Discrete Symmetries and Torsion in F-Theory

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Discrete symmetries often arise in the context of theoretical particle physics

They offer an elegant tool for model building, e.g. The MSSM matter parity

They are also useful probes of ultraviolet physics

General quantum gravity considerations imply a non-trivial spectrum of states must accompany discrete symmetries

Consider a 4-dimensional U(1) gauge theory broken to Z_p

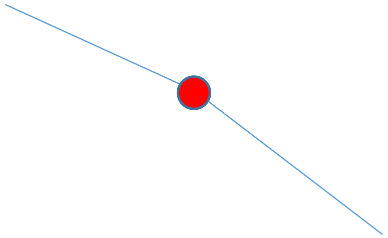
Electric formulation: $(d\phi - pA_1)\wedge\star(d\phi - pA_1)$ $\qquad A_1 \to A_1 + d\lambda \quad , \quad \phi \to \phi + p\,\lambda$

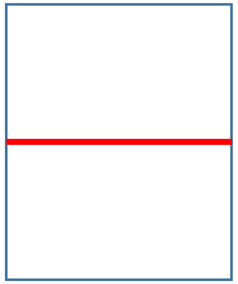
Magnetic formulation: $(dV_1 - pB_2)\wedge\star(dV_1 - pB_2)$ $\qquad B_2 \to B_2 + d\Lambda_1 \quad , \quad V_1 \to V_1 + p\,\Lambda_1$

There are two types of associated operators in the theory

$$W_A\left(\Sigma_1, n_A\right) = \exp\left[in_A\left(\phi\left(\partial\Sigma_1\right) + p\int_{\Sigma_1} A_1\right)\right] \qquad W_B\left(\Sigma_2, n_B\right) = \exp\left[in_B\left(V_1\left(\partial\Sigma_2\right) + p\int_{\Sigma_2} B_2\right)\right]$$

It is possible to associate these operators to the world-line/world-sheet of Z_p charged probe particles/strings





It is conjectured that in quantum gravity these probe particles must be actual states

[Banks,Seiberg '10]

If we raise the probe particle mass to decouple it from the theory it forms a charged black hole which Hawking radiates to a stable charged state

What is the origin and relation of these states to the symmetries in the ultraviolet theory?

String theory provides a candidate ultraviolet theory where black hole type states can be well understood in terms of branes and the geometry of the extra dimensions

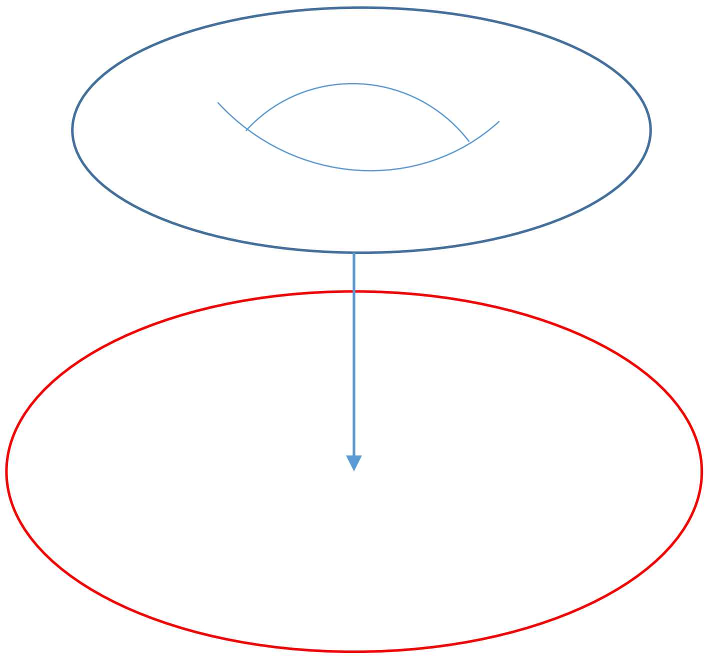
M-theory and its dual F-theory provide a geometric uplift of open string physics

Studying discrete symmetries in F-theory therefore has two motivations:

1) To understand how discrete symmetries that are used in model building can be constructed from a top-down perspective

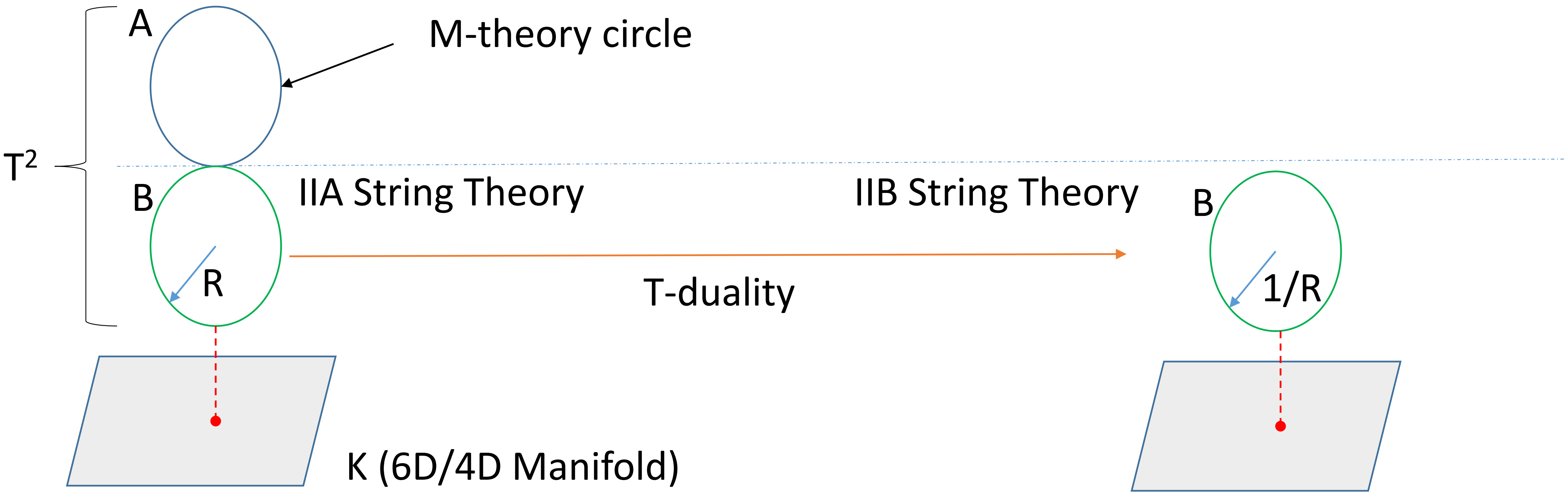
2) The ‘geometrization’ of particle physics may allow an explicit connection between discrete particle physics symmetries and black hole states

Consider M-theory compactified on a Calabi-Yau fourfold/threefold which is a torus fibration over a six/four-dimensional base



F-theory can be defined as the limit where the volume of the torus vanishes

Fibre-wise picture:



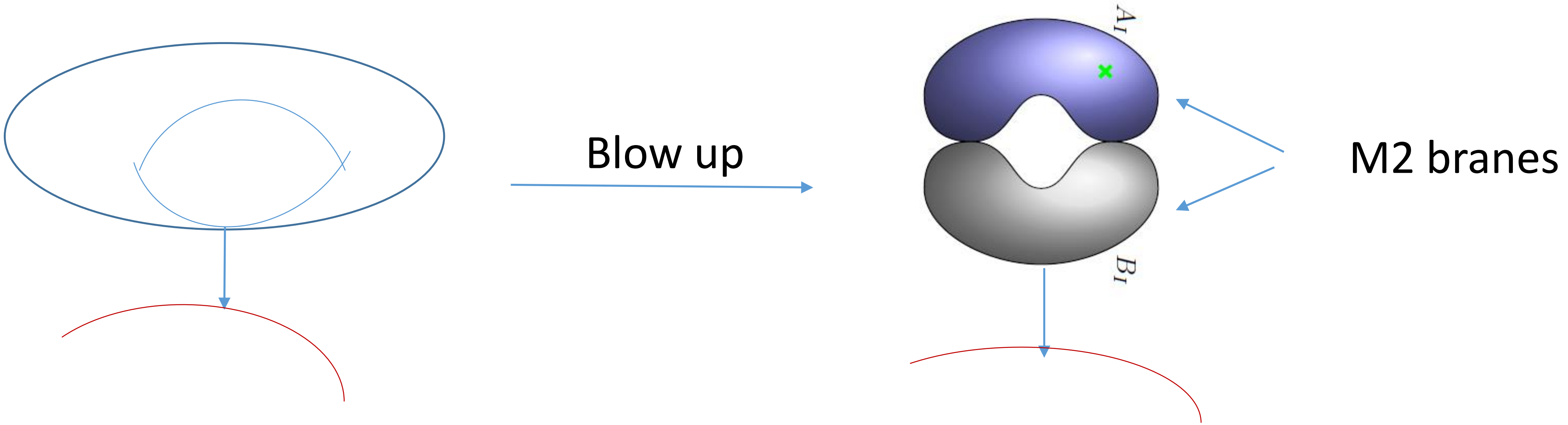
So in the geometric supergravity regime the physics is dual to type IIB string theory on K times a circle of small size, while in the F-theory limit the circle expands to infinity and we recover a 4/6-dimensional type IIB vacuum.

In the M-theory geometric regime (3D/5D) U(1) gauge symmetries are associated to divisors (6/4 cycles)

$$S_0 \leftrightarrow \omega_0 \qquad\qquad C_3 = A^0 \wedge \omega_0$$

There is always one divisor associated to the gravitational U(1) symmetry from the metric component along the circle direction

Matter fields are associated to singular (co-dimension 2) loci in the CY



Out starting point is a torus fibration which has two U(1) divisors (sections) and two matter loci

[Morrison,Park '12]

$$Bl^1P_{[1,1,2]} = sw^2+b_0s^2u^2w+b_1suvw+b_2v^2w+c_0s^3u^4+c_1s^2u^3v+c_2su^2v^2+c_3uv^3 = 0$$

$$(u,v,w,s) \simeq (\lambda u, \lambda\mu v, \lambda^2\mu\, w, \mu\, s)$$

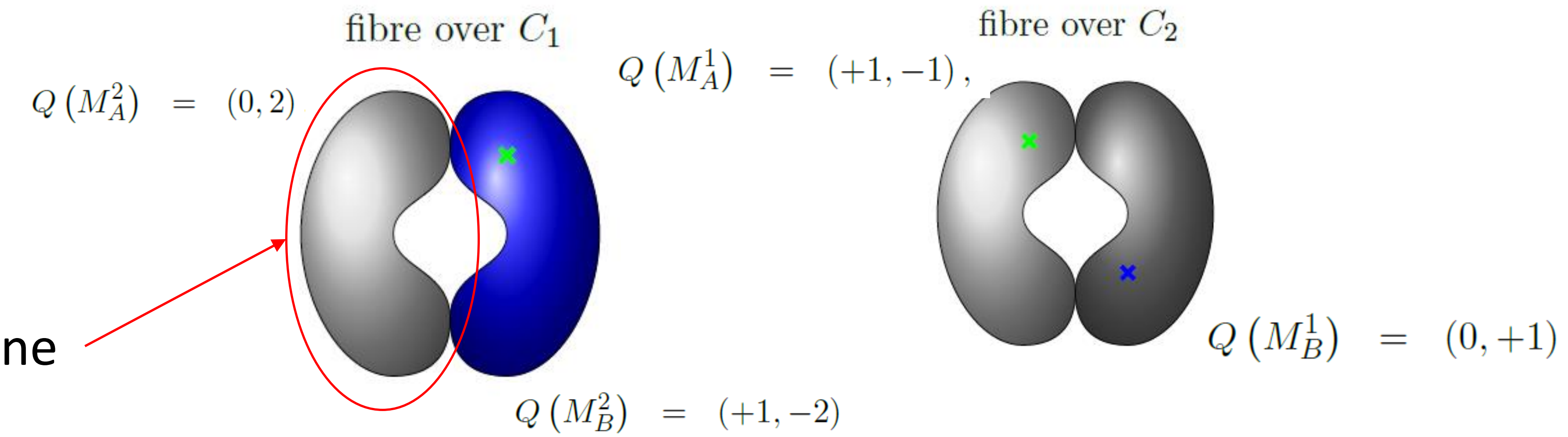
The 2 U(1)s are given explicitly by the divisors:

$$\mathrm{Sec}_0 : u = 0 \;\; : \quad Bl^1P_{[1,1,2]}\Big|_{u=0} = w\left(sw + b_2v^2\right)$$

$$\mathrm{Sec}_1 : s = 0 \;\; : \quad Bl^1P_{[1,1,2]}\Big|_{s=0} = v^2\left(b_2w + c_3uv\right)$$

There are two types of blow-up singular curves carrying matter and their charges under the U(1)s are deduced by their intersection with the associated divisors

Wrapped M2 brane

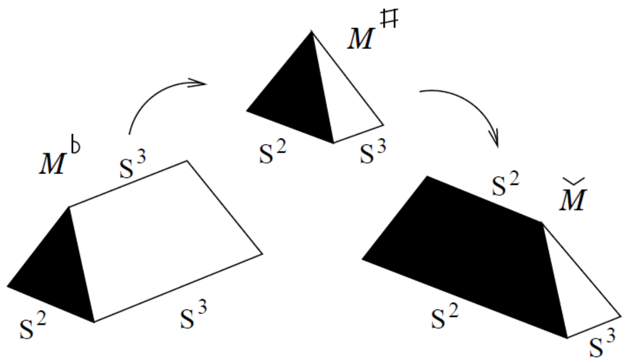


The states over curve C_1 have charges 2 under a U(1) and so we can try to induce a discrete symmetry in the theory by Higgsing them

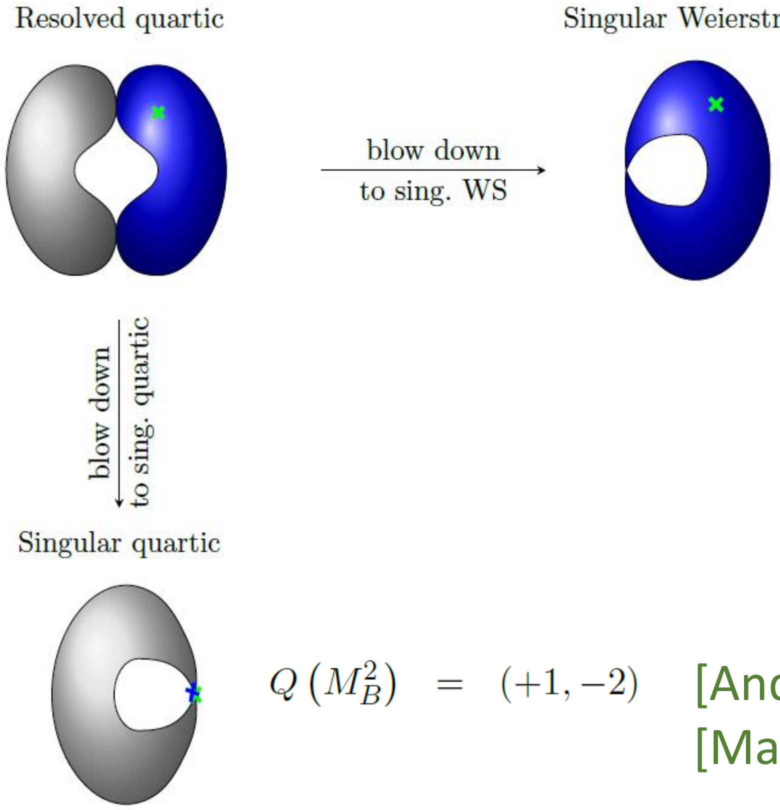
Before Higgsing the state must be made massless which implies blowing down the sphere that the M2 wraps

Then the Higgsing has a geometric formulation as a deformation of the geometry

Overall the whole process is a topology-changing conifold transition



We have 2 choices for how to perform this transition by shrinking either of the components of the fibre



$$Q\left(M_A^2\right) = (0,2)$$

$$Q\left(M_B^2\right) = (+1,-2)$$

[Anderson,García-Etxebarria,Grimm,Keitel ‘14]
[Mayrhofer,EP,Till,Weigand ‘14]

Only one choice gives a remnant discrete symmetry since a unimodular transformation gives

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

More generally for a p-discrete symmetry in the singular F-theory limit, there are p inequivalent smooth M-theory geometries which together form the Tate-Shafarevich group of the fibration

In terms of the field theory this amounts to Higgsing p different Kaluza-Klein modes of the 3-dimensional Higgs field

$$Q\left(M_B^2\right) = (+1,-2) \qquad Q\left(M_A^2\right) = (0,2)$$

KK charge

The conifold transition which geometrises the Higgsing is described by mapping the fibration to Weierstrass form, and then deforming

$$P_W = y^2 - x^3 - f x z^4 - g z^6$$

$$f = e_1\, e_3 - \frac{1}{3} e_2^2 - 4 e_0\, e_4 ,$$
$$g = -e_0 e_3^2 + \frac{1}{3} e_1 e_2 e_3 - \frac{2}{27} e_2^3 + \frac{8}{3} e_0 e_2 e_4 - e_1^2 e_4$$

$$\Big(\quad e_0 = -c_0 + \frac{1}{4} b_0^2, \qquad e_1 = -c_1 + \frac{1}{2} b_0 b_1,$$
$$e_2 = -c_2 + \frac{1}{2} b_0 b_2 + \frac{1}{4} b_1^2, \qquad e_3 = -c_3 + \frac{1}{2} b_1 b_2,$$
$$e_4 = -c_4 + \frac{1}{4} b_2^2.$$

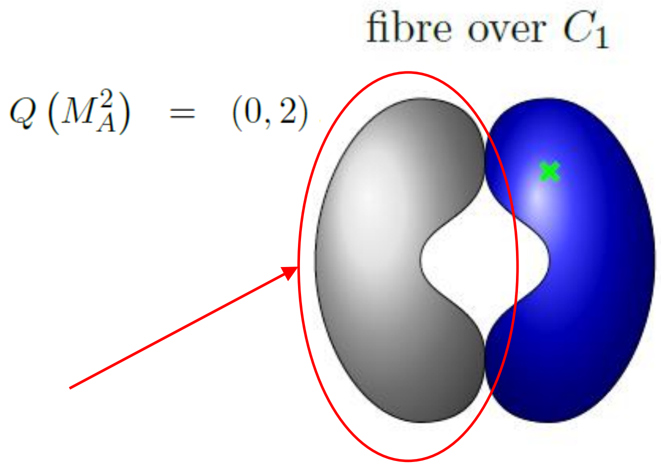
[Morrison,Braun ‘14]
[Morrison,Taylor ‘14]

The parameter c_4 can be identified with the Higgs vev

Will henceforth restrict to 6D F-theory compactifications or 5D M-theory ones

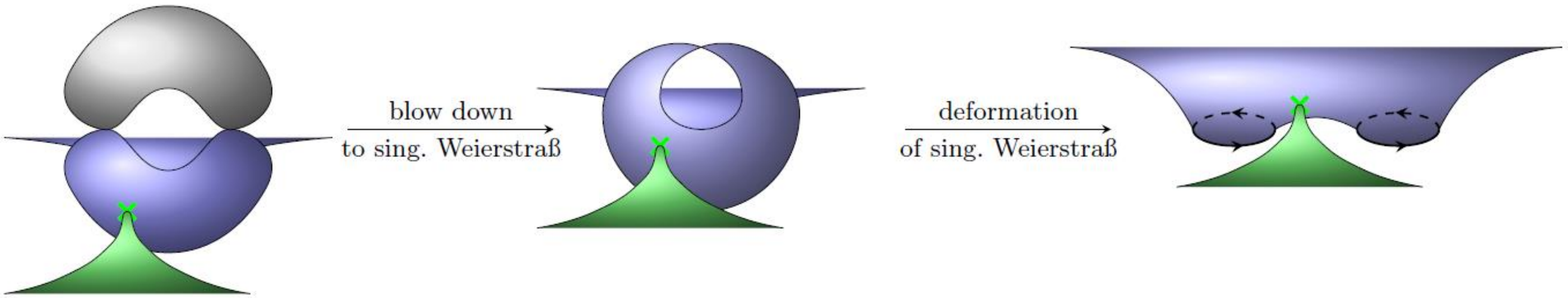
The conifold transition is crucially modified due to the charge 2 of the state, which implies it has an intersection number 2 with the divisor associated to that U(1)

$$D \cdot B_I^i = 2.$$



After the transition the 2-spheres become 3-sphere boundaries to the 4-chain S

$$2\,\Gamma = \partial \hat{D}, \qquad \Gamma = \sum S_3^i.$$



The resulting geometry is that of a torsional 3-cycle where 2 times the cycle is trivial in integer homology

$$2\,\Gamma = \partial \hat{D}, \qquad \Gamma = \sum_i S_3^i.$$

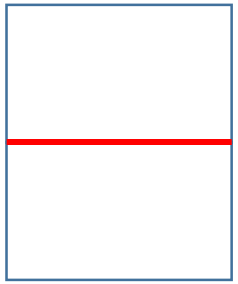
There is a known relation between discrete torsion in the extra dimensions of string theory and the black hole states expected from a discrete symmetry (from breaking closed-string U(1)s)

[Camara,Ibanez,Marchesano ’11]

[Berasaluce-Gonzalez,Ibanez,Soler,Uranga ‘11][Berasaluce-González,Ramírez, Uranga ’13]
[Berasaluce-Gonzalez,Camara,Regalado,Marchesano,Uranga ’12]

Adapting their analysis to the case at hand we uncover the following structure

In 5D the magnetic states are p membranes ending on a string



4D + 1

The strings are M5 branes wrapping the 4-chain $\hat{D}$

The membranes are M5-branes wrapping the torsional 3-cycle Γ so 2 of them end on a string

We can also identify the electric states

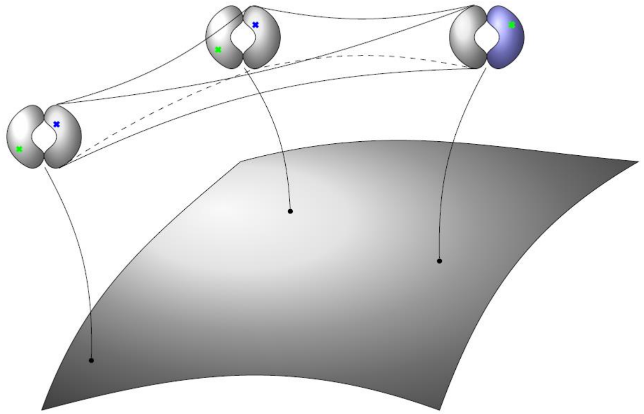
The intersection numbers

	A_I	B_I	A_{II}	B_{II}
S	-1	2	0	1
U	1	0	1	0

imply in homology $2B_{II} = B_I$

Therefore there are is a 3-chain Ω with boundary

$$\partial\Omega = 2B_{II} - B_I$$



After the transition the boundary B_I is lost, leaving a torsional 2-cycle B_{II}

The particles are therefore M2 branes wrapping B_{II} while the instantons are M2 branes wrapping the chain Ω

We can also see the massive U(1) symmetry through the resulting torsional cohomology

$$dw^{(2)} = k\alpha^{(3)} \qquad d\beta^{(3)} = k\tilde{\omega}^{(4)} \qquad C_3 = A \wedge w^{(2)}$$

[Grimm,Kerstan,EP,Weigand '11]
[Camara,Ibanez,Marchesano '11]

Finally we can generalise this analysis including a further SU(5) GUT symmetry and matter resulting in phenomenologically appealing Z_2 selection rules. Including a realisation of matter parity.

See talk by Weigand

Summary

Discrete gauge symmetries are realised in M-theory/F-theory as torsional homology

Manifestly exhibit the charged states conjectured by quantum gravity arguments

The analysis performed is expected to generalise to 4-dimensions and higher Abelian discrete symmetries

[Cvetic,Klevers,Pena,Oehlmann,Reuter ‘15]

An understanding of non-Abelian discrete symmetries in F-theory remains to be fully developed for global compactifications

[Antoniadis,Leontaris ‘13][Karozas,King,Leontaris,Meadowcroft ’14 ‘15][Grimm,Pugh,Regalado ‘15]

Possible applications of the geometry beyond particle physics, for example generalising inflation models based on massive Wilson lines

[Marchesano,Shiu,Uranga ‘14]

Thanks