03)*

**Moduli-dependent threshold corrections**



D7-branes

D7-branes

$$\Delta = -4b_{N=2} \ln \eta(iU)$$



**Dedekind function**

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**The gaugino condensation on D7-branes (SU(L) SYM)**

$$W = A e^{-8\pi^2 / L g^2} = A e^{-\frac{2\pi T}{L} - \frac{2b_{N=2}}{L} \ln \eta(iU)}$$

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$$\eta(iU) \rightarrow e^{-\frac{\pi}{12}U} \left[ 1 - \mathcal{O}(e^{-2\pi U}) \right], \text{Re } U \gg 1$$

$$W = A e^{-\frac{2\pi T}{L}} e^{-\frac{b_{N=2}\pi}{6L}U}$$



# Type IIB string on toroidal orientifold and orbifold with D3/D7-branes

$$T^6/(Z_2 \times Z_2)$$

## Superpotential

$$W = A e^{-\frac{2\pi T}{L}} e^{-\frac{b_{N=2}\pi}{6L} U}$$



Integrate out the moduli except for  $\text{Im } U$

## Axion potential

$$V_{\text{eff}} = \Lambda^4 \left( 1 - \cos \left( \frac{b_{N=2} \pi \sqrt{2} \langle \text{Re } U \rangle}{6L} \phi \right) \right)$$

$$f \simeq \frac{1.4 L}{b_{N=2} \langle \text{Re } U \rangle}$$

$$\phi \simeq \frac{\text{Im } U}{\sqrt{2} \langle \text{Re } U \rangle}$$

In the case of  $L/b_{N=2} > 1$  and  $\langle \text{Re } U \rangle \simeq 1$ ,  
we can realize the **trans-Planckian axion decay constant**.

# Moduli stabilization

We consider the scenario similar to the KKLT scenario.

## Moduli

$S$  ··· dilaton,  $T$  ··· Overall Kähler moduli (for simplicity)

$U_i$  ··· Three complex structure moduli

(One of  $\text{Im}(U_i)$  is the inflaton.)

- i) Flux compactification to stabilize the linear combination of  $S$  and  $U$ .

$$W_{\text{flux}}(U, S) = \int G_3 \wedge \Omega$$

- ii) Non-perturbative effects for  $S$  and  $T$   
(racetrack superpotential)

- iii) Uplifting potential (E.g., F-term uplifting)

# Outline

- i) Introduction
- ii) Trans-Planckian axion decay constant
- iii) Natural inflation with modulations  
(type IIB string)**
- iv) Conclusion**

# Natural inflation with modulatoins

## Superpotential

$$W = \underline{A e^{-\frac{2\pi}{L}T} \eta(iU)^{2b_{N=2}/L}} + \dots$$

$$\Downarrow \quad \eta(iU) \rightarrow e^{-\frac{\pi}{12}U} \left[ 1 - e^{-2\pi U} - \mathcal{O}(e^{-4\pi U}) \right], \text{Re } U > 1$$

$$A e^{-\frac{2\pi T}{L}} e^{-\frac{b_{N=2}\pi}{6L}U} (1 - e^{-2\pi U})$$

## Inflaton potential

*T. Kobayashi and F. Takahashi ('10)*

$$V_{\text{eff}} \simeq \Lambda - \Lambda_1 \cos(\lambda_1 \phi) + \boxed{\Lambda_2 \cos(\lambda_2 \phi)} \quad \text{Modulation term}$$

$$\lambda_1 = \frac{b_{N=2} \pi \sqrt{2} \langle \text{Re } U \rangle}{6L}$$

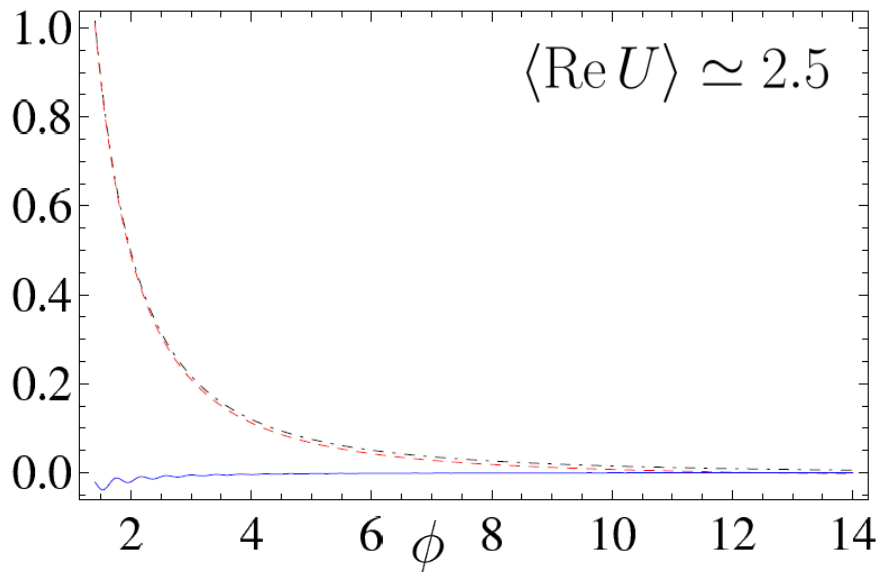
$$\Lambda_2 = \Lambda_1 \frac{2b}{L} e^{-\left(2\pi + \frac{b_{N=2}\pi}{6L}\right) \langle \text{Re } U^2 \rangle} < \Lambda_1$$

$$\lambda_2 = \left(2\pi + \frac{b_{N=2}\pi}{6L}\right) \sqrt{2} \langle \text{Re } U \rangle$$

# The behavior of the slow-roll parameters

$$\epsilon = \frac{1}{2} \left( \frac{\partial_\phi V_{\text{eff}}}{V_{\text{eff}}} \right)^2 \quad \eta = \frac{\partial_\phi \partial_\phi V_{\text{eff}}}{V_{\text{eff}}} \quad \xi = \frac{\partial_\phi V_{\text{eff}} \partial_\phi \partial_\phi \partial_\phi V_{\text{eff}}}{V_{\text{eff}}^2}$$

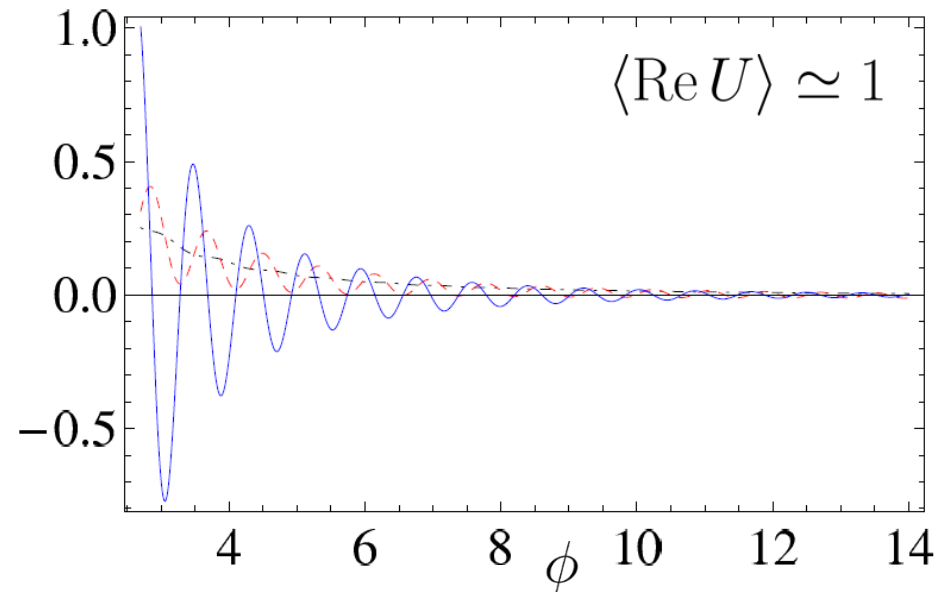
**Without modulations**



$$V_{\text{eff}} \simeq \Lambda_1 (1 - \cos(\lambda_1 \phi))$$

$$\eta(iU) \rightarrow e^{-\frac{\pi}{12}U} \left[ 1 - \mathcal{O}(e^{-2\pi U}) \right]$$

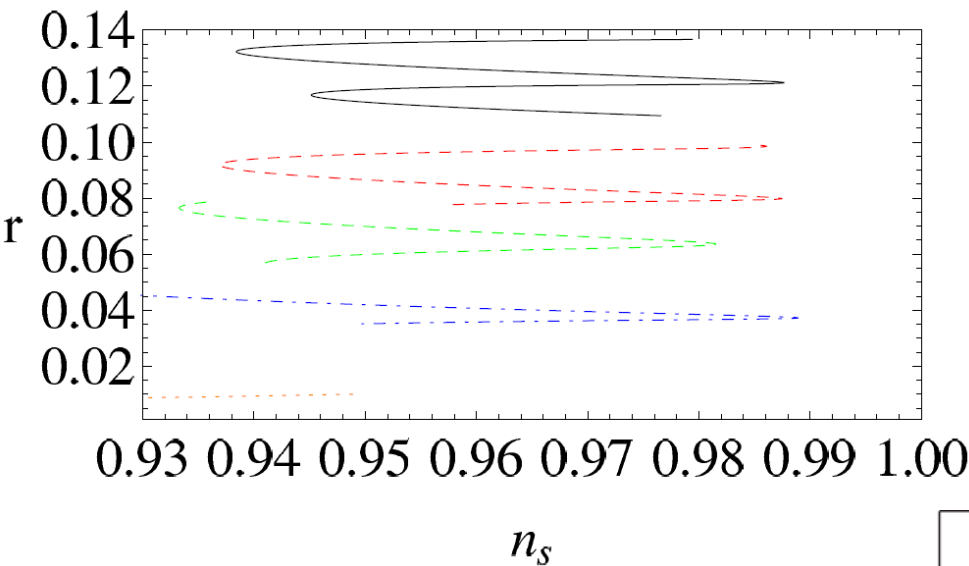
**With modulations**



$$V_{\text{eff}} \simeq \Lambda - \Lambda_1 \cos(\lambda_1 \phi) + \Lambda_2 \cos(\lambda_2 \phi)$$

$$\eta(iU) \rightarrow e^{-\frac{\pi}{12}U} \left[ 1 - e^{-2\pi U} - \mathcal{O}(e^{-\frac{4\pi}{13}U}) \right]$$

$$V_{\text{eff}} \simeq \Lambda - \Lambda_1 \cos(\lambda_1 \phi) + \boxed{\Lambda_2 \cos(\lambda_2 \phi)} \quad \text{Modulation term}$$



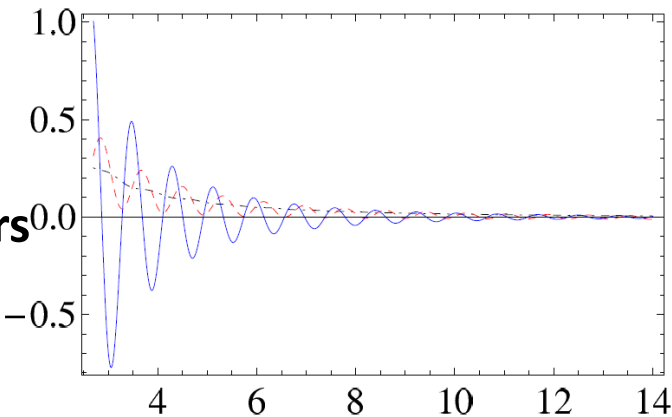
$$n_s = 1 + 2\eta - 6\epsilon + \dots$$

$$r = 16\epsilon$$

$$\lambda_1 = \frac{b_{N=2} \pi \sqrt{2} \langle \text{Re } U \rangle}{6L}$$

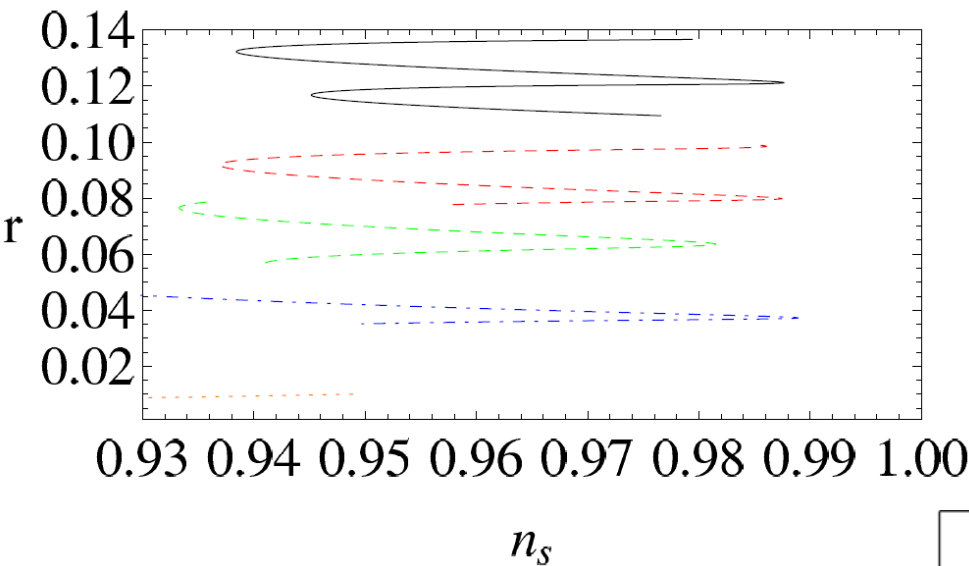
$$\lambda_2 = \left( 2\pi + \frac{b_{N=2} \pi}{6L} \right) \sqrt{2} \langle \text{Re } U \rangle$$

Slow-roll  
parameters



$b/L$	$\langle \text{Re } U^2 \rangle$	$N_e$	$n_s$	$r$	$dn_s/d \ln k$
1/10	1.3	50	0.96	0.14	-0.0008
1/10	1.3	57	0.96	0.12	-0.012
1/5	1.2	55	0.96	0.08	-0.002
1/5	1.2	60	0.96	0.08	-0.001
1/4	1.2	53	0.96	0.07	-0.002
1/4	1.2	58	0.96	0.06	-0.001
1/3	1.1	54	0.96	0.04	-0.002
1/3	1.1	60	0.96	0.04	-0.001
1/2	1.1	50	0.95	0.01	-0.00034

$$V_{\text{eff}} \simeq \Lambda - \Lambda_1 \cos(\lambda_1 \phi) + \boxed{\Lambda_2 \cos(\lambda_2 \phi)} \quad \text{Modulation term}$$

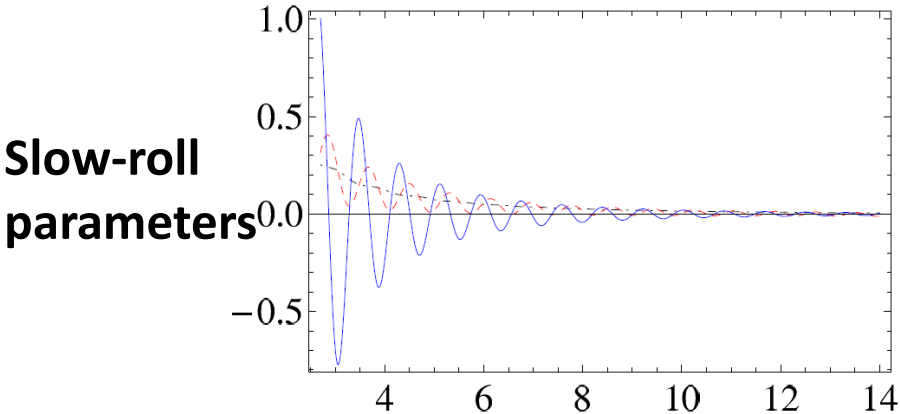


$$n_s = 1 + 2\eta - 6\epsilon + \dots$$

$$r = 16\epsilon$$

$$\lambda_1 = \frac{b_{N=2} \pi \sqrt{2} \langle \text{Re } U \rangle}{6L}$$

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1/3	1.1	60	0.96	0.04	-0.001
1/2	1.1	50	0.95	0.01	-0.00035

# Conclusion

i) We propose the single-field natural inflation w/ and w/o modulations (type IIB).

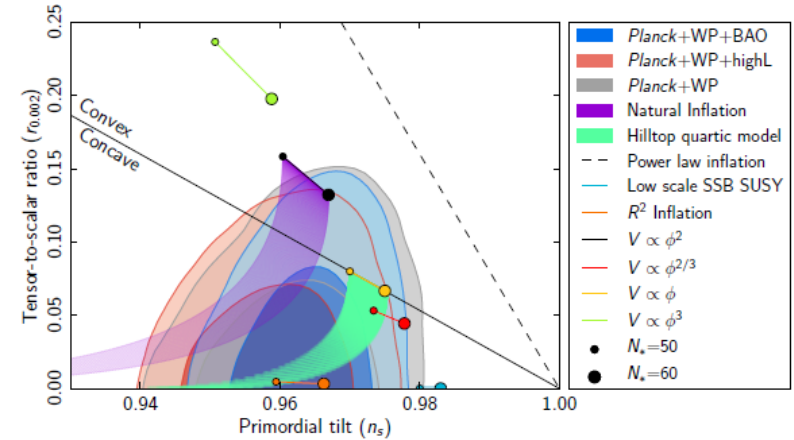
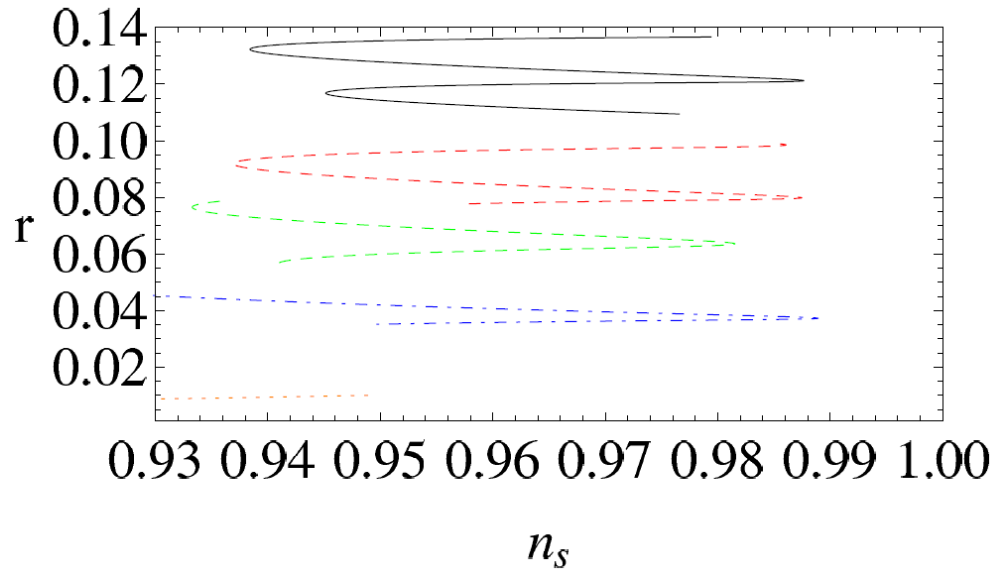


Fig. 1. Marginalized joint 68% and 95% CL regions for  $n_s$  and  $r_{0.002}$  from Planck in combination with other data sets compared to the theoretical predictions of selected inflationary models.

ii) The axion decay constant is enhanced by the inverse of loop factor.

iii) String threshold corrections include the modulation terms.



# Appendix

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) - \sum_i \ln(U_i + \bar{U}_i)$$

$$W_{flux} = w_1 + iw_2(U_1 - U_2) + iw_3U_3 + iw_4S \\ + w_5SU_3 + (U_1 - U_2)(w_6U_3 + w_7S + iw_8SU_3)$$



$$W_{flux} = w_1 + iw_2(U_4) + iw_3U_3 + iw_4S \\ + w_5SU_3 + (U_4)(w_6U_3 + w_7S + iw_8SU_3)$$

**Only  $S$ ,  $U_3$  and  $U_4 = U_1 - U_2$  appear in the superpotential.  
One of complex structure moduli is massless.**

$$K = -\ln(U_2 + \bar{U}_2) - \ln(U_4 + \bar{U}_4 + U_2 + \bar{U}_2) \\ - 3 \ln(T' + \bar{T}' - c(U_2 + \bar{U}_2))$$

$$W_{non} = A \exp[-8\pi^2 f_1 / N_1] + B \exp[-8\pi^2 f_2 / N_2] \\ + C \exp[-8\pi^2 f_3 / N_3] + D \exp[-8\pi^2 f_4 / N_4]$$

$$f_1 = f_2 = T / (2\pi) + b_{N=2} U_2 / (48\pi) = T' / (2\pi)$$

$$f_3 = f_4 = S / (2\pi)$$

**S and  $T'$  are stabilized and Re  $U_2$  is also stabilized as**

$$K_{U_2} W = 0$$

**Inflaton potential**

$$W = w_0 + A' \exp[-2b_{N=2} \ln \eta(iU_2) / L]$$