D-brane moduli stabilisation and linear equivalence

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Based on
[1410.0209] with F. Marchesano and G. Zoccarato

Outline

- For concreteness, D6-branes in IIA CY orientifolds. Geometric moduli.
- Usual counting of moduli (ignoring backreaction).
- Taking backreaction into account.
- Linear equivalence.
- Superpotential analysis.

Counting D6-brane moduli in CY

- In Type II D-brane models, we typically ignore the backreaction of D-branes on the background ($g_s \to 0$).

- Worldsheet or worldvolume supersymmetry analysis (ignoring WS instantons).

D6-branes wrap SLags π_3 with flat bundles $\mathcal{F}$:

[Becker et al.'95]
[Ooguri et al. '96]
[Bergshoeff et al. '97]
[Kapustin et al. '03]

$$J|_{\pi_3} = 0, \qquad \mathcal{F} = 0$$
$$\mathrm{Im}\,(e^{i\theta}\Omega)|_{\pi_3} = 0$$

- Moduli: deformations that preserve this $\longrightarrow$ $b^1(\pi_3)$ complex scalars.

[McLean '98]

This talk : Does this change if we take into account the backreaction?

Setting up the problem

- Equations for 4d N=1 Minkowski vacua.

$$ds^2 = e^{2A} ds^2_{\mathbb{R}^{1,3}} + ds^2_{X_6}$$

$$d(3A-\phi) = H_3 + i dJ = 0, \qquad F_0 = \tilde{F}_4 = \tilde{F}_6 = 0$$

$$d(e^{2A-\phi}\mathrm{Im}\,\Omega) = 0, \qquad d(e^{-\phi}\mathrm{Re}\,\Omega) = -J \wedge F_2$$

[Tomasiello '07]

- Bianchi identities.

$$dF_2 = \delta(\Pi), \qquad [\Pi] = 0, \qquad \Pi = \sum_\alpha [\delta(\pi_3^\alpha) + \delta(\pi_3^{\alpha *})] - 4\delta(\pi_3^{O6})$$

- Quantisation of the field strength.

$$\int_{\pi_2} F_2 \in \mathbb{Z}, \quad \forall \pi_2 \in H_2(X_6 - \Pi, \mathbb{Z})$$

Tool: Linear equivalence

[Hitchin '99]

- Field strength coupled to magnetic sources and not electric

$$dF_2 = \delta(\Pi), \qquad d^\dagger F_2 = 0 \qquad \text{with} \quad [\Pi] = 0$$

- We can use Hodge's decomposition for the field strength

$$F_2 = \alpha_2 + d^\dagger \gamma_3 + d\beta_1 \qquad \text{with } \alpha_2 \text{ harmonic}$$

- The quantization condition fixes the harmonic piece

We say that Π is linearly equivalent to zero iff $\alpha_2 = 0$

Alternatively, iff $\displaystyle\int_{\Sigma_4} \omega_4 = 0 \qquad \forall \omega \in \mathcal{H}^4(X_6, \mathbb{Z}) \qquad \text{with} \quad \partial\Sigma_4 = \Pi$

(depends on the metric)

Supersymmetric configurations (geometry)

- Supersymmetry implies that

$$\underset{\substack{\uparrow \\ \text{SUSY}}}{d(e^{-\phi}\mathrm{Re}\,\Omega) = -J \wedge F_2} \quad \Longrightarrow \quad Q \equiv \int_{X_6} F_2 \wedge J \wedge \omega_2 = 0 \quad \forall \text{ closed } \omega_2$$

$$\Big\downarrow \text{ Lefschetz duality}$$

- Using Bianchi + Quantisation:

$$dF_2 = \delta(\Pi), \quad \Pi = \partial\Sigma_4$$

F_2 is quantised, $dJ = 0$

$$\Longrightarrow \quad Q = \int_{\Sigma_4} J \wedge \omega_2 = 0 \quad \forall \text{ closed } \omega_2$$

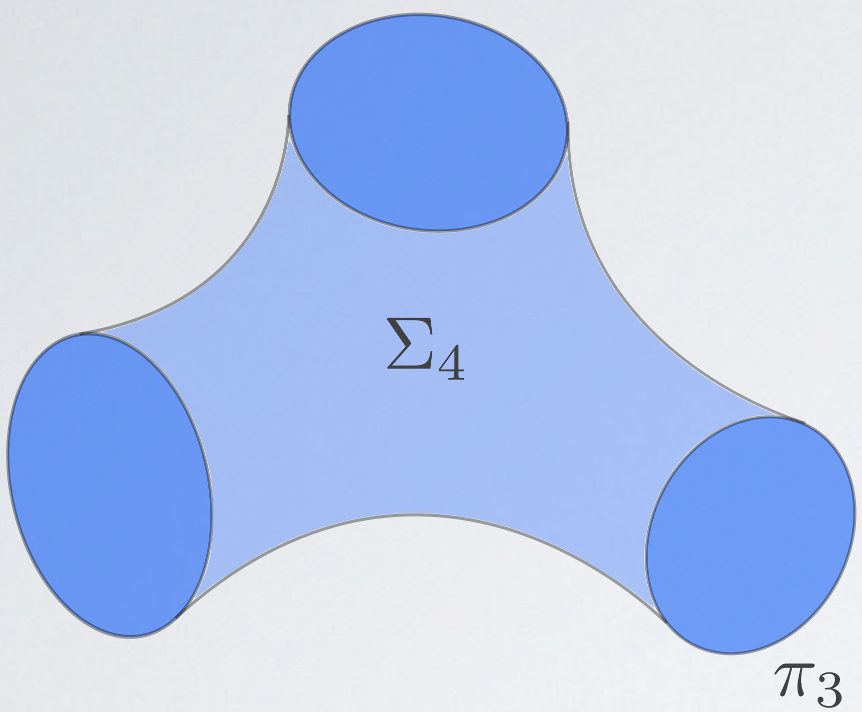
- If Π is Lagrangian, then only harmonic ω_2 are relevant

[Hitchin '99]
[Marchesano et al.'14]

$$Q = \int_{\Sigma_4} J \wedge \omega_2 = 0 \quad \forall \text{ harm. } \omega_2 \quad \Longrightarrow$$ Π is linearly equivalent to zero

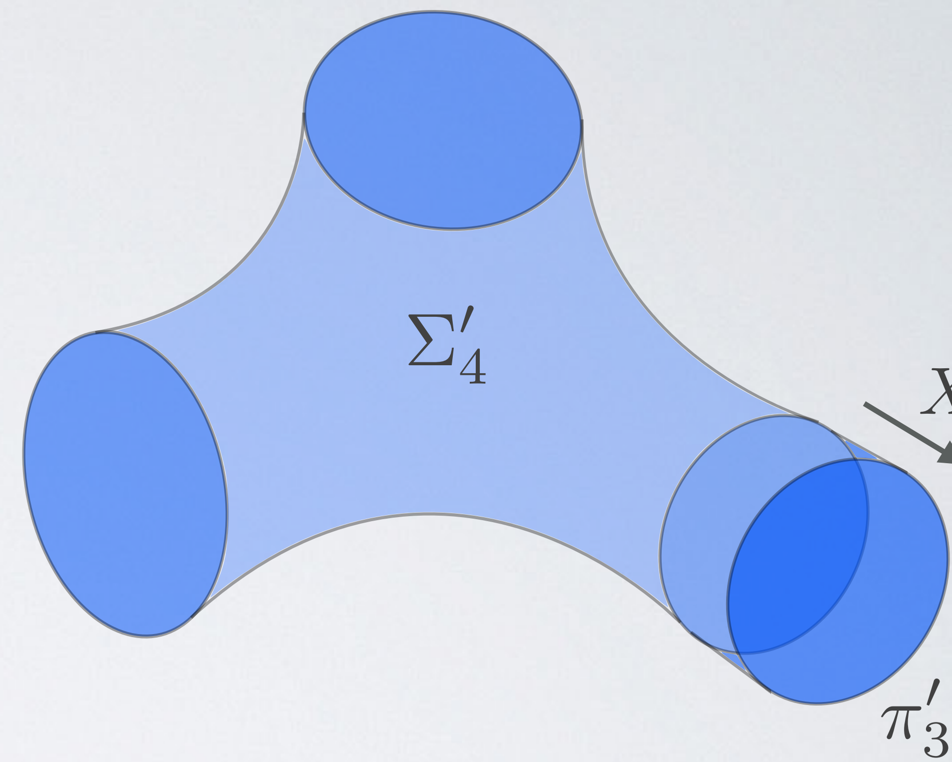
(wrt the CY metric)

Obstruction to deformations



Deformation along normal vector X

$\longrightarrow$



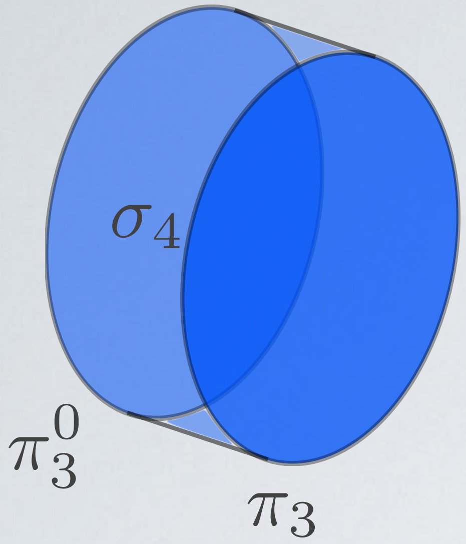
$$Q = \int_{\Sigma_4} J \wedge \omega_2 \underset{\substack{\uparrow \\ \text{SUSY}}}{=} 0 \quad \xrightarrow{J|_{\pi_3} = 0} \quad \delta Q = \int_{\pi_3} \iota_X J \wedge \omega_2$$

- If X preserves the SLag condition: $\delta Q = \displaystyle\int_{\pi_2^X} \omega_2$ [McLean '98]

- SUSY is broken if $\delta Q \neq 0 \iff [\pi_2^X]$ is non-trivial in $H_2^-(X_6, \mathbb{Z})$

[Hitchin '99]

(Potential for position moduli)



Superpotential analysis

[Martucci '06]
[Thomas '01]

$$\Delta W(\pi_3, A) = \int_{\sigma_4} (J_c + F)^2, \qquad \partial\sigma_4 = \pi_3 - \pi_3^0$$

- Critical points:

$$\delta \Delta W = 2 \int_{\pi_3} (J_c + F) \wedge (\iota_X J_c + \delta A) = 0 \quad \Longrightarrow \quad (J_c + F)|_{\pi_3} = 0 \quad \text{(Lagrangian)}$$

- Full superpotential: $\quad W(J_c, \Pi, A) = \int_{\Sigma_4} (J_c + F)^2$

$$\delta W = 2 \int_{\Pi} (J_c + F) \wedge (\iota_X J_c + \delta A) + 2 \int_{\Sigma_4} (J_c + F) \wedge \delta J_c = 0$$

(Lagrangian) (D-brane linear equivalence)

[Marchesano et al. '14]

- Close to SUSY:

$$W = m_j^a \, T_a \, \Phi^j, \quad m_j^a = \int_{\pi_3} \omega_2^a \wedge \zeta_j \qquad J_c = T_a \, \omega_2^a \qquad \iota_X J_c + \delta A = \Phi^j \, \zeta_j$$

[Escobar et al. '15]
See Aitor Landete's talk

Summary and outlook

- We have revisited the counting of moduli associated to D6-branes in CY.
- Extra condition: D6-branes and O6-planes are linearly equivalent to zero. Mechanism to lift position moduli and Wilson lines.
- Simple topological criterion (non-trivial 2-cycles in the D6 which are non-trivial in the bulk).
- We have recovered this result from known superpotential.
- Something similar occurs for magnetised D7-branes.
- Extend this to other configurations.
- Applications to model buiding: reduce the number of moduli, inflation…

Thank you!