Higher Derivative Terms in M-Theory Reductions

based on
1408.5136 and 1412.5073 with
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Outline

- Introduction
- M-theory Vacua with 3d N=2 Supersymmetry
- The 3d Effective Theory
- Conclusions

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INTRODUCTION

- Reductions of M-theory to three dimensions present many interesting features.
- In general the vacua include fluxes and warping, which require higher derivative corrections to the 11d action for global consistency.
- These reductions represent the M-theory duals of F-theory reductions to 4d and are important for deriving the F-theory effective action.
- Higher derivative corrections in M-theory can induce corrections to the 3d effective theory and the 4d F-theory lift.

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The Higher Derivative Vacua

- The 11d action and field equations are given as an expansion in the 11d plank length
- So the solutions we will study exist as an expansion in $\alpha \sim (\kappa_{11})^{\frac{2}{3}} \sim l_p^3$

$$d\hat{s}^2 = e^{\alpha^2 Z}(e^{-2\alpha^2 W^{(2)}}\eta_{\mu\nu}dx^\mu dx^\nu + 2e^{\alpha^2 W^{(2)}} g_{m\bar{n}}dy^m dy^{\bar{n}})$$

$$g_{m\bar{n}} = g^{(0)}_{m\bar{n}} + \alpha^2 g^{(2)}_{m\bar{n}}$$

$$\hat{G}_{m\bar{n}r\bar{s}} = \alpha G^{(1)}_{m\bar{n}r\bar{s}} \qquad \hat{G}_{\mu\nu\rho m} = \epsilon_{\mu\nu\rho}\partial_m e^{-3\alpha^2 W^{(2)}}$$

- Where 3d N=2 supersymmetry at lowest order in α implies [Becker, Becker]

$$dJ^{(0)} = d\Omega^{(0)} = 0 \quad G^{(1)} \wedge J^{(0)} = 0 \quad G^{(1)} \in H^{2,2}(Y_4)$$

$\mathcal{O}(\alpha^2)$ Constraints on the Vacua

- And at second order in α the solution is constrained by the warp factor equation

$$d^\dagger dW^{(2)} + G^{(1)} \wedge G^{(1)} + X_8 = 0$$

- And the internal space part of the metric field equation [Becker, Becker] [Lu, Pope, Stelle, Townsend]

$$R^{(2)}_{m\bar{n}} = \partial^{(0)}_m \bar{\partial}^{(0)}_{\bar{n}} Z \qquad Z = *^{(0)}(J^{(0)} \wedge c_3^{(0)})$$

$$c_3^{(0)} = i\mathrm{Tr}(\mathcal{R}^{(0)} \wedge \mathcal{R}^{(0)} \wedge \mathcal{R}^{(0)})$$

- This is solved by [Grimm, TP, Weissenbacher]

$$c_3^{(0)} = Hc_3^{(0)} + i\partial^{(0)}\bar{\partial}^{(0)}F$$

$$g^{(2)}_{m\bar{n}} = \nabla^{(0)}_m \bar{\nabla}^{(0)}_{\bar{n}} *^{(0)} (F \wedge J^{(0)} \wedge J^{(0)})$$

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Perturbations for Kähler Moduli

- We make an ansatz for the perturbations that correspond to Kähler moduli where

$$\delta g^{(0)}_{m\bar{n}} = i\delta v^i \omega^{(0)}_{im\bar{n}} \qquad \delta\hat{G} = F^i \wedge \omega^{(0)}_i$$

$$F^i = dA^i \qquad d *^{(0)} \omega^{(0)}_i = d\omega^{(0)}_i = 0$$

- The metric shift induces a shift of the higher derivative parts of the ansatz

$$\delta Z = i\delta v^i Z^{\bar{n}m} \omega^{(0)}_{im\bar{n}} + \dots$$

- And induces a shift of the warping

$$W^{(2)}| \to W^{(2)}| + \partial_i W^{(2)}|\delta v^i + \dots$$

(See Martucci's Talk)

Reducing The 11D Action

- Next we reduce the action on the background described

$$\boxed{S = S^{(0)} + \alpha^2 S^{(2)}_{\hat{R}^4} + \alpha^2 S^{(2)}_{\hat{G}^2 \hat{R}^3} + \alpha^2 S^{(2)}_{(\hat{\nabla}\hat{G})^2 \hat{R}^2}}$$

$$+\mathcal{O}(\hat{G}^3\alpha^2) + \mathcal{O}(\alpha^3) \qquad \text{[Liu, Minasian]}$$

- We derive the effective action to second order in 3d derivatives, α , δv^i and A^i. This is sufficient to determine the higher derivative contributions to 3d kinetic terms and mass terms.
- This results in an effective action given in terms of

$$Z_{m\bar{n}r\bar{s}} = (\epsilon_8^{(0)} \epsilon_8^{(0)} R^{(0)\,3})_{m\bar{n}r\bar{s}}$$

$$Z_{m\bar{m}} = i Z_{m\bar{m}n}{}^{n} = (*^{(0)} c_3^{(0)})_{m\bar{m}} \qquad Z_m{}^m = Z$$

Kinetic Terms in The Effective Action

- Substituting into the 11d action and integrating we find

$$S_{\rm kin} = \frac{1}{2\kappa_{11}} \int_{\mathcal{M}_3} \Big[R * 1 - (G^T_{ij} + \mathcal{V}_T^{-2} K^T_i K^T_j) Dv^i \wedge * Dv^j - \mathcal{V}_T^2 G^T_{ij} F^i \wedge * F^j \Big]$$

- Where

$$G^T_{ij} \sim \frac{1}{\mathcal{V}_0} \int_{Y_4} \Big[e^{3\alpha^2 W^{(2)} + \alpha^2 Z} \omega^{(0)}_{im\bar{n}} \omega^{(0)\,\bar{n}m}_j + \alpha^2 Z_{m\bar{n}r\bar{s}} \omega^{(0)\,\bar{n}m}_j \omega^{(0)\,\bar{s}r}_i \Big] *^{(0)} 1$$

$$K^T_i = \int_{Y_4} \Big[e^{3\alpha^2 W^{(2)} + \alpha^2 Z} \omega^{(0)\,m}_{im} - Z_{m\bar{n}} \omega^{(0)\,\bar{n}m}_i \Big] *^{(0)} 1$$

$$\mathcal{V}_T = \int_{Y_4} e^{3\alpha^2 W^{(2)} + \alpha^2 Z} *^{(0)} 1$$

Scalar Mass Terms

- The additional mass terms for the scalar fluctuations cancel

$$\int \hat{t}_8 \hat{t}_8 \hat{R}^4 \hat{*} 1|_{\text{mass}} = -\int_{\mathcal{M}_3} \delta v^i \delta v^j *_3 1 \int_{Y_4} \nabla_r \nabla^r Z \omega^{(0)}_{im\bar{n}} \omega_j^{(0)\,\bar{n}m} *_8 1$$

$$\int \hat{R} \hat{*} 1|_{\text{mass}} = \alpha^2 \int_{\mathcal{M}_3} \delta v^i \delta v^j *_3 1 \int_{Y_4} \nabla_r \nabla^r Z \omega^{(0)}_{im\bar{n}} \omega_j^{(0)\,\bar{n}m} *_8 1$$

- So that the only potential terms are

$$S_{\text{pot}} = \frac{\alpha^2}{4\kappa_{11}} \int_{\mathcal{M}_3} *_3 1 \int_{Y_4} \left(G^{(1)} \wedge *' G^{(1)} - G^{(1)} \wedge G^{(1)} \right)$$

- These come with the associated Chern-Simons terms for the vectors

$$S_{CS} = \alpha \int_{\mathcal{M}_3} A^i \wedge F^j \int_{Y_4} G^{(1)} \wedge \omega_i \wedge \omega_j$$

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Conclusions

- Effective theories including higher derivative corrections for 3d moduli may be derived in the way we have described.
- This process involves corrections to the vacua, ansatz and action parameterized by the 11d Planck length.
- These computations required significant use of computer algebra packages such as xAct.
- The masses of the 3d fluctuations are not altered.
- All corrections may be written in terms of $Z_{m\bar{n}r\bar{s}} = (\epsilon_8^{(0)}\epsilon_8^{(0)}R^{(0)\,3})_{m\bar{n}r\bar{s}}$ and the warp factor.

Thank You